Discussion Paper No. 1171

DISCIPLINED COALITIONS AND REDISTRIBUTION: THE EFFECT OF THE VOTE OF CONFIDENCE PROCEDURE ON LEGISLATIVE BARGAINING*

by

Daniel Diermeier[1]
and
Timothy J. Feddersen[2]
October 1996

* Formerly "Voting Cohesion in Presidential and Parliamentary Legislatures." The authors wish to thank David Baron, Michele Boldrin, John Carey, Gary Cox, John Huber, Ken Janda, Roger Myerson, Wolfgang Pesendorfer, Al Slivinski and participants of the 1996 CEPR Summer Workshop in Theoretical Economics, the 1996 Annual Conference of the W. Allen Wallis Institute of Political Economy at the University of Rochester as well as seminars at Stanford University and Columbia University for valuable comments and suggestions. All errors are our own.

[1] Graduate School of Business, Stanford University, email: dnld@gsb-pound.stanford.edu

[2] J. L. Kellogg Graduate School of Management, Northwestern University,
email: tfed@casbah.acns.nwu.edu

Abstract

One of the distinguishing features of parliamentary democracies is the ability of the prime minister to link a vote on bill with a vote on the government. This feature is known as the vote of confidence procedure. We use a legislative bargaining model to analyze the effects of the vote of confidence procedure on voting behavior and the distribution of benefits. We show that the procedure permits legislative majorities to behave in a more disciplined fashion and to capture more of the legislative rents from the minority. Our results are consistent with the high levels of coalitional cohesion typical of parliamentary systems.

Introduction

Parliamentary systems may be distinguished from other legislative systems both in terms of institutional features and legislative behavior. Characteristic institutional features of parliamentary systems include: cabinet government; the ability of the prime minister to call for early elections; the ability of the prime minister to link votes on bills with a vote on the government (the vote of confidence procedure); the ability of minorities to call for a vote of no-confidence; strong political parties; and extra-legislative bargaining mechanisms. In terms of legislative behavior, parliamentary systems are characterized by a minimal role for the minority (the minority is by definition not included in the cabinet) and high levels of party and coalitional cohesion. On the other hand, non-parliamentary legislatures (like the U.S. congress) are characterized by weak parties and a committee system in which members of the minority party play a key role. The U.S. Congress lacks the vote of confidence and vote of no-confidence procedures as well as the ability of the governing majority to call for early elections. Policy coalitions are issue specific, change frequently, and often cross party lines.

The focus of this paper is the vote of confidence procedure. In contrast to a motion of no-confidence or censure, the vote of confidence procedure is initiated by the prime minister. It allows the governing coalition to propose any bill under a closed rule and link the adoption of the bill with the survival of the government.[1] In a recent paper, Huber (1996) has focused on the question of when prime ministers will use the vote of confidence procedure. Huber argues that confidence procedures enable the prime minister to choose any final policy outcome from the set of policies that the majority prefers to the status quo even if the procedure is not invoked explicitly (p. 279).

In this paper we develop a model of repeated legislative bargaining to illustrate that many of the behavioral differences between parliamentary systems and the U.S. congress can be linked directly to the vote of confidence procedure. We show how the vote of

[1] While confidence procedures are typically authorized by the constitution the may also be used by convention. The key question for our purposes is whether in the latter case confidence procedures actually constitute a commitment to step down in case of defeat. This is a question of constitutional law. In the case of Britain, for instance, constitutional lawyers generally agree that votes of confidence are binding. Defeat of a motion of confidence or no confidence has been treated as fatal since 1832 (e.g. Brazier 1988). Huber (1996, table 1) gives a detailed overview of confidence procedures in 18 countries.

confidence permits legislative majorities to capture almost all the distributional benefits of political competition and therefore motivates coalitional cohesion, strong parties, extra-legislative bargaining mechanisms and lower minority participation within the legislature.

This paper consists of five sections. In the first section we discuss key concepts in the study of legislative coalitions. In the second section we develop a finite multi-stage voting model that endogenizes the formation of legislative coalitions. We demonstrate that the vote of confidence procedure generates cohesive voting blocs and increases the majority's ability to capture the benefits of legislation. In the third section we discuss the implications of our results for the comparative study of legislatures. In the final section we offer a brief conclusion. Proofs and formal statements of all propositions can be found in the appendix.

Legislative Institutions and Coalitional Cohesion.

One of the defining characteristics of legislative voting in parliamentary systems is coalitional cohesion. A group of legislators constitutes a cohesive legislative coalition if they persistently vote together on both questions of legislative organization and on the string of policy proposals subsequently generated. In contrast, legislative voting in the U.S. Congress is disciplined, i.e. the appointment of a speaker and the membership on the committees, while coalitions are constantly shifting on policy (Cox and McCubbins 1993). There is an extensive literature that documents the differing levels of voting cohesion (e.g. Loewenberg and Patterson 1979, Mezey 1979,von Beyme 1985). It will be useful, both for understanding our model and the previous literature on cohesion, to differentiate between three types of legislative coalitions--*governing, procedural* and *policy.*

In the study of Western European governments a governing coalition is a group of legislators or parties that hold portfolios. While cabinets are complicated institutions performing a variety of legislative and executive functions (e.g. specialization, control of the bureaucracy), we here focus on a very small aspect of their role in the legislative process: their procedural prerogatives. In parliamentary democracies ministers have privileged access to the floor in their respective policy domain. In terms of procedural prerogatives cabinets are thus very similar to committees in the U.S. Congress (Rogowski

1990). The set of legislators who may be recognized to make a policy proposal is the *governing coalition.*[2]

Governing coalitions can range in size from universal coalitions to one-player coalitions but they must always be supported by a chamber majority. That is, following Bagehot (1963 [1867]) we interpret cabinets as well as the congressional committee system as particular forms of legislative organization that must be supported in a vote on the organization of the chamber. The group of legislators that votes to install a governing coalition is a *procedural coalition.*

Finally, there are *policy coalitions,* i.e. groups of legislators that support a particular bill. Since governing, procedural and policy coalitions are found in almost all legislative systems, we can compare the effects of different constitutional features i.e., the vote of confidence procedure, using a single formal framework. In two party systems, for instance, procedural coalitions are comprised by a single party. Cox and McCubbins (1993) suggest that party leaders may use this control over the organization of the chamber to maintain party discipline on policy votes. Conceptually, however, legislative parties and procedural coalitions are distinct, as is immediately obvious from the case of multi-party governments in parliamentary settings.

We can now use the definitions of governing, procedural and policy coalitions to define somewhat more precisely coalitional cohesion. Legislative voting behavior is said to be *cohesive* when the procedural and policy coalitions are identical over almost all votes.

Voting cohesion in legislatures has often been used as evidence of strong parties (e.g. Collie 1985). The standard argument is that variation in legislative cohesion is a consequence of variation in the similarity of the legislators' (party- induced) preferences over policy, which in turn depends on different levels of party discipline. While legislators may have genuine policy preferences, they are also interested in holding office. This office motivation will lead them to modify their voting behavior accordingly. In particular, representatives in parliamentary democracies will mainly follow their party leadership, while members of the U.S. Congress have to balance the wishes of their leadership with the demands of diverse reelection constituencies.

---

[2] Of course, in most parliamentary democracies ministers have only privileged, not exclusive access to the

The main difficulty for the standard view, however, is the presence of cohesion in multi-party parliamentary democracies. Any general theory of cohesive coalitions must account for the fact that cohesive coalitions are a feature of British legislative politics *and* parliamentary (multi-party) democracies. Theories of cohesion based on party discipline can, at most, account for *intra-party* cohesion. But in Western European democracies cohesion is not only maintained *across parties*, it is also significantly higher than the level of cohesion among members *of the same party* in the U.S. Congress. This *inter-party* cohesion cannot be accounted for by disciplined parties.

In contrast to the standard view our approach is more closely connected to the traditional comparative politics literature that has previously suggested the government's ability to call for votes of confidence as one of the distinguishing features of parliamentary government (e.g. Epstein 1964). We demonstrate below that the presence of a vote of confidence procedure alone is sufficient to explain cohesion. Formally, the vote of confidence procedure amounts to a commitment by the governing coalition to resign when its policy proposal is defeated. The vote of confidence procedure causes a vote on a bill to be a vote on the government thus ensuring that those who vote for or against the government will vote for or against the bill. As we show below this institutional feature has important distributive consequences. Governing coalitions in parliamentary democracies capture almost all the distributive benefits leaving almost nothing for the minority. Whereas, when the vote of confidence procedure is not available, the minority shares in the spoils although they receive less than the governing coalition.

Model

We model the formation of legislative coalitions using a legislative bargaining model in the spirit of Baron and Ferejohn (1989). We construct two 3-player, finite T+1 period games denoted G_{us} and G_{uk} with obvious intended interpretations. We let N denote the set of legislators. In both games of either type there are two kinds of periods: organizational (O) and bill (B). Each period of each type can be divided into two stages: a proposal stage (P) and a voting stage (V). Thus in any period t the game is in one of the following stages:

floor (Doering 1995). Our assumption emphasizes this asymmetry.

organizational proposal stage, organizational voting stage, bill proposal stage, or bill voting stage. Each stage may include moves by nature as well as the legislators. In each period we denote a generic legislator by j, the chosen proposer by i, and legislators other than i by k and k'.

We construct the multi-period games from a finite sequence of organizational and bill periods. Each game differs only in the transition rules from period to period. Strategies in multi-period games can depend on the history of play. We define the set of possible histories prior to period $t \in \{T, T-1, \ldots, 0\}$ that lead to an organizational period t as $H(O, t)$ and the set of histories of play that lead to a bill period as $H(B, t)$. Define the set of all possible histories leading to period t as H(t). A generic history is denoted by h. If necessary, we write h^t for a history that leads to period t. Note that periods are indexed in reverse order of their occurrence. That is, the index t indicates the number of periods remaining *after* the current period rather than the number of periods that have already occurred.

First, consider an organizational period. In the organizational proposal stage nature draws a proposer $i \in N$ with probability 1/3. The proposer then proposes a governing coalition C^i with $\varnothing \neq C^i \subseteq N$. A strategy for legislator i in an organization period t proposal stage is a map from the set of possible histories into the set of possible governing coalitions conditional on nature's selection of a proposer. All players other than the chosen proposer play a common dummy action. We denote an organizational proposal strategy for player j in period t as

$$op_j^t : H(O,t) \times N \to \begin{cases} \text{Pot}(N) \setminus \{\varnothing\} \text{ if } j = i \\ \{o\} \text{ otherwise} \end{cases}.$$

We restrict ourselves to pure strategies.

A proposal stage is always followed by a voting stage. In an organizational period voting stage legislators vote to accept or reject the proposed governing coalition where 1 denotes acceptance and 0 rejection. Coalition C^i is approved if and only if a majority of legislators accept it. An organizational voting stage strategy is a function mapping the set of possible histories along with nature's choice of proposer and the proposer's proposed governing coalition into a decision to accept or reject. We denote an organizational voting strategy by legislator j in period t as:

$$ov_j^t : H(O,t) \times N \times Pot(N) \setminus \{\varnothing\} \rightarrow \{0,1\} \; \forall j \in N .$$

The players that vote for a given C^i thus constitute a procedural coalition. Figure 1 illustrates the sequence of play in an organizational period.

<<Figure 1 about here>>

The structure of a bill period is very similar to the structure of an organizational period (see Figure 2). A bill period consists of a proposal stage followed by a voting stage. In the proposal stage nature chooses a proposer $i \in C$ from a given governing coalition with probability $1/|C|$. Nature also chooses a period reservation outcome $x \in X$ where

$$X \equiv \left\{ x \in [0,1]^n \mid \sum_{i \in N} x_i = 1 \right\}$$

according to the uniform density function

$$f(x) = \begin{cases} n-1 & \forall x \in X \\ 0 & \text{otherwise} \end{cases} .$$

Random variables are indicated by bold face letters. Once we consider multiple periods, the moves by nature in each bill period are nothing else than i.i.d. draws of a random vector. Assuming i.i.d. draws stacks the deck against voting cohesion because the order of legislators in terms of period reservation values changes each period. We will show how the ability to maintain persistent policy coalitions despite these random changes will crucially depend on the presence of a vote of confidence procedure.

<<Figure 2 about here>>

Nature's choice of $\mathbf{x}$ is observed by all the legislators. Intuitively, $\mathbf{x}$ represents the consequences if a bill proposal is rejected. The recognized proposer $i=\mathbf{i}$ then proposes a bill, that is some division of the dollar $b^i \in X$. A bill period proposal strategy conditioned upon being recognized is a map from the set of possible histories leading to t and the period reservation outcome x into an element of X. Legislators that are not recognized play a dummy action o.

$$bp_j^t : H(B,t) \times N \times X \to \begin{cases} X \text{ if } i = j \\ \{o\} \text{ otherwise} \end{cases}.$$

In the bill voting stage legislators vote to accept or reject the bill with 1 denoting acceptance and 0 rejection. As in the organizational voting stage, a bill is accepted if and only if at least (n+1)/2 legislators vote to accept and rejected otherwise. We formalize the voting strategies as follows:

$$bv_j^t : H(B,t) \times N \times X \times X \to \{0,1\} \ \forall j \in N$$

We denote strategy profiles in an organizational period t as (op^t, ov^t), where $op^t = (op_1^t, op_2^t, \ldots, op_n^t)$ and $ov^t = (ov_1^t, ov_2^t, \ldots, ov_n^t)$. For bill periods we write (bp^t, bv^t), respectively. A strategy profile for period t then is given by the string $\sigma^t = \left\{op^\tau, ov^\tau, bp^\tau, bv^\tau\right\}_{\tau=t}^{0}$, with $\sigma := \sigma^T$.

For a given strategy profile we can define per-period payoffs. In the case of an organizational period, $u_j(.)$ is completely determined once h, i, and σ are given. So, we simply write $u_j(h,i,\sigma)$. The period payoff to all legislators for an organizational period is always zero.

In contrast to organizational periods, bill periods may result in non-zero period payoffs to some legislators. In addition to $h \in H(B,t)$, i, and σ, these payoffs also depend on x. When $h \in H(B,t)$ the bill period t payoff for legislator j is:

$$u_j(x,h,i,\sigma) = \begin{cases} b_j^i & \text{if } \sum_{k \in N} bv_k^t(h,i,x,b^i) \geq \dfrac{n+1}{2} \\ x_j & \text{otherwise} \end{cases} \quad \text{with } b^i := bp^t(h,i,x).$$

That is, in each bill period the legislator receives a period payoff as allocated by the bill if the bill is accepted or her period reservation value if it is rejected. Let C be the accepted governing coalition from the most recent organizational period. Then, for each h and σ, we can calculate the expected bill period payoff for any j conditional on i being recognized as $EP_j(h, \sigma | \boldsymbol{i} = i)$. The ex-ante expected payoff per bill period is simply

$$EP_j(h,\sigma) := \begin{cases} (1/|C|)EP_j(C,h,\sigma|\boldsymbol{i} = j) + (1 - 1/|C|)EP_j(C,h,\sigma|\boldsymbol{i} \neq j) & \text{if } j \in C \\ EP_j(h,\sigma|\boldsymbol{i} \neq j) & \text{if } j \notin C \end{cases}$$

Finally, we make the following two technical assumptions[3]:

A.1 All legislators have a lexicographical preference for voting to accept a proposal.

A.2 Each legislator i lexicographically prefers to make a proposal that will be accepted by legislator k to one that will be accepted by legislator k' if $i<k<k'<n+1$ or $0<k'<i<k<n+1$ or $0<k<k'<i$.

Assumption A.2 implies that proposers make proposals in a "round-robin" fashion. Finally, let Σ^t be the set of partial strategy profiles for all subsequent periods including t satisfying assumptions A1, A.2 and the iterated elimination of weakly dominated strategies from period t onward.

Now we construct the two multi-stage games G_{us} and G_{uk} as a finite sequence of organizational and bill periods. First consider G_{us}. The game begins in period T with an organizational period. Each succeeding period is an organizational period until a governing coalition is accepted or there are no more periods. When a governing coalition is accepted, all of the remaining periods are bill periods in which only members of the governing coalition are recognized to make proposals. In a bill period legislators get a period payoff determined by the bill (if it is accepted) or the period reversion outcome. The structure of G_{us} is portrayed in Figure 3.

<<Figure 3 about here>>

The game G_{uk} differs from G_{us} only in the consequences of rejecting a bill in the bill period. In the previous game, once a governing coalition is accepted it remains in place for the rest of the game. In contrast, if a bill is rejected in G_{uk} then the next period is an organization period not a bill period. This feature establishes a link between a vote on a bill and a vote on the governing coalition. This game is portrayed in Figure 4.

[3] Both assumptions are standard in the legislative bargaining literature (Baron and Ferejohn 1989) and allow us to restrict attention to pure strategies. The assumptions are technical in the following sense: if we permitted the use of mixed voting strategies, then the behavior implied by A.1 would occur in subgame-perfect equilibrium. Similarly, allowing for mixed proposal strategies would generate "balanced" proposal behavior. That is, every member with symmetric characteristics would have an equal chance of being included in the proposal. Assuming A.1 and A.2 allows us to dispense with equilibrium analysis altogether and instead only use the iterated elimination of weakly dominated strategies as our solution concept.

<<Figure 4 about here>>

We formalize the concept of cohesion by a cohesion index γ^t: the probability that the procedural coalition (that voted to install the governing coalition) is identical to the policy coalition (that voted for the bill).

Definition 1. *Let C be the governing coalition in bill period t, $C^i \subseteq N$ be the procedural coalition that voted for C, and $h \in H(B,t)$. Then the cohesion index for a bill period t is*

$$\gamma^t := Pr(bv_j^t(h,i,x,b^i) = 1 \Leftrightarrow j \in C^i)$$

Note that with 3 players and i.i.d. draws from a uniform distribution we must have $1/2 \leq \gamma^t \leq 1$.

Next we analyze the effects of the iterated elimination of weakly dominated strategies on legislative cohesion in a 3-legislator game for large T starting in the last period and then proceeding by backward induction. First we consider the US-case[4].

Proposition 1. *For the game G_{us} and for any period t>2:*

1. *Minimum winning governing coalitions are proposed and accepted in the first period.*
2. *The cohesion index, γ^t, equals* 1/2.
3. *The expected per-period payoff for members of the governing coalition is a constant* k_C, *for non-coalition members* k_{NC} *with* $k_C > k_{NC} > 0$.

The proof of Proposition 1 proceeds in three steps. First we show that bill voting strategies are history independent. This is fairly easy to see since if period t is a bill period then every subsequent period is also a bill period. In the last period, independent of history, legislators will vote to accept any bill that gives them at least their period reversion value and reject anything else. Thus, the last period proposer will propose a bill that pays off the

[4] The reader may wonder why we did not choose to model the president explicitly as a further players in the U.S. case. Proposition 1 answers this question. The president would have to be modeled as a further player with veto rights at the bill stage, but no proposal rights. Hence each proposer would have to pay the president exactly his reservation value reducing the available total pay-off by this amount. Nothing else changes.

cheapest other legislator and keep the rest for herself. It follows that in the next to the last bill period play will again be history independent. Thus, play in all bill periods is history independent. It follows immediately that the cohesion index will be 1/2, since a proposer in a bill period picks her governing coalition partner only if this member is cheaper than the non-coalition member. This occurs with probability 1/2.

The bill period behavior guarantees that play in any organizational period will be history independent as well. If there are more than 2 periods left any proposer in an organizational period will propose a minimum winning coalition.[5] To see this observe that if the coalition of the whole were proposed it would always be accepted because the continuation value for rejecting a coalition is always (t-1)/3 whereas the payoff for accepting a coalition of the whole is t/3. However, the continuation value to a member of any proposed governing coalition that is less than universal will, if accepted, be strictly greater than t/3 while those left out will have a continuation value of strictly less than (t-1)/3. Thus, the only non-universal coalition that will be accepted is minimum winning.

The only difference between G_{US} and G_{UK} is that rejection of a bill implies that the next period will be an organizational period. This, however, has dramatic consequences both for cohesion and the distribution of benefits.

Proposition 2. *For the game G_{UK}:*

1. *Minimum winning governing coalitions form in the first period.*
2. *For t>2 the cohesion index, γ^t, equals* 1.
3. *For t>5, the expected per-period payoff for members of the governing coalition* $k_C \approx 1/2$, *for non-coalition members* $k_{NC} \approx 0$.

The linkage between bill and organizational periods implies that a legislator's future expected payoff depends on the current voting decision. Rejection of a bill not only implies the loss of a bill period, but also the chance that the current governing coalition will be replaced. So, each legislator's voting behavior will depend on the current period reservation value x and the net continuation value from accepting a bill[6] (V_j^t). In general, the net continuation values differ between members of the governing coalition and other legislators. Iterated weak dominance implies that admissible voting and proposal strategies can be characterized by the parameters x and V_j^t, but are otherwise history independent. In particular, (using A.1), legislators accept a proposal if and only if $b_j^i \geq x_j - V_j^t$. So, in contrast to G_{US}, legislators vote on the basis of period reservation *and* the net acceptance continuation values.

Given this voting behavior, it is a strictly dominant strategy for the proposer to offer the "cheapest" non-proposer just enough to make him indifferent and keep the rest of the payoff to himself.[7] A legislator k is cheaper than legislator k' if and only if $x_k - V_k^t < x_{k'} - V_{k'}^t$. Note that for $x_k < V_k^t$, k would accept every feasible proposal (k is "free") and i will proposes $b^i = (1,0,0)$ where $b^i = (b_i^i, b_k^i, b_{k'}^i)$. Otherwise i must offer $b^i = \left(1 + V_k^t - x_k, x_k - V_k^t, 0\right)$. All such offers are accepted by assumption A.1. The proposer keeps his own reservation value plus the reservation of the member not included in the policy coalition plus the portion of the policy coalition member's reservation value that was not needed to ensure the bill's passage.

A key difference between G_{UK} and G_{US} is the fact that minimum winning governing coalitions lead to different net continuation values between members of the governing coalition and other legislators. The higher a player's net continuation value, the more likely he is free. This, however, means that the chosen proposer's conditional expected payoff increases as well. Since only members of the governing coalition can be recognized, their continuation values increase, but decreases for non-coalition members. This can be seen in the following figure illustrating a bill period t=1 when the governing coalition size is |C|=2.

---

[5] The fact that our model implies minimum-winning coalitions should not be over-interpreted. It is a direct consequence of the divide-the-dollar framework and the take-it-or-leave- it offer (Baron and Ferejohn 1989). Relaxing either assumption typically leads to different coalition sizes.

[6] Net continuation values are formerly defined in the appendix.

[7] The reader may wonder if such a bill will always exist. Indeed, if $x_k > 1 - V_{k'}^t$, no feasible offer would be accepted. Thus, no legislator would be "affordable". Iterated weak dominance, however, guarantees that this case will never occur.

<<Figure 5 about here>>

As in the US, both non-coalition members have the same net-continuation value, namely 1/12. The graph shows the per-period pay-offs for all possible values of $x_k, x_{k'}$. We can distinguish three regions. In the dotted region at least one legislator is free. So, the proposer keeps the entire pay-off. Otherwise, the proposer has to offer a positive payoff to the cheaper member. Each non-proposer has the same conditional expected payoff. But since only members of the governing coalition may be recognized, their net continuation value increases; the net continuation value of other legislators decreases.

Now consider bill period t=2 in which the governing coalition is of size |C|=2. Figure 6 illustrates the asymmetry between governing coalition members and other members created by different net continuation values.

<<Figure 6 about here>>

The non-coalition member k's net continuation value is already negative. To see this recall that in the case of a rejected bill the game proceeds to an organizational period in t=1. Continuation values in organizational periods depend on the total expected payoff from accepting a particular coalition. As in G_{US}, it is a strictly dominant strategy for each legislator in t=1 to accept any proposed governing coalition. But then the proposer has a strictly dominant strategy to always propose a coalition that includes only herself. Hence, since last period bill-proposals will always be accepted, each legislator has an ex ante expected payoff of 1/3 before the proposer in organizational period t=1 is determined. The continuation value from rejecting a bill at t=2 thus equals 1/3. On the other hand, the continuation value for accepting the bill for legislator k is strictly less than 1/3.

In contrast, the continuation value for the coalition member k' is strictly larger than 1/3. Therefore k' is more likely to be included in the policy coalition than k. Furthermore, as illustrated by the large dotted region in Figure 6, the non-proposing coalition member often receives a payoff of zero and still votes for the proposal. Thus, the expected payoff conditional on not being the proposer is rather small. But, since members of the governing coalition also may be proposers, the ex ante expected per period payoff for each member of the governing coalition is close to 1/2.

Now consider a bill period t=3 where |C|=2 illustrated in Figure 7. The difference between coalition member k' and non-coalition member k net continuation values has grown so large that k can never be cheaper than k'. Hence, every policy coalition will be composed exclusively of members of the governing coalition. As in t=2, the non-coalition member would strictly prefer to reject the proposal and move to an organizational period. Hence, we have perfect cohesion in this and all bill periods with t>2. Note that the period 2 proposer appropriates even more of the expected dollar to herself than the period 1 proposer. Indeed, it can easily be shown that for t>4, the proposer can keep the entire dollar and still be accepted.

<<Figure 7 about here>>

The proposition is proven by showing that a minimal winning coalition will indeed be proposed and accepted in the first period. It is immediately obvious that a proposer in an organizational period will have a higher expected pay-off in a minimum winning rather than a universal coalition. As we show in the appendix, governing coalitions of size 1 will always be rejected for t>1 since the expected per-period payoff for non-coalition members is so low that they are willing to sacrifice the loss of a single bill period in exchange for a chance to be a member of the governing coalition formed in the next period.[8]

Despite the striking difference between the U.S. and the U.K. in bill periods, both cases are rather similar when it comes to organizational periods. The following corollary captures these similarities.

---

[8] The reader may wonder whether these results change in the presence of discounting, e.g. to capture the effects of an exogenously probability that the game will terminate. Let $\delta \in (0,1)$ be a discount factor with the interpretation that δ is the probability the game continues to the next period. In this case it can be shown that for a game of fixed length T there is a sufficiently low discount factor such that in both G_{US} and G_{UK} the predicted size of the governing coalition changes from minimum winning to minority (i.e., from size 2 to size 1) and cohesion is reduced to zero. The intuition is as follows. When the discount factor is very low agents care exclusively about what they receive in the current period (if it is a bill period) and in obtaining a positive payoff from the next period (if it is an organizational period). In a bill period proposers therefore cannot exploit coalition members in the UK and will form the cheapest minimum winning coalitions in each bill period. In organizational periods the proposer recognizes that for any approved coalition the proposer will always form the cheapest minimum winning coalition in the following bill period. Further, the proposer and all the other legislators only care about the comparison between their expected payoff in the next bill period (if the coalition is approved) and the expected payoff in the organizational period (zero). Therefore the proposer in the organizational period will propose a minority coalition of only herself which will be approved by all the legislators. Thus cohesion is zero in this game because in every bill period someone votes against the bill proposed while all legislators voted for the governing coalition.

Corollary 1. *In both games for any governing coalition C approved at organizational period t>2 it is the case that members of the procedural coalition are strictly better off by maintaining the governing coalition than by making the next period an organizational period whereas non-coalition members would strictly prefer the next period to be an organizational period.*

In both games the members of the procedural coalition would vote to maintain the current coalition should this need arise. In this sense the procedural coalition that originally supported the governing coalition implicitly persists in subsequent periods. In the U.S. case this is true, even though the policy coalitions may change depending of the realization of **x**. This features is consistent with the previously mentioned observation that voting is very disciplined even in the U.S. when it comes to matters of organizing the chamber.

The next corollary compares the governing coalition's expected payoffs for the two games.

Corollary 2. *For t>2 the expected per-period payoff for each member of the governing coalition in* G_{uk} *is strictly higher than in* G_{us} .

Corollary 2 suggests that a governing coalition would always choose to commit itself to step down if its bill is defeated. This indicates that the availability of such a commitment mechanism in parliamentary democracies marks a key difference between parliamentary democracies and the United States Congress. To investigate this question further we compare our result with some of the traditional concept from the comparative government literature.

Implications for the Comparative Study of Legislatures

We showed above that the ability of the prime minister to call for a vote of confidence is sufficient to generate cohesion. We now use our model to consider, informally, some other institutional features commonly associated with parliamentary systems and to establish the unique role of the vote of confidence procedure.

Votes of Censure, Constructive Vote of No-Confidence, Votes of Investiture. One of the most frequently mentioned differences between parliamentary and presidential democracies is the fact that the governing coalition is elected by the chamber and can be voted out of office by a procedural majority at any time.[9] In our model this election corresponds to a choice of governing coalitions and votes of censure correspond to a transition from a bill period to an organizational period. Notice, however, that *in this sense* presidential and parliamentary systems are very similar. A collapse of the government (in the precise sense of a governing coalition) in a parliamentary systems corresponds to changing the organizational structure of Congress. Corollary 1 above establishes that in both systems procedural coalitions have a strict incentive to maintain the current governing coalition. The presence of votes of censure or votes of investiture is thus neither sufficient, nor necessary for voting cohesion.[10]

Electoral mechanisms. Confidence vote procedures constitute only one possible mechanism sufficient to generate cohesion. Other mechanisms are imaginable, especially if one deviates from our assumption of i.i.d. draws of reservation values. As an example of an electoral mechanism consider a two-party legislature where each party leader has complete control over the electoral renomination process. By threatening to deselect a member, the leader can thus ensure the loyalty of each party member. But then the (induced) preferences of a party leader and member are identical for every possible bill and reservation value. So, perfect cohesion follows trivially in the two-party case. If control over deselection is only partial, cohesion may vary as a function of the degree of control.

---

[9] See already Bagehot (1963[1867]) "The House of Commons is an electoral chamber; it is the assembly which chooses our president" (p.115) and later "...the principle of Parliament is obedience to leaders. Change your leaders if you will, but obey No.1 while you serve No.1, and obey No.2 when you have gone over to No.2..."(p.125).

[10] Some parliamentary democracies such as Germany and Spain, use constructive votes of no-confidence instead of a vote of censure. The difference to a normal vote of censure is that a government can not be voted out of office, but only replaced by an alternative governing coalition. In our model, this would mean that governing coalitions could be replaced directly without moving to an organizational stage first. Proposition 2, however, implies that no member of the current coalition has a strict incentive to vote for such a replacement for $t>2$.

The key point is that in the absence of institutional mechanisms that ensure high cohesion, the level of cohesion will vary as a consequence of the correlation between reservation values. Consequently, our model also implies higher variance of the level of cohesion in the U. S. Congress than in parliamentary democracies, but lower average levels of cohesion. Previous research on Congress, for instance, has indicated such changes in cohesion (e.g. Brady, Cooper and Hurley 1979, Collie 1985, Cox and McCubbins 1993, Rohde 1991). Our model suggests that in order to understand these variations one should mainly focus on the changes in reservation values that may be caused e.g., by changes in the electoral domain.

France's Fourth Republic. The formal analysis has shown that confidence vote procedures are sufficient for cohesion, but are they necessary? One may, for instance, imagine that a governing coalition such as the leadership in the United States Congress may publicly announce to step down in case a proposed bill is defeated. Corollary 2 implies that governing coalitions would always strictly prefer to make such a commitment if possible, since the expected payoff to each member of the governing coalition is strictly higher in G_{uk} than G_{us}. The question is, whether such an announcement would be credible. We certainly do not wish to claim that this is never possible in the U.S.. Our model suggests, however, that in contrast to the U.S., parliamentary constitutions always offer such a commitment device: the vote of confidence procedure.

To investigate the question of necessity further we would e.g., need a constitution with a vote of censure, a vote of investiture, but no ability of the government to link votes on bills with the survival of the government. Fortunately, there exists such a case: the French Fourth Republic. This polity presents a notorious puzzle for the comparative study of legislatures. Although it seemingly fulfilled all constitutional prerequisites for a stable parliamentary democracy, it was plagued (among other things) by low levels of cohesion. Our result suggests to investigate the details of the vote of confidence procedure and find out whether they force the government to step down in case of defeat.

While the French Fourth Republic did have a vote of confidence procedure, Huber (1996) recently pointed out that it was deficient in a critical way: abstentions counted against the bill, but for the government. Thus, through the strategic use of abstentions legislators could vote down the bill, but avoid the resignation of the government. Hence, a vote on a bill could *not* be linked to the survival of the government. This example emphasizes that the vote of confidence procedure is an instrument of the government that forces legislators to support the cabinet. The strategic use of abstention in the French Fourth Republic, however, allowed the legislators to untie these knots and only vote on the bill. In this case, our model predicts low cohesion. This surprising finding suggests that rather subtle differences in the constitutional structure correlate with cohesion in precisely the direction predicted by the game-theoretic analysis.

Pay-Off Distributions and the Design of Constitutions

So far the discussion of our results has only focused on the explanation of observed voting behavior. Corollary 2, however, points out that the design of constitutions may also have consequences for the distribution of expected pay-offs. In particular, our results imply that governing coalitions in parliamentary systems capture almost all the spoils leaving almost nothing for the minority. Whereas, when the vote of confidence procedure is not available, the minority shares in the spoils although they receive less than the governing coalition.

This result is important in the context of constitutional design, since it points out trade-offs between different goals in designing a constitution, e.g. between stable and predictable policy coalitions and pay-off equity. This is especially important if majorities do not change frequently enough from election to election as could be expected e.g. if social, political, and economic cleavages coincide (Lijphard 1977). In this case, the majority in the population may have an incentive to systematically exploit a permanent minority which may be encouraged to rebel. Northern Ireland would provide an example. In these cases Lijphard (1977) has suggested the use of constitutional means such as veto mechanisms that encourage a more egalitarian distribution of pay-offs.

On the other hand the higher pay-off to the governing coalition in Britain may provide theoretical leverage for the investigation of related questions in comparative politics. Our analysis suggests an explanation for variation in the level of party discipline between parliamentary and presidential systems, even when the electoral rules used are identical as in the case of the British House of Commons and the U.S. House of

Representatives. To see how this may work consider Cox and McCubbins' (1993) approach to parties. Here, parties provide a collective good to legislators, a "brand label", that provides information to risk-averse voters and thus increases reelection probabilities. On the other hand legislators also have an incentive to maintain individual reputations. Maintaining the brand label thus creates a collective action problem. Parties are viewed as self-enforcing mechanisms that solve this problem by providing selective incentives to legislators. Corollary 2 implies that the expected pay-off difference between being a member of the majority and the minority is always strictly higher in parliamentary systems, thus providing stronger incentives to maintain a collective reputation.

Another significant difference between parliamentary systems and the U.S. Congress occurs on the dimension of legislative organization. In parliamentary systems legislative bargaining is mainly conducted by government party leaders and ministers. In contrast, bargaining occurs within congressional committees in the U.S. Congress. Only members of the governing coalition are in the cabinet whereas congressional committees are typically bipartisan with the minority exercising significant influence. Suppose, following Gilligan and Krehbiel (1987), that committees are understood to provide a collective good in the form of information to the legislature as a whole. In that case there is a greater incentive under a system without the vote of confidence to include members of the minority on the committees. This follows since the minority can expect to gain as well as the majority from better information. The same cannot be said for parliamentary systems. In such systems the vote of confidence procedure permits any informational gains to be entirely exploited by the majority alone thus reducing the incentive for the minority to usefully contribute any information. If the minority is not going to contribute anything by being involved in the policy formation stage then there is little reason to give them much voice either within the cabinet or on the floor.

Finally, the fact that the majority can so effectively exploit the minority under the vote of confidence procedure suggests an incentive for minorities to seek extra-legislative remedies. Minorities might gain leverage by engaging in protest activities that constrain the government. The greater incentive to engage in disruptive activities would naturally lead to greater incentives to set up extra-legislative mechanisms to diffuse such crises. Thus, our model may provide some leverage for explaining the greater incidence extra-legislative bargaining mechanisms found more frequently under parliamentary systems (Lijphard 1977).

Conclusion

In this paper we developed a model to study the formation of legislative coalitions and voting cohesion in parliamentary and congressional legislatures. Our model implies low cohesion in systems without the vote of confidence procedure, like the United States, and high cohesion in systems with the procedure even if the latter are characterized by many parties. The key feature that leads to cohesion is the fact that votes on bills are treated as votes on government. We identified the vote of confidence procedure as one constitutional feature sufficient for cohesion. The fact that cabinets are accountable to the assembly, however, is neither necessary nor sufficient for cohesion.

Other constitutional features that may be investigated within a similar framework include multi-cameralism, presidents with proposal power, and the endogenous dissolution of chambers. Such features may be investigated to answer questions about what incentives they present not only for legislative organization and voting behavior but also for party formation and the creation of extra-legislative institutions.

Appendix

Derivation of Continuation Values

First we recursively define the total expected payoff at the beginning of each period for the remaining game as the sum of the expected payoff in each period So, for $h \in H(B,0) \cup H(O,0)$ we have

$$W_j(h^0,\sigma) = EP_j(h^0,\sigma),$$

and for $h^t \in H(B,t) \cup H(O,t)$. For $h^{t+1} \in H(B,t+1) \cup H(O,t+1)$ where h^t is consistent with h^{t+1} we have[11]

$$W_j(h^{t+1},\sigma) = EP_j(h^t,\sigma) + W_j(h^t,\sigma).$$

Then for each period t>0 we can define continuation values, that is the expected payoff given σ after t is concluded. For an organizational period t with $h \in H(O,t)$ we write the acceptance continuation value as $AO_j^t(h,i,C^i,\sigma) = W_j(h^{t-1},\sigma)$, the (rejection) continuation value as $RO_j^t(h,i,C^i,\sigma) = W_j(h^{t-1},\sigma)$. Similarly for bill periods reached by $h \in H(B,t)$ we write $AB_j^t(h,i,x,b^i,\sigma) = W_j(h^{t-1},\sigma)$ if the bill is accepted, and $RB_j^t(h,i,x,b^i,\sigma)$ $= W_j(h^{t-1},\sigma)$ if it is rejected. It will be convenient to state our argument in terms of the net continuation value for accepting a bill defined as

$$V_j^t(h,i,x,b^i,\sigma) := AB_j^t(h,i,x,b^i,\sigma) - RB_j^t(h,i,x,b^i,\sigma).$$

For fixed h,i,σ,x,b^i we suppress the arguments of these functions.

Proposition 1. *For the game G_{US} and any $\sigma^t \in \Sigma_{us}^t$, and $j \in N$:*

1.) If $t<T$, $h \in H(B,t)$ and $(x,b^i) \in X^2$, then

(i) $bv_j^t(h,i,x,b^i) = 1 \Leftrightarrow b_j^i \geq x_j$

and for all x such that $x_k < x_{k'}$

(ii) $bp_i^t(h,i,x) = (b_i^i, b_k^i, b_{k'}^i) = (1 - x_k, x_k, 0)$

---

[11] By "consistent" we mean that h^t is identical to h^{t+1} in all periods up to t.

2.) If $h \in H(O,t)$, $C^i \in Pot(N)\backslash\{\emptyset\}$ and $1<t\leq T$, then

(i) $ov_j^t(h,i,C^i) = 1 \Leftrightarrow j \in C^i$

$$(ii)\ op_i^t(h,i) = \begin{cases} \{i,i+1\} & \text{if } i<3 \\ \{i-1,i\} & \text{otherwise} \end{cases}.$$

Proof:

1.) The result follows by induction on t. Consider the last period *t*=0. From weak dominance and A.1 (i) follows immediately for each i, x, and $h \in H(B,0)$. That is voting behavior in the last period is history independent. But then, the iterated elimination of weakly dominated strategies implies that the proposer will always form the cheapest minimum winning coalition given *x*. This proves (i) and (ii) for the induction basis. Further it follows that for all $h, h' \in H(B,0)$ we have

$$W_j(h,\sigma) = W_j(h',\sigma).$$

To prove the induction step, suppose that (i), (ii) hold for t, and we have for all $h, h' \in H(B,t)$

$$W_j(h,\sigma) = W_j(h',\sigma).$$

Then by A.1 and the iterated elimination of weakly dominated strategies we have

$$bv_j^{t+1}(h,i,x,b^i) = 1 \text{ iff}$$

$$b_j^i + AB_j^{t+1}(h,i,x,b^i,\sigma^{t+1}) \geq x_j + RB_j^{t+1}(h,i,x,b^i,\sigma^t) \text{ iff}$$

$$b_j^i \geq x_j\ .$$

Thus (i) follows immediately for each i, x, and $h \in H(B,t+1)$. But then (ii) is optimal for each i, x, and $h, h' \in H(B,t+1)$. But this implies $W_j(h,\sigma^{t+1}) = W_j(h',\sigma^{t+1})$.

2.) Again the proof proceeds by induction on t. First we show that the claim holds for t=2. From 1.) we know that in any policy period the proposer will form the cheapest minimum winning coalition and offer those in the coalition exactly their period reservation payoff. In any period there is one proposer and two non-proposers. The ex-ante expected period payoff to a non-proposer in any period is the expected value of his reservation payoff conditional on the reservation payoff being less than the reservation payoff of the other non-proposer:

$$EP_k(h,\sigma|\boldsymbol{i}=i)=EP_{k'}(h,\sigma|\boldsymbol{i}=i)=2\left[\int_{x=0}^{\frac{1}{2}} x\int_{y=x}^{1-x} dydx\right]=1/12.$$

The expected payoff to a proposer is just:

$$EP_i\,(h,\sigma|\boldsymbol{i}=i)=1-2EP_k(h,\sigma|\boldsymbol{i}=i)=5/6.$$

Therefore the expected period payoff to a coalition member is:

$$EP_j(h,\sigma)=\frac{1}{|C|}EP_j\,(h,\sigma|\boldsymbol{i}=j)+(1-\frac{1}{|C|})EP_{k'}(h,\sigma|\boldsymbol{i}\neq i)=\frac{3}{4|C|}+\frac{1}{12}$$

and the expected period payoff to a non-coalition member is

$$EP_j(h,\sigma)=1/12.$$

For t=0 there is nothing to prove. Consider t=1. Then for all j: $RO_j^t(h,i,C^i,\sigma)=0$.

However, the continuation value for accepting a coalition is

$$AO_j^1(h,i,C^i,\sigma)=\begin{cases}\dfrac{3}{4|C|}+\dfrac{1}{12} & \forall j\in C^i\\ \dfrac{1}{12} & \forall j\notin C^i\end{cases}.$$

The iterated elimination of weakly dominated strategies implies $ov_j^1(h,i,C^i)=1\ \forall j\in N$.

Now given this voting behavior it is a strictly dominant strategy for any proposer *i* to propose a governing coalition consisting of herself alone. So, $op_i^1(h,i)=\{i\}$ and $ov_j^1(h,i,\{i\})=1\ \forall j\in N$.

Consider organizational period 2. The acceptance continuation value for any proposed coalition C^i is as follows

$$AO_j^2(h,i,C^i,\sigma)=\begin{cases}2\left(\dfrac{3}{4|C^i|}+\dfrac{1}{12}\right) & \forall j\in C^i\\ \dfrac{2}{12} & \forall j\notin C^i\end{cases}.$$

The rejection continuation value is

$$RO_j^2(h,i,C^i,\sigma)=1/3\ \forall j\in N.$$

Now the iterated elimination of weakly dominated strategies implies that legislators will vote against any proposed coalition if and only if they are not included in it. Therefore

$ov_j^2(h,i,C^i) = 1 \Leftrightarrow j \in C^i$. It follows that the only coalitions that will be accepted are those that include 2 or more legislators. Given assumption A.2 and the iterated elimination of weakly dominated strategies at the proposal stage we get the result on proposing behavior:

$$op_i^2(h,i) = \begin{cases} \{i,i+1\} & \text{if } i<3 \\ \{i-1,i\} & \text{otherwise} \end{cases}$$

The claim then follows by induction on t. We have already established the claim for t≤2. To conclude the proof assume that voting and proposing strategies satisfy the supposition in all periods through period t. Then the acceptance continuation values in period t+1 is:

$$AO_j^{t+1}(h,i,C^i,\sigma) = \begin{cases} (t+1)\left(\dfrac{3}{4|C|}+\dfrac{1}{12}\right) & \forall j \in C^i \\ \dfrac{t+1}{12} & \forall j \notin C^i \end{cases}.$$

The rejection value is

$$RO_j^{t+1}(h,i,C^i,\sigma) = t/3 \ \forall j \in N.$$

This follows because each legislator is equally likely to be in the accepted minimum winning coalition in organizational period *t*. The result follows by the iterated elimination of weakly dominated strategies, A.1 and A.2.

Proposition 2.

For the game G_{uk} for all $\sigma^t \in \Sigma_{uk}^t$, and $j \in N$:

1.) If $h \in H(B,t)$, $(x,b^i) \in X^2$, and 2<t<T, then

(i) $pv_j^t(h^t,i,x,b^i) = 1 \Leftrightarrow j \in C$;

(ii) If $k' \notin C$, then for all $i \in C$: $b_{k'}^i = 0$.

2.) If $h^t \in H(O,t)$, $C^i \in Pot(N) \setminus \{\emptyset\}$ and 1<t≤T

(i) $ov_j^t(h^t,i,C^i) = 1 \Leftrightarrow j \in C^i$;

(ii) $op_i^t(h^t,i) = \begin{cases} \{i,i+1\} & \text{if } i<3 \\ \{i-1,i\} & \text{otherwise} \end{cases}.$

Proof:

Again the proof proceeds by induction on t. First we show the claim for the induction basis, here t=3.

Induction Basis.

Period t=0

Let $h \in H(B,0)$.

Suppose that |C|=3. That is all legislators are members of the coalition. Then by the same argument as in Proposition 1 and 2 voting and proposing is history independent and we have $EP_k(h,\sigma|\boldsymbol{i} = i) = EP_{k'}(h,\sigma|\boldsymbol{i} = i)$, and $EP_i(h,\sigma|\boldsymbol{i} = i) = 1 - 2EP_k(h,\sigma|\boldsymbol{i} = i)$.

Hence, $EP_j(h,\sigma) = (1/3)EP_j(h,\sigma|\boldsymbol{i} = j) + (2/3)EP_j(h,\sigma|\boldsymbol{i} \neq j) = 1/3$.

Suppose that |C|=2. Then as in the proof of Proposition 2 iterated weak dominance implies history independent behavior and we have

$$EP_j(h,\sigma) = \begin{cases} 11/24 & \forall j \in C \\ 1/12 & \forall j \notin C \end{cases}.$$

Suppose that |C|=1. Then as in the proof of Proposition 2 we have independently of history

$$EP_j(h,\sigma) = \begin{cases} 5/6 & \forall j \in C \\ 1/12 & \forall j \notin C \end{cases}$$

For $h \in H(O,0)$, we always have $EP_j(h,\sigma) = 0$.

Period t=1

Let $h \in H(B,1)$.

Suppose that |C|=3. From the previous step (t=0) we know that $RB_j^1(h,i,x,b^i,\sigma) = 0$ and $AB_j^1(h,i,x,b^i,\sigma) = 1/3$. Then, since $V_j^1 = 1/3$ by the same argument as in t=0, $EP_j(h,\sigma) = 1/3$.

Suppose that |C|=2. From the previous step (t=0) we know that $RB_j^1(h,i,x,b^i,\sigma) = 0$ and

$$AB_j^1(h,i,x,b^i,\sigma) = \begin{cases} 11/24 & \forall j \in C \\ 1/12 & \forall j \notin C \end{cases}.$$

Hence

$$V_j^1 = \begin{cases} 11/24 \cong 0.29 + 1/6 \text{ if } j \in C \\ 1/12 \cong 0.41 - 1/3 \text{ if } j \notin C \end{cases}.$$

By A.1 and iterated weak dominance we know that a legislator j will vote in favor of any bill that satisfies $b_j^i \geq x_j - V_j^t$. Then for all x such that $x_{k'} - V_{k'}^t < x_k - V_{k'}^t$

$$b^i = (b_i^i, b_{k'}^i, b_k^i) = \begin{cases} (1 - x_{k'} + V_{k'}^t, x_{k'} - V_{k'}^t, 0) & \text{if } 0 < x_{k'} - V_{k'}^t \\ (1,0,0) & \text{if } 0 > x_{k'} - V_{k'}^t \end{cases}$$

It follows that, for a given draw of reservation values, coalition members are cheaper than non-coalition members and, therefore, are more likely to be included in a policy coalition. Hence for k'∈C, k∉C:

$$EP_{k'}(h, \sigma | \boldsymbol{i} = i\ ) = 2\left[\int_{V_{k'}}^{\frac{1}{2}+\frac{V_{k'}-V_k}{2}} (x_{k'} - V_{k'}) \int_{x_{k'}-V_{k'}+V_k}^{1-x_{k'}} dx_k dx_{k'} \right]$$

$$= 2\left[\int_{11/24}^{\frac{1}{2}+\frac{9}{48}} (x_{k'} - 11/24) \int_{x_{k'}-9/24}^{1-x_{k'}} dx_k dx_{k'} \right] \cong 0.008$$

$$EP_k(h, \sigma | \boldsymbol{i} = i\ ) = 2\left[\int_{1/12}^{\frac{1}{2}-\frac{9}{48}} (x_k - 1/12) \int_{x_k+9/24}^{1-x_k} dx_{k'} dx_k \right] \cong 0.008, \text{ and}$$

$$EP_i(h, \sigma | \boldsymbol{i} = i\ ) = 1 - EP_k(h, \sigma | \boldsymbol{i} = i\ ) - EP_{k'}(h, \sigma | \boldsymbol{i} = i\ ) \cong 0.984$$

Then we can calculate the expected per period payoff as

$$EP_j(h, \sigma) \cong \begin{cases} 0.496 \text{ if } j \in C \\ 0.008 \text{ if } j \notin C \end{cases}$$

Suppose that |C|=1. From the previous step (t=0) we know that $RB_j^1(h, i, x, b^i, \sigma) = 0$ and

$$AB_j^1(h, i, x, b^i, \sigma) = \begin{cases} 5/6 \ \forall j \in C \\ 1/12 \ \forall j \notin C \end{cases}.$$

Then we have

$$V_j^1 = \begin{cases} 5/6 \text{ if } j \in C \\ 1/12 \text{ if } j \notin C \end{cases}$$

As in the case of |C|=2 voting and proposing behavior is history independent by iterated weak dominance and A.1 and we have

$$EP_{k'}(h,\sigma|i=i\ ) = EP_k(h,\sigma|i=i\ ) = 2\left[\int_{1/12}^{\frac{1}{2}} (x_k - 1/12) \int_{x_k}^{1-x_k} dx_{k'}dx_k\right] \cong 0.048 \text{ and}$$

$$EP_j(h,\sigma) \cong \begin{cases} 0.904 \text{ if } j \in C \\ 0.048 \text{ if } j \notin C \end{cases}.$$

For $h \in H(O,1)$ we know from the analysis at t=0 that $RO_j^1(h^1,i,C^i,\sigma) = 0$. However, for all j and h, i and C^i $AO_j^1(h,i,C^i,\sigma) > 0$. The iterated elimination of weakly dominated strategies thus implies $ov_j^1(h,i,C^i) = 1\ \forall j \in N$. Now given this voting behavior it is a strictly dominant strategy for any proposer *i* to propose a governing coalition consisting of herself alone. So $op_i^1(h,i) = \{i\}$ and $ov_j^1(h,i,\{i\}) = 1\ \forall j \in N$.

Period t=2

Let $h \in H(B,2)$.

Suppose that |C|=3. From t=1 it follows that

$$V_j^2 = AB_j^2(h\ ,i,x,b^i,\sigma) - RB_j^2(h\ ,i,x,b^i,\sigma) = 2/3 - 1/3 = 1/3$$

Hence, again $EP_j(h,\sigma) = 1/3$.

Suppose that |C|=2. From t=1 we know that

$$V_j^2 \cong \begin{cases} 0.496 + 11/24 - 1/3 = 0.62 \cong 0.28 + 2/6 \text{ if } j \in C \\ 0.008 + 1/12 - 1/3 = -0.24 \cong 0.42 - 2/3 \text{ if } j \notin C \end{cases}$$

Then we have

$$EP_k(h,\sigma|i=i\ ) \cong 2\left[\int_0^{0.07} (x_k + 0.24) \int_{x_k+0.85}^{1-x_k} dx_{k'}dx_k\right] \cong 0.003$$

$$EP_{k'}(h,\sigma|i=i\ ) \cong 2\left[\int_{0.621}^{0.85} (x_{k'} - 0.621) \int_0^{1-x_{k'}} dx_k dx_{k'} + \int_{0.85}^{0.93} (x_{k'} - 0.621) \int_{x_{k'}-0.85}^{1-x_{k'}} dx_k dx_{k'}\right] \cong 0.014$$

and therefore $EP_i(h,\sigma|i=i\ ) \cong 0.857$

Then we can calculate the expected per period payoff as

$$EP_j(h,\sigma) \cong \begin{cases} 0.4985 \text{ if } j \in C \\ 0.003 \text{ if } j \notin C \end{cases}$$

Suppose that |C|=1. From the previous step (t=1) we know that

$$V_j^2 \begin{cases} >1 & \text{if } j \in C \\ \cong -0.2 & \text{if } j \notin C \end{cases}.$$

Then we have

$$EP_{k'}(h,\sigma|\boldsymbol{i} = i\ ) = EP_k(h,\sigma|\boldsymbol{i} = i\ ) \cong 2\left[\int_0^{\frac{1}{2}} (x_k + 0.2) \int_{x_k}^{1-x_k} dx_{k'} dx_k\right] \cong$$

$$\cong 1/2(0.2 + 1/6) = 0.18 \cong EP_j(h,\sigma) \text{ for } j \notin C \text{, and}$$

$$EP_j(h,\sigma) \cong \begin{cases} 0.64 \text{ if } j \in C \\ 0.18 \text{ if } j \notin C \end{cases}$$

Note that in all three cases voting and proposing behavior is history independent by the same arguments as in period 1

Let $h \in H(O,2)$. Then

$$RO_j^2(h,i,C^i,\sigma) = RB_j^2(h,i,C^i,\sigma) = 1/3 \text{ and}$$

$$ov_j^2(h,i,,C^i) = 1 \Leftrightarrow AO_j^2(h,i,C^i,\sigma) \geq RO_j^2(h,i,C^i,\sigma)$$

$$\Leftrightarrow AB_j^2(h,i,C^i,\sigma) \geq RB_j^2(h,i,C^i,\sigma) \Leftrightarrow V_j^2 \geq 0$$

Now let $|C^i|$=3. Now from the previous period $V_j^2 = 1/3$. Let $|C^i|$=2. Then,

$$V_j^2 \cong \begin{cases} 0.62 & \text{if } j \in C \\ -0.24 & \text{if } j \notin C \end{cases}.$$

For $|C^i|$=1 we have

$$V_j^2 \begin{cases} >1 & \text{if } j \in C \\ \cong -0.2 & \text{if } j \notin C \end{cases}$$

Therefore $ov_j^2(h^2,i,C^i) = 1 \Leftrightarrow j \in C^i$. It follows that the only coalitions that will be accepted are those that include 2 or more legislators. Since 0.62>1/3, the chosen proposer will offer a coalition of size |C|=2. Given assumption A.2 and the iterated elimination of weakly dominated strategies at the proposal stage we have:

$$op_i^2(h^2,i) = \begin{cases} \{i,i+1\} & \text{if } i<3 \\ \{i-1,i\} & \text{otherwise} \end{cases}.$$

Period t=3

Let $h \in H(B,3)$.

Suppose that |C|=3. From t=2 it follows that

$$V_j^3 = AB_j^3(h\ ,i,x,b^i,\sigma) - RB_j^3(h\ ,i,x,b^i,\sigma) = 2/3 - 1/3 = 1/3$$

Hence, again $EP_j(h,\sigma) = 1/3$.

Suppose that |C|=2. From t=1 we know that

$$V_j^3 \cong \begin{cases} 0.621 + 0.4985 \text{ - } 1/3 = 0.77 \cong 0.28 + 3/6 \text{ if } j \in C \\ \text{-}0.24 + 0.003 \text{ - } 1/3 = \text{-}0.57 \cong 0.43 \text{ - } 3/3 \text{ if } j \notin C \end{cases}.$$

But then $|V_{k'}^3 - V_k^3| > 1$. Therefore if $j \notin C$, j is never included in the policy coalition.

Hence for $k' \in C, EP_k(h,\sigma|\boldsymbol{i} = i\ ) = 0$

$$EP_{k'}(h,\sigma|\boldsymbol{i} = i\ ) \cong 2\left[\int_{0.775}^{1}(x_{k'} - 0.775)\int_{0}^{1-x_k} dx_{k'}dx_k\right] \cong 0.004 \text{ and}$$

$$EP_j(h,\sigma) \cong \begin{cases} 0.5 \text{ if } j \in C \\ 0 \text{ if } j \notin C \end{cases}.$$

Suppose that |C|=1. From the previous step (t=2) we know that

$$V_j^3 \begin{cases} >1 & \text{if } j \in C \\ \cong 1/2(0.2 + 1/6) - 0.2 - 1/3 \cong -0.35 & \text{if } j \notin C \end{cases}$$

Then as in t=2, for $j \notin C$ $EP_j(h,\sigma) \cong 1/2(0.35 + 1/6) = 0.26$. Note that again in all three cases voting and proposing behavior is history independent.

Let $h \in H(O,3)$. Then from the analysis of t=2, $RB_j^3(h,i,C^i,\sigma) = 2/3$

and as in period 2 $ov_j^2(h^2,i,C^i) = 1 \Leftrightarrow j \in C^i$. So,

$$op_i^2(h^2,i) = \begin{cases} \{i,i+1\} & \text{if } i<3 \\ \{i-1,i\} & \text{otherwise} \end{cases}.$$

<u>Induction Step</u>.

Assume the net continuation values for the bill period *t* depend only on C and are given as:

$$V_j^t \begin{cases} \cong 1/3 & j \in C,\ |C| = 3 \\ \cong 0.28 + t/6 & j \in C,\ |C| = 2 \\ \cong 0.43 - t/3 & j \notin C,\ |C| = 2 \\ > 1 & j \in C,\ |C| = 1 \\ \in (-1/2, -1/4) & j \notin C,\ |C| = 1 \end{cases} \tag{3.1}$$

Furthermore, the continuation value for rejecting the bill in every period $1 < t' \le t$ is

$$RB_j^{t'} = (t' - 1)/3. \tag{3.2}$$

We complete the proof by showing that the period strategies given in the proposition are optimal in period t when the above conditions are met and that the continuation values generated for period t + 1 satisfy (3.1) and (3.2).

Suppose t is an organizational period. By the induction hypothesis and iterated weak dominance we know that legislator j will vote for any coalition such that

$$ov_j^t(h, i,, C^i) = 1 \Leftrightarrow V_j^t \ge 0$$

It follows from 3.1 and $t \ge 5$ that

$$ov_j^t(h^t, i, C^i) = 1 \Leftrightarrow j \in C^i$$

Hence, only coalitions of size 2 and 3 will be accepted. The proposer in this period will always offer a coalition of size $|C| = 2$ since $\frac{t}{6} + .28633 > \frac{1}{3}$. Since future play depends only on C and x, A.2 and iterated weak dominance imply

$$op_i^t(h^t, i) = \begin{cases} \{i, i+1\} & \text{if } i < 3 \\ \{i-1, i\} & \text{otherwise} \end{cases}$$

Suppose that t is a bill period and $|C| = 3$. By A.1 and iterated weak dominance we know that a legislator j will vote in favor of any bill that satisfies $b_j^i \ge x_j - V_j^t = x_j - 1/3$. Then for $x_k < x_{k'}$

$$b^i = (b_i^i, b_k^i, b_{k'}^i) = \begin{cases} (1 - x_k + 1/3, x_k - 1/3, 0) & \text{if } x_k > 1/3 \\ (1,0,0) & \text{otherwise} \end{cases}$$

it is also trivial to see that a bill will be proposed that wins in the period and that the net continuation value for bill period t + 1 when $|C| = 3$ is

$$V_j^{t+1} = EP_j^t(h, \sigma) + V_j^t + RB_j^t - RB_j^{t+1} = \frac{1}{3}$$

Suppose $t \in H(B,t)$ and $|C| = 2$. Now for $t \geq 5$ the voting and proposing behavior specified in the proposition hold because for any $j \in C$

$$b_j^i \geq 0 > 1 - V_j^t \geq x_j - \frac{t}{6} - .28633.$$

Thus $j \in C$ will accept every proposal by i, while for $j \notin C$

$$b_j^i \leq 1 < -V_j^t \leq x_j - .42733 + \frac{t}{3},$$

which means that $j \notin C$ will reject every proposal by i.

It follows that $b^i = (1,0,0)$ and

$$EP_j^t(h,\sigma) = \begin{cases} 1/2 & j \in C \\ 0 & j \notin C \end{cases}$$

and therefore

$$V_j^{t+1} = EP_j^t(h,\sigma) + V_j^t + RB_j^t - RB_j^{t+1} = \begin{cases} \frac{1}{2} + \frac{t}{6} + .28 - \frac{1}{3} = \frac{t+1}{6} + .28 & j \in C \\ 0 + .43 - \frac{t}{3} - \frac{1}{3} = .43 - \frac{t+1}{3} & j \notin C \end{cases}$$

Suppose $t \in H(B,t)$ and $|C| = 1$. From (3.1) we know that if $x_{k'} > x_k$ then we know that $1/2 > x_k$ and that the cheapest minimum winning coalition is $\{i, k\}$ and i maximizes his payoff with $b^i = \left(1 + V_k^t - x_k, x_k - V_k^t, 0\right)$. It follows from $1/2 > x_k$ and $V_k^t \in (-\tfrac{1}{2}, -\tfrac{1}{4})$ that b^i is feasible and that legislator k will vote for this bill independent of history. To see that legislator i will also support the bill note that $1 + V_k^t - x_k > 0 > x_i - V_i^t$, by $V_i^{t-1} > 1$. Thus the proposing and voting behavior specified in the proposition are satisfied. Now we show that

$$V_j^{t+1} \begin{cases} \in \left(-\tfrac{1}{2}, -\tfrac{1}{4}\right) & j \notin C, |C| = 1, V_j^t \in \left(-\tfrac{1}{2}, -\tfrac{1}{4}\right) \\ > 1 & j \in C, |C| = 1, V_k^t \in \left(-\tfrac{1}{2}, -\tfrac{1}{4}\right), V_j^t > 1 \end{cases}$$

First consider a non-coalition member k

$$V_k^{t+1} = EP_k^t(h,\sigma) + V_k^t + RB^t - RB^{t+1} = EP_k^t(h,\sigma) + V_k^t - \frac{1}{3}, \text{where}$$

$$EP_k^t(h,\sigma) = 2\left[\int_{x_k=0}^{1/2} (x_k - V_k^t) \int_{y=x_k}^{1-x_k} dy dx\right] = \frac{1}{2}(-V_k^t + \frac{1}{6}), \text{ and}$$

$$V_k^{t+1} = \frac{1}{2}(-V_k^t + \frac{1}{6}) + V_k^t - \frac{1}{3} = \frac{1}{2}V_k^t - \frac{1}{4}.$$

Then $V_k^{t+1} \in \left(-\frac{1}{2}, -\frac{1}{4}\right)$ follows from (3.1).

The continuation value for the proposer *j* is

$$V_i^{t+1} = EP_i^t(h,\sigma) + V_k^t + RB^t - RB^{t+1} = (1 - 2EP_k^t(h,\sigma)) + V_i^t - \frac{1}{3}$$

which can be rewritten as

$$V_j^{t+1} = \left(1 - \left(-V_k^t + \frac{1}{6}\right)\right) + V_j^t - \frac{1}{3} = \frac{1}{2} + V_j^t + V_k^t.$$

Hence $V_j^{t+1} > 1$

Thus if the continuation values satisfy (3.1) and (3.2) in period *t*, then the behavior in period *t* satisfies the proposition and the continuation values in period *t* + 1 satisfy the same conditions. This completes the proof by induction.

References

Bagehot, Walter. 1963 [1867]. *The English Constitution.* Ithaca, NY: Cornell University Press.

Baron, David P. and John Ferejohn. 1989. "Bargaining in Legislatures" *American Political Science Review* 83:1181-1206.

Brady, David W., Joseph Cooper, and Patricia Hurley. 1979. "The Decline of Party Voting in the U.S. House of Representatives." *Legislative Studies Quarterly* 4:381-400.

Brazier, Rodney. 1988. *Constitutional Practice.* Oxford: Oxford University Press.

Collie. Melissa B. 1985. "Voting Behavior in Legislatures" in Gerhard Loewenberg, Samuel C. Patterson, and Malcolm E. Jewell, eds. *Handbook of Legislative Research.* Cambridge: Harvard University Press.

Cox, Gary W., and Mathew D. McCubbins. 1993. *Legislative Leviathan: Party Government in the House.* Berkeley: University of California Press.

Doering, Herbert, ed. 1995. *Parliaments and Majority Rule in Western Europe.* New York: St. Martin's Press

Epstein, Leon D. 1964. "A Comparative Study of Canadian Parties." *American Political Science Review* 57:46-59.

Huber, John. 1996. "The Impact of Confidence Votes on Legislative Politics in Parliamentary Systems." *American Political Science Review* 90:269-282

Gilligan, Thomas W., and Keith Krehbiel. 1987. "Collective Decision-Making and Standing Committees: An Informational Rationale for Restrictive Amendment Procedures." *Journal of Law, Economics, and Organization* 3:287-335

Lijphard, Arend. 1977. *Consocional Democracy.* Yale: Yale University Press.

Loewenberg, Gerhard, and Samuel C. Patterson. 1979. *Comparing Legislatures.* Boston: Little, Brown.

Mezey, Michael L. 1979. *Comparing Legislatures.* Durham: Duke University Press.

Rogowski, Ronald. 1990. "The Information-Economizing Organization of Legislatures". mimeo. UCLA

Rohde, David W. 1991. *Parties and Leaders in the Postreform House.* Chicago: University of Chicago Press.

Von Beyme, Klaus. 1985. *Political Parties in Western Democracies.* New York: St. Martin's Press.

Figure 1

Organizational Period

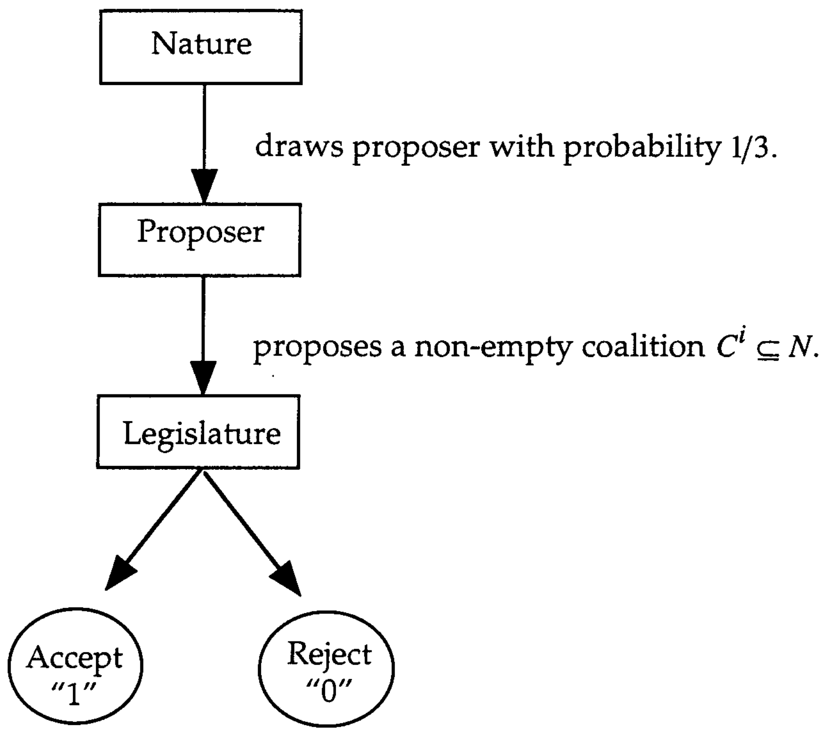


Figure 2

Bill Period

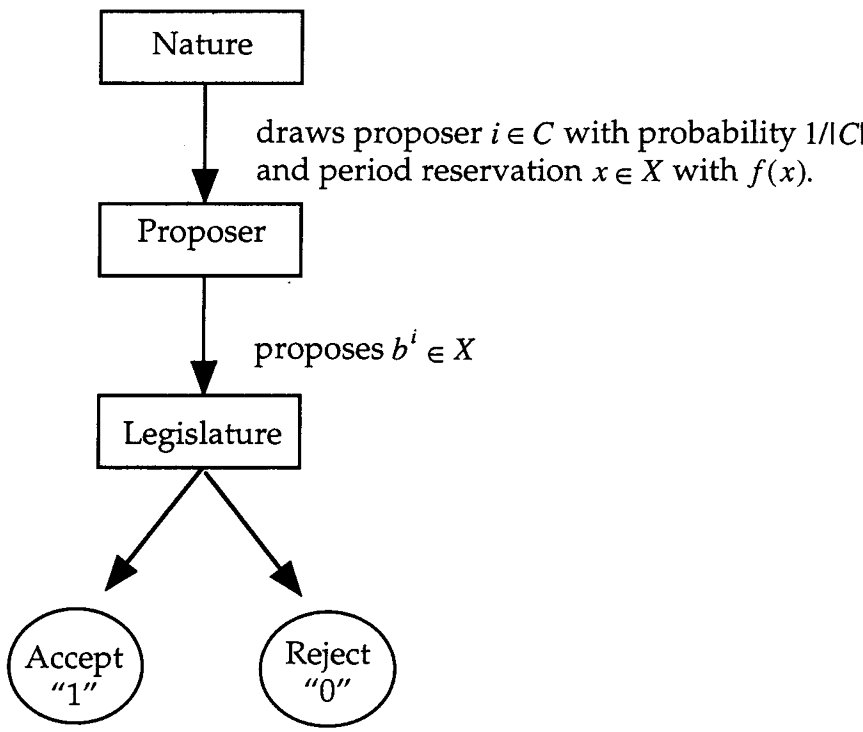
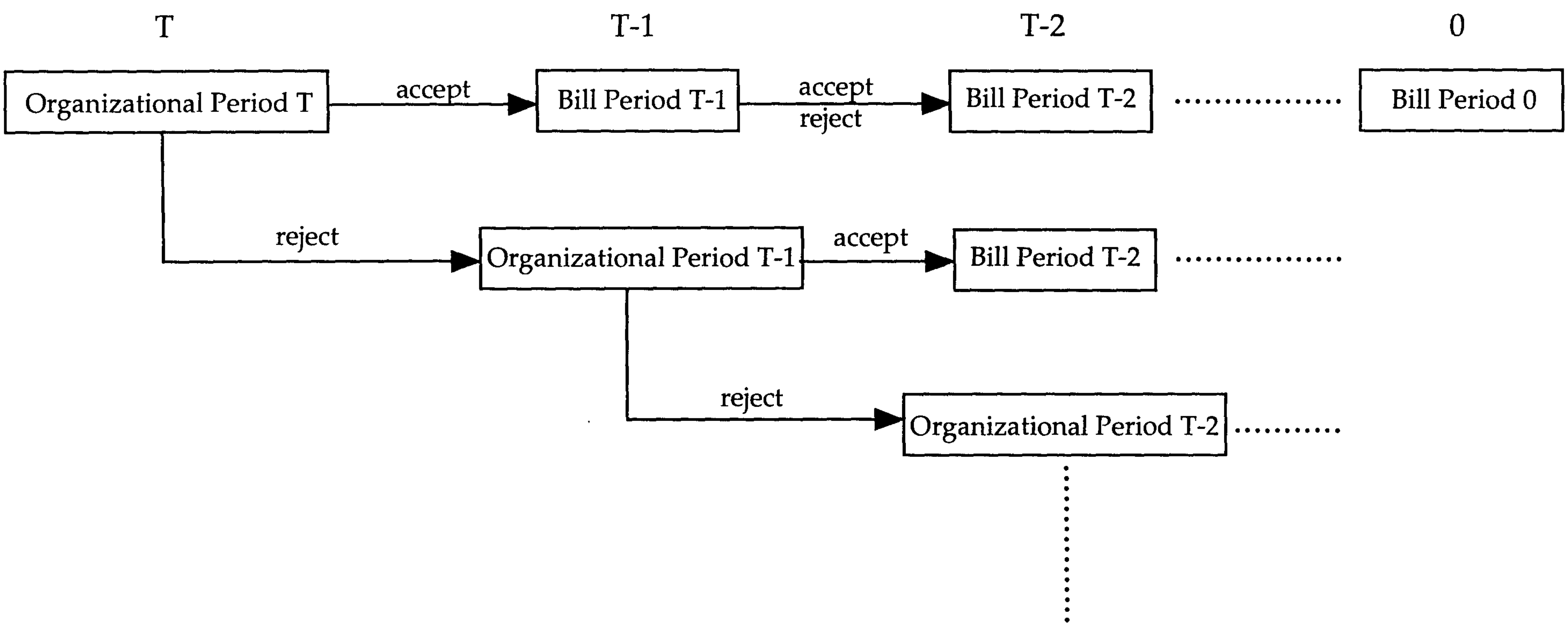


Figure 3

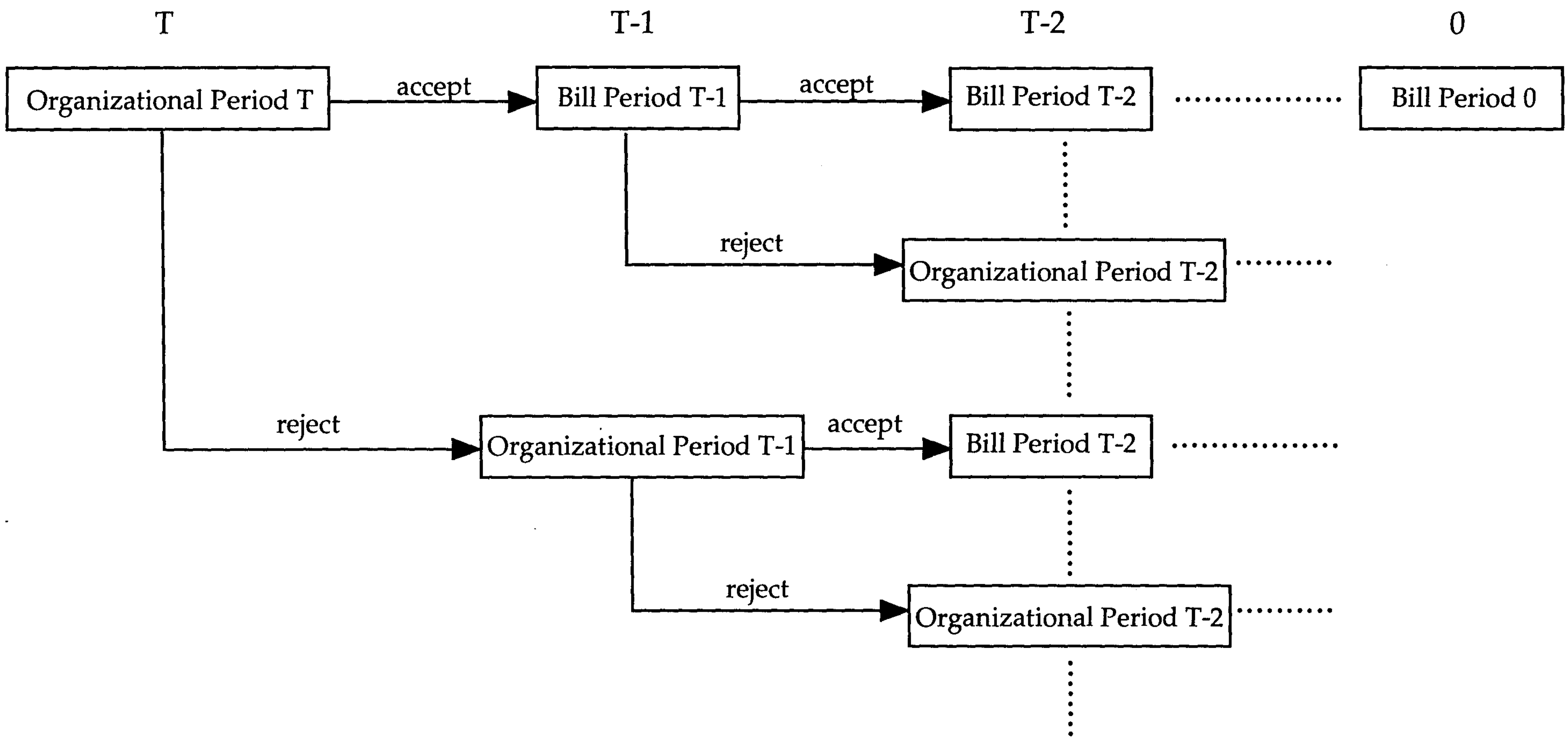
U.S.



T
T-1
T-2
0
Organizational Period T
accept
Bill Period T-1
accept
reject
Bill Period T-2
Bill Period 0
reject
Organizational Period T-1
accept
Bill Period T-2
reject
Organizational Period T-2

Figure 4

U.K.



T
T-1
T-2
0
Organizational Period T
accept
Bill Period T-1
accept
Bill Period T-2
Bill Period 0
reject
Organizational Period T-2
reject
Organizational Period T-1
accept
Bill Period T-2
reject
Organizational Period T-2

Figure 5

$t = 1, \quad |C| = 2$

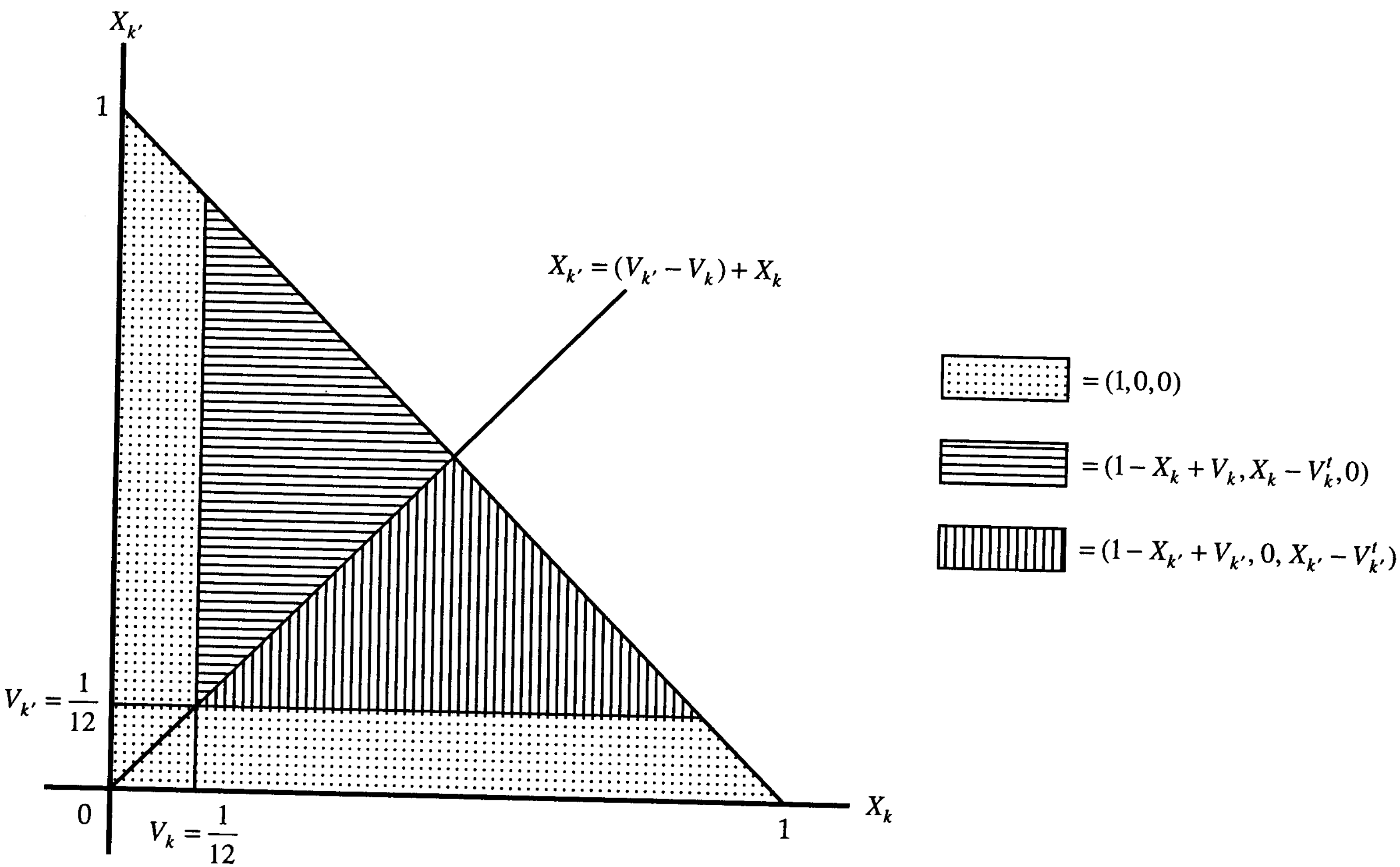


Figure 6

$t = 2, \quad |C| = 2$

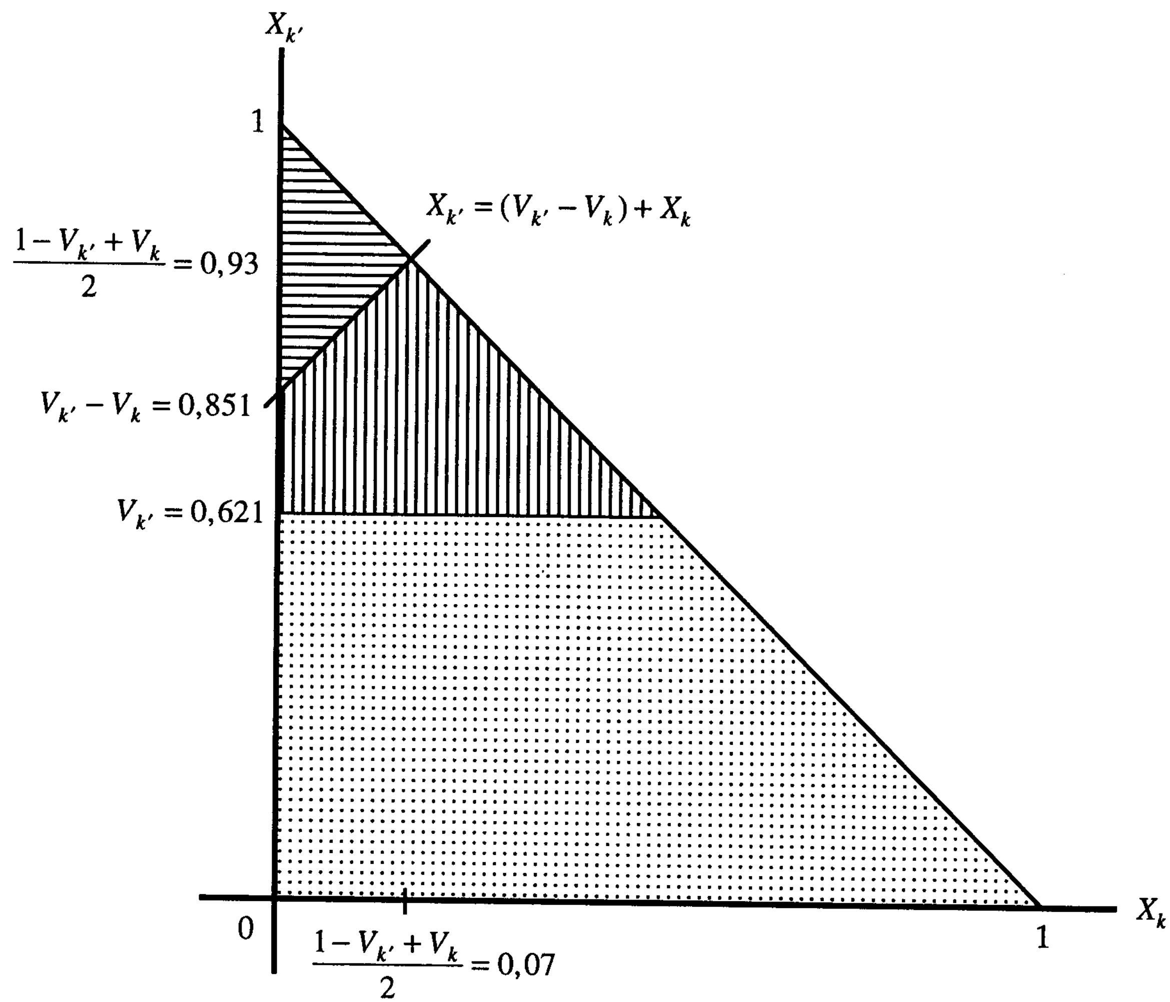


Figure 7

$t = 3, \quad |C| = 2$