Discussion Paper No. 1170

CONVICTING THE INNOCENT: THE INFERIORITY OF UNANIMOUS JURY VERDICTS

by

Timothy Feddersen[1]
and
Wolfgang Pesendorfer[2]
November 1996

[1] MEDS Department, J.L. Kellogg Graduate School of Management, Northwestern University, Evanston, Illinois, 60208-2009. E-mail: tfed@casbah.acns.nwu.edu.

[2] Economics Department, Northwestern University, Evanston, Illinois, 60208-2400. E-mail: wpes@nwu.edu.

Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts

Timothy Feddersen and Wolfgang Pesendorfer*

November 1996

Abstract

It is often suggested that requiring juries to reach a unanimous verdict reduces the probability of convicting an innocent defendant while increasing the probability of acquitting a guilty defendant. We construct a model that demonstrates how strategic voting by jurors undermines this basic intuition. We show that unanimity rule may lead to high probabilities of **both** kinds of errors and that the probability of convicting an innocent may actually *increase* with the size of the jury. Finally, we demonstrate that a wide variety of voting rules, including simple majority rule, lead to much lower probabilities of *both* kinds of errors.

1 Introduction and Literature Review

It is often suggested that requiring juries to reach a unanimous verdict reduces the probability of convicting an innocent defendant while increasing the probability of acquitting a guilty defendant. We construct a model that demonstrates how strategic voting by jurors undermines this basic intuition. We show that unanimity rule may lead to high probabilities of **both** kinds of errors and that the probability of convicting an innocent may actually *increase* with the size of the jury. Finally, we demonstrate that a wide variety of voting rules, including simple majority rule, lead to much lower probabilities of *both* kinds of errors.

*We thank Roger Myerson for providing the argument given in Appendix A.

There is a large literature on juries and jury decision making.[1] One of the central arguments for juries is that a group will make a better decision than an individual.[2] This is the central argument developed in the literature on Condorcet's jury theorem.[3] The jury theorem literature formalizes jury decision making by assuming that jurors possess both public and private information e.g., about the guilt or innocence of the defendant. The public information comes from the fact that all jurors observe the same evidence presented at the trial. The formalization that jurors have private information captures the fact that jurors interpret evidence differently by virtue of different life experiences and competencies. While jurors possess private information, all are assumed to prefer to acquit innocent defendants and convict guilty ones. Since no juror knows with certainty if the defendant is guilty or innocent, jury decisions made by voting aggregate both the public and private information. Thus, juries reach a better result than any individual.

Until recently, the literature on voting by juries has assumed that jurors will vote to convict or acquit without taking into account how the other jurors are voting. We call such behavior sincere voting. Several recent papers have challenged this assumption and demonstrated that the use of voting rules creates an incentive for jurors to vote strategically.[4] The incentive to vote strategically arises because a juror's vote only matters when a vote is pivotal and because the information possessed by other jurors is relevant for a juror's decision. For example, under the unanimity rule a vote is pivotal only when all the other jurors have voted to convict. The fact that all the other jurors have voted to convict reveals additional relevant information about the guilt of the defendant: it reveals, at least in part, the other jurors' private information. Such information may overwhelm the juror's private assessment of the case and cause a juror otherwise inclined to vote for acquittal to vote for conviction instead.

In this paper we examine the implications of strategic voting by jurors and demonstrate that basic intuitions about the consequences of different jury decision rules derived from the assumption that jurors will vote sincerely may

[1] See for example: Klaven and Zeisel (1966); McCart (1965); Levine (1992); and Adler (1994).

[2] See, for example, Klaven and Zeisel (1966, page 8): "it is argued that 12 heads are inevitably better than one."

[3] See Klevorick et. al. (1984), Ladha (1992), Miller (1986), and Young (1988,1994).

[4] See Austen-Smith and Banks (1996); Feddersen and Pesendorfer (1994, 1996a, 1996b); Myerson (1994); Wit (1996); and McLennan (1996).

be dramatically wrong. Below we construct a model of jury decision making which incorporates private information and strategic voting. We show that the requirement of a unanimous verdict to convict may actually result in a significantly higher probability of convicting an innocent defendant than, e.g., simple majority rule. We conclude the paper with few brief remarks on the implications of our results for jury reform.

2 Model

We assume there are n jurors who decide the fate of a defendant using a voting rule. In order to make things simple we assume there are two states of the world: the defendant is either guilty or innocent.[5] We denote the state of the world in which the defendant is guilty by G and innocent by I. We assume that each state occurs with equal probability.[6] A voting rule is described by a threshold $\hat{k}$ such that the defendant will be convicted if and only if the number of jurors that vote to convict is greater than or equal to $\hat{k}$. Thus if $\hat{k} = n$ a unanimous verdict is required to convict, whereas, if $\hat{k} = \frac{n+1}{2}$, then a conviction is obtained by a simple majority vote. There are two possible outcomes of the jury's vote: either the defendant is convicted, denoted C, or he is acquitted, denoted A.

Jurors do not know for sure if the defendant is guilty or innocent. We assume that each juror gets a signal g or i that is correlated with the true state. The parameter $p \in (.5, 1)$ is the probability that juror receives signal j in state J.[7] We assume that the signal is private information. This assumption implies that communication between jurors is limited. Given that juries observe the same facts at trial and engage in deliberation prior to taking the final vote it may seem inappropriate to assume that jurors have private information. However, there are multiple impediments standing in the way of complete disclosure of private information through the deliberation process. For example, some jurors may have technical knowledge that is relevant for the decision but which cannot be fully communicated in the

[5] Feddersen and Pesendorfer (1996b, 1996c) generalize the simple two state model to multiple states. Adding additional states will not fundamentally alter our results.

[6] This assumption can be easily relaxed without significantly changing our results.

[7] The assumption that the probability of recieving signal i in state I is identical to the probability of receiving signal g in state G is can be relaxed without significantly changing our results.

limited amount of time practically available. Jurors may also have different standards of guilt and innocence i.e., different preferences over the outcomes. Different preferences will make the sharing of all information very difficult if not impossible. Of course, if there is no private information and no preference diversity then all voting rules will produce the same outcome and the voting rule is unimportant.

Each juror must choose how to vote based on her private signal. The standard assumption is that jurors will vote sincerely. A juror votes sincerely by voting to convict if she receives a guilty signal and voting to acquit otherwise.

A compelling argument for the requirement of a unanimous verdict, ($k = n$), is that it lowers the probability of convicting an innocent defendant. It is thought that the additional protection of the innocent comes at the expense of acquitting the guilty and that such a trade-off is appropriate. This argument has merit if jurors vote sincerely. When a unanimous decision is required to convict and when jurors vote sincerely the probability of convicting an innocent defendant is minimized.[8]

The trouble with intuitions derived from sincere voting is that sincere voting may not be rational. Suppose all jurors have preferences which are given by $u(A, I) = u(C, G) = 0$, $u(C, I) = -q$ and $u(A, G) = -(1-q)$ where $q \in (0, 1)$. If a guilty defendant is convicted or an innocent defendant is acquitted then each juror's payoff is zero. If an innocent defendant is convicted then the jurors' payoff is $-q$ whereas if a guilty defendant is acquitted then the juror's payoff is $-(1-q)$. The parameter q exactly characterizes what constitutes reasonable doubt for the jurors. A juror who believes the defendant is guilty with probability higher than q will prefer the defendant to be convicted. The larger the value of q the less concern jurors have for acquitting a guilty defendant relative to convicting an innocent. We assume that jurors employ the same standard of reasonable doubt, i.e., q is identical for all jurors. This assumption is made purely for technical convenience. Our results generalize to the case in which jurors preferences are represented by

[8] The probability an innocent defendant is convicted under unanimity rule given sincere voting is $(1-p)^n$. It is easy to see that this probability is smaller than $\sum_{j=k}^{n} \binom{n}{j}(1-p)^j p^{n-j}$, the probability of convicting an innocent defendant under any rule that requires only k out of n votes to convict. The probability of acquitting a guilty defendant is strictly higher under unanimity rule. This probability is $1-p^n$ which is strictly larger than the probability $\sum_{j=k}^{n} \binom{n}{j}(1-p)^{n-j} p^j$ using any other rule.

different values of q.[9]

Each juror must vote either to convict (C) or acquit (A). Given the voting rule $\hat{k}$, a juror's behavior can be described by a strategy, $\sigma : \{g, i\} \to [0, 1]$, that maps the set of signals into a probability of voting to convict, $\sigma(s)$.

Let $\beta(k, n)$ denote the posterior probability that the defendant is guilty conditional on k of n guilty signals (and, therefore, $n - k$ signals i):

$$\beta(k, n) = \frac{p^k(1-p)^{n-k}}{p^k(1-p)^{n-k} + (1-p)^k p^{n-k}}$$

If $\beta(k, n) > q$ then, given all the information available to the jury, the defendant is guilty beyond a reasonable doubt. Therefore, the optimal outcome from the jurors' point of view is to convict. Similarly, if $\beta(k, n) < q$ then the optimal outcome for the jurors is to acquit. We assume that there is a k^* with $n \geq k^* \geq 1$ such that

$$\beta(k^* - 1, n) < q < \beta(k^*, n).$$

This assumption implies that if the jurors know that all received the guilty signal then they always want to convict the defendant. On the other hand, if the jurors believe that everyone has received an innocent signal they always want to acquit.

As has been shown in the literature cited above, jurors may have an incentive to behave strategically. Therefore, the natural benchmark to use when evaluating the performance of various voting rules is Nash equilibrium. We define a *voting equilibrium* as a symmetric Nash equilibrium.[10]

Under any voting rule there is a voting equilibrium in which all jurors vote the same way independent of their signal. Since no juror can influence the outcome this is always an equilibrium.[11] In the following we will ignore such equilibria and instead focus on voting equilibria where jurors sometimes

[9] See Feddersen and Pesendorfer (1994, 1996a and b) for examples of strategic voting under preference diversity.

[10] A symmetric strategy profile is one in which all jurors who receive the same signal take the same (possibly mixed) action.

[11] Even when the unanimity rule is used the profile in which all jurors vote to acquit regardless of their signal is a voting equilibrium. Ruling out weakly dominated strategies does not eliminate this equilibrium. This equilibrium is even trembling hand perfect if $p < q$. Note that in this case a single juror, if he is pivotal, and if he has no information about the other jurors' signals would vote to acquit. On the other hand, if $p > q$, then the equilibrium where all jurors always vote to acquit is not trembling hand perfect.

change their vote as a function of their private information. We call such equilibria *responsive*.

2.1 Unanimity

In this section we examine voting equilibria under the unanimity rule, i.e., $\hat{k} = n$.

First consider the case where $k^* = n$. Thus $\beta(n-1, n) \leq q$ and $\beta(n, n) > q$. In this case sincere voting is a voting equilibrium under unanimity rule. To see this suppose that all jurors vote to convict if and only if they receive the signal g. If a juror is pivotal and receives the signal i then he knows that $n-1$ of n jurors received the signal g. Therefore, he believes that the defendant is guilty with probability $\beta(n-1, n) \leq q$, i.e., not guilty beyond a reasonable doubt, and hence he (weakly) prefers that the defendant is acquitted. Thus a vote A is optimal. Conversely, if the juror receives the signal g then he believes that the defendant is guilty with probability $\beta(n, n) > q$ and hence he prefers that the defendant is convicted. Thus if the preferences of the jurors are such that their threshold q is larger than $\beta(n-1, n)$ then the standard intuition is correct and the unanimity rule minimizes the probability that an innocent defendant is convicted.

In the following we will focus on the case where

$$\beta(n-1, n) > q. \tag{1}$$

This assumption says that if a juror could observe all the signals and if $n-1$ of the n signals are g, the juror prefers to convict the defendant. We believe that this is a weak assumption which is satisfied in a large fraction of juries. Furthermore, for any fixed q there is an $\bar{n}$ such that (1) is true for any $n > \bar{n}$.[12]

Suppose that (1) is satisfied and a juror believes that the other jurors are voting sincerely. How should such a juror vote? Recent papers by Feddersen and Pesendorfer (1994,1996a) and Austen-Smith and Banks (1996) have demonstrated that a rational juror will condition her vote not only on her private information but also on what she believes others must know in the event her vote is pivotal. Under unanimity rule a vote is pivotal only when all the other jurors have voted to convict. A juror who receives an innocent

[12] To see this observe that $\beta(n-1, n) = \frac{p^{n-1}(1-p)}{p^{n-1}(1-p)+(1-p)^{n-1}p} = \frac{1}{1+\left(\frac{1-p}{p}\right)^{n-1}p}$ and, since $p > 1/2$ it follows that $\lim_{n\to\infty} \beta(n-1, n) = 1$.

signal and believes that the other jurors are voting sincerely *must* believe that the probability the defendant is guilty is exactly $\beta(n-1,n)$. But since $\beta(n-1,n) > q$ such a juror will ignore her private signal and vote to convict. Therefore, if (1) holds sincere voting cannot be a Nash equilibrium.

Given that sincere voting is not an equilibrium, any responsive voting equilibrium must be in mixed strategies. More precisely, each juror must both vote to convict and vote to acquit with positive probability whenever she receives a signal i. When the juror receives the signal g she votes to convict with probability 1.[13]

We define such a responsive voting equilibrium by the probability that a juror with signal i votes to convict, i.e., $\sigma_i \in (0,1)$. Given σ_i, the probability that a juror votes to convict if the defendant is guilty is given by

$$g_G = p + (1-p)\sigma_i$$

whereas the probability that a juror votes to convict if the defendant is innocent is given by

$$g_I = (1-p) + p\sigma_i.$$

For a mixed strategy profile to be an equilibrium, a juror who receives an innocent signal must be indifferent between voting to acquit and voting to convict. This occurs when, conditional on $n-1$ others voting guilty and the juror receiving signal i, the probability that the defendant is guilty is exactly equal to q. By Bayes' law we get the following equilibrium condition for unanimity rule

$$\frac{(1-p)(g_G)^{n-1}}{(1-p)(g_G)^{n-1} + p(g_I)^{n-1}} = q. \tag{2}$$

Therefore, we have that

$$\sigma_i = \frac{\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1}{n-1}} p - (1-p)}{p - \left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1}{n-1}} (1-p)} \tag{3}$$

[13] It is straightforward to verify that there are no mixed strategy equilibria in which jurors who get the signal g vote with positive probability to acquit. In order to support such a mixed strategy equilibrium it is necessary that those who receive a guilty signal are indifferent between convicting and acquitting given their vote is pivotal. In such a case those who receive an innocent signal strictly prefer to acquit. But then the only jurors who vote to convict are those who receive the guilty signal. Under unanimity a vote is only pivotal when all others vote guilty. Thus, such a strategy profile is not a Nash equilibrium by the same argument as was used to show sincere voting is not an equilibrium.

Since σ_i is a mixed strategy it must be that $\sigma_i \leq 1$. Examining equation (3) we see that this is satisfied as long as $q \geq 1 - p$. If $q < 1 - p$ then there does not exist a responsive voting equilibrium. Instead, if $q < 1 - p$ the voting equilibrium is

$$\sigma_i = 1. \tag{4}$$

Thus, if $q < 1 - p$ then jurors vote to convict independent of the signal.[14]

To see why this is the equilibrium note that in this equilibrium the fact that another juror voted to convict does not provide any information about the guilt of the defendant. Thus, a juror who receives the signal i believes that the defendant is guilty with probability $1 - p$ conditional on his vote being pivotal. Since the juror favors conviction of the defendant if he is guilty with probability $1 - p > q$, it follows that $\sigma_i = 1$ in this case.

To understand why there cannot be a responsive equilibrium in this case note that (in equilibrium) a guilty vote of some other juror can never be information in favor of the innocence of the defendant. Hence, conditional on his vote being pivotal, each juror must believe the defendant to be guilty with probability at least $1 - p$ if he receives the signal i and hence each juror must vote to convict even if he receives the signal i.

Now, it is fairly straightforward to compute the probability of making each type of error in equilibrium. When $q \geq 1 - p$ the probability that an innocent defendant is convicted is given by:

$$l_I(p, q, n) = (g_I)^n = \left(\frac{(2p-1)\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1}{n-1}}}{p - (1-p)\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1}{n-1}}} \right)^n$$

and the probability of acquitting a guilty defendant is:

$$l_G(p, q, n) = 1 - (g_G)^n = 1 - \left(\frac{(2p-1)}{p - (1-p)\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1}{n-1}}} \right)^n$$

When $q < 1 - p$ then all defendants are convicted and hence the probability of convicting an innocent defendant is one and the probability of acquitting a guilty defendant is zero.

[14] As we argued above there is another (non-responsive) symmetric Nash equilibrium in which all jurors vote to aquit independent of their signal. However, for $q < 1 - p < p$ this equilibrium is not trembling hand perfect. In contrast, the equilibrium in which all jurors vote to convict independent of their signal is trembling hand perfect for $q < 1 - p$.

We summarize our findings in the following proposition.

Proposition 1 *If $q \geq 1-p$ the unique responsive voting equilibrium for unanimity rule is given by (3). Moreover, $\sigma_i \to 1$ as $n \to \infty$ and*

$$\begin{aligned} \lim_{n\to\infty} l_I(p,q,n) &= \left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{p}{2p-1}} \\ \lim_{n\to\infty} l_G(p,q,n) &= 1-\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1-p}{2p-1}} \end{aligned}$$

If $q < 1-p$ then there is a voting equilibrium given by (4) and $l_I(p,q,n) = 1$, $l_G(p,q,n) = 0$.

Proof. We demonstrated in the text above that the unique responsive voting equilibrium under unanimity rule is given by (3). In appendix A we show that

$$\lim_{n\to\infty} l_I(p,q,n) = \lim_{n\to\infty}\left(\frac{(2p-1)\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1}{n-1}}}{p-(1-p)\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1}{n-1}}}\right)^n = \left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{p}{2p-1}}.$$

The proof that $\lim_{n\to\infty} l_G(p,q,n) = 1-\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1-p}{2p-1}}$ is analogous. ■

The results in Proposition 1 present a stark contrast to the results under sincere voting. In a voting equilibrium the probability of convicting an innocent defendant, $l_I(p,q,n)$, stays bounded away from zero for all n. Similarly, the probability of acquitting a guilty defendant also stays bounded away from zero. In the event that $q < 1-p$ *all defendants are convicted* regardless of the probability of their guilt or innocence. This is true independent of the size of the jury.

Proposition 1 also implies that the probability that a guilty defendant is convicted $(1-l_G)$ is bounded away from zero for all n. This is again in contrast to the case of sincere voting where the probability of conviction converges to zero as $n \to 0$ independent of whether the defendant is guilty or innocent.[15] Thus, a second implication of strategic voting is that the probability of a guilty verdict is much larger than under sincere voting.

[15] Note however, that under sincere voting the probability of convicting an innocent defendant *conditional* on a conviction converges to zero. As Proposition 1 shows this is not the case for voting equilibria.

To provide an intuition for Proposition 1 first observe that Equation (3) implies that $\sigma_i \to 1$ as $n \to \infty$. As a consequence g_G (the probability that a juror votes to convict if the defendant is guilty) and g_I (the probability that a juror votes to convict if the defendant is innocent) both converge to one. This is not enough to show that the probability of convicting an innocent defendant, $(g_I)^n$, stays bounded away from zero. In Appendix A we demonstrate that for large n, g_I can be approximated by

$$1 + \frac{1}{n-1}\left(\frac{p}{2p-1}\ln f\right)$$

where $f = \frac{(1-q)(1-p)}{qp}$ and hence $(g_I)^n$converges to $f^{\frac{p}{2p-1}}$ which is the bound given in Proposition 1.[16]

The convergence to the bounds given in Proposition 1 is fast and hence the limit formula allows us to approximate the probabilities of each kind of error even for small juries. Figure 1 illustrates the convergence of $l_I(p, q, n)$ for the values $p = 0.7, q = 0.5$.

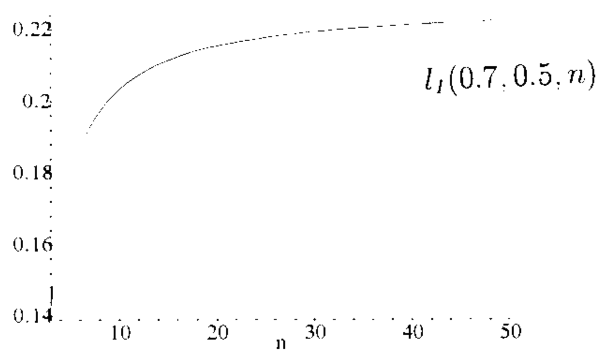

Figure 1

Figure 1 is quite startling for several reasons. First, the limit probability of convicting an innocent defendant is quite large at 22%. Second, when there are only 12 jurors the probability of convicting an innocent is 21%. There is only a 1% difference between the probability of convicting an innocent defendant with a jury size of $n \geq 12$ and the limit probability. Third, the probability of convicting an innocent defendant is *increasing* in the size of the jury.

[16] Recall that $e^x = \lim_{n\to\infty}(1 + \frac{x}{n})^n$.

Figure 2 shows the probability of acquitting a guilty defendant for the values $p = 0.7, q = 0.5$.

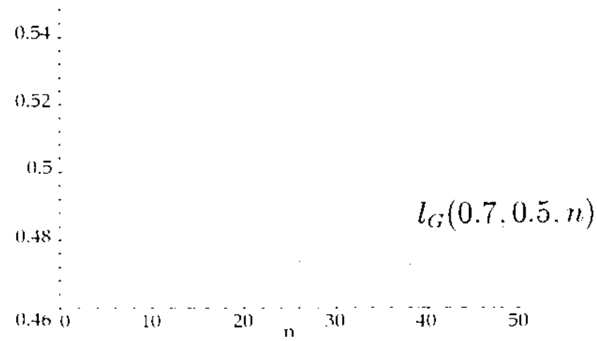

Figure 2.

The limit probability of acquitting a guilty defendant is 47%. Once again the limit probability is a very good estimate of the actual probability of this type of mistake even for small juries. Note that the probability of acquitting a guilty defendant is actually *decreasing* in the size of the jury.

The assumption that $q = .5$ in the above examples may seem unreasonable. It implies that jurors are very willing to convict. Indeed, they are willing to convict if they believe the defendant is more likely guilty than innocent. We certainly hope that jurors would have a higher threshold of reasonable doubt than $q = .5$, however, the rationale for the unanimity rule is to protect innocent defendants from such unreasonable juries.

Figure 3 shows the limit errors $l_I(p, q) = \lim_{n \to \infty} l_I(p, q, n)$ and $l_G(p, q) = \lim_{n \to \infty} l_G(p, q, n)$ for the value $p = 0.7$ as a function of q.

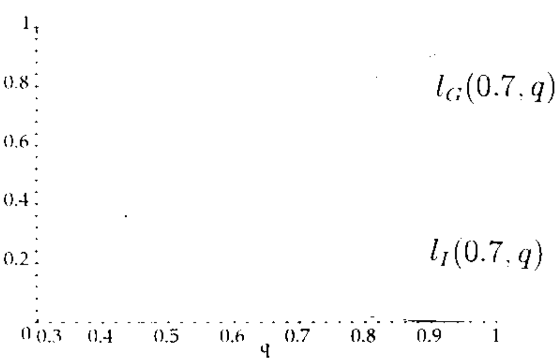

Figure 3

Figure 3 demonstrates that for large juries the probability of convicting an innocent is decreasing in q while the probability of acquitting a guilty defendant is increasing in q. Thus, the unanimity rule does a poor job in protecting innocent defendants from unreasonable juries. While, if the jury is responsible (e.g., $q = .9$), the innocent defendant is protected at the cost of a high probability of acquitting the guilty. In the next section we show that unanimity rule is a uniquely bad voting rule in terms of the probabilities of both kinds of errors it induces.

2.2 Non-unanimous rules

To compare the unanimity rule to other voting rules we must compute the voting equilibrium for general rules $\hat{k}$. In Appendix B we compute the responsive voting equilibrium for all $n, \hat{k}$. This allows us to compute the probabilities of convicting an innocent defendant, $l_I(p, q, n, \hat{k})$, and the probability of acquitting a guilty defendant, $l_G(p, q, n, \hat{k})$ as a function of the rule $\hat{k}$.

We use these results to investigate a particular class of rules: Consider the rule which requires $\hat{k} = \alpha n$ votes to convict where $0 < \alpha < 1$. (In the following we always assume that αn is an integer.) Thus, it takes an α fraction of jurors to convict the defendant. For a fixed α consider a sequence of voting equilibria corresponding to an increasing jury. In the following proposition we show that as $n \to \infty$ the probability of making either of the two kinds of error converges to zero.

Proposition 2 *Fix any α with $0 < \alpha < 1$. (1) There is a n' such that for $n > n'$ there is a responsive voting equilibrium. (2) For any sequence of responsive voting equilibria the probability of convicting an innocent defendant, $l_I(p, q, n, \alpha n)$, and the probability of acquitting a guilty defendant, $l_G(p, q, n, \alpha n)$ both converge to zero.*

Proof. See Appendix B. ■

Proposition 2 holds in much more general environments. Feddersen and Pesendorfer (1994) prove the analogous result for an environment that includes preference diversity and a much broader range of information environments. Myerson (1995) proves a similar result for the case of simple majority rule.

Proposition 2 shows that for any $\alpha \in (0, 1)$ the probability of convicting an innocent defendant converges to zero and the probability of acquitting a guilty defendant converges to zero for large juries. This is in sharp contrast to the result of Proposition 1 where it was shown that both types of mistakes stay bounded away from zero for the unanimity rule. What accounts for this difference?

As we noted above, under the unanimity rule the probability that a juror votes to convict converges to one independent of the signal. This can never happen for the rules considered in Proposition 2. Consider a profile with $\sigma_g = 1$ and suppose $\sigma_i \to 1$. If a juror is pivotal it must be that an $\alpha-$fraction of the remaining jurors have voted to acquit. A vote to acquit occurs with probability $p(1-\sigma_i)$ if the defendant is innocent and with probability $(1-p)(1-\sigma_i)$ if the defendant is guilty. If σ_i is close to one it is very unlikely that an $\alpha-$fraction jurors votes to acquit both when the defendant is guilty and when the defendant is innocent. However, for large n, since $\frac{p(1-\sigma_i)}{(1-p)(1-\sigma_i)} = \frac{p}{(1-p)} > 1$ it is much more likely that an $\alpha-$fraction of jurors voted to acquit if the defendant is innocent than if the defendant is guilty.[17] For large n the juror must therefore conclude that being pivotal implies that the defendant is innocent with probability arbitrarily close to one. Since $q < 1$ the juror therefore has a strict preference for voting to acquit. Hence σ_i must be zero and we have a contradiction to $\sigma_i \to 1$. Thus we have demonstrated that σ_i must stay bounded away from one. (An analogous argument shows that σ_g must stay bounded away from zero.) As a consequence, even in the limit as $n \to \infty$, a juror is more likely to vote to convict if the defendant is guilty than if the defendant is innocent. In the appendix we demonstrate that the expected fraction of guilty votes must be smaller than α if the defendant is innocent and larger than α if the defendant is guilty. This implies that both l_I and l_G converge to zero since for large n the actual fraction of guilty votes converges in probability to the expected fraction of guilty votes.[18]

Propositions 1 and 2 imply that the unanimity rule is *uniquely bad* for large juries. A second conclusion that can be drawn from Proposition 2 is that under non-unanimous rules the size of a jury is more important in determining the probability of making a mistake in the verdict than the voting rule. Therefore, if the probability an innocent defendant is convicted is considered to be too large, then the natural remedy is to fix a rule, say 2/3 majority, and then increase the size of the jury to the appropriate level.

[17] The ratio of these two conditional probabilities is approximately $\left(\frac{p}{1-p}\right)^{\alpha n}$ which converges to infinity as $n \to \infty$.

[18] This is an immediate consequence of the law of large numbers.

In the next section we provide examples which indicate that convergence is fast and hence our limit results are indeed relevant for relatively small juries.

2.3 Example

In the following we consider 12 person jury. We set the parameter for reasonable doubt at $q = 0.90$, i.e., jurors need to believe that the defendant is guilty with probability 0.9 in order to convict. We assume that $p = 0.8$, i.e., the probability of receiving a guilty signal if the defendant is guilty is 0.8.

The probability of convicting an innocent defendant, $l_I(\hat{k})$, is given by

$$l_I(\hat{k}) = \sum_{j=\hat{k}}^{n} \binom{n}{j} \left(g_I(\hat{k})\right)^j (1 - g_I(\hat{k}))^{n-j}$$

where

$$g_I(\hat{k}) = (1-p)\sigma_g(\hat{k}) + p\sigma_i(\hat{k})$$

is the probability that any juror votes for conviction if the defendant is innocent.

Similarly, the probability of acquitting a guilty defendant, $l_G(\hat{k})$, is given by

$$l_I(\hat{k}) = 1 - \sum_{j=\hat{k}}^{n} \binom{n}{j} \left(g_G(\hat{k})\right)^j (1 - g_G(\hat{k}))^{n-j}$$

where

$$g_G(\hat{k}) = p\sigma_g(\hat{k}) + (1-p)\sigma_i(\hat{k})$$

Table 1 gives the probability of making mistakes as a function of the rule $\hat{k}$ when $\hat{k} \geq 7$:

$\hat{k}$	7	8	9	10	11	12
l_I	0.004	0.0011	0.0025	0.0045	0.0066	0.0069
l_G	0.019	0.066	0.135	0.245	0.42	0.654
σ_i	0	0.023	0.143	0. 277	0. 423	0. 575

Table 1

At $\hat{k} = 7$ sincere voting is the equilibrium. Thus every juror who receives a signal g votes to convict and every juror who receives a signal i votes to acquit.. For all $\hat{k} > 7$ the jurors who receive the signal i mix between voting to convict and voting to acquit while those who receive signal g always vote to convict.

As the table shows, unanimity has the largest probability of convicting an innocent defendant when $\hat{k} \geq 7$. In addition, all rules $\hat{k} \geq 7$ have the property that they lead to a lower probability of acquitting a guilty defendant.

The next table shows the probability of making an error for $\hat{k} < 7$:

$\hat{k}$	1	2	3	4	5	6
l_I	0	0.0159	0.034	0.027	0.019	0.010
l_G	1	0.78	0.41	0.21	0.095	0.036
σ_g	0	0.091	0.306	0.512	0.704	0.879

Table 2

For $\hat{k} \leq 6$ the equilibrium strategies are such that a juror who receives the signal i always votes to acquit while a juror who receives the signal g mixes between voting to convict and voting to acquit.

For $\hat{k} = 1$ the defendant is never convicted, i.e., $l_G = 1$. To see why this is the case suppose that no juror ever votes to convict. In this case, each juror is always pivotal since one guilty vote is enough for a conviction. But this implies that the only information the juror has conditional on his vote being pivotal is his own signal. Conditional on having received signal g the juror believes that the defendant is guilty with probability 0.8. Since the reasonable doubt threshold is 0.9 each juror votes to acquit.

The probability of convicting an innocent defendant reaches a maximum at $\hat{k} = 3$ while the probability of acquitting a guilty defendant is monotonically decreasing for $\hat{k} \leq 6$ and the probability of convicting an innocent defendant is zero at $\hat{k} = 1$.[19]

Taken together our results raise serious questions about the appropriateness of the unanimity rule. Unanimity rule results in a strictly positive probability both of acquitting the guilty and convicting the innocent even in the limit. Increasing the size of the jury does not help, it may actually increase the probability of a mistake. Unanimity rule is almost never optimal from the perspective of the jurors. Finally, given even modestly large juries unanimity rule is uniquely inferior to a variety of other rules.

[19] In this case, conditional on a vote being pivotal the juror knows that all other jurors have voted to acquit. In equilibrium this can never be information in favor of the guilt of the defendant. If the juror receives a guilty signal he therefore believes the defendant to be guilty with probability at most p. Since $p < q$ the juror votes to acquit.

3 Conclusion

We have demonstrated that strategic behavior dramatically alters our intuitions about the consequences of jury voting rules. It is appropriate to conclude with a note of caution. We remind readers that our results depend upon the twin assumptions of private information and strategic voting. The degree to which either of these assumptions characterizes actual juries is ultimately an empirical question and, therefore, beyond the scope of this paper.

For example, consider an alternative model in which jurors are not privately informed and all the information is common knowledge. Suppose further that jurors differ in their threshold for reasonable doubt. In such a setting, the unanimity rule implies that the juror with the highest threshold must be convinced. Any other rule would only serve to lower the threshold for conviction. Thus, the assumption of private information is essential for our conclusions.

The note of caution is appropriate because jury reform is not an abstract proposition. In California a group calling itself "Citizens for a Safer California" is sponsoring the "Public Safety Protection Act of 1996". This act would eliminate the requirement of unanimous jury verdicts in all but capital murder cases and replace it with a rule requiring only 10 of 12 jurors to convict.[20] The jury reform proposal enjoys the support of the current governor of California, Pete Wilson, as well as presidential candidate Lamar Alexander.[21] Clearly, our results lend some support to such initiatives. On the other hand, our results suggest that retaining the unanimity rule in capital cases is exactly the wrong thing to do. Presumably, the motive for retaining unanimity rule in capital cases is to protect against the terrible consequences of convicting an innocent. If our model is correct it would be better to combine a super-majority rule with a larger jury for cases in which it is desirable to reduce the probability of convicting an innocent.

[20] The Citizens for a Safer California claim that a "broad coalition of crime victims, law enforcement and concerned citizens" support the Public Safety Act of 1996. They also claim that in independent poll showed that 71% of Californians support 10:2 jury verdicts. From material downloaded from the internet at http://taren.ns.net/cdaa.htm.

[21] See Kinley (1996) and a press release by the Alexander campaign reported at the PoliticsNow web site (http://politicsusa.com/ PoliticsUSA/campaign96/candidates/ lamar/1019la02.html.cgi).

4 Appendix A

We now show that

$$\lim_{n\to\infty}\left(\frac{1}{\frac{p}{2p-1}\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1}{n-1}}-\frac{(1-p)}{2p-1}}\right)^{n}=\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{p}{2p-1}}.$$

Let

$$h=\frac{p}{2p-1}\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{1}{n-1}}-\frac{(1-p)}{2p-1}.$$

Suppose $\infty>\lim_{n\to\infty}h^{n}>0$ then $\lim_{n\to\infty}h^{-n}=\frac{1}{\lim_{n\to\infty}h^{n}}$. Now let

$$f=\frac{(1-q)(1-p)}{qp}$$

We use the following facts:

$$\lim_{n\to\infty}\left(1+\frac{z}{n}\right)^{n}=e^{z} \tag{5}$$

and, given $f\in(0,1)$

$$1+\frac{-1}{n}\ln f\geq f^{\frac{1}{n-1}}\geq 1+\frac{-1}{n-1}\ln f. \tag{6}$$

From (6) we know

$$h\geq\frac{p}{2p-1}\left(1+\frac{-1}{n-1}\ln f\right)-\frac{(1-p)}{2p-1}.$$

Some simple algebra shows that

$$\frac{p}{2p-1}\left(1+\frac{-1}{n-1}\ln f\right)-\frac{(1-p)}{2p-1}=1+\frac{1}{n-1}\frac{-p}{2p-1}\ln f$$

Thus

$$\lim_{n\to\infty}(h)^{n}\geq\lim_{n\to\infty}\left(1+\frac{1}{n-1}\frac{-p}{2p-1}\ln f\right)^{n}$$

and from (5) we get

$$\lim_{n\to\infty}\left(1+\frac{1}{n-1}\frac{-p}{2p-1}\ln f\right)^{n}=\left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{p}{2p-1}}.$$

We can use an identical argument using (6) to establish that

$$\lim_{n\to\infty}(h)^n \leq \lim_{n\to\infty}\left(1+\frac{1}{n}\frac{-p}{2p-1}\ln f\right)^n = \left(\frac{(1-q)(1-p)}{qp}\right)^{\frac{p}{2p-1}}.$$

■

5 Appendix B

First we compute the equilibrium for a general $\hat{k}$. Denote the probability that a juror votes to convict in state I as

$$g_I = (1-p)\,\sigma_g + p\sigma_i$$

and the probability that a juror votes to convict in state G as

$$g_G = p\sigma_g + (1-p)\sigma_i.$$

When $1 > \sigma_i$ we must have

$$\frac{1}{1+\frac{p(g_I)^{\hat{k}-1}(1-g_I)^{n-\hat{k}}}{(1-p)(g_G)^{\hat{k}-1}(1-g_G)^{n-\hat{k}}}} \leq q \tag{7}$$

with equality holding if $1 > \sigma_i > 0$. Similarly, when $\sigma_g > 0$ it must be true that

$$\frac{1}{1+\frac{(1-p)(g_I)^{\hat{k}-1}(1-g_I)^{n-\hat{k}}}{p(g_G)^{\hat{k}-1}(1-g_G)^{n-\hat{k}}}} \geq q \tag{8}$$

with equality holding if $1 > \sigma_g > 0$.

We now show that $1 > \sigma_g > 0$ implies $\sigma_i = 0$ (an identical exercise shows that $1 > \sigma_i > 0$ implies $\sigma_g = 1$). Suppose $1 > \sigma_g > 0$ then (8) implies

$$\frac{(1-q)p}{q(1-p)} = \frac{(g_I)^{\hat{k}-1}(1-g_I)^{n-\hat{k}}}{(g_G)^{\hat{k}-1}(1-g_G)^{n-\hat{k}}}.$$

Since $p > 1/2$ we can rewrite (7) as

$$\frac{1}{1+\frac{(1-q)}{q}\left(\frac{p}{1-p}\right)^2} = q\frac{p^2-(2p-1)}{p^2-q(2p-1)} < q$$

which implies $\sigma_i = 0$.

Thus, in any responsive equilibrium we must have either $\sigma_i = 0$ and $\sigma_g > 0$ or $\sigma_i < 1$ and $\sigma_g = 1$. If

$$\frac{(1-p)\,(p)^{k-1}\,(1-p)^{n-k}}{p\,(1-p)^{k-1}\,p^{n-k}+(1-p)p^{k-1}\,(1-p)^{n-k}} = \frac{1}{1+\frac{p(p)^{k-1}(1-p)^{n-k}}{(1-p)(p)^{k-1}(1-p)^{n-k}}} \le q \quad (9)$$

and

$$\frac{(p)^{k}\,(1-p)^{n-k}}{(1-p)^{k}\,p^{n-k}+p^{k}\,(1-p)^{n-k}} = \frac{1}{1+\frac{(1-p)(p)^{k-1}(1-p)^{n-k}}{p(p)^{k-1}(1-p)^{n-k}}} \ge q \qquad (10)$$

then the unique responsive voting equilibrium is $\sigma_i = 0$ and $\sigma_g = 1$. (Recall that a voting equilibrium is a symmetric Nash equilibrium).

To see why this is the unique responsive voting equilibrium observe that the left hand side of Equation (7) is strictly decreasing in σ_i. Together with (9) this implies that whenever $\sigma_i > 0$ (and $\sigma_g = 1$) every voter has a strict incentive to vote to acquit. Similarly, the left hand side of Equation (8) is strictly decreasing in σ_g. This together with (10) implies that whenever $\sigma_g < 1$ (and $\sigma_i = 0$) every voter has a strict incentive to vote to convict.

If one of the conditions (9) or (10) does not hold then there are two cases to consider.

Case 1: Suppose

$$\frac{(p)^{k}\,(1-p)^{n-k}}{(1-p)^{k}\,p^{n-k}+p^{k}\,(1-p)^{n-k}} < q$$

then $\sigma_i = 0$ is the equilibrium and the equilibrium condition for σ_g is defined by (8) with equality holding. This yields

$$\frac{1}{1+\frac{(1-p)(g_I)^{k-1}(1-g_I)^{n-k}}{p(g_G)^{k-1}(1-g_G)^{n-k}}} = q$$

which we can rewrite

$$\begin{aligned}
\frac{(1-q)p}{q(1-p)} &= \frac{(g_I)^{k-1}(1-g_I)^{n-k}}{(g_G)^{k-1}(1-g_G)^{n-k}} \\
&= \frac{((1-p)\,\sigma)^{k_n-1}(1-(1-p)\,\sigma)^{n-k_n}}{(p\sigma)^{k_n-1}(1-p\sigma))^{n-k_n}} \\
&= \left(\frac{1-p}{p}\right)^{k-1}\left(\frac{1-(1-p)\sigma}{1-p\sigma}\right)^{n-k}
\end{aligned}$$

Therefore we get

$$\frac{(1-q)}{q}\left(\frac{p}{1-p}\right)^{k} = \left(\frac{1-(1-p)\sigma}{1-p\sigma}\right)^{n-k}$$
$$\frac{(1-\sigma+p\sigma)}{(1-p\sigma)} = \left(\frac{(1-q)}{q}\left(\frac{p}{1-p}\right)^{k}\right)^{\frac{1}{n-k}}$$

This yields

$$\sigma_g = \frac{h-1}{p(h+1)-1}$$

where

$$h = \left(\frac{(1-q)}{q}\left(\frac{p}{1-p}\right)^{k}\right)^{\frac{1}{n-k}}.$$

Clearly, since σ_g is the unique solution of (8) in this case, there is a unique responsive voting equilibrium in this case.

Case 2: Suppose

$$\frac{(1-p)(p)^{k-1}(1-p)^{n-k}}{p(1-p)^{k-1}p^{n-k}+(1-p)p^{k-1}(1-p)^{n-k}} > q$$

In this case $\sigma_g = 1$ and the equilibrium condition is given by:

$$\frac{(1-p)(g_G)^{k-1}(1-g_G)^{n-k}}{p(g_I)^{k-1}(1-g_I)^{n-k}+(1-p)(g_G)^{k-1}(1-g_G)^{n-k}} \leq q$$

with equality holding whenever $\sigma_i \in (0,1)$.

A straightforward calculation shows that for an interior solution in this case

$$\sigma_i = \frac{p(1+f)-1}{p-f(1-p)}$$

where

$$f = \left(\frac{(1-q)}{q}\left(\frac{(1-p)}{p}\right)^{n-k+1}\right)^{\frac{1}{k-1}}.$$

Again, since σ_i is the unique solution of 7 in this case, there is a unique responsive voting equilibrium in this case.

Whenever σ_i as defined by the previous two equations is less than zero then $\sigma_i = 0$ and whenever σ_i as defined by the previous two equations is larger than 1 then $\sigma_i = 1$.

Proof of Proposition 2 First we demonstrate that for sufficiently large n there exists a responsive voting equilibrium. To see this we first compute the limit equilibrium as $n \to \infty$ for the case where $\hat{k}/n = \alpha$. In case 2 we have

$$\sigma_i = \frac{p(1+f)-1}{p-f(1-p)}$$

where

$$f = \lim_{n\to\infty}\left(\frac{(1-q)}{q}\left(\frac{(1-p)}{p}\right)^{n-\alpha n+1}\right)^{\frac{1}{\alpha n-1}} = \left(\frac{1-p}{p}\right)^{\frac{1-\alpha}{\alpha}}.$$

and therefore we have

$$\sigma_i = \frac{p(1+f)-1}{p-f(1-p)} = \frac{p(1+\left(\frac{1-p}{p}\right)^{\frac{1-\alpha}{\alpha}})-1}{p-\left(\frac{1-p}{p}\right)^{\frac{1-\alpha}{\alpha}}(1-p)}$$

It is easily checked that $\sigma_i \geq 0$ for $\alpha \geq 1/2$ with $\sigma_i \to 1$ as $\alpha \to 1$. Similarly, in case 1 we have that

$$\sigma_g = \frac{\left(\frac{p}{1-p}\right)^{\frac{\alpha}{1-\alpha}}-1}{p\left(\left(\frac{p}{1-p}\right)^{\frac{\alpha}{1-\alpha}}+1\right)-1}$$

and again it can easily be checked that $0 < \sigma_g \leq 1$ for $0 < \alpha \leq 1/2$ with $\sigma_g \to 0$ as $\alpha \to 0$.

Together this implies that for any $0 < \alpha < 1$ there is a responsive limit equilibrium. Now a simple continuity argument implies that for sufficiently large n the solution to equations (7) and (8) must be arbitrarily close to the limit solution and hence it follows that for sufficiently large n there is a responsive voting equilibrium.

To prove part (2) of Proposition 2 observe that in the limit as $n \to \infty$ we have that

$$g_I = 1-p+p\frac{p(1+\left(\frac{1-p}{p}\right)^{\frac{1-\alpha}{\alpha}})-1}{p-\left(\frac{1-p}{p}\right)^{\frac{1-\alpha}{\alpha}}(1-p)}$$

and

$$g_G = p + (1-p)\frac{p(1+\left(\frac{1-p}{p}\right)^{\frac{1-\alpha}{\alpha}}) - 1}{p - \left(\frac{1-p}{p}\right)^{\frac{1-\alpha}{\alpha}}(1-p)}.$$

Note that for $0 < \alpha < 1$ this implies that

$$g_G > g_I - \varepsilon \tag{11}$$

for some $\varepsilon > 0$ which depends on α.

Next we show that

$$g_I < \alpha < g_G. \tag{12}$$

To see why this is sufficient to prove proposition 2 note that by the law of large numbers the actual share of guilty votes converges to the expected share of guilty votes in each state. Hence the share of guilty votes if the defendant is innocent converges to $g_I < \alpha$ in probability and hence the defendant is acquitted with probability close to one for large n. Similarly, if the defendant is guilty the share of guilty votes converges to $g_G > \alpha$ in probability and hence the defendant is convicted with probability close to one for large n.

Suppose (12) is violated and $g_G > g_I \geq \alpha$. From the equilibrium conditions we know that for all n

$$\frac{1}{1 + \frac{(1-p)(g_{ni})^{\alpha n-1}(1-g_{ni})^{(1-\alpha)n}}{p\left(g_{ng}^n\right)^{\alpha n-1}\left(1-g_{ng}^n\right)^{(1-\alpha)n}}} \geq q \tag{13}$$

(where g_{ns} denotes the equilibrium probability of a guilty vote in state s with n jurors). Note that $x^\alpha(1-x)^{(1-\alpha)}$ is a single peaked function of x with a maximum at $x = \alpha$. If $g_G > g_I \geq \alpha$, then $\frac{(g_I)^\alpha(1-g_I)^{(1-\alpha)}}{(g_G)^\alpha(1-g_G)^{(1-\alpha)}} > 1$ and hence

$$\left(\frac{(g_I)^\alpha (1-g_I)^{(1-\alpha)}}{(g_G)^\alpha (1-g_G)^{(1-\alpha)}}\right)^n \to \infty$$

and so the left hand side of (13) must converge to zero as $n \to \infty$ and hence inequality (13) cannot hold. An analogous argument can be made if $\alpha \geq g_G > g_I$. ■

References

[1] Adler, Stephan J. 1994. *The Jury: Trial and Error in the American Courtroom.* New York: Times Books.

[2] Austen-Smith, David and Jeffrey S. Banks. 1996. "Information Aggregation, Rationality and the Condorcet Jury Theorem" *American Political Science Review*, 90:1 34-45.

[3] Feddersen Timothy J. and Wolfgang Pesendorfer. 1994. "Voting Behavior and Information Aggregation in Elections with Private Information", forthcoming in: *Econometrica.*

[4] Feddersen, Timothy J. and Wolfgang Pesendorfer. 1996a. "The Swing Voter's Curse". *American Economic Review*, 86:3 408-24.

[5] Feddersen Timothy J. and Wolfgang Pesendorfer. 1996b. "Abstention and Common Values". Unpublished manuscript.

[6] Kinley, Dean. 1996. "The governor reflects on law and the legal profession". Web page. Internet address: http://www.calbar.org/2cbj/96apr/art21.htm.

[7] Klaven, Harry and Hans Zeisel. 1966. *The American Jury.* Boston: Little, Brown.

[8] Klevorick, Alvin K., Michael Rothschild and Christopher Winship. 1984. "Information Processing and Jury Decision making." *Journal of Public Economics* 23:245-78.

[9] Ladha, Krishna K. 1992. "The Condorcet Jury Theorem, Free Speech, and Correlated Votes". *American Journal of Political Science* 36:3 617-34.

[10] Levine, James P. 1992. *Juries and Politics.* Pacific Grove, Calif.: Brooks/Cole Pub. Co.

[11] McCart, Samuel W. 1964. *Trial by Jury: A Complete Guide to the Jury System.* Philadelphia: Chilton Books.

[12] Miller, N. 1986. "Information, Electorates, and Democracy; some Extensions and Interpretations of the Condorcet Jury Theorem," in: B. Grofman and G. Owen, ed. *Information Pooling and Group Decision Making*. Greenwich Ct. JAI Press.

[13] Myerson, Roger. 1994a. "Population Uncertainty and Poisson Games", mimeo, Northwestern University.

[14] Myerson, Roger. 1994b. "Extended Poisson Games and the Condorcet Jury Theorem", mimeo, Northwestern University.

[15] Wit, Jorgen. "Rational Choice and the Condorcet Jury Theorem", mimeo, Caltech.

[16] Young, Peyton. 1988. "Condorcet's Theory of Voting", *American Political Science Review* 82(4): 1231-44.