Discussion Paper No. 1157

Time-Consistent Protection of an Infant-Industry: The Symmetric Oligopoly Case

by

Eugenio J. Miravete

First version: Nov. 15, 1994
This version: May 8th, 1996

Time–Consistent Protection of an Infant–Industry: The Symmetric Oligopoly Case

Eugenio J. Miravete [†] [*]

Department of Economics
Northwestern University

First version: November 15th, 1994
This version: May 8th, 1996

Abstract

This paper discusses the design of an optimal, time–consistent tariff to protect an infant–industry in the presence of learning effects. Firms decide how much to produce, taking into account learning effects induced by their current production and the tariff policy, while the government decides on the level of tariff protection. In order to ensure time consistency we solve the symmetric case without spillovers where learning leads to lower fixed costs. Assuming that domestic and foreign products are imperfect substitutes for each other but perfect substitutes within each group, we use a complete linear demand system to represent domestic consumers' preferences. The analytic Markov Perfect Equilibria of this game is derived by solving a linear quadratic differential game. It is shown that in equilibrium, only a declining tariff over time can be regarded as a time consistent tariff policy JEL: C73, F12, F13.

[†] Department of Economics, Northwestern University, 2003 Sheridan Road, Evanston, IL 60208-2600, U.S.A. Internet: *miravete@merle.acns.nwu.edu*

[*] I wish to thank Jeffrey Campbell, Kiminori Matsuyama, John Panzar, Robert Porter, and especially Kyle Bagwell and Pierre Regibeau for helpful comments and suggestions. I also thank Stephanie Lofgren for her assistance. An earlier version of this article was presented at the XIII Latin American Meeting of the Econometric Society, Caracas and the 1st Conference on Empirical Investigations in International Trade, Purdue University. Financial support from the Fundación Ramón Areces and the Ministerio de Educación y Ciencia, Spain is gratefully acknowledged.

:/.../spring96/mcwp.tex

1 Introduction

The aim of this paper is to study whether there may exist a time–consistent tariff policy to protect an infant industry when learning effects are important. Time consistency requires that the goverment be able to commit to a particular protection policy. Governments are generally unable to precommit to a particular tariff policy over time, but this problem is circumvented in this paper by modeling the tariff as contingent on industry's performance so that the resulting protection is the government's dynamic best response over an infinite horizon game.

Many models have addressed the possibility that government protection leads to higher levels of domestic welfare because the intervention makes Stackelberg leadership a credible strategy for domestic firms [Brander and Spencer (1983)]. This is Krugman's (1984) known argument of exports promotion through imports protection. As many policy makers claim, Krugman shows that in the presence of learning by doing (also with static economies of scale or international R&D competition), a future cost advantage may induce government protection. The negative effect of a protection policy of this kind is that firms behavior may become more collusive in this environment, and as a consequence, firms respond to protection in a manner that frustrates government efforts to promote higher domestic production in order to reduce costs through learning [Gruenspecht (1988)]. However, the optimal tariff policy derived in the present model does not suffer from this caveat. The government does not commit to any particular level of protection that firms may take as given while colluding. Instead, the tariff policy is credibly contingent on industry performance. Excessive collusion that leads to a low rate of cost reduction will be compensated by lower levels of tariff protection in order to increase competition.

The present model studies the features of an optimal protection policy from a developing country's point of view. We will show in a very simplified framework that time consistent tariff protection may serve to ensure development of the infant industry and to enhance domestic welfare. However, not all policies are valid, and only decreasing tariff profiles can be considered in order to compensate for learning exhaustion with increasing competition. In the present model, we take the domestic industry structure as given and we concentrate on the tariff design. The monopoly case is embodied in the general formulation but the analysis of entry/exit policies in order to promote different industry configurations falls far beyond the objective of the present study.

There exists a growing literature that addresses either the strategic interaction among governments [Bagwell and Staiger (1990)] or between governments and the industry [Anderson (1992); Dixit and Kyle (1985); Staiger and Tabellini (1987)]. Within this literature, the work of Matsuyama (1990) is the closest to ours. Firms and the government play a repeated bargainig game where firms ask for protection in order to develop a cost reducing investment, and the government wants to liberalize to maximize welfare. An especially nice feature of Matsuyama's model is the recursivity of the game. After one period, if the government chose to protect and the firm did not invest, the game is identical to the one played one period before. However, if learning by doing effects are considered, there exists at least one state variable (the level of unit cost and/or the accumulated output) that differs from the previous period due to production. Therefore, we cannot work with time-independent strategies when we include the existence of dynamic economies of scale.

The paper attempts to build a dynamic model of optimal tariff design when learning economies exist and strategies are state contingent. The model's technical requirements are kept to a minimum. For simplicity, we choose a very particular specification for demand and the learning curve. This particular model structure allows us to solve the model in closed form and to use a result from dynamic programming on the solutions of a linear-quadratic differential game to prove time-consistency of the model's solution. This approach permits the characterization of a set of time-consistent tariff policies. Most significantly, we show that only decreasing tariff policies can credibly be considered as time consistent policies. Furthermore, for the case of exhausted learning, we also characterize the unique time-consistent tariff within the space of continuous strategies.

The paper is organized as follows. In section 2 the model and its assumptions are described. In section 3 the optimality conditions and features of the Markov Perfect Equilibria (MPE) are derived. In section 4 we proceed with the calibration of the model in order to carry out some comparative statics on changes in the model's structural parameters. In section 5 we summarize conclusions.

2 A Linear–Quadratic Differential Game of Industry Protection with Dynamic Economies of Scale

The game consists of $n+1$ players: n firms and the government of a small country. The problem to be addressed is protection of an infant-industry. Firms ask for protection to have time to reduce total costs and to be better positioned latter to compete with foreign firms. Total cost declines due to learning by doing.

The model has a continuous time specification. The only state variable is the vector of accumulated outputs for each firm in the industry. Denote the realization of this vector at time t as y^t. The level of total cost is assumed to depend on accumulated output. For each player, time t strategies are contingent on the state of the game. Production is the firm's control variable. The only choice variable for the government is the tariff level. Each firm's objective in each period is to maximize its expected discounted profits. The government maximizes the sum of consumer surplus, total profits and tariff revenues. For simplicity, we will assume that foreign firms, who produce a slightly differentiated good, behave competitively. In addition we assume that they have exhausted their respective learning processes. This assumption allows us to ignore strategic effects between domestic and foreign firms as well as foreign firms' investment consideration of output decisions.

Domestic firms supply a differentiated good in a monopolistically competitive regime and are subject to learning effects.

Consider the existence of dynamic economies of scale through the process of learning by doing. We allow for the existence of more than one firm in the industry but the industry configuration is taken as given in order to concentrate our atention on the design of the optimal tariff. There is neither entry or exit into or out of this industry[1]. The existence of more than one firm requires that we specify the industry's conduct and the nature of the solution [Helpman and Krugman (1989, §8)], and so we solve the benchmark case of Cournot competiton among firms with the government playing as the Stackelberg leader who defines the tariff policy. We concentrate on the symmetric case since it points out all features of the model avoiding unnecessary analytical complexity. We also assume that there is no learning spillover effects in order to avoid learning externalities of a quite different nature. All these assumptions lead to an n-vector of state variables with identical accumulated output along the equilibrium path, or alternatively the industry's level of accumulated output.

For the purpose of the present model, an equilibrium is time consistent if it fulfills Markov perfection over an infinite horizon. This modeling choice is made to avoid that endpoint conditions have any influence on equilibrium strategies. Given the lack of commitment of the government's tariff policy, the equilibrium outcome of the game after the final period may determine the equilibrium strategies in the last period of the game, and therefore the features of players equilibrium strategies over the finite horizon of the game. Depending on how players discount future payoffs, finite horizons of different length lead to different equilibrium strategies. An infinite horizon game is the correct approach to games with so long finite horizons that endpoint outcomes have no influence on the equilibrium strategies. The derived infinite horizon game equilibrium strategies are such that players do not need any additional source of commitment for them to be equilibrium strategies. Since these strategies are the best response at any time over an infinite horizon, they are time consistent.

The main objective of this paper is to show that there may exist a time consistent tariff policy that does not require any source of commitment[2], and study its features. To achieve this goal, technical requirements of our differential game force us to assume that learning does not affect marginal cost. Marginal cost is assumed to be constant: learning only reduces fixed cost. Limiting learning to fixed costs allows us to work with strategies which are linear in the state of the game and that enable us to solve the dynamic game in

---

[1] This assumption is also made only for the sake of tractability. Entry and exit may be addressed by computational methods as the Pakes and McGuire's (1994) algorithm, but then we loose the ability to address time consistency, which is the objective of this paper.

[2] In a recent empirical study, Head (1994, §5) shows that the actual protection policy of the U.S. steel rail industry between 1867 and 1913 was welfare maximizing, but fails to shows that the government had any credible commitment to that policy.

closed form. The standard case of marginal cost reduction due to learning can be solved numerically but its features will respond to particular formulations valid only for finite horizon games[3]. The importance of fixed costs reductions due to learning in the steel industry has been pointed out by Pratten (1971) who affirms that fixed costs reductions exceeds variable costs reductions. Theoretically, fixed costs reductions may be justified by lower organizational costs of the firms as their experience increases, and/or because technical improvements are embodied in sucessive vintages of capital at lower costs [Salter (1966)].

Fixed costs reductions induce similar externalities to marginal costs reductions. Protection in the early stages of development of an industry enhances welfare by ensuring lower costs but also larger variety in later periods. Intuitively the effect of externalities appear as follows. The government is interested in protecting the industry in its early stages of development in order to enhance consumer variety and promote competition. Given the parameters of the model, let n^* define the number of potential firms such that the present value of the industry's profit is zero. The government has interest in promoting competition given the number of potential firms in this industry. Suppose that the actual number of firms in a particular period t is $n_t > n^*$, so that without government intervention, firms make losses and will eventually exit the market. This is the usual case of an infant–industry. Government protection allows firms to produce in early stages by ensuring a high enough domestic price for firms to cover total costs. This policy temporarily reduces consumer surplus because it induces high prices for both domestic and imported goods. However it permanently increase variety, allowing domestic production to be profitable. Since fixed costs decline with production, the tariff level necessary to ensure profitability also falls. If tariffs do not drop off, firms will profit from the excessive market power allowed by tariff protection. Since this policy also allows all domestic firms to remain in the market, competition among them is enhanced, which ensures the lowest sustainable domestic price for a particular industry configuration.

In the following sections we solve for MPE. Markov strategies are state dependent and therefore they embody the idea of a protection policy that is contingent on the industry's performance. Given the government's optimal tariff policy (defined on a vector of state variables), firms choose their optimal output paths symmetrically (which also depends on the same vector of state variables). For these strategies to be an MPE, the government's strategy must also be a best response to firms' strategies as described above.

---

[3] Stokey (1986) studies the dynamics of an industry under the assumption of complete spillovers in learning which reduces marginal cost instead of fixed cost. It is shown in this infinite horizon environment that there exists a unique symmetric Nash Equilibrium within the space of continuous strategies. But nothing can be said about time consistency of the suggested compensating policy to favour production in early stages of the industry's lifecycle because of the existence of learning externalities. Shifting learning effects from marginal to fixed cost, as the present model does, enables us to find a closed form solution for the equilibrium strategies and to derive some propositions on features of one possible compensating policy.

2.1 Demand System

Assume that domestic and foreign products are considered imperfect substitutes for each other by domestic consumers but perfect substitutes within each category. Let $X^t = \sum_{i=1}^{n} x_i^t$ denote the domestic industry production and let M^t denote imports at time t. Assume a quadratic utility function with symmetric cross–effects for domestic consumers such that own effects dominate cross effects, that is, a strictly concave function of the form:

$$U(X^t, M^t) = Q_0^t + a_x X^t + a_m M^t - \frac{1}{4}\left[b_x (X^t)^2 + b_m (M^t)^2 + 2k X^t M^t\right]$$

where all parameters a_x, a_m, b_x, b_m, k are strictly positive. The sufficient condition $DetH[U(X^t, M^t)] = \frac{1}{4}(b_x b_m - k^2) > 0$ ensures the utility function to be strictly concave. At each time, t, consumers maximize $U(X^t, M^t)$ subject to the monetary constraint $I^t = Q_0^t + \tilde{P}_x X^t + \tilde{P}_m M^t$, where $U(X^t, M^t)$ is a money valued utility function and Q_0^t represents the aggregate consumption of a competitive numeraire good. Let $\tilde{P}_x^t$ and $\tilde{P}_m^t$ represent the domestic market price for domestic goods and imports in each period. Since we consider the case of a specific import tariff, τ^t, we have:

$$\tilde{P}_x^t = P_x^t \tag{1}$$

$$\tilde{P}_m^t = P_m^t + \tau^t \tag{2}$$

where P_x^t and P_m^t are the world prices of the domestic and the imported good respectively. From the FOC's of the consumer problem we derive the demands for domestically produced goods and imports as functions of the tariff level:

$$X^t(P^t, \tau^t) = X^t(P^t, 0) + \frac{2k\tau^t}{b_x b_m - k^2} = X^t(P^t, 0) + \mu_x \tau^t \geq 0 \tag{3}$$

$$M^t(P^t, \tau^t) = M^t(P^t, 0) - \frac{2b_x \tau^t}{b_x b_m - k^2} = M^t(P^t, 0) - \mu_m \tau^t \geq 0 \tag{4}$$

with $\mu_x > 0$, $\mu_m > 0$. Finally, consumer surplus is given by:

$$CS(X^t, M^t) = U(X^t, M^t) - Q_0^t - \tilde{P}_x^t X^t - \tilde{P}_m^t M^t \tag{5}$$

Observe that demand does not induce any dynamic effect because of its stationary linear specification. Welfare gains from protection may be higher than those highlighted by this model if, in addition, learning induces marginal cost reductions, and/or if demands grows over time.

2.2 Cost Function

We assume that the learning effect only reduces firms' fixed costs. Marginal cost is also independent of current production level, so that it remains constant over time. The fixed cost is a positive, strictly decreasing, and strictly convex function defined on $[0, y^*]$ and constant on $[y^*, \infty)$, for some large level of accumulated output y^*. In particular, we adopt the following additively separable specification:

$$C_i^t(y_i^t, x_i^t) = \begin{cases} c_0 + c_1 y_i^t + \frac{1}{2}c_2(y_i^t)^2 + c_3 x_i^t & \text{if} \quad y_i^t \leq y^* \\ c_0 + c_1 y^* + \frac{1}{2}c_2(y^*)^2 + c_3 x_i^t & \text{if} \quad y_i^t \geq y^* \end{cases} \tag{6}$$

where x_i^t represents output and y_i^t represents accumulated output of firm i at time t. Assumptions on the shape of the fixed cost function allow us to impose two restrictions on the admissible values of the parameters of this function: $c_1 < 0$ and $c_2 > 0$. These parameter signs capture the intuition that protection could be welfare improving in the early stages of development of an industry because protection would promote domestic production which leads to lower costs and enhances variety. The convexity of this function ensures that when the industry becomes mature, protection is not longer the optimal policy because it induces important consumer welfare loss but only achieves minor cost reductions.

2.3 The Firm's Problem

In an infinite horizon game, each firm's problem is to maximize the present value of its own profits given its competitors' behavior and the government's tariff, while considering the learning effects induced by current production. This problem can be stated as:

$$\max_{x_i^t} \quad u_i = \int_0^\infty \pi_i^t(y^t, x^t, \tau^t)e^{-rt}dt$$

$$\text{s.t.} \qquad \dot{y}_i^t = \frac{dy_i^t}{dt} = x_i^t \qquad : \qquad y_i(0) = y_i^0$$

Given the production decisions of the rest of the competitors this is a standard linear quadratic differential game. Hence, we have a dynamic programming problem that can be solved using Pontryagin's maximum principle. The necessary conditions for this problem depend on $\hat{x}_i^t = \hat{x}_i^t(y_1^t, \ldots, y_n^t)$ and $\hat{\tau}^t = \hat{\tau}^t(y_1^t, \ldots, y_n^t)$, the optimal control for firm i and government's optimal tariff respectively, at time t. The optimal controls capture the interaction of firms' strategies and the government's policy over the game horizon[4]. This effect makes firm i's co-state variable depend on the government's tariff policy and competitors'

[4] The complete derivation of players optimal strategies is presented in the appendix.

actions. In order to simplify and aggregate these first order conditions, we further impose Cournot conjectures for firms and equilibrium conjectures for the government. These conjectures also remains constant over the horizon of the game:

$$\frac{\partial x_j^t}{\partial x_i^t} = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{if } j \neq i \end{cases} \qquad : \qquad \frac{\partial x_i^t}{\partial \tau^t} = \frac{\mu_x}{n} > 0 \quad \forall i$$

The last expression is obtained by imposing symmetry on consumers' domestic demands (3), so that the conjecture of the government's optimal tariff choice on each firm's output decision is the same and, in fact, equal to equilibrium beliefs. The government and firms decide simultaneously on their respective control variables but firms consider the Stackelberg leader role of the government in establishing the tariff policy. Therefore, the firms' necessary conditions can be written as follows after rearranging and aggregating:

$$0 = a_x - \frac{1}{2}\left[b_x(1+\frac{1}{n}) + k\frac{\mu_m}{\mu_x}\right]X^t - \frac{1}{2}kM^t(P^t,\tau^t) - c_3 + \frac{\lambda^t}{n} \tag{7a}$$

$$\dot{\lambda}^t = r\lambda^t - (nc_1 + c_2Y^t) + \frac{1}{2}b_x(n-1)X^t\frac{\partial \hat{X}^t}{\partial Y^t} + \frac{1}{2}k\mu_m(X^t + \tau^t\mu_x)\frac{\partial \hat{\tau}^t}{\partial Y^t} \tag{7b}$$

2.4 The Government's Problem

The government's problem is to maximize the present value of the sum of consumer surplus, industry profits and tariff revenues, given the optimal industry production strategy and considering the aggregate learning effects induced by its tariff policy:

$$\max_{\tau^t} \quad u_{n+1} = \int_0^\infty \left[CS^t(y^t,x^t,\tau^t) + \Pi^t(y^t,x^t,\tau^t) + R^t(y^t,x^t,\tau^t)\right] e^{-rt}dt$$

$$\text{s.t.} \qquad \dot{Y}^t = \frac{dY^t}{dt} = X^t \qquad : \qquad Y(0) = Y^0$$

Because of the demand and cost assumptions, this is also consistent with a standard linear quadratic differential game structure given the production decisions of the industry. Now, denote by $\hat{X}^t(Y^t) = \sum_{i=1}^n \hat{x}_i^t(y^t)$ the optimal choice of X^t for the industry as a whole at time t. Given the industry optimal production decision, the optimal government's policy must satisfy the following generalized Hamilton–Jacobi conditions:

$$0 = \Gamma_0(X^t(P^t,0), M^t(P^t,0)) + \Gamma_1\tau^t + \mu_x\tilde{\lambda}^t \tag{8a}$$

$$\dot{\tilde{\lambda}}^t = r\tilde{\lambda}^t + c_1 + \frac{c_2}{n}Y^t \tag{8b}$$

As it is shown in the appendix, dynamic optimality conditions for the government are equivalent to the standard one player case. This holds because the game state variable is the result of direct firms' decisions while the government does not have its own state variable. Therefore, the government's necessary conditions to establish the optimal tariff are the same for either the *open loop* or the *closed loop* equilibrium of this problem.

3 Theoretical Results

We chose the simplest specification to show that there may exist a time consistent protection policy in presence of dynamic economies of scale. An MPE always exists for a finite horizon game, but time consistency cannot be addressed within this framework as it was explained before. An infinite horizon is required in order to avoid any posibility for the government to deviate from its equilibrium strategy in any later period. Therefore, in this section we first determine whether a finite horizon MPE may also constitute an infinite horizon MPE. Next, the resulting condition is used to characterize a time-consistent tariff policy. Finally, one case with a unique, infinite horizon, time-consistent MPE is isolated.

An MPE is a Subgame Perfect Equilibria in Markov strategies, that is, strategies that only depend on the state of the game, and which capture the interaction among players over the game horizon. In this case, co-state variables depend on opponents' actions [Başar and Olsder (1995, §6.5); Fudenberg and Tirole (1991, §13.3.2)]. The solution to equations (7) and (8) provide the MPE directly. Therefore, the optimal government and induced industry strategies can be found from conditions (7) and (8). Using (3)–(4) they may be written as:

$$\hat{X}^t = \frac{2a_x - kM^t(P^t, \tau^t) - 2c_3 + \frac{2}{n}\lambda^t}{b_x\left(2 + \frac{1}{n}\right)} \tag{9a}$$

$$\hat{\tau}^t = \frac{\Gamma_0(X^t(P^t, 0), M^t(P^t, 0)) + \mu_x\tilde{\lambda}^t}{-\Gamma_1} \tag{9b}$$

The solution to this linear–quadratic game is found by assuming that the co-state variables are linear in the state, so that there exists a closed–form strategy equilibria of the game:

$$\lambda^t(Y^t) = o_0 + o_1 Y^t \tag{10a}$$

$$\tilde{\lambda}^t(Y^t) = \tilde{o}_0 + \tilde{o}_1 Y^t \tag{10b}$$

As a consequence, the optimal strategies $\hat{X}^t(Y^t)$ and $\hat{\tau}^t(Y^t)$, are also linear in the state. The MPE is found solving the two coupled nonlinear Riccati equations associated to (9)–(10). The Riccati equations are derived in detail in the appendix.

A general proof of the existence of this solution is provided by Lukes (1971). Given existence, we still may wonder whether there may exist another nonlinear MPE. Because the linear–quadratic structure of the game, given any particular transversality condition, the linear specifications of $\hat{X}^t(Y^t)$ and $\hat{\tau}^t(Y^t)$ suffice for this linear solution to be unique within the space of analytic functions of the state variable for the finite time horizon.

$T < \infty$ [Papavassilopoulos and Cruz (1979)]. But an infinite–horizon, linear–quadratic differential game might have multiple MPE even though every finite horizon version of it has a unique MPE [Papavassilopoulos and Olsder (1984)]. However, it is possible to show that the unique linear solution for the finite horizon case may also constitute an MPE for the infinite horizon case.

PROPOSITION 1: *Equilibrium strategies $\hat{X}^t(Y^t)$ and $\hat{\tau}^t(Y^t)$ constitute an infinite horizon MPE if $o_1 \leq 0$ and $\tilde{o}_1 \leq 0$.*

PROOF: See appendix[5].

In the infinite horizon case an MPE must also satisfy the following transversality conditions:

$$\lim_{t \to \infty} \lambda^t(Y^t)e^{-rt} = \lim_{t \to \infty} \tilde{\lambda}^t(Y^t)e^{-rt} = 0 \tag{11}$$

By (10), these conditions hold whenever Y^t is bounded, but this is not the case in this model. However, it is sufficient to assume that the optimal accumulation output path Y^t is a function of exponential order less than the discount rate r. Then the transversality conditions (11) are fulfilled by implicitly imposing an upper bound for each period's production relative to the actual accumulated output, $X^t < rY^t$. This is a reasonable restriction that will always hold if the learning effect is exhausted after some level of accumulated output. After learning is exhausted, and provided the model's demand system is stationary, each period's production will be constant and the ratio X^t/Y^t decreases as production continues, and eventually reaching a value lower than r. Therefore, Proposition 1 together with the transversality conditions allows us to characterize the set of time-consistent tariff policies.

PROPOSITION 2: *An increasing tariff policy cannot be time consistent in an infinite horizon game.*

PROOF: Suppose that the tariff is increasing. Differentiating the optimal tariff $\hat{\tau}^t(Y^t)$, yields:

$$\frac{\partial \hat{\tau}^t(Y^t)}{\partial Y^t} = \frac{\mu_x \tilde{o}_1}{-\Gamma_1}$$

For this derivative to be positive, $\text{sign}(\tilde{o}_1) \neq \text{sign}(\Gamma_1)$. Since $\Gamma_1 < 0$, in order for the optimal tariff to increase with accumulated output, it must be the case that $\tilde{o}_1 > 0$. But then, the conditions of Proposition 1 are violated. Hence, this tariff policy cannot be an MPE in the infinite horizon case. ■

---

[5] The linear–quadratic structure of the differential game makes it possible to account for this result because the matrix of net effects of state variables over control variables becomes diagonal. For non–symmetric solutions of the model, other, more complex characterizations of time consistency in terms of the parameters of the model may be found.

This proposition proves the intuition behind many protectionist theories. As long as there exist important learning effects, a temporal protection of the industry may be optimal from the developing country perspective. This policy will ensure to benefit from important cost reductions at early stages of development of the industry. It will enhance welfare later by allowing competition and because of the lower cost of domestic production due to learning. Even when many assumptions are needed to show the time consistency of the tariff policy, the intuition behind Proposition 2 is simple. If a government announces an increasing tariff, this is clearly a non optimal strategy in the long run. The objective of a tariff protection policy is to improve welfare by reducing costs of production of domestic firms. Once firms have reduced their costs by increasing production above the static equilibrium output, the government has no interest in intensify this policy because it will hurt consumers more than producers could gain from little remain costs reduction. When learning effects fall below some threshold, switching to a less restrictive trade policy is always a dominant strategy because it promotes competition and increases welfare by increasing consumer surplus despite a second order producer surplus loss.

From the proof of Proposition 1, it is obvious that Proposition 2 is also true regardless of the symmetric oligopoly assumption of the actual solution. However, since the game does not have a unique endpoint in an infinite horizon set up, there may be other non–linear equilibrium strategies. Proposition 2 has proved that within the space of linear strategies, an increasing tariff policy will never be a time–consistent policy in the infinite horizon case, although it may constitute an MPE for some finite horizon cases. But although time consistency of a non linear strategy cannot be characterized in general, it is possible to isolate an interesting case. For the exhausted learning case, the result of Proposition 1 allows ua to characterize the unique non–linear MPE strategies for the infinite horizon game. If costs stop decreasing after some periods of learning, the game has a unique endpoint regardless its infinite horizon.

Proposition 2 applies also to the non–exhausted learning case, $y_i^\infty < y^*$. However in most industries, learning is eventually exhausted or not significative after some level of accumulated production. Let $\hat{t} = \min_t\{t \mid y_i^t \geq y^*\}$. Beyond this point, investment considerations of output decisions dissapear and the relevant state is y^*, $\forall t \geq \hat{t}$. Solving the firms' and government's problems for $y_i^t > y^*$, the MPE strategies are:

$$\hat{X}^t(Y^t) = X^t(Y^*), \qquad \tau^t(Y^t) = \tau^t(Y^*), \quad \forall t \geq \hat{t} \tag{12}$$

Proposition 3: *If the learning process is eventually exhausted, $y_i^\infty > y^*$, and if an MPE for the finite horizon $[0, \hat{t}]$ involves a non–increasing tarif policy, then there exists a unique MPE for the infinite horizon game.*

Proof: First, consider the case where $t \geq \hat{t}$. Since the relevant state of the game becomes constant once learning has been exhausted, the MPE reduces to the infinitely repeated NE whose equilibrium strategies are represented by (12). Any other subgame perfect equilibrium (trigger–strategy) for this infinitely repeated game does not qualify for Markov perfection. This is due to the fact that the payoff–relevant history is the same over $[\hat{t}, \infty)$. Hence, it is not possible to have different payoffs in each period when the state is common and strategies are state–contingent. Therefore, industry output and the optimal tariff/subsidy are constant over time and are completely determined by the model's parameters so the MPE is unique over $[\hat{t}, \infty)$. Second, if there exists at least one non-increasing tariff policy equilibrium for $[0, \hat{t}]$, it is a linear function of the state because of the linear–quadratic structure of the model. If there exists more than one non–increasing tariff for the finite horizon game, only one will be welfare maximizing since the problem is strictly concave. This unique non–increasing tariff policy equilibrium solves the system of partial differential equations (7) – (8). It is straightforward to show that for any initial state of the game, (12) is the limit of the equilibrium strategies $\hat{X}^t(Y^t)$ and $\hat{\tau}^t(Y^t)$ as $t \rightarrow \hat{t}$. ∎

The solution of the model works as follows. The Riccati equations that solve the MPE generate a fourth degree polynomial in $\tilde{o}_1$. Therefore, there exists four possible values of o_0, o_1, $\tilde{o}_0$, and $\tilde{o}_1$ that may solve the MPE for a given transversality condition. None of these solutions needs to be such that $o_1 \leq 0$ and $\tilde{o}_1 \leq 0$. In this case no time–consistent solution would exists. However if there exists a unique, non–increasing tariff, linear MPE for the finite horizon in which learning occurs, Proposition 3 proves that there is also a unique, time–consistent tariff policy for the infinite horizon game within the space of continuous strategies if learning is exhausted. Observe that in this particular case the MPE strategies are continuous and generally non linear over $[0, \infty)$. Unless production and the tariff remain constant over the whole period, these continuous MPE strategies are kinked at $t = T$. If more than one solution to the Riccati equations satisfy the signs of Proposition 1, the linear quadratic structure of the game ensures that only one of the time consistent MPE will be welfare maximizing over $[0, \hat{t}]$ and therefore, given the static strategies followed after $\hat{t}$, also over $[0, \infty)$.

Finally, our model provides an analytic characterization of firms' strategies over an infinite horizon MPE. It seems plausible to expect that when learning effects are higher, production will increase faster than when learning is exhausted. However this does not need to be so. Observe that differentiating the optimal industry's output strategy $\hat{X}^t(Y^t)$ we get:

$$\hat{x}_1 = \frac{\partial \hat{X}^t(Y^t)}{\partial Y^t} = \frac{k\mu_m\mu_x \dfrac{\tilde{o}_1}{\Gamma_1} + \dfrac{2}{n} o_1}{b_x\left(2 + \dfrac{1}{n}\right)}$$

Given its linear–quadratic structure, the model generates a constant rate of change of industry production with respect to accumulated output. It follows that industry output will always decrease with the state when $o_1 < 0$ and $\tilde{o}_1 > 0$ (not time–consistent) or when $\tilde{o}_1 < 0$ and o_1 is negative enough[6] (time consistent). Therefore, even for an infinite horizon, time consistent and linear MPE industry production shows ambiguous dynamics. Still, we can rule out increases in industry output over time. The intuition for this result is that firms try to profit from large cost reductions in their early stages of development by increasing initial production. However as cost reductions are smaller in latter periods, production decreases in order to exploit market power. Therefore the domestic price has an increasing path. Government intervention induces an increase in production in the early stages of development of the industry, and it promotes competition once learning has been exhausted. Other things equal, a reduction in the degree of substitution between domestic and foreign goods, k, makes it more likely that the above derivative will be negative.

4 Numerical Comparative Statics

In order to explore the qualitative features of the model, we have calculated dynamic equilibria for different values of the structural parameters. These structural parameters have been chosen such that firms must produce above the instantaneous equilibrium level to exhaust learning. The calibration is therefore carried out such that without tariff protection the price of the domestic good is $P_x = 65$ and that of the foreign good $P_m = 40$ for the static equilibrium. Demand parameters are the following: $a_x = 100$; $b_x = 3$; $k = 1.85$; $a_m = 80$; and $b_m = 5$. Costs parameters are: $c_0 = 210$; $c_1 = -9$; $c_2 = 0.2$; and $c_3 = 7$. In order to run these comparative statics exercises, the model has been evaluated at $Y = 150$. With the present parameterization, learning is exhausted at $Y^* = 450$. Therefore, the chosen level of accumulated output is an intermediate level such that there still remains important cost reductions due to learning. For such a given scenario, domestic firms earn profits but the optimal tariff is at prohibitive levels, so that imports are still not allowed. It is straightforward to show that with such parameterization the sufficient condition for the utility function to be concave holds ($2.89437 > 0$). It is necessary to specify two additional parameters to complete this particular version of the model: the discount rate $r = 0.05$, and the number of firms in the industry, $n = 10$.

INSERT TABLE 1

Table 1 illustrates the model solutions. Parameters o_0, o_1, $\tilde{o}_0$, and $\tilde{o}_1$ are the unknowns of the Riccati equations as defined in the appendix. Two solutions involve complex values for these parameters but they are not of interest because the real part of the value of o_1 is always positive. Only the first real solution is such that o_1 and $\tilde{o}_1$ are both negative. Therefore, Proposition 3 applies so that the present parameterization of the model ensures a unique time–consistent tariff policy for the infinite horizon game.

[6] Specifically $o_1 < -nk\mu_m\mu_x\tilde{o}_1/(2\Gamma_1) < 0$. Observe that a negative sign for this derivative will ensure the fulfillment of transversality conditions (11).

The time consistency of the optimal policy design has been extensively studied over the past few years. The work of Staiger and Tabellini (1987) addresses the issue of the time consistency of optimal tariff protection policies. In a different framework, they conclude that the optimal trade policy must be time inconsistent, since it provides unexpected protection in order to maximize redistributive effects in favor of individuals with high marginal utility of income. Protection results from the government's inability to precommit to free trade. Moreover, they show that any time-consistent policy involves an excessive amount of protection.

It is worth noting several differences between this work and our model. The present model does not deal with the distributive effects of tariff protection, even when they may be modeled as exogenous determinants of the government's objective function. Instead, this paper presents a very particular situation where there exists an infant-industry that shows important learning effects in the early stages of development. In this case, and in contrast with most of the works dealing with time consistency of optimal policies, there may be an optimal time consistent tariff policy that does not require the government's precommitment to future liberalization. By contrast, the future liberalization is a feature of the optimal strategy of the Government. This result derives from the fact that the tariff policy is contingent on industry performance, that is, Government's best response is a Markov strategy that depends on one state variable which captures firms' learning effects. We have also established the limits of any tariff policy to be time-consistent in an infinite horizon framework. Therefore, we obtain the opposite result in which the policy provides excessive protection compared to Staiger and Tabellini's work. In our case a time inconsistent tariff policy provides excessive protection as compared to a time-consistent one. Where the former shows an increasing tariff path, the later will (generally) decrease.

The government need not commit to future liberalization when he enforces the time consistent strategy since that liberalization is already embodied in that policy. However, this does not mean that the government's lack of commitment has no cost. If the government could commit to any tariff policy, he would choose that one that achieves the maximum welfare independently if this policy qualify for time-consistency or not. Within this framework the government would enforce the tariff policy implied by the fourth solution of Table 1. This non-consistent tariff policy is increasing over time and would lead to a welfare level of 1734.57 at $Y = 150$, while the time consistent policy reaches a welfare level that is only 11.23% of the non consistent case as shown in Table 2. The cost of the government's lack of commitment is important and it increases with Y, since the non-consistent policy is increasing while the time-consistent tends to open the domestic economy as learning declines.

To complete the analysis, we must address how changes in the model's structural parameters affect the optimal production decision and tariff policy. All the comparative statics are monotone. Table 2 summarizes these comparative statics results and shows the arc-elasticity of each item with respect to each structural parameter evaluated in a neighborhood of $\pm 1\%$ about their values. The first column shows the value for each item corresponding to the time-consistent solution. All money valued items are specified in domestic currency units. The solutions of the Riccati equations are independent of the value of the state variable, but the accumulated output enters into the optimal strategies and welfare components. The signs between parenthesis below the arc elasticities of $\hat{X}(Y)$, $\hat{\tau}(Y)$, $CS(Y)$, $\Pi(Y)$, and $W(Y)$ show how these elasticities are related to Y.

INSERT TABLE 2

A shift in the demand for domestic goods, a_x, increases domestic market profitability so that optimal production is also higher. It also raises the optimal tariff by a larger proportion than the increase in demand in order to mantain the increase in demand served by domestic firms and to extend the learning effect. Consumers and producers are better off so that welfare incrases with a_x.

Given that $b_x > 0$, the steeper is the demand for domestic goods, the lower is consumer willingness to pay. Consumers tend to substitute domestic for foreign goods, so that the optimal tariff must rise. We should expect a reduction of domestic production, but at this level of accumulated output, since tariff increases above the prohibitive level, it induces enough learning effects to increase domestic production, which leads to increases in consumer surplus, profits, and welfare.

An increase of k, the degree of substitution between domestic and foreign produced goods, also decreases consumer's marginal willigness to pay for domestic goods and the optimal domestic output decreases. The shift in demand towards foreign goods is so important that the optimal tariff must decline to allow imports in order for consumer surplus to partially compensate for the reduction in profits. However, the tariff reduction is not enough to allow imports at all. The combined effect is a sensitive reduction in welfare due to reduction in learning because of the lower domestic production, and lower consumer surplus.

Changes in the demand intercept for foreign goods have just the opposite effect of its counterpart for domestic goods. An increase in a_m raises the willingness to pay for foreign goods. Since the demand system has only two goods, this implies that it lowers the willingness to pay for domestic goods. Domestic production, the optimal tariff, consumer surplus and profits change in the same way as they do when k increases. The same argument applies to changes in b_x. An increase in this parameter reduces the marginal willingness to pay for imports, therefore increasing the marginal willigness to pay for domestic goods and the optimal domestic production. It follows that consumer surplus, profits, and welfare also increase. The increase of the tariff reinforces these effects by enhancing learning.

The following set of parameters refers to the cost function. All of them are inversely related to the optimal tariff decision, but effects vary for output, and therefore for consumer surplus, profits, and welfare. The abnormal positive effect of c_1 and c_3 on $\hat{X}(Y)$ may be explained by the absence of imports since tariffs always remain above the prohibitive level.

When the speed of learning declines ($c_1 < 0$ becomes greater) tariff also falls. The optimal tariff will be lower in order to compensate for slow learning and to reduce the monopoly power of the domestic firms. Welfare increases significantly mostly due to higher profits associated with domestic firms' higher market power. Results are similar when marginal cost, c_3, rises. These results also hold when c_2 increases but the magnitude of the effects is larger. With a more convex fixed cost function, learning lasts less time and firms reduce domestic production in order to delay learning exhaustion and to profit from domestic market protection. The optimal strategy for the government is then to reduce the tariff significantly in order to induce imports.

Finally, the interest rate is positively related to production. As an increase in the interest rate promotes domestic production, tariffs must be raised for the economy to profit from learning. This is the same case as increased profitability of domestic production through a shift in a_x. Welfare and its components also increase as they are positively related to domestic production.

5 Concluding Remarks

The main contribution of this paper is showing that there may exist an optimal, time consistent tariff policy which ensures maximization of a discounted welfare function when there exist learning effects. Furthermore, the Government's optimal policy does not need any external source of precommitment since it is contingent on industry's performance.

Assuming that learning is limited to fixed cost reduction and that demand follows a simple linear structure, the optimal equilibrium strategies have been derived in closed form. This setup allows us to prove the intuitive result that any time–consistent tariff policy must involve a decreasing tariff in order to compensate for the exhaustion of the learning process with foreign competition. The optimal policy balances the actual loss in consumer surplus with future gains from lower costs, and when learning is exhausted, the excessive monopoly power of the domestic firms is offset by higher foreign competition. The model could be generalized in different ways in order to capture features of particular cases more realistically, but in most cases we loose the ability to characterize time–consistency.

Observe that the intuition of Matsuyama's (1990) model is retained in the present dynamic framework. The government wants to liberalize in order to maximize domestic welfare. But, due to learning effects, welfare maximization over time requires the establishment of a tariff to protect the domestic industry. The optimal tariff will depend on industry performance. Given a low learning effect, the optimal policy will reduce the tariff to increase competition and avoid excessive domestic monopoly power. On the other hand, domestic firms prefer a monopolistic position, but the possibility of foreign competition induces them to increase production above the static profit maximization level in order to reduce cost and be able to compete later. The dynamics have their origin only in the decreasing speed of learning induced by a downward slopping convex fixed cost function over accumulated output because demand is stationary in the present formulation.

Appendix

Derivation of Optimal Controls

In games like this one, the state follows a Markov process in the sense that the probability distribution over next period's state is a function of the current state and actions, and hence, the history at t can be summarized by y^t. In solving this model, we assume perfect information, which implies that each player knows the history of the game, i.e., the previous realizations of the state vectors, $y^s \in R^n$, and control variables, $(x^s, \tau^s) \in R^{n+1}$, $\forall s \leq t$. Markov strategies depend only on the state of the system and player's information sets includes only the payoff-relevant history [Maskin and Tirole (1993)]. Markov perfection requires that these strategies be a perfect equilibria for any time and state [Fudenberg and Tirole (1986, §2b)].

Focusing on smooth equilibria [Starr and Ho (1969)], a differential game equilibrium of the model is a set of functions $\{a_i^t(y^t)\} = \{(\ldots, x_i^t(y^t), \ldots), \tau^t(y^t)\}$ such that for any time and state, a player's strategy maximizes its payoff from that time on. Applying dynamic programming, the differential game equilibrium solves a set of generalized Hamilton–Jacobi conditions, *i.e.*, a system of partial differential equations which are the first order conditions of the corresponding Hamiltonian for each player. Such a system is not easily solved except in the case of some particular functional specifications such as the linear–quadratic case of this model in which the players are the government and any representative symmetric firm of the industry. Furthermore, the motion equation must be linear and the objective function must be quadratic in the state and control variables. Therefore, given equations (1)–(4) and (6), each firm's stage profit function is:

$$\pi_i^t = (a_x - \frac{1}{2}b_x \sum_{j=1}^{n} x_i^t - \frac{1}{2}k M^t)x_i^t - (c_0 + c_1 y_i^t + \frac{1}{2}c_2(y_i^t)^2 + c_3 x_i^t)$$

So that, the current Hamiltonian for firm i is:

$$H_i^t = (a_x - \frac{1}{2}b_x \sum_{j=1}^{n} x_i^t - \frac{1}{2}k M^t)x_i^t - (c_0 + c_1 y_i^t + \frac{1}{2}c_2(y_i^t)^2 + c_3 x_i^t) + \lambda_i^t x_i^t$$

Observe that the coefficient of $(x_i^t)^2$ is $-\frac{1}{2}b_x < 0$ which ensures that player i's optimal control is well defined. The solution must satisfy the following generalized Hamilton–Jacobi conditions:

$$\frac{\partial H_i^t}{\partial x_i^t} = \left(a_x - \frac{1}{2}b_x X^t - \frac{1}{2}kM^t\right) - \frac{1}{2}b_x\left(\sum_{j=1}^{n}\frac{\partial x_j^t}{\partial x_i^t}x_i^t\right) + \frac{1}{2}k\mu_m\frac{\partial \tau^t}{\partial x_i^t}x_i^t - c_3 + \lambda_i^t = 0$$

$$\dot{\lambda}_i^t = r\lambda_i^t - \frac{\partial H_i^t}{\partial y_i^t} - \sum_{j\neq i}^{n}\frac{\partial H_i^t}{\partial x_j^t}\frac{\partial \hat{x}_j^t}{\partial y_i^t} = r\lambda_i^t - (c_1 + c_2 y_i^t)$$

$$-\sum_{j\neq i}^{n}\left[-\frac{1}{2}b_x\left(x_i^t\sum_{h=1}^{n}\frac{\partial x_h^t}{\partial x_j^t} + X^t\frac{\partial x_i^t}{\partial x_j^t}\right)\frac{\partial \hat{x}_j^t}{\partial y_i^t}\right] + \frac{1}{2}k\mu_m\left(x_i^t + \tau^t\frac{\partial x_i^t}{\partial \tau^t}\right)\frac{\partial \hat{\tau}^t}{\partial y_i^t}$$

Imposing symmetry, $x_i^t = x_j^t, \forall i, j, t$ enables us to use the following identities:

$$X^t = nx_i^t \qquad Y^t = ny_i^t \qquad \lambda^t = n\lambda_i^t$$

Moreover, given a fixed number of firms n, symmetry also implies:

$$\dot{Y}^t = n\dot{y}_i^t = nx_i^t = X^t \qquad ; \qquad \dot{\lambda}^t = n\dot{\lambda}_i^t$$

$$\frac{\partial \hat{X}^t}{\partial Y^t} = \frac{\partial(n\hat{x}_i^t)}{\partial y_i^t}\frac{\partial y_i^t}{\partial Y^t} = \frac{\partial \hat{x}_i^t}{\partial y_i^t} \qquad ; \qquad \frac{\partial \hat{\tau}^t}{\partial Y^t} = \frac{\partial \hat{\tau}^t}{\partial y_i^t}\frac{\partial y_i^t}{\partial Y^t} = \frac{1}{n}\frac{\partial \hat{\tau}^t}{\partial y_i^t}$$

Using these expressions and firms' conjectural variation parameters, equation (7) is obtained by aggregation of the above necessary conditions.

We must also address the welfare function. The components of the government's objective function are written as follows:

$$\begin{aligned} CS^t(y^t, x^t, \tau^t) =& \frac{1}{2}(b_x X^t + kM^t)X^t + \frac{1}{2}(kX^t + b_m M^t)M^t \\ &- \frac{1}{4}\left[b_x(X^t)^2 + b_m(M^t)^2 + 2kX^tM^t\right] \\ \Pi^t(y^t, x^t, \tau^t) =& n\pi_i^t(y^t, x^t, \tau^t) = (a_x - \frac{1}{2}b_x X^t - \frac{1}{2}kM^t)X^t \\ &- (nc_0 + c_1 Y^t + \frac{1}{2n}c_2(Y^t)^2 + c_3 X^t) \\ R^t(y^t, x^t, \tau^t) =& \tau^t M^t \end{aligned}$$

Therefore, the current Hamiltonian for the government becomes:

$$\begin{aligned} H_{n+1}^t =& \frac{1}{2}(b_x X^t + kM^t)X^t + (a_x - \frac{1}{2}b_x X^t - \frac{1}{2}kM^t)X^t + \frac{1}{2}(kX^t + b_m M^t)M^t \\ &- \frac{1}{4}\left[b_x(X^t)^2 + b_m(M^t)^2 + 2kX^tM^t\right] - \left[nc_0 + c_1 Y^t + \frac{1}{2n}c_2(Y^t)^2 + c_3 X^t\right] \\ &+ \tau^t M^t + \tilde{\lambda}^t X^t \end{aligned}$$

Differentiate this Hamiltonian to obtain:

$$0 = \frac{\partial H_{n+1}^t}{\partial \tau^t} = \mu_x(a_x - c_3) - [\frac{1}{2}b_x\mu_x]X^t(P^t, 0) + \mu_x\tilde{\lambda}^t$$
$$+ [1 - \frac{1}{2}b_m\mu_m]M^t(P^t, 0) + [-\frac{1}{2}k\mu_x\mu_m + \mu_m(\frac{1}{2}\mu_m b_m - 2)]\tau^t$$

$$\dot{\tilde{\lambda}}^t = r\tilde{\lambda}^t - \frac{\partial H_{n+1}^t}{\partial Y^t} - \frac{\partial H_{n+1}^t}{\partial X^t}\frac{\partial \hat{X}^t}{\partial Y^t} = r\tilde{\lambda}^t - \frac{\partial H_{n+1}^t}{\partial Y^t} - \frac{\partial H_{n+1}^t}{\partial \tau^t}\frac{\partial \tau^t}{\partial X^t}\frac{\partial \hat{X}^t}{\partial Y^t}$$

where $\Gamma_0(X^t(P^t, 0), M^t(P^t, 0))$ and Γ_1 are implicitly defined by the first optimality condition. Then, equation (8) follows from the above specification of the government's Hamiltonian. Observe that the Envelope Theorem can be used to simplify the second generalized Hamilton–Jacobi condition by substituting with the the first one, so that the government's co-state variable does not depend directly on firms actions.

For this problem to be well defined (concave in τ^t), Γ_1, the coefficient of $(\tau^t)^2$ must be negative. Substituting (3)–(4) into the current Hamiltonian expression and grouping terms, it can be shown that this condition holds whenever:

$$\frac{b_x b_m}{b_x b_m - k^2} > 1$$

Which is ensured to hold by the concavity condition of the utility function and the assumed signs of demand parameters.

Riccati Equations

Substituting (10) into (9), the optimal controls are:

$$\hat{X}^t(Y^t) = \frac{2a_x - kM^t(P^t, 0) - k\mu_m\dfrac{\Gamma_0(\cdot) + \mu_x(\tilde{o}_0 + \tilde{o}_1 Y^t)}{\Gamma_1} - 2c_3 + \dfrac{2}{n}(o_0 + o_1 Y^t)}{b_x\left(2 + \dfrac{1}{n}\right)}$$

$$\hat{\tau}^t(Y^t) = \frac{\Gamma_0(\cdot) + \mu_x(\tilde{o}_0 + \tilde{o}_1 Y^t)}{-\Gamma_1}$$

To solve the game, differentiate the proposed solution (10) making use of the fact that $\dot{Y}^t = X^t$. Later, substitute (10) into the right hand side of (7b) and (8b) using $\hat{X}^t(Y^t)$, $\hat{\tau}^t$, $\frac{\partial X^t}{\partial Y^t}$, and $\frac{\partial \tau^t}{\partial Y^t}$ according to the above expressions. These substitutions produce two sets of two linear equations in Y^t. Equating the coefficients of the corresponding equations generates four, nonlinear Riccati equations that determine o_0, o_1, $\tilde{o}_0$, and $\tilde{o}_1$. These equations must be satisfied by any linear MPE production–tariff path.

For convenience, define the following parameters:

$$\sigma_1 = \frac{2a_x - kM^t(P^t,0) + k\mu_m \dfrac{\Gamma_0(X^t(P^t,0), M^t(P^t,0))}{\Gamma_1} - 2c_3}{b_x\left(2 + \dfrac{1}{n}\right)}$$

$$\sigma_2 = \frac{nk\mu_m\mu_x}{b_x(1+2n)\Gamma_1} \quad : \quad \sigma_3 = \frac{2}{b_x(1+2n)}$$

$$\upsilon_0 = \frac{b_x(n-1)}{2} \quad : \quad \upsilon_1 = \frac{k\mu_m\mu_x}{2\Gamma_1}$$

Observe that using these coefficients the above expressions for the optimal controls are:

$$\hat{X}^t(Y^t) = \sigma_1 + \sigma_2(\tilde{o}_0 + \tilde{o}_1 Y^t) + \sigma_3(o_0 + o_1 Y^t)$$

$$\frac{\partial \hat{X}^t(Y^t)}{\partial Y^t} = \sigma_2\tilde{o}_1 + \sigma_3 o_1$$

First, differentiate the proposed solution for the co state variables (10):

$$\dot{\lambda}^t(Y^t) = o_1\dot{Y}^t = o_1 X^t \quad : \quad \dot{\tilde{\lambda}}^t(Y^t) = \tilde{o}_1\dot{Y}^t = \tilde{o}_1 X^t$$

Substituting the optimal controls and making use of the above notation we get:

$$\dot{\lambda}^t(Y^t) = \sigma_1 o_1 + \sigma_2 o_1\tilde{o}_0 + \sigma_2 o_1\tilde{o}_1 Y^t + \sigma_3 o_1 o_0 + \sigma_3 o_1^2 Y^t$$

$$\dot{\tilde{\lambda}}^t(Y^t) = \sigma_1\tilde{o}_1 + \sigma_2\tilde{o}_1\tilde{o}_0 + \sigma_2\tilde{o}_1^2 Y^t + \sigma_3\tilde{o}_1^2 o_0 + \sigma_3\tilde{o}_1 o_0 + \sigma_3\tilde{o}_1 o_1 Y^t$$

Now substitute (10) into (7*b*) and (8*b*) using again the optimal controls and their derivatives. This leads to:

$$\begin{aligned}
\dot{\lambda}^t = & r o_0 + r o_1 Y^t - n c_1 - c_2 Y^t + \frac{1}{2} b_x n\left[2 - \frac{1}{n} - 1\right]\left[\sigma_1 + \sigma_2(\tilde{o}_0 + \tilde{o}_1 Y^t)\right. \\
& \cdots + \left.\sigma_3(o_0 + o_1 Y^t)\right]\left[\sigma_2\tilde{o}_1 + \sigma_3 o_1\right] - \frac{1}{2}k\mu_m\left[\sigma_1 + \sigma_2(\tilde{o}_0 + \tilde{o}_1 Y^t)\right. \\
& \cdots + \sigma_3(o_0 + o_1 Y^t) - \mu_x \left.\frac{\Gamma_0 + \mu_x(\tilde{o}_0 + \tilde{o}_1 Y^t)}{\Gamma_1}\right]\frac{\mu_x\tilde{o}_1}{\Gamma_1}
\end{aligned}$$

$$\dot{\tilde{\lambda}}^t = r\tilde{o}_0 + r\tilde{o}_1 Y^t + \alpha^t c_1 + \alpha^t\frac{c_2}{n}Y^t$$

Equating coefficients gives the following set of Riccati nonlinear equations that must be satisfied by any MPE:

$$
\left.\begin{array}{ll}
\sigma_1 o_1 + \sigma_2[1 - \upsilon_0\sigma_3]o_1\tilde{o}_0 + \sigma_3[1 - \upsilon_0\sigma_3]o_1 o_0 & \\
\quad \cdots + \upsilon_1[\sigma_1 - \mu_x \frac{\Gamma_0}{\Gamma_1}][\upsilon_0\sigma_2 - \upsilon_1]\sigma_3 o_0 \tilde{o}_1^2 & \\
\quad \cdots + [\upsilon_1(\sigma_2 - \frac{\mu_x^2}{\Gamma_1}) - \upsilon_0\sigma_2^2]\tilde{o}_0\tilde{o}_1 - r o_0 & = \upsilon_0\sigma_1 - n c_1 \\
[\sigma_2 - 2\upsilon_0\sigma_2\sigma_3 + \upsilon_1\sigma_3]o_1\tilde{o}_1 + \sigma_3[1 - \upsilon_0\sigma_3]o_1^2 & \\
\quad \cdots + [\upsilon_1(\sigma_2 - \frac{\mu_x^2}{\Gamma_1}) - \upsilon_0\sigma_2^2]\tilde{o}_1^2 - r o_1 & = -c_2 \\
\sigma_1\tilde{o}_1 + \sigma_2\tilde{o}_1\tilde{o}_0 + \sigma_3\tilde{o}_1 o_0 - r\tilde{o}_0 & = c_1 \\
\sigma_2\tilde{o}_1^2 + \sigma_3\tilde{o}_1 o_1 - r\tilde{o}_1 & = c_2/n
\end{array}\right\}
$$

Proof of Proposition 1

For convenience, denote by e_i a $(n+1)\times 1$ vector with all its elements equal to zero except the i–th, which is one. Similarly V_i denotes a null $(n+1)\times(n+1)$ matrix with a unit element in the i–th position of the diagonal:

$$
e_i' = (0, \cdots, 0, 1, 0, \cdots, 0)\,, \qquad V_i = e_i \cdot e_i'
$$

Drop the time superscripts for notational simplicity. Any finite horizon linear quadratic differential game can be written in matrix form as:

$$
u_i = \int_0^T \left(\frac{1}{2} y' Q_i y + \frac{1}{2}\sum_{j=1}^{n+1} x_j' R_{ij} x_j + \frac{1}{2}\sum_{j=1}^{n+1} r_{ij}' x_j + q_i' y + f_i \right) e^{-rt} dt + \frac{1}{2} y'(T) S_i y(T)
$$

$$
\dot{y} = Ay + \sum_{j=1}^{n+1} B_j x_j
$$

The proposed linear solution (10) in matrix form is:

$$
\begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ \lambda_{n+1} \end{pmatrix} = \begin{pmatrix} o_{01} \\ \vdots \\ o_{0n} \\ \tilde{o}_0 \end{pmatrix} + \begin{pmatrix} o_{11} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & o_{1n} & 0 \\ 0 & \cdots & 0 & \tilde{o}_1 \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \\ Y \end{pmatrix} = \Omega_0 + \Omega_1 \begin{pmatrix} y \\ Y \end{pmatrix}
$$

Next, define the following matrix of net effects of the state variables over the control variables:

$$Z = A - \sum_{i=1}^{n+1} B_i R_i^{-1} B_i' \Omega_i$$

Each of these matrices may be identified for the present model. Matrix A is an $(n+1) \times (n+1)$ null matrix. Matrix R_i is diagonal because each player only has one control variable; its elements are equal to $-b_x < 0$ for $i = 1, \ldots, n$, and $\Gamma_1 < 0$ for $i = n+1$. Matrix $B_i = v_i$ in the symmetric case for $i = 1, \ldots, n$, and matrix $B_{n+1} = \mu_x / n > 0$ by symmetry and equation (3). Finally, $\Omega_i = \Omega_1 V_i = o_{1i} V_i$ for $i = 1, \ldots, n$. Therefore:

$$Z = \sum_{i=1}^{n} \frac{o_{1i}}{b_x} V_i - \frac{\mu_x}{n} \Gamma_1 \tilde{o}_1 V_{n+1} = \begin{pmatrix} \frac{o_{1i}}{b_x} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \frac{o_{1n}}{b_x} & 0 \\ 0 & \cdots & 0 & -\frac{\mu_x}{n} \Gamma_1 \tilde{o}_1 \end{pmatrix}$$

Papavassilopoulos, Medanic, and Cruz (1979) have shown that if $\hat{x}^t$ and $\hat{\tau}^t$ satisfy the Riccati equations and the real parts of all eigenvalues of matrix Z are negative, an MPE for the finite horizon is also an MPE for the infinite horizon. In this model, Z is a diagonal matrix with o_{1i} on its diagonal multiplied by some positive constant. Then this asymptotic stability condition is equivalent to all co state variables being negatively related to the accumulated output, *i.e.*, $o_{1i} < 0$, for $i = 1, \ldots, n$, and $\tilde{o}_1 < 0$. Since the symmetric model collapses into a two players differential game (the government and any representative firm) with $n = 1$, this condition requires that $o_1 < 0$ and $\tilde{o}_1 < 0$. ■

References

ANDERSON, J.E. (1992): "Domino Dumping, I: Competitive Exporters". *American Economic Review*, 82, 65–81.

BAGWELL, K. AND R.W. STAIGER (1990): "A Theory of Managed Trade". *American Economic Review*, 80, 779–795.

BAŞAR, T. AND G.J. OLSDER (1995): *Dynamic Noncooperative Game Theory*, 2nd Ed. Academic Press.

BRANDER, J.A. AND B.J. SPENCER (1983): "International R&D Rivalry and Industrial Strategy". *Review of Economic Studies*, 50, 707–722.

DIXIT, A.K. AND A.S. KYLE (1985): "The Use of Protection and Subsidies for Entry Promotion and Deterrence". *American Economic Review*, 75, 139–152.

FUDENBERG, D. AND J. TIROLE (1991): *Game Theory*. MIT Press.

GRUENSPECHT, H.K. (1988): "Dumping and Dynamic Competition". *Journal of International Economics*, 25, 225–248.

HEAD, K. (1994): "Infant Industry Protection in the Steel Rail Industry", *Journal of International Economics*, 37, 141–165.

HELPMAN, E. AND P.R. KRUGMAN (1989): *Trade Policy and Market Structure*. MIT Press.

KRUGMAN, P.R. (1984): "Import Protection as Export Promotion: International Competition in the Presence of Oligopolies and Economies of Scale", in H. Kierzkowski (ed.): *Monopolistic Competition and International Trade*. Oxford University Press.

LUKES, D.L. (1971): "Equilibrium Feedback Control in Linear Games with Quadratic Costs", *SIAM Journal on Control*, 9, 234–252.

MASKIN, E. AND J. TIROLE (1993): "Markov Perfect Equilibrium", Mimeo, Harvard University.

MATSUYAMA, K. (1990): "Perfect Equilibria in a Trade Liberalization Game", *American Economic Review*, 80, 480–492.

PAKES, A. AND P. MCGUIRE (1994): "Computing Markov Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model", *Rand Journal of Economics*, 25, 555–589.

PAPAVASSILOPOULOS G.P., AND J.B. CRUZ, JR. (1979): "On the Uniqueness of Nash Strategies for a Class of Analytic Differential Games", *Journal of Optimization Theory and Applications*, 27, 309–314.

PAPAVASSILOPOULOS, G.P., J.V. MEDANIC, AND J.B. CRUZ, JR. (1979): "On the Existence of Nash Strategies and Solutions to Coupled Riccati Equations in Linear Quadratic Games", *Journal of Optimization Theory and Applications*, 28, 49–76.

PAPAVASSILOPOULOS G.P., AND G.J. OLSDER (1984): "On the Linear–Quadratic, Closed Loop, No Memory Nash Game", *Journal of Optimization Theory and Applications*, 42, 551–560.

PRATTEN, C.A. (1971): *Economics of Scale in Manufacturing Industry*. Cambridge University Press.

SALTER, W.E.G. (1966): *Productivity and Technical Change*, 2nd Ed. Cambridge University Press.

STAIGER, R.W. AND G. TABELLINI (1987): "Discretionary Trade Policy and Excessive Protection", *American Economic Review*, 77, 823–837.

STARR, A. AND Y.C. HO (1969): "Nonzero-Sum Differential Games", *Journal of Optimization Theory and Applications*, 3, 183–206.

STOKEY, N.L. (1986): "The Dynamics of Industrywide Learning", in W.P. Heller, R.M. Starr, and D.A. Starret (eds.): *Equilibrium Analysis, Essays in Honor of Kenneth J. Arrow*, Volume II. Cambridge University Press.

Table 1

o_0	o_1	$\tilde{o}_0$	$\tilde{o}_1$
244.634	-4.848	354.374	-2.070
-3475.585 -3159.025i	1.326 +2.957i	-254.815 +262.194i	-0.011 -0.180i
-3475.585 +3159.025i	1.326 -2.957i	-254.815 -262.194i	-0.011 +0.180i
399.217	7.352	320.931	1.838

Table 2. Comparative Statics.

	Solution	a_r	b_r	k	a_m	b_m	c_1	c_2	c_3	r
ϕ_0	244.634	0.0075	0.1610	-0.1150	-0.0022	0.0581	0.0367	-0.1041	0.0036	-0.0039
ϕ_1	-4.848		0.0026	0.0523		-0.0261		0.0712		-0.0425
$\tilde{\phi}_0$	354.374	0.1689	0.2334	-0.4027	-0.0500	0.2165	-0.0061	-0.0420	-0.0052	0.0349
$\tilde{\phi}_1$	-2.070		0.1604	-0.2291		0.1146		0.0542		-0.0083
$\hat{X}(Y)$	14.206	0.0041	0.0837	-0.1478	-0.0012	0.0742	0.0201	-0.1726	0.0019	0.0669
		(+)	(+)	(-)	(+)	(+)	(+)	(-)	(+)	(+)
$\hat{\tau}(Y)$	57.089	0.6467	0.3555	-0.7922	-0.1914	0.4444	-0.0233	-0.3422	-0.0120	0.1614
		(+)	(+)	(-)	(-)	(+)	(-)	(-)	(-)	(+)
$CS(Y)$	151.349	0.0083	0.1675	-0.2956	-0.0024	0.1485	0.0401	0.3452	0.0039	0.1338
		(+)	(+)	(-)	(-)	(+)	(+)	(-)	(+)	(+)
$\Pi(Y)$	43.422	0.0680	1.3807	-2.4445	-0.0201	1.2256	0.3306	-2.8471	0.0321	1.1035
		(?)	(?)	(?)	(?)	(?)	(?)	(?)	(?)	(?)
$W(Y)$	194.771	0.0280	0.5743	-0.9840	-0.0083	0.5087	0.1364	-1.1442	0.0132	0.4577
		(+)	(+)	(-)	(-)	(+)	(+)	(-)	(+)	(+)