Discussion Paper No. 1153

EFFICIENCY WAGES AND THE HOURS/UNEMPLOYMENT TRADE-OFF

by
Boaz Moselle

March, 1996

Efficiency Wages And The Hours/Unemployment Trade-Off

Abstract: It is commonly argued that reducing the number of hours worked by the employed would lead to lower unemployment, since firms will respond by hiring more workers. This paper examines the relationship between hours worked and unemployment, in the context of an efficiency wage model where involuntary unemployment occurs owing to imperfect monitoring of worker effort. The first part of the paper presents a partial equilibrium model where the number of hours worked per week is determined exogenously. The model makes standard assumptions about effort costs (i.e., increasing marginal disutility of work), and also allows for the presence of daily 'set-up costs' for the worker. It is shown that under these assumptions the equilibrium level of unemployment, viewed as a function of hours worked, is 'U-shaped' (or simply increasing, if the set-up costs are zero). The paper then moves to a general equilibrium framework where hours are determined endogenously; it is shown that in this case the free market choice of hours is always greater than the unemployment-minimizing level, so that 'work-sharing' could indeed lower unemployment. However the paper also presents a powerful and surprising welfare result: conditional on being unemployed, a representative worker is always best off under the free market outcome. Nonetheless, starting from the free market equilibrium there is always an hours-reduction policy which reduces unemployment and increases the expected utility of a currently unemployed worker. The currently employed are always made worse off by such a policy.

Thanks to Mathew Ellman, Roger Myerson, Juliet Schor, Martin Weitzman and seminar participants at Harvard University, Northwestern University and the Bank of England.

Introduction

Proponents of 'work-sharing' argue that a reduction in the number of hours worked by those in employment will lead to lower unemployment through a simple substitution effect: firms will hire more workers to compensate for getting less hours from each individual worker. During periods of high unemployment this argument has received considerable attention. In the United States, the need to create jobs was used to justify the imposition of a mandatory overtime premium in the Fair Labor Standards Act of 1938.* More recently, work sharing has been high on the policy agenda in many European countries, attracting support from the European Commission (*White Paper on Growth, Competitiveness, and Employment*, 1993) and from politicians in France, Italy and elsewhere (*New York Times*, Nov. 22, 1993).

Economists are typically skeptical about the ability of such policies to lower unemployment (see for example Ehrenberg and Schumann, 1982). Those who argue for work-sharing usually rely on the 'lump of work' fallacy (Layard, Nickell and Jackman, 1991), i.e., the assumption that the number of person-hours used by a firm will remain constant as the number of hours is changed, thus ignoring the presence of 'quasi-fixed labor costs' which make people and hours imperfect substitutes. Beyond pointing out some of the attendant fallacies and pitfalls, however, there has been little work on the subject. There is one major empirical study, due to Ehrenberg and Schumann (1982), who estimated the effects of raising the mandated overtime premium from time-and-a-half to double-time (they predict an increase in employment of 0.3% to 4.0%, but note a number of reasons for skepticism). However, their underlying model of the labor market is a standard market-clearing one, essentially assuming away involuntary unemployment.

* Legislation to raise the overtime premium is occasionally introduced in Congress, most recently in 1993, although it seems clear that a simple redefinition of the 'basic rate' would enable most firms to attain legal compliance with no real effect (as discussed in Trejo, 1991).

The contribution of this paper is to use microeconomic theory to address rigorously the issues raised by work-sharing. In particular it uses a model of the labor market (efficiency wage theory) which provides a proper microeconomic foundation for the existence of involuntary unemployment. The paper investigates the relationship between hours worked and the equilibrium level of unemployment in such a model, answering both the positive question of whether hours-reduction would decrease or increase unemployment and the normative questions concerning the welfare effects of such policies.

The paper proceeds as follows. In the first section, we introduce an efficiency wage model of the labor market (based on that of Shapiro and Stiglitz, 1984), with work hours an exogenously fixed parameter. Section II examines the comparative statics of this model, determining how equilibrium unemployment changes as the length of the work day varies. The main conclusion from this exercise is given in proposition 1: as a function of the number of hours in the work period, equilibrium unemployment is U-shaped. It follows from this result that legislation to shorten the work day could either lower or raise unemployment, depending on the initial level (and the extent of the reduction). In section III the model is completed by allowing for work hours to be determined endogenously, with firms choosing the number of hours in the employment contract so as to minimize their production costs. Proposition 2 shows that this market choice of work hours is always strictly greater than the unemployment-minimizing level, so that it is always possible for the government to reduce unemployment by a work-sharing policy. Section IV discusses the welfare implications of such a policy. Perhaps the most surprising result is proposition 3, which shows that the level of hours chosen by the free market actually maximizes the welfare of a representative unemployed worker (in a comparative static sense). Hours reduction therefore, while it benefits those unemployed workers for whom it creates jobs, lowers the welfare of those who remain unemployed after the policy is put into effect. Nonetheless, the unemployed will support some degree of work-sharing (proposition 5). Currently employed workers, however, are unambiguously made worse off by hours reduction, and can be expected to oppose it (proposition 4). Sections V and VI discusses some possible extensions to the model, and section VII concludes.

Moving beyond the issue of work-sharing, the approach taken here provides a rigorous setting in which to consider a number of related issues. Work hours are just one dimension of 'observable intensity' in an employment relationship characterized by the presence of moral hazard. Others include the pace of work in a 'continuous flow' production process (for instance the speed of an assembly belt), the number of machines that a single worker is in charge of, and the amount of land to be cultivated in a share-cropping arrangement. It seems that many of these intensities vary across time and/or between different countries to a rather puzzling extent. The differences in hours worked between Europe, the United States and East Asia are well known, but the work of Clark suggests that these are dwarfed by the cross-country variance in intensity of effort exerted by workers, as measured by for example the number of looms controlled by each worker in a cotton mill (Clark, 1987). While these apparent anomalies remain unexplored, this paper at least provides a framework in which they might be addressed.

This issue is largely unexplored in the economic literature. Schor (1992) observes that in an efficiency wage model the 'job rent' varies with the number of hours worked, but does not investigate the nature of this relationship. Lewin (1994) uses a related framework to discuss the effects on work-hours of a posited economy of scale in monitoring (with respect to hours worked). Rosen's work on the 'supply of work schedules' (Rosen, 1978) is also relevant, but assumes market-clearing. Some discussion of work hours in the context of non-market clearing models of the labor market can be found in the work of European economists, including Hoel (1985), Dreze (1986), and Houpis (1993).

I. The Model

Workers and the no-shirking constraint

There are many identical firms, many identical workers, and a single consumption good. Each period ($t = 0, 1, \ldots$) a worker can be employed or unemployed, and if employed can be working or shirking. For the moment, we take the length of the workday to be exogenously fixed at h hours. The period t wage is denoted w_t, and the disutility of working h hours is given (in terms of the consumption good) by $e(h)$. We will assume throughout that $e(.)$ is increasing and convex ($e' > 0, e'' > 0$) and that $e(0) = 0$. Workers live forever, and lifetime utility is the discounted sum of per-period utilities:

$$V = \sum_{t=0}^{\infty} \frac{v_t}{(1+r)^t}$$

where

$$v_t = \begin{cases} w_t - e(h) & \text{if working in period } t; \\ w_t & \text{if shirking in period } t; \\ 0 & \text{if unemployed in period } t. \end{cases}$$

For the moment, we make the simplest possible assumptions about the monitoring technology: a worker who shirks faces a probability q of being caught and therefore fired, while one who works will never be fired (since the monitoring technology never gives 'false positives' about shirking, it is clearly optimal for firms to fire shirkers, on the 'shoot the agent' principle). There is also an exogenous probability b (the 'quit rate'), for all workers, of leaving their job each period. For notational convenience we assume that q is actually the probability a shirker is caught conditional on not being layed off, so that the probability of dismissal for a shirker is $b + q$. Section V discusses extensions of the monitoring technology in more detail.

Assume that we are in a steady state, so that wages and unemployment are the same each period. Let V^N represent the expected lifetime utility of a currently employed worker who chooses to work not shirk. He gets utility $w - e(h)$ today, and tomorrow will either be unemployed, with probability b, or still employed (and by assumption still not shirking). This gives

$$V^N = w - e(h) + \frac{1}{1+r}[bV^U + (1-b)V^N] \tag{1}$$

Similarly, if V^S is the expected lifetime utility of a currently employed shirker, then

$$V^S = w + \frac{1}{1+r}[(b+q)V^U + (1-b-q)V^S] \tag{2}$$

For an employee to chose not to shirk requires $V^N \geq V^S$, or equivalently, using equations (1) and (2)

$$q\frac{(V^N - V^U)}{1+r} \geq e(h) \tag{3}$$

This condition has a natural interpretation: $(V^N - V^U)/(1+r)$ is the 'job rent' J, i.e. the present value of the cost of getting fired (the $1+r$ term enters because shirking gives immediate benefits but delayed costs). Thus the left-hand side represents the expected cost of shirking, which for incentive compatibility to hold must exceed the expected benefits of shirking, $e(h)$. Solving (3) with (1) gives the efficiency wage or 'no-shirking condition'

$$w \geq \hat{w}(h, V^U) \equiv \frac{rV^U}{1+r} + \left(\frac{r+b+q}{q}\right)e(h) \tag{4}$$

Firms are competitive, i.e. take V^U to be fixed, so that each firm views the above as the wage it has to pay to get workers to work for h hours, and chooses hours and employment to maximize profits subject to (4). Of course, V^U is itself endogenous, determined by the aggregate of all firms' choices of hours and employment. In order to get an aggregate version of (4), we need to determine V^U. By analogy with (1) and (2), we have

$$V^U = \frac{1}{1+r}\left[aV^N + (1-a)V^U\right] \tag{5}$$

where a is the 'job acquisition rate'. In steady-state we must have equal flows into and out of the pool of unemployed, so if we normalize the total workforce to 1 and denote by u the proportion of the workforce which is unemployed, then we get $au = b(1-u)$. Using this to replace a in (5) and solving with (3), which must hold with equality in equilibrium, gives an aggregate no-shirking condition

$$w \geq w(h,u) = \left(\frac{r - q + b/u}{q}\right)\epsilon(h) \tag{6}$$

$w(h,u)$ represents the minimum wage which will ensure that workers do not shirk when the unemployment rate is u and the work day consists of h hours.

Production

We now turn to the production side of the economy. Our model assumes a particularly simple form for the production function. Each worker has a daily set-up cost $s \geq 0$ and constant marginal product $\alpha >> \epsilon'(0)$, and output is proportional to the number of workers. Thus l workers each working for h hours produce output

$$y(h,l) = (\alpha h - s)l$$

The assumption of constant returns to scale in number of workers is convenient, but not essential to the results. The presence of a set-up cost captures the notion that there may be efficiency reasons for working longer hours: we will see that there is a trade-off between the benefit provided by longer hours in spreading out this set-up cost, and the costs of rising marginal disutility, and it is this trade-off which determines the relationship between work-hours and unemployment.

It is also worth noting that in equilibrium workers will take all the output, so that it makes no difference if the set-up cost appears to be paid by the worker rather than the firm (e.g., commuting costs).

II. The Hours/Unemployment Trade-off

Given the constant returns technology above, equilibrium must require zero profits, so the equilibrium wage is given by

$$w = \alpha h - s$$

Substituting into (6) gives the *hours-unemployment schedule*

$$\frac{b}{[u(h)]} = \frac{q}{[e(h)/(\alpha h - s)]} - r - q \qquad (7)$$

Informally, equation (7) tells us that equilibrium unemployment $u(h)$ varies as does the 'effort per unit output' function $e(h)/(\alpha h - s)$. Formally, we need the following assumptions in order to get our first result.

Assumptions. *Suppose that (1) in the first best case (no unobservable shirking) it is strictly efficient to work a finite, strictly positive number of hours; and (2) In the second best case (i.e., the model under consideration) it is possible to induce a strictly positive number of hours.*

These assumptions are very minimal: the case where (1) fails to hold is simply not interesting, while (2) rules out cases where, for instance, the detection technology is so poor (q is so small) that whatever values h and u take on, workers will always choose to shirk.

Proposition 1. *Under these assumptions, there is a range of values $\underline{h} \leq h \leq \overline{h}$ which are compatible with equilibrium. If $s = 0$ then $\underline{h} = 0$ and equilibrium unemployment $u(h)$ is always increasing* *in h. If $s > 0$ then $u(h)$ is U-shaped: as h varies within $[\underline{h}, \overline{h}]$, equilibrium unemployment is first decreasing in h, then increasing, with a unique minimand.*

Proof. see appendix. ¶

In other words, when work hours are long, an exogenously imposed reduction in their length will reduce unemployment ($u'(h) > 0$), but when they are already short, it will increase it. This result is driven by a robust economic intuition. Recall the incentive compatibility condition, which can be rewritten in terms of the 'hourly job rent'

$$\frac{J(h)}{h} = \frac{V^N - V^U}{h(1-r)} \geq \frac{\epsilon(h)}{qh} \tag{3'}$$

i.e., the hourly expected cost of shirking should outweigh the hourly cost of effort (not shirking). Of course the hourly cost of effort is increasing by the standard assumptions on the shape of $\epsilon(h)$, so as we increase h, we must simultaneously increase the hourly job rent J to preserve incentive compatibility. Now J depends on the wage w and the unemployment rate u. However, absent set-up costs (i.e., if $s = 0$) the hourly wage is constant, so in order to increase $J(h)/h$ it is necessary to raise the unemployment level u. In the presence of set-up costs, if h is small then the hourly wage $\alpha - s/h$ increases rapidly with h, and unemployment can actually fall as h rises. When h is large however, the s/h term becomes negligible and the hourly wage is essentially constant, so that again unemployment must rise with h to preserve equilibrium.

III. The 'Natural Length' Work Day

In the previous section we treated h as an exogenously fixed parameter. Now, however, we allow firms to choose the number of hours they offer to employees. Recall that individual firms face a minimum incentive-compatible wage given by

$$\hat{w}(h, V^U) = rV^U + \left(\frac{r+b+q}{q}\right) e(h) \tag{4}$$

and that the assumption of perfect competition means that firms take V^U to be fixed. Of course in equilibrium all firms make zero profits, but as usual we can use a cost-minimization condition. The firm's problem then is

$$\min \quad \hat{w}(h, V^U)l \qquad \text{s.t.} \quad (\alpha h - s)l = y$$

and eliminating h this problem becomes equivalent to minimizing the wage per unit of output $\hat{w}(h, V^U)/(\alpha h - s)$, giving a first-order condition

$$\frac{\partial \hat{w}}{\partial h}(h, V^U) = \frac{\alpha \hat{w}(h, V^U)}{\alpha h - s}$$

Since in equilibrium we have $\hat{w}(h, V^U) = w(h) = \alpha h - s$, the equilibrium number of hours is given by

$$\frac{\partial \hat{w}}{\partial h} = \alpha \tag{8}$$

Proposition 2. *The equilibrium length of the workday, denoted h^*, is strictly greater than the unemployment-minimizing level, denoted $h_{\min}$.*

Proof. From (4) and (8) we know that h^* is given by

$$e'(h^*) = \frac{\alpha q}{r+b+q}$$

(so $\underline{h} < h^* < \bar{h}$ from the proof of proposition 1). Now from (7) we have

$$\frac{-b}{u^2(h^*)}\frac{du}{dh} = \frac{q}{e^2(h^*)}(\alpha e(h^*) - (\alpha h^* - s)e'(h^*))$$

so substituting in for $e(h^*)$ from (7) and for $e'(h^*)$ from above gives

$$\begin{aligned}\alpha e(h^*) - (\alpha h^* - s)e'(h^*) &= \frac{\alpha q(\alpha h^* - s)}{r + q + b/u(h^*)} - \frac{\alpha q(\alpha h^* - s)}{r + q + b} \\ &= \alpha q(\alpha h^* - s)\left[\frac{1}{r + q + b/u(h^*)} - \frac{1}{r + q + b}\right] \\ &< 0\end{aligned}$$

so $u(h^*) > 0$ as claimed. ¶

A proper understanding of this result depends on the welfare results in the next section. In brief preview, it is not hard to see that for h below the unemployment-minimizing level, the equilibria corresponding to different h are Pareto-ranked: if $h_1 < h_2 \leq h_{\min}$ then everyone is better off in the equilibrium corresponding to h_2 than to h_1. Employed workers are better off because the job rent is higher, and unemployed workers are better off because unemployment is lower and the employed are better off. Proposition 3 below shows that the free market chooses a Pareto efficient level of h, so it follows that it must choose $h \geq h_{\min}$, and in fact this inequality must be strict by the usual sort of argument (increasing h slightly from $h_{\min}$ increases the welfare of employed workers to first order and affects unemployment only to second order).

IV. Welfare Implications

In this section we examine how the lifetime utilities V^N, V^U of employed and unemployed workers change as we move along the no-shirking-zero-profit schedule defined by (7). We already have expressions for V^N and V^U in terms of h, u and substituting in for u via (7) gives

$$V^U(h) = (1 + r^{-1})\left[\alpha h - \left(\frac{q + r + b}{q}\right)e(h)\right] \tag{9}$$

$$V^N(h) = (1 + r^{-1})\left[\alpha h - \left(\frac{q+b}{q}\right)\epsilon(h)\right] \tag{10}$$

[INSERT FIGURE 1]

Proposition 3 ('First Welfare Theorem '). *The welfare of the unemployed $V^U(h)$ is strictly highest at the free market outcome h^*.*

Proof. Immediate from comparing the first-order condition for maximizing (9) with the free market conditions (4) and (8). ¶

This rather surprising result has a simple explanation.* Suppose that some other value of h, say h', has $V^U(h') > V^U(h^*)$. By definition, workers will work and not shirk h' hours for $w(h') = \hat{w}(h', V^U(h'))$ in the h' equilibrium. So in the h^* equilibrium, where unemployment is a stronger disincentive $(V^U(h^*) < V^U(h'))$, they will work h' hours for less:

$$\hat{w}(h', V^U(h^*)) < \hat{w}(h', V^U(h')) = \alpha h' - s$$

so a single firm could deviate from the h^* equilibrium by offering a contract of h' hours for $\hat{w}(h', V^U(h^*))$, and make a positive profit, which is of course incompatible with equilibrium.

We can now explain the result in Proposition 2, that the free market chooses a level of work-hours higher than that which minimizes unemployment. Notice that until unemployment starts to rise with h we must have $(V^N)'(h) > 0$ since longer hours mean a higher job rent while working and less time spent in unemployment. In this region, then, we can always make a Pareto improvement by increasing h, and in particular the unemployed will be better off since raising h raises both the chances of becoming employed a and the returns to being employed V^N.

Other welfare properties are captured in the next proposition:

* which I thank Mathew Ellman for suggesting to me.

Proposition 4. *The welfare of employed workers,* $V^N(h)$*, and of unemployed workers,* $V^U(h)$*, are both concave in* h*. They are increasing for low values of* h*, and* $V^U(h)$ *has an interior maximum* $(\underline{h} < h^* < \overline{h})$ *while the maximum value of* $V^N(h)$ *can be either interior or at* $\overline{h}$*, depending upon the parameters of the model.*

Proof. Concavity is immediate from (9) and (10), and the previous proposition shows that $V^U(h)$ has an interior maximum. The first-order condition for maximizing $V^N(h)$ is $\epsilon'(h) = \alpha q/(q+b)$, so its maximand, say, h^N, is greater than h^* (which has $\epsilon'(h^*) = \alpha q/(r+q-b) < \epsilon'(h^N)$). It is easy to check graphically that the problem of maximizing $V^N(h)$ over $\underline{h} \leq h \leq \overline{h}$ has an interior solution for small r and a corner solution for large r. ¶

Clearly, employed workers will oppose any move to shorten hours below the market level, since this lowers $V^N(h)$. The attitude of unemployed workers is more ambiguous, however. Suppose that, starting from laissez-faire, the government proposes a law restricting work hours to some $h < h^*$, where h is chosen so that $V^N(h) > V^U(h^*)$ (if h is too low, then it can be Pareto-worsening, since being employed with the law in place may be worse than being unemployed under laissez-faire!). This will create new jobs, and we assume that they are distributed amongst the unemployed according to some lottery, which need not be uniform or 'fair'.

Proposition 5. *Suppose that the government proposes a reduction in work hours to some* $h < h^*$*, and some probabilistic mechanism to distribute the newly created jobs amongst the currently unemployed. Assume that the mechanism gives everyone a positive (bounded below by some* $\epsilon > 0$*) probability of obtaining one of these jobs. Then if the proposed hours reduction* $h^* - h$ *is sufficiently small the unemployed will support it.*

Proof. Take a typical unemployed worker, and suppose that the job distribution mechanism gives this particular individual a probability ρ of getting each of the new jobs. Then the number of newly created jobs is $u(h^*) - u(h)$ and her chance of gaining employment as a result of the measure is $\rho(u(h^*) - u(h))$. She will support the measure if and only if it raises her *ex ante* expected utility, i.e., if and only if

$$\rho(u(h^*) - u(h))V^N(h) + (1 - \rho)(u(h^*) - u(h)))V^U(h) > V^U(h^*)$$

or equivalently

$$V^N(h) - V^U(h) > \frac{V^U(h^*) - V^U(h)}{\rho(u(h^*) - u(h))}$$

Now for h close to h^* the right side of the last inequality is of order $h^* - h$, since we have shown that $(V^U)'(h^*) = 0$ while $u'(h^*) > 0$, so there is some $\Delta = \Delta(\rho) > 0$ such that the inequality is satisfied for $h^* - h < \Delta$, and since by assumption ρ is uniformly bounded below across all workers, there is a $\Delta > 0$ such that $h^* - h < \Delta$ makes all workers support the measure. ¶

V. Hours and Monitoring Technology

In introducing the model we followed Shapiro and Sitglitz (1984) in interpreting b as the exogenous 'quit rate' and q as the probability of being caught shirking. A more standard approach in the moral hazard literature would be to suppose that the employer receives a noisy signal of the employee's performance, and that the signal can take on one of two values, say 'good' or 'bad'. Then one would interpet b as the probability of the signal taking on the value 'bad' conditional on the employee not shirking, and $b + q$ as the probability of that event conditional on shirking.*

In the model we have considered up to now, this reinterpretation changes nothing. Formally the analysis is identical, while informally it is easy to see that anything the employer might do

* One could also combine the two cases and have both an exogenous quit rate and a positive probability of workers being incorrectly reported as shirking.

to lower the probability of dismissing a non-shirker increases the returns to shirking more than to working, thus breaking the incentive compatibility constraint. However, it does make a difference if we also alter the model in some other way.

One such alteration we might want to consider is the following. So far we have assumed that q is independent of h, but it might make more sense to think that shirking over a long day is more likely to be detected than shirking over a short day, so that $q = q(h)$ with $q'(h) > 0$. Then at least in our second interpretation, it also makes sense to have $b = b(h)$. How does this affect our previous results? Differentiating (7) now gives

$$\left[\frac{b}{u^2(h)}\frac{du}{dh} = \frac{q(h)}{[\epsilon(h)/(\alpha h - s)]^2}\frac{d}{dh}\left(\frac{\epsilon(h)}{\alpha h - s}\right) - \left(\frac{\alpha h - s - \epsilon(h)}{\epsilon(h)}\right)\frac{dq}{dh} + \frac{1}{u(h)}\frac{db}{dh}\right]$$

so the effect of setting $q'(h) > 0$ is to lower $u'(h)$. This is certainly intuitive: if longer hours make the detection technology more effective then they lessen the moral hazard problem and therefore lead to a lower level of unemployment. Conversely, if $b'(h) > 0$ then longer hours have a second, deleterious effect on the detection technology by making it more likely to give 'false positives', and this effect tends to make unemployment higher. One could also however imagine a situation in which longer hours lead to more accuracy in detecting both shirking ($q'(h) > 0$) and non-shirking ($b'(h) < 0$), giving an unambiguous reduction in unemployment.

To summarize, in so far as longer hours make the detection technology more effective they will lead to lower unemployment than that predicted in the previous sections. If this effect is sufficiently powerful then it might be the case that $u(h)$ is always decreasing with h, so that work-sharing would necessarily lead to higher unemployment.

VI. Continuous Shirking

A second natural variant on our model would be to allow for shirking to be a continuous time decision rather than an all-or-nothing one. Suppose for example that an employee chooses the number of hours y that she will shirk out of a workday of h hours, and that this leads to disutility of effort $e(h-y)$, and a probability $q(y)$ of detection. The problem then becomes significantly more complex: in this section we derive equilibrium conditions and argue that our earlier results are reasonably robust, although they certainly need not hold for arbitrary specifications of the monitoring technology and cost-of-effort function (it would be very surprising if they did.)

As before, we define $V(y)$ to be the expected lifetime utility of an employed worker who shirks for y hours, and analogously to equation (1) we have

$$V(y) = w - e(h-y) + \frac{1}{1+r}\left[(b+q(y))V^{U} + (1-b-q(y))V(y)\right] \tag{11}$$

where as before $b > 0$ represents the exogenous quit rate. Our first observation is that under reasonable assumptions about the monitoring technology, no shirking will occur in equilibrium. The reason is essentially that any contract which induces a positive amount of shirking is dominated by one which turns that shirking into free time (without changing the daily wage). Given our assumptions above about the nature of the monitoring technology, this change does not lead to a breach of incentive compatibility. It does however make workers strictly better off, as it lowers the turnover rate. They would therefore be willing to accept a lower wage in return for the concession making both sides strictly better off.

Proposition 6. *Suppose that the monitoring technology is concave, i.e., that $q' > 0$ and $q'' < 0$. Then any incentive-compatible contract can be replaced with one which induces zero hours of shirking and the same amount of output per worker, makes workers better off and lowers the unit cost of production for the firm.*

Proof. see appendix. ¶

If the monitoring technology is non-concave then this result need not hold. However, the assumption of concavity seems reasonable. For example, one could model the employee as controlling a process where errors occur according to a Poisson process which has intensity λ_N if she is working and $\lambda_S > \lambda_N$ if she is shirking. Then in general the optimal contract between employer and employee would depend on the number of errors, significantly complicating the analysis.* However, the probability of detection is certainly concave in y, and the special case where $\lambda_N = 0$ fits into our framework, since again the 'shoot the agent' principle applies and it is optimal to dismiss with certainty anyone who allows an error to occur.

It follows from Proposition 6 therefore that any equilibrium will have to satisfy an incentive compatibility condition of the form $0 \in \arg\max V(y)$. Unfortunately, we cannot say anything very general about the shape of $V(y)$. Rewriting (12) as

$$V(y) - V^U = \frac{(1+r)\,(w - e(h-y)) - rV^U}{r + b + q(y)}$$

we see that the right hand side is the ratio of two increasing concave functions of y. Intuitively, this corresponds to the fact that both the marginal benefit of shirking, $e'(h-y)$, and the marginal cost of shirking are decreasing with y (in the latter case this is because the marginal increase in the probability of detection is decreasing under our assumptions on $q(y)$). Its main implication is that $V(y)$ need not be either concave or convex, so that the constraint $0 \in \arg\max V(y)$ can bind locally or at a distance (see figure 2), and we have to consider each case in turn (the appendix gives numerical examples of both cases).

[INSERT FIGURE 2]

---

* Another kind of complication arises if the employee becomes aware of an error as soon as it occurs, while the employer observes only the number of errors that have occurred over the whole day (compare Holmstrom and Milgrom, 1987). Here we assume that both parties observe only the end of day error count.

Case A: IC constraint binds locally

In this case obviously $V'(0) = 0$, and this produces a reasonably tractable equilibrium condition. It turns out that the comparative static for equilibrium unemployment as h varies is controlled by the ratio of the marginal disutility of work to the net surplus produced by that work, $\epsilon'(h)/(\alpha h - s - \epsilon(h))$.

Proposition 7. *Suppose that $q(y)$ is concave, and that in equilibrium the IC constraint binds at 0. Then equilibrium unemployment is given by the condition*

$$q'(0)\frac{\alpha h - s - \epsilon(h)}{\epsilon'(h)} = r + \frac{b}{u(h)}$$

It is therefore increasing in h if and only if marginal disutility is growing at a faster rate than net surplus, i.e., if and only if

$$\frac{\epsilon''(h)}{\epsilon'(h)} > \frac{\alpha - \epsilon'(h)}{\alpha h - s - \epsilon(h)}$$

The conclusion that $u(h)$ is U-shaped therefore remains at least plausible, since it is not unreasonable to imagine that marginal disutility might have an increasing growth rate and net surplus a decreasing one. In fact it is easy to check that this conclusion holds for quadratic effort costs*:

Example 8. *If disutility of effort is quadratic in hours worked ($\epsilon(h) = \gamma h^2$) then equilibrium unemployment $u(h)$ is U-shaped as a function of h, with a unique minimum at $h = \sqrt{s/\gamma}$.*

---

* and the example in the appendix shows that case A can occur when effort costs are quadratic. It should also be noted that proposition 7 holds even if $q(y)$ is not concave, provided that equilibrium involves no shirking.

Case B: IC constraint binds at a distance

The second case is more complex, and it becomes impossible to say anything very definite. However, it is worth recording the equilibrium condition

Proposition 9. *Suppose that $q(y)$ is concave, and that in equilibrium there is some $\hat{y} = \hat{y}(h)$ such that $V(0) = V(\hat{y})$. Then equilibrium unemployment satisfies*

$$q(\hat{y}(h))\left[\frac{\alpha h - s - e(h)}{e(h) - e(h - \hat{y}(h))}\right] = \frac{b}{u(h)} + r$$

Proof. see appendix. ¶

As one might expect, the function $\hat{y}(h)$ depends in a complicated way on the interactions between hours worked, changes in the marginal cost of effort, and the monitoring technology, so that in general this condition is not terribly useful. However, some insight can be gained by considering the special case where $\hat{y}(h)$ is locally constant, so that unemployment varies with work hours according to

$$\frac{e(h) - e(h - \hat{y}(h))}{\alpha h - s - e(h)}$$

i.e., the ratio of the cost (in utility) of working to the net surplus produced by working. Again this ratio must be decreasing when h is small (provided the set-up cost $s > 0$), and one might reasonably expect it to be increasing when h is large, so that $u(h)$ might still tend to be U-shaped. However, the most one can conclude is that this result remains plausible.

VII. Conclusions

Efficiency wages provide a plausible explanation for the existence of involuntary unemployment. This paper uses a simple model of imperfect monitoring and efficiency wages to argue that an exogenously imposed lowering of the number of hours worked by the employed could lead to a reduction in the equilibrium level of involuntary unemployment, although such a reduction comes at the expense of lowering the welfare of both employed workers and those who remain unemployed. Despite the simplicity of the model, the results seem quite robust. In particular, we showed that they are not invalidated by allowing for quasi-fixed labor costs of employment, and that they remain at least plausible when we allow for a richer set of shirking strategies and more complicated detection technology.

Although these conclusions would be welcomed by supporters of work-sharing, it is worth pointing out that the underlying economics is, at least from their perspective, paradoxical. The free market equilibrium involves longer hours than those which minimize unemployment, but this happens because competition in the labor market leads to an equilibrium which maximizes the welfare of a representative unemployed worker. Moreover, once below the unemployment-minimizing level of hours further reductions in work hours make everyone worse off, employed or unemployed.

Of course in the end these are empirical questions. As noted in the introduction, this is a topic which is often at least on the margin of the public policy agenda, and accordingly deserves further investigation.

APPENDIX

Proof of Proposition 1.

Assumption 1 (just before Proposition 1) implies that $\epsilon(h)$ intersects $\alpha h - s$ exactly twice: since $\epsilon(h)$ is convex, it must intersect $\alpha h - s$ 0, 1 or 2 times, and 0 would mean that a positive amount of work was inefficient, while 1 would mean that either that work was only weakly efficient (i.e., there is a h such that $\alpha h - s = \epsilon(h)$, but no more) or that the returns to work increase indefinitely with h, so that it is efficient to work forever (e.g., if $\epsilon(h) = \alpha(h + e^{-h} - 1)$). Using (6), we see that assumption 2 requires $\epsilon(h)$ to intersect $q(\alpha h - s)/r + q + b)$ exactly twice, since the fact that we can induce some positive number of hours of work means that for some $s > 0$ and $u < 1$

$$w \geq \left(\frac{r + q + b/u}{q}\right)\epsilon(h) > \frac{r + q + b}{q}\epsilon(h)$$

and $w = \alpha h - s$. So $\epsilon(h)$ has to intersect $q(\alpha h - s)/r + q + b)$ at least once, and in order to satisfy assumption (12), i.e., in order to intersect $\alpha h - s$ twice, it must cross it again. We now define $\underline{h}$ and $\overline{h}$ to be the smaller and larger of the two solutions to

$$\epsilon(h) = \frac{q(\alpha h - s)}{r + q + b}$$

Now, from the definitions, we have

$$\begin{cases} \epsilon'(\underline{h}) < \dfrac{\alpha q}{r + b + q} < \epsilon'(\overline{h}) \\ \epsilon(\underline{h}) = \dfrac{q}{r + b + q}(\alpha \underline{h} - s) \\ \epsilon(\overline{h}) = \dfrac{q}{r + b + q}(\alpha \overline{h} - s) \end{cases}$$

giving

$$\epsilon(\underline{h})/\epsilon'(\underline{h}) > \underline{h} - s/\alpha$$

$$\epsilon(\overline{h})/\epsilon'(\overline{h}) < \overline{h} - s/\alpha$$

Now we take the hours/unemployment schedule

$$\frac{b}{u(h)} = \frac{q(\alpha h - s)}{\epsilon(h)} - r - q$$

and differentiate with respect to h to get

$$u'(h) = -\frac{qu^2}{b}\frac{\alpha\epsilon'(h)}{\epsilon^2(h)}\left[\frac{\epsilon(h)}{\epsilon'(h)} - \left(h - \frac{s}{\alpha}\right)\right]$$

and from the expressions above we know that the term in square brackets is positive for $h = \underline{h}$ and negative for $h = \overline{h}$.

It follows from this result that $u'(h) = 0$ whenever $u(h)$ has a local minimum on $[\underline{h}, \overline{h}]$. Suppose now that this occurs twice, say at h_1 and h_2, with $h_1 < h_2$. We know from (7) that $u'(h) = 0$ iff $\epsilon'(h)/\epsilon(h) = \alpha/(\alpha h - s)$, so we must have

$$\alpha\epsilon(h_i) = (\alpha h_i - s)\epsilon'(h_i) \quad i = 1, 2$$

Applying this together with the mean value theorem, we know that for some $\hat{h}$ with $h_1 < \hat{h} < h_2$

$$\begin{aligned}\alpha\epsilon'(\hat{h}) &= \frac{\alpha\epsilon(h_2) - \alpha\epsilon(h_1)}{h_2 - h_1} \\ &= \frac{\alpha(h_2 - h_1)\epsilon'(h_2) + (\alpha h_1 - s)[\epsilon'(h_2) - \epsilon'(h_1)]}{h_2 - h_1} \\ &> \alpha\epsilon'(h_2)\end{aligned}$$

since $\epsilon'' > 0$. But this is a contradiction since $\epsilon'' > 0$! Finally, the result for $s = 0$ is immediate from (7) since $\epsilon(h)/h$ is increasing in h (just as a convex cost function with no fixed costs exhibits increasing average costs), and it is easy to check that $u(\underline{h}) = (\overline{h}) = 1$. ¶

Proof of Proposition 6.

Without loss of generality we may assume that $q(0) = 0$ (by adjusting b if necessary). Now suppose there is an incentive compatible contract which induces $\bar{y}$ hours of shirking out of a total of h contracted hours, where $0 < y < h$. We know (from solving (11) for $V(y)$) that expected lifetime utility from shirking for y hours under this contract is given by

$$V(y) = \frac{(1+r)(w - e(h-y))}{r+b+q(y)} + \left(1 - \frac{r}{r+b+q(y)}\right) V^U \tag{12}$$

Similarly, we know that expected lifetime utility from shirking for $\hat{y}$ hours in a contract which involves only $h - \bar{y}$ hours on the job for the same daily wage w, is given by

$$\hat{V}(\hat{y}) = \frac{(1+r)(w - e(h - \bar{y} - \hat{y}))}{r+b+q(\hat{y})} + \left(1 - \frac{r}{r+b+q(\hat{y})}\right) V^U$$

Obviously $\hat{V}(0) > V(\bar{y})$, i.e. workers are strictly better off shirking 0 hours under the second contract than shirking $\bar{y}$ hours under the first, since the only difference between the two is that the turnover rate (per period probability of job loss) falls from $b + q(\bar{y})$ to b. By definition we have $\bar{y} \in \arg\max V(y)$, and we will use this to show that $0 \in \arg\max \hat{V}(\hat{y})$, so that shirking 0 hours under the second contract is also incentive-compatible. To do so, observe that the two equations above give

$$[V(\bar{y} + \hat{y}) - V^U] = \phi(\hat{y})[\hat{V}(\hat{y}) - V^U]$$

where

$$\phi(\hat{y}) = \frac{r+b+q(\hat{y})}{r+b+q(\bar{y}+\hat{y})}$$

and that the assumptions $q' > 0$, $q'' < 0$ mean that $\phi'(\hat{y}) > 0$. Suppose that $0 \notin \arg\max \hat{V}(\hat{y})$, so there exists some $\hat{y}$ with $\hat{V}(\hat{y}) > \hat{V}(0)$. Then

$$\frac{V(\bar{y}+\hat{y})-V^U}{V(\bar{y})-V^U} = \frac{\phi(\hat{y})}{\phi(0)}\frac{\hat{V}(\hat{y})-V^U}{\hat{V}(0)-V^U} > \frac{\phi(\hat{y})}{\phi(0)} > 1$$

which is impossible since $V(y+\hat{y}) \leq V(y)$. So no shirking is indeed optimal under the second contract. Since this makes workers strictly better off at no cost to the firm, a slight reduction in w would lead to a contract which strictly dominates the first one, and we may accordingly rule out the first contract.¶

Examples for section VI.

As the discussion after Proposition 6 suggests, if the probability of detection is close to linear while disutility of effort is strictly and significantly convex then we might expect $V(y)$ to be decreasing in y, giving case A (locally binding constraints). In fact it is easy to check numerically that the following functional forms and parameters give that case: take a quadratic efort function $\epsilon(h) = 2h^2$, and a probability of detection function $q(y) = (y/h)^{.999}$. Then with $\alpha = 5, s = .5, \dot{h} = 1, b = .01, r = .25$ and a somewhere between .35 and .40 we get case A.

Case B is given by taking the same parameter values (with $a = .35$) and the following functional forms for effort and detection probability: let $\epsilon(h) = h$ $(h \leq .3)$ and $\epsilon(h) = .3 + 5(h - .3)$ $(h > .3)$; let $q(y) = 10(y/h)$ $(y \leq .2)$ and $y(h) = (2/h) + \mu(y - .2)/h$ $(y > .2)$. Then μ somewhere between 2 and 2.2 gives case B.

Proof of Proposition 7.

Differentiating (12) and setting $V'(0) = 0$ gives

$$\frac{(1-r)\epsilon'(h)}{(1+r)(w-\epsilon(h)) - rV^U} = \frac{q'(0)}{r+b} \tag{13}$$

We then use equation (12) together with

$$V^U = \frac{1}{1+r}\left[aV(0) + (1-a)V^U\right] \tag{14}$$

to solve simultaneously for $V(y)$ and V^U, and substitute in for V^U in (13). As before the job acquisition rate a satisfies $a = b(1-u)/u$ in steady state, while the equilibrium wage must satisfy $w = \alpha h - s$, giving the equilibrium condition

$$q'(0)\frac{\alpha h - s - \epsilon(h)}{\epsilon'(h)} = r + \frac{b}{u(h)}$$

as claimed.¶

Proof of Proposition 9.

Here we essentially mimic the derivation given in section I. Equation (1) still holds, while equation (2) becomes

$$V^S = w - \epsilon(h - \hat{y}(h)) + \frac{1}{1+r}\left[(b + q(\hat{y}(h)))V^U + (1 - b - q(\hat{y}(h)))V^S\right] \tag{2'}$$

and the same formal procedure gives a new 'no-shirking condition'

$$w \geq \hat{w}(h, V^U) \equiv \frac{rV^U}{1+r} + \epsilon(h - \hat{y}(h)) + \left(\frac{r + b + q(\hat{y}(h))}{q(\hat{y}(h))}\right)(\epsilon(h) - \epsilon(h - \hat{y}(h))) \tag{4'}$$

and using (5) together with the zero-profit condition $w = \alpha h - s$ as before gives the result as claimed.¶

References

Clark, G., (1987). 'Why Isn't the Whole World Developed? Lessons from the Cotton Mills.' *Journal of Economic History* **47**, pp. 141-74.

Commission of the European Communities (1993). *White Paper on Growth, Competitiveness, and Employment* COM(93) 700 final, Brussels.

Dreze, J.H., (1986). 'Work-Sharing: Some Theory and Recent European Experience.' *Economic Policy* **3**, pp. 562-619.

Ehrenberg, R. G. and P.L. Schumann, (1982). *Longer Hours or More Jobs? An Investigation of Amending Hours Legislation to Create Employment.* Ithaca: Cornell University.

Hoel, M., (1985). 'Employment and Allocation Effects of Reducing the Length of the Workday.' *Economica* **53**, pp. 75-85.

Holmstrom, B. and Milgrom, P. (1987). 'Aggregation and Linearity in the Provision of Intertemporal Incentives.' *Econometrica* **55**, pp. 303-28.

Houpis, G., (1993). 'The Effects of Lower Hours of Work on Wages and Employment.' *LSE Centre for Economic Performance Discussion Paper 131.*

Layard, R., S. Nickell and R. Jackman, (1991). *Unemployment: Macroeconomic Performance and the Labour Market.* Oxford: Oxford University Press.

Lewin, S., (1994). 'Discipline and Excessive Work-Hours', mimeo, Harvard University.

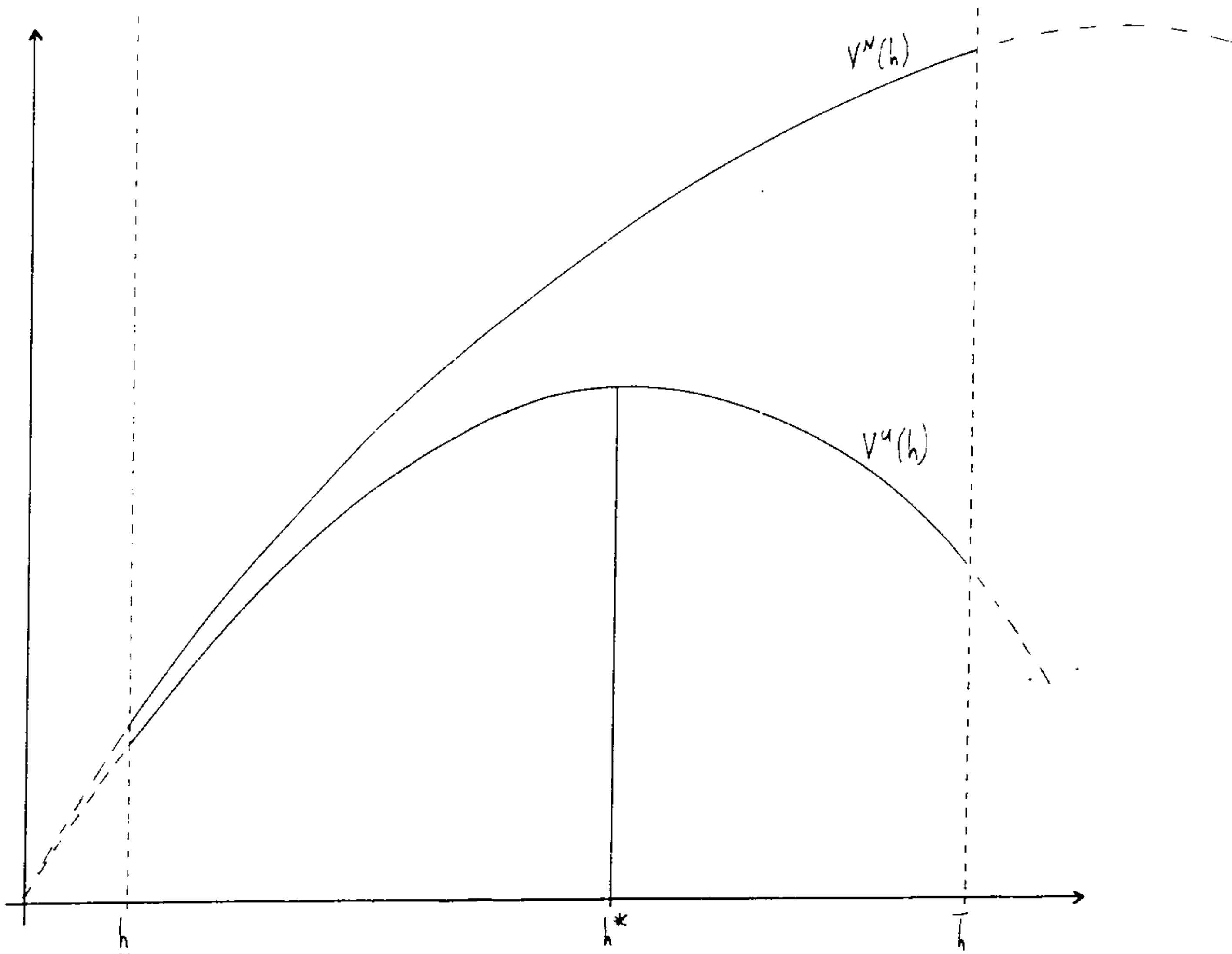
Rosen, S., (1978). 'The Supply of Work Schedules and Employment', in *Work Time and Employment*, Washington D.C.: National Commision for Manpower Policy.

Schor, J.B., (1992). *The Overworked American*. New York: Basic Books.

Shapiro, C. and J.E. Stiglitz, (1984), 'Equilibrium Unemployment as a Worker Discipline Device,' *American Economic Review* **74**, pp. 433-444.

Trejo, S.J., (1991), 'The Effects of Overtime Pay Regulation on Worker Compensation,' *American Economic Review* **81**, pp. 719-40.

FIGURE 1



$V^N(h) \equiv$ expected lifetime utility of currently employed worker
$V^U(h) \equiv$ expected lifetime utility of currently unemployed worker
$h^* \equiv$ free market choice of number of work hours

FIGURE 2a: IC binds locally

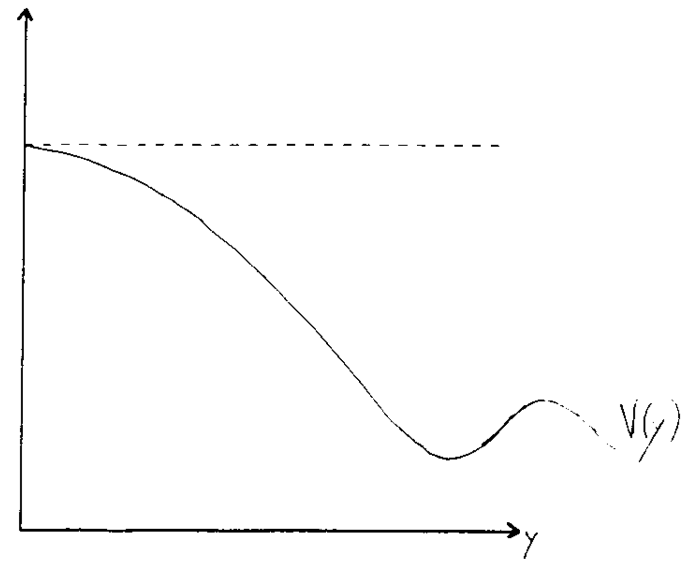
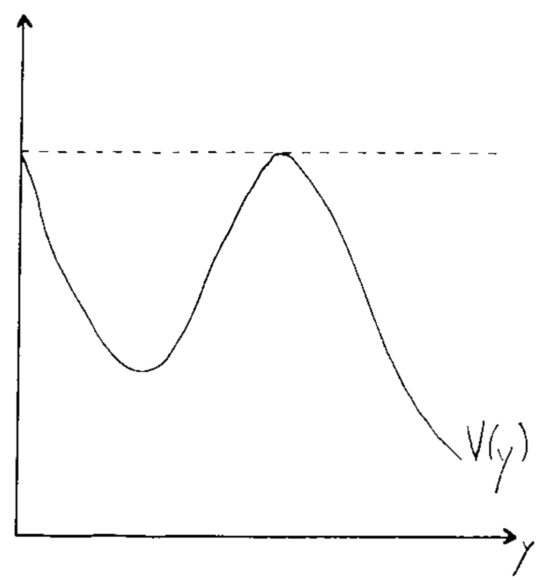


FIGURE 2b: IC binds at a distance



y≡ hours shirked