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COORDINATION ECONOMIES, SEQUENTIAL SEARCH AND ADVERTISING

by

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ABSTRACT

This paper considers pricing, cost-reducing investment and dissipative advertising by firms when consumers acquire price information via two information channels, observation of advertising and sequential price search. We find that advertising guides consumers to the lowest prices in the market, even when consumers have the option to search. The threat of search by advertising-uninformed consumers introduces price competition among firms, giving short-and long-run resolutions to the Diamond paradox. Higher concentration raises welfare as a consequence of coordination econornies. An extension to loss-leader advertising is developed.

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1. Introduction

In retail markets, consumers may acquire price information through a variety of channels. One possibility is that consumers may themselves actively seek out price information by engaging in search. Alternatively, firms may communicate price information through their advertising activities. Advertising may communicate direct as well as indirect price information. For example, consumers may infer price information from the fact that a firm advertises, or from the intensity with which it advertises. In any event, the channel through which consumers acquire price information is important, since different information channels lead to distinct market outcomes.

An extensive literature analyzes the case in which firms are unable to advertise and consumers can acquire price information only via *sequential price search*. In a classic paper, Diamond (1971) assumes that firms are identical and observes the paradoxical outcome that all firms choose the monopoly price, no matter how small is the level of positive search costs. Bagwell and Ramey (1994a,b) explore an alternative information channel in which consumers are unable to search but firms attempt to communicate price information indirectly, through the choice of *dissipative advertising expenditures*. This argument hinges on the presence of *coordination economies*: consumers and active firms collectively benefit from a concentration of sales at fewer firms, since a firm that expects larger market share makes greater investments in cost–reducing technologies and is attracted to lower prices. When consumers respond positively to advertising, they then may infer that a high–advertising firm expects greater market share and therefore selects low prices, even if the advertisements themselves contain no direct price information.

In this paper, we combine these two information channels by allowing consumers to obtain price information via both sequential search and advertising. As in our earlier work, we assume the existence of coordination economies and explore how firms might communicate price information using dissipative advertising expenditures. We depart from our previous papers, however, in allowing that consumers might also acquire price information through sequential search. Thus, we assume that some consumers are *informed* and observe the advertising activities of firms, while other consumers are *uninformed* and do not observe these activities. Informed consumers are able to acquire price information through both advertising and search information channels, whereas uninformed consumers can obtain price information only through search.

Our central finding is that advertising continues to guide informed consumers to the lowest prices in the market, even when they have the option of using the sequential search channel. The scope for search by uninformed consumers does, however, introduce important new effects as far as equilibrium advertising and pricing behavior. Consider a consumer who is uninformed of firms' advertising activities. If such a consumer happens initially to visit a firm with low advertising and a relatively high price, then the consumer can now credibly threaten to search again, inspired by the hope of finding a lower–price firm on the next draw. In contrast to our earlier models, firms with low market shares are thus driven to compete in price, selecting prices below their monopoly levels as a consequence of the sequential–search threat of uninformed consumers. Moreover, price competition becomes more severe as search costs are reduced, since a lower price is then required to prevent uninformed consumers from searching again.

Thus, as our second main finding, we offer a resolution to Diamond's monopoly–pricing paradox, based on the hypothesis that firms choose cost–reducing investments and advertising along with prices. The Diamond paradox is resolved on two levels. First, for the *short–run* case in which the number of firms is fixed, we find that equilibrium prices are reduced continuously as the level of search costs is lowered. In the limit as search costs approach zero, each firm prices below its monopoly level with probability one, selecting the full–market monopoly price, i.e. the price that would be charged by a firm that expected to capture all informed consumers. A discontinuity still arises in the short–run case, however, at the point where search costs actually become zero, since the ensuing Bertrand outcome cannot be approximated by outcomes arising under small positive search costs.

Second, we consider an alternative *long–run* benchmark, in which both pricing behavior and industry structure are allowed to vary with the level of search costs. When the number of firms is determined by free entry, and sunk entry costs are positive, the zero–search–cost Bertrand outcome yields only a *single* entrant, choosing the full–market monopoly price. We establish conditions under which the equilibrium of our model approaches this prediction continuously as the level of search costs is reduced toward zero.

A third contribution of this paper rests on the analysis of the welfare consequences of more concentrated markets. Our model highlights an interesting tradeoff. On the one hand, when there are multiple firms, small firms are driven to reduce their prices in order to induce consumers not to search again. This *price–competition effect* suggests that the optimal market structure might involve multiple firms. On the other hand, as the number of firms in the market is reduced, each firm expects greater market share and thus invests more in cost reduction. This *increasing–returns effect* suggests that equilibrium prices will be lower when the market is more concentrated. We find that the resolution of the two effects is unambiguous: the increasing–returns effect *always* dominates the price–competition effect, so that welfare is highest when a single firm monopolizes the market.

With the relation between concentration and welfare established, we are able to evaluate the welfare consequences of advertising restrictions in retail markets. When advertising is allowed, we discover that the market is more concentrated than when advertising is prohibited. Our model therefore predicts that prices will be lower and welfare will be higher when advertising is allowed, even if advertising itself relates no direct price information. We develop similar predictions in our earlier work, and argue that the predictions are consistent with Benham's (1972) empirical analysis of the retail eyeglass industry. In comparison to our earlier work, the present paper demonstrates that uninformative advertising directs consumers to the best deals in the market and improves market performance, even when consumers have the option of obtaining direct price information themselves through sequential search.

Finally, we further extend our analysis by considering *loss–leader advertisements* that may be employed by multi–product firms. Loss–leader advertisements represent an additional direct channel through which consumers might acquire price information. In this case, a firm's advertisements also contain indirect information as to its expected market share and the corresponding pricing policy for its nonadvertised products. Two interesting effects emerge. First, by expanding the firm's market share, a low loss–leader price induces greater investment in cost reduction, which implies that consumers will obtain low prices on the firm's other goods as well. When search costs are sufficiently low, however, smaller firms are constrained by the need to deter consumer search, and a second effect arises: lower prices on the loss leader correspond to *higher* prices on nonadvertised goods. The latter effect is consistent with the common perception that low loss–leader prices portend higher prices on other products. Nevertheless, the interplay of these two effects is once again unambiguous: we show that sensitivity to loss–leader pricing is completely consistent with rational consumer behavior, as consumers can do no better than to visit the firm that advertises the lowest loss–leader price.

Our modeling approach follows Varian (1980) in that only part of the consumer population is assumed to be able to observe the informational variable that generates interfirm rivalry, while the remainder of the consumers are unable to observe the variable. We also focus on symmetric mixed–strategy equilibria on the part of the firms. Varian, however, allows informed consumers directly to observe price, and he does not consider advertising. Stahl (1989) introduces sequential search on the part of uninformed consumers in Varian's model, and he shows that pricing approximates the Bertrand outcome as the search cost approaches zero. A similar route is pursued by Robert and Stahl (1993), who consider sequentially–searching consumers in the context of the price–advertising model of Butters (1977). They find that reducing search costs leads equilibrium prices to decline, but prices remain bounded away from unit cost as search costs approach zero if the marginal cost of sending price messages is itself bounded away from zero.

The papers mentioned above make strong use of the hypothesis that firms are able directly to communicate all price information to a subset of consumers. Reinganum (1979) pursues an alternative approach, in which the threat of sequential search is sustained by exogenously–specified heterogeneity in firms' production technology, which implies heterogeneity in monopoly prices. In common with the present paper, Reinganum finds that sufficiently low search costs lead high–cost firms to reduce their prices below monopoly levels, and as search costs approach zero, all firms must choose the monopoly price of the lowest–cost firm in the market. Our results may be viewed as extending those of Reinganum by endogenizing the determination of technology, as well as highlighting the key role played by advertising in generating the heterogeneity needed to support credible search threats.

We develop the model in section two, and section three constructs an advertising equilibrium in which consumers purchase from the first firm that they visit. In section four we consider equilibria in which consumers must make multiple searches with positive probability, which arises when the number of firms is large relative to the proportion of consumers who cannot utilize advertising information. Section five introduces free entry, section six extends the model to incorporate multi–product firms and loss–leader advertising, and section seven concludes.

2. Model

In this section, we develop a basic modeling framework. We begin by presenting our assumptions and developing the concept of coordination economies. We then define the advertising game and the equilibrium concept. Finally, we consider a benchmark case, where advertising is not allowed, and characterize the equilibria of the associated game.

a. Assumptions

Our basic model is comprised of N firms and a large number of consumers that trade a single homogeneous good in a single period. Our assumptions regarding consumer preferences are minimal, with the principal requirement being that consumers each possess a common downward–sloping demand function. Specifically, we assume that consumers are uniformly distributed on the unit interval, with unit mass, where each consumer obtains utility $U(Q) - PQ$ from purchasing Q units at the price P. The utility function is restricted as follows:

Assumption 1. $U' > 0 > U''$, $\lim_{Q \to 0} U'(Q) = \infty$ and $\lim_{Q \to \infty} U'(Q) = 0$.

With preferences defined in this way, we may let $D(P)$ denote a consumer's utility–maximizing level of Q, which under Assumption 1 is strictly positive, strictly decreasing, and satisfies $\lim_{P \to \infty} D(P) = 0$. The maximized level of utility is denoted as $W(P)$, which is a strictly decreasing function of P.

With respect to the behavior of firms, we assume that each firm chooses its price P along with its level of cost–reducing investment K. The production technology, which is the same for all firms, exhibits constant unit costs of C(K), where the unit cost function is restricted as follows:

Assumption 2. $C' < 0 < C''$, $\lim_{K \to \infty} C(K) > 0$, $\lim_{K \to 0} C'(K) = -\infty$ and $\lim_{K \to \infty} C'(K) = 0$.

The requirement that greater investment lowers unit costs is our key assumption. We also impose the regularity conditions that there are diminishing returns to investment and that unit costs cannot be reduced to zero via an arbitrarily large investment. The boundary conditions on marginal unit cost reduction are made to ensure an interior solution to the firm's profit–maximization problem.

Our emphasis on cost–reducing investments for retail firms can be motivated as follows. Retail firms can reduce costs with investments in information technologies, such as electronic–scanner check–out systems and satellites, as well as investments in production and distribution systems, such as privately–owned trucks and warehouses. As we argue in our earlier papers, investments of this kind have figured prominently in the strategies pursued by large retailers, including Wal*Mart, Toys–R–Us and others.

A firm's profit–maximmizing price and investment selections depend upon the number of consumers to whom the firm expects to sell. To explore this dependence, it is useful first to consider the selections that would be made by a monopolist facing an exogeneous expected market share level of M. With the cost of investment normalized to be one per unit, we may define the firm's profit function by:

$$\Pi(Z|M) = (P - C(K))MD(P) - K \tag{1}$$

where $Z = (P,K)$ indicates the vector of choice variables for the firm. The profit function is restricted by the following assumption:

Assumption 3. Π is strictly concave Z, with unique maximizer $Z^*(M)$ for $M > 0$.

Observe that the monopoly selection $Z^*(M) = (P^*(M),K^*(M))$ is not well–defined when zero market share is expected. We thus define $Z^*(0)$ as the limit of $Z^*(M)$ as M approaches zero. Finally, we define a firm's monopoly profit function as $\Pi^*(M) = \Pi(Z^*(M)|M)$.

With this notation in place, we now examine the relationship between a firm's expected market share and its monopoly price, investment and profit levels. Our findings are captured in the following lemma, the proof of which may be established via straightforward analysis of (1):

Lemma 1. (a) Π^* is strictly increasing in M.
(b) P^* is strictly decreasing in M.
(c) K^* is strictly increasing in M.

As we will see below, the behavior of monopoly selections and profits plays an important role in the construction of equilibria when consumers face positive search costs.

We turn now to the interpretation of Lemma 1. From part (a) we have that a firm's monopoly profits increase when it receives a greater share of consumers. This *better profit property* is an easy consequence of the fact that the monopoly markup must be strictly positive under our assumptions. We have from part (b) that a firm also chooses a lower monopoly price when it anticipates a larger market share. We call this the *better deal property*, since consumers' utility level $W(P^*(M))$ is strictly higher when more of them patronize the given firm. The better deal property is explained by the *complementarity* that exists between low prices, high market shares and high levels of cost–reducing investment: as the firm's market share rises, total sales increase at a given price, and so the firm earns a greater marginal return from lowering its unit costs. Thus, as part (c) confirms, an increase in M raises K^*; correspondingly, P^* is reduced as a consequence of lower unit costs. For markets in which firms possess market power, we may interpret the combination of these two properties as indicating the presence of *coordination economies*, since active firms and consumers collectively benefit from concentrating sales at fewer firms.

Finally, in the construction of equilibria below, it is often true that a firm prices below its monopoly level, and so it becomes important to characterize the optimal investment level for any arbitrary price and expected market share level. This motivates the following definition:

$$\tilde{K}(P,M) = \arg\max_K \Pi(P,K \mid M) \tag{2}$$

We now add our final assumption:

Assumption 4. For all P and M:

$$\frac{d}{dM}\Pi_P(P,\tilde{K}(P,M) \mid M) < 0$$

For any given initial price, this assumption indicates that a firm investing optimally is less inclined to raise its price following an increase in its expected market share. Two effects can be identified. On the one hand, a higher expected market share raises the optimal investment level, which in turn lowers unit costs and makes higher prices less attractive. On the other hand, the loss from below–monopoly pricing is amplified as the expected market share increases, making higher prices more attractive. With Assumption 4, we require that the former effect dominates. We explain the role of this assumption for our equilibrium construction in Section 3.

b. Advertising, Consumer Search and Equilibrium

With the assumptions of the model now presented, we turn next to a description of the information channels available to consumers. Consider first the advertising activities of firms. In addition to its choice of price and investment, each firm chooses a level of *advertising expenditures*, denoted as A. Advertising expenditures have no direct effect on demand, and so they may be regarded as being purely dissipative. Firms' advertising expenditures are observed by a proportion $I \in (0,1)$ of consumers, and we say that these consumers are *informed*. The remaining proportion $U = 1 - I$ of consumers are said to be *uninformed*, since they cannot observe advertising. With respect to price information, we assume that all consumers are able to make direct price observations by engaging in *sequential search*. In other words, a consumer can observe a firm's price by visiting the firm, and the consumer incurs a cost of $c > 0$ in utility terms for each visited firm. After each new price is observed, the consumer may choose to stop searching and make his purchases at the lowest of the previously–observed prices.

We are now prepared to define the advertising game. We consider the following two–stage version of the Diamond search model:

Stage 1. The N firms simultaneously choose their prices, investments and advertising expenditures.

Stage 2. Given their respective information as to firms' first–stage advertising selections, informed and uninformed consumers engage in sequential search and make their purchase selections.

An important feature of this game is that the order in which informed consumers search among firms can be influenced by the advertising selections of firms.

For our equilibrium concept, we employ a version of sequential equilibrium (Kreps and Wilson (1982)) that restricts the behavior of consumers in the following way. First, after each search, consumers choose whether or not to make another search in a manner that maximizes their expected utility, given the previously–observed prices and their conjecture as to the distribution of prices at the unsearched firms, conditional on any observed information. Second, a consumer's conjecture of a given firm's strategy agrees with the firm's equilibrium strategy, no matter what choices by other firms the consumer has previously observed. In particular, the consumer does not conjecture that a firm has deviated from its equilibrium strategy after observing an advertising or price deviation by another firm.[1] As for the firms, it suffices to specify that their choices constitute a Nash equilibrium, given the search strategies used by consumers. Finally, we will focus throughout on *symmetric* equilibria, in which all firms choose the same strategy and consumers do not discriminate between firms that have the same observable attributes; in particular, uninformed consumers will be taken to *randomize uniformly* among unsearched firms whenever they to select a firm to visit.

In equilibrium, the uninformed consumers will determine whether to continue or stop searching based on a *reservation utility rule*: for a given utility level $\underline{W} \geq 0$, if a consumer observes the price P at the currently–visited firm, then he stops searching and

[1] The first restriction on consumer strategies is an application of Kreps and Wilson's concept of *sequential rationality*, while the second restriction comprises the notion of *independent price conjectures* that is discussed in Bagwell and Ramey (1994a, note 12); the latter may be thought of as a version of Kreps and Wilson's *consistent beliefs*.

purchases from the currently–visited firm provided that $W(P) \geq \underline{W}$, while he searches again if $W(P) < \underline{W}$. As we will see, the reservation utility level is determined by the search cost c as well as the equilibrium pricing distribution. Informed consumers do not need to use a reservation utility rule, to the extent that they will be able to infer prices from the firms' observed advertising choices. Instead, we will construct *advertising equilibria* in which informed consumers search optimally by using an *advertising search rule* that directs their initial search to the firm(s) choosing the highest level of advertising expenditures. The informed consumers will buy from the high–advertising firm(s), unless a deviant price is encountered. In this event, they infer the equilibrium price selected by the next highest–advertising firm, compare the price savings to the cost of an additional search, and either purchase at the initial firm or search again.

Before constructing advertising equilibria, we establish as a benchmark the symmetric equilibrium outcome when there is no advertising.

Proposition 1. There exists a symmetric equilibrium of the no–advertising game, and in any symmetric equilibrium the firms' strategy is given by:

$$\hat{Z} = Z^*(1/N) \tag{3}$$

Proof. Given in the Appendix.

Here we have a version of the well–known result of Diamond (1971) that the threat of sequential search does not generate any actual price rivalry among firms, no matter how small the level of search costs. In the present case, the fact that consumers randomize uniformly in their search decisions means that each firm captures a $1/N$ market share, and price and investment choices are at monopoly levels relative to this market share. Note in particular that a rise in industry concentration, as reflected by a fall in N, actually *increases* consumer welfare, by inducing firms to make greater investments in cost reduction; thus, consumers benefit when concentration is high, as a result of coordination economies.[2]

3. Advertising Equilibria with Many Uninformed Consumers per Firm

With the no–advertising benchmark in place, we now return to our original focus and allow firms to engage in advertising activities that are observed by the informed consumers prior to search. We argue that uninformed consumers' threat to search rival firms generates price rivalry among firms, in the form of a departure from monopoly behavior: in order to keep its uninformed consumers, a low–advertising firm must offer a deal that gives greater consumer surplus than would a monopolist that expects the same number of buyers. We also find that rivalry becomes more intense as search costs fall. Comparing the results developed in this section with the no–advertising benchmark (Proposition 1), we conclude that Diamond's monopoly–pricing paradox is averted when firms "compete" via dissipative advertising expenditures. Throughout the section we will assume that the market has two or more firms, $N \geq 2$.

a. Profit–Maximization by Firms

We first derive the profit–maximizing symmetric price, investment and advertising strategies of the firms for given consumer search rules. With informed consumers using the advertising search rule, it follows that firms compete for the highest advertising level, and so pure–strategy equilibria fail to exist for the usual reasons. We

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2 There also exist asymmetric equilibria in the no–advertising case in which consumers refuse to visit some of the firms. Here $P = P^*(1/N')$ gives the equilibrium price for each of the N' visited firms.

therefore consider equilibria in which firms randomize over Z and A. Let $\hat{F}(A)$ denote the probability distribution that represents the firms' mixed advertising strategy, which for symmetric equilibria may be taken to be a continuous function.[3] Our construction of symmetric advertising equilibria is carried out by considering two cases in turn. As our first case, we construct equilibria in which the equilibrium mixed strategy determining firms' prices satisfies $W(P) \geq \underline{W}$ with probability one, so that the uninformed consumers will locate an acceptable price on their first search. In other words, we consider as our first case equilibria in which all consumers purchase from the first firm that they visit; it will be shown that equilibria of this form exist when the number of uninformed consumers per firm, given by U/N, is large. We consider the second case, in which U/N is small and $W(P) < \underline{W}$ with positive probability, in the next section.

Let us begin by considering the profit–maximization problem of the firms. In this portion of the analysis, we constrain firms to choose prices such that $W(P) \geq \underline{W}$, where $\underline{W}$ is fixed. When a firm expects market share M, it then solves the following problem:

$$(4) \qquad \bar{P}(M,\underline{W}) = \arg\max_{P} \Pi(P,\bar{K}(P,M)|M) \text{ subject to } W(P) \geq \underline{W}$$

where the expected–profit–maximizing investment level $\tilde{K}$, defined in (2), has been chosen.

The optimal pricing and investment behavior associated with (4) may be characterized as follows. Let us next define $\underline{P}$ by $W(\underline{P}) = \underline{W}$, so that prices below $\underline{P}$ satisfy the constraint in (4). Using Lemma 1, it follows that if market share exceeds some critical value $\underline{M}$, where $\underline{M}$ is defined by $\underline{P} = P^{*}(\underline{M})$, then (4) is uniquely solved when the

[3] In models without sequentially–searching consumers, Varian (1980) and Bagwell and Ramey (1994b) verify that symmetric equilibrium strategies must be continuous nondegenerate probability distributions having connected support, which are uniquely defined. This uniqueness argument for symmetric equilibria extends immediately to the present model for the case of large U/N, which is one of the two cases considered below.

associated monopoly price is selected: $\tilde{P}(M,\underline{W}) = P^*(M)$ for $M \geq \underline{M}$. Using Assumption 3, the investment choice in this event is given by $\tilde{K}(P^*(M),M) = K^*(M)$. On the other hand, for smaller market shares $M < \underline{M}$, we have that the monopoly selections violate the constraint, and so the firm must reduce its price below the monopoly level. Here we have that $\tilde{P}(M,\underline{W}) = \underline{P} < P^*(M)$, with $\tilde{K}(\underline{P},M)$ giving the associated investment choice. Summarizing, (4) has a unique solution for every M, and this solution is given by:

$$\tilde{P}(M,\underline{W}) = \begin{cases} P^*(M), & M > \underline{M} \\ \underline{P}, & M \leq \underline{M} \end{cases} \tag{5}$$

In the sequel, $P = \tilde{P}(M,\underline{W})$ will give the equilibrium price choice when the expected market share is M and the equilibrium reservation utility level is $\underline{W}$.

Given that uninformed consumers use the reservation utility rule while informed consumers use the advertising search rule, equilibrium expected market shares are determined as follows. By choosing $W(P) \geq \underline{W}$, a firm obtains a $1/N$ share of the uninformed, who search randomly and purchase from the first firm they visit. Further, by spending A on advertising, the firm induces the informed consumers to visit it first with probability $\hat{F}(A)^{N-1}$, which is the probability that it chooses the highest advertising level, given the advertising strategies of the other firms. Thus, when the firm chooses A and prices according to $\tilde{P}$, its expected market share is $M = U/N + \hat{F}(A)^{N-1}I$.

Let the maximized profit function associated with (4) be given by $\tilde{\Pi}(M,\underline{W})$. The next lemma expresses a key property of this function.

Lemma 2. Suppose the following holds:

$$\tilde{\Pi}(U/N,\underline{W}) \geq 0 \tag{6}$$

Then $\tilde{\Pi}(M,\underline{W})$ is strictly increasing in M for $M \in [U/N, U/N + I]$.

Proof. Given in the Appendix.

Condition (6) ensures that, when a firm is certain that it will not capture the informed, and so its market share is $M = U/N$, it still earns nonnegative profits. As we will see below, when consumer search behavior is endogenized, it will necessarily be the case that $\underline{W} < W(P^*(U/N + I)) - c$; hence, regardless of the level of search costs, (6) will hold in the equilibrium derived here if a firm receiving only U/N consumers can make nonnegative profits when it charges a price close to $P^*(U/N + I)$. Intuitively, this will be the case if U/N is large and thus significant coordination economies are achieved even by relatively small firms. It is also sufficient that the coordination economies associated with a large firm are modest (e.g., $P^*(1) > C(0)$).

We characterize next the firms' profit–maximizing advertising strategy. Given (6), a firm's markup is positive for every $M \in [U/N, U/N + I]$, so that a rise in market share will add to its profits. Thus, as long as (6) holds, we may define the equilibrium advertising distribution $\hat{F}$ by:

$$\tilde{\Pi}(U/N + \hat{F}(A)^{N-1}I, \underline{W}) - A = \tilde{\Pi}(U/N, \underline{W}) \tag{7}$$

Observe that (7) defines $\hat{F}$ as a continuous function, which is strictly increasing in view of Lemma 2. A firm that expects to capture the informed with probability zero chooses zero advertising, and its profits are $\tilde{\Pi}(U/N, \underline{W})$. For higher levels of advertising, the increased probability of capturing the informed is exactly offset by the higher advertising expenditures. Thus, firms are indifferent between all advertising levels in the support

$[0,\overline{A}]$, where $\overline{A}$ is determined by $\hat{F}(\overline{A}) = 1$. Moreover, any $A > \overline{A}$ must give negative profits, since expected market share cannot rise above $U/N + I$.

One issue remains in verifying that $\tilde{P}$ gives the profit–maximizing prices. Specifically, we must be sure that a firm does not deviate to some P with $W(P) < \underline{W}$. When a firm deviates in this way, it obtains a zero share of the uninformed, who under the reservation utility rule will search again and locate a firm with $W(P) \geq \underline{W}$ on their next visit. The informed consumers, in contrast, will not necessarily visit another firm when $W(P) < \underline{W}$, and a firm choosing A such that $\tilde{P}(U/N + \hat{F}(A)^{N-1}I,\underline{W}) = \underline{P}$ may benefit by shading price upward: in the event that this firm has the highest advertising level, the informed can infer that the expected market shares of all other firms lie below $U/N + \hat{F}(A)^{N-1}I$, and so other firms charge $P = \underline{P}$ under the equilibrium pricing strategy $\tilde{P}$; thus, the highest price $\overline{P} > \underline{P}$ that dissuades the informed from searching again is given by $W(\overline{P}) = W(\underline{P}) - c$, and correspondingly, the best deviant price among prices with $W(P) < \underline{W}$ is $P^d = \min\{\overline{P},P^*(\hat{F}(A)^{N-1}I)\}$.

The next lemma establishes that firms will not in fact prefer to choose P^d under our assumptions.

Lemma 3. If (6) holds, then for all $M \geq U/N$ such that $M < \underline{M}$:

$$\tilde{\Pi}(M,\underline{W}) \geq \Pi(P^d,\tilde{K}(P^d,M - U/N)\,|\,M - U/N) \tag{8}$$

with strict inequality for $M > U/N$.

Proof. Given in the Appendix.

Using (8), it follows at once that firms prefer $\tilde{P}$ to P^d when $\tilde{P} = \underline{P}$. Assumption 4 plays a key role in the proof of Lemma 3, by assuring that firms with higher expected market shares have reduced incentives to choose the higher price P^d. Note finally that $\tilde{P}$ is obviously preferred to any P when $\tilde{P} = P^*$.

b. Utility–Maximization by Consumers

Having determined the firms' optimal strategy given the consumers' search rules, let us now verify that the consumers' search rules are in turn best responses to the firms' strategy. To accomplish this, we fix $\underline{M}$ and assume that firms choose prices according to the strategy $\bar{P}$. Since the firm(s) choosing the highest advertising level will also choose the lowest price under this pricing strategy, it is clear that the informed consumers maximize their utility by using the advertising search rule. Now consider the uninformed consumers. Their reservation utility rule maximizes expected utility if the reservation utility level is set equal to the expected value obtainable from an extra search (Rothschild (1973)). Our next step, therefore, is to determine the expected value of an additional search when firms price according to $\bar{P}$.

To this end, consider a consumer that visits a firm that happens to have chosen the advertising level A. This consumer observes the price $P = \tilde{P}(U/N + \hat{F}(A)^{N-1}I, \underline{W})$. Thus, the probability distribution of prices that a consumer faces is induced by the probability distribution of expected market shares, $M = U/N + \hat{F}(A)^{N-1}I$; the latter distribution is determined as follows:

$$(9)\quad \hat{G}(M) = \mathrm{Prob}\{U/N + \hat{F}(A)^{N-1}I \le M\} = \mathrm{Prob}\{\hat{F}(A) \le [(M - U/N)/I]^{1/(N-1)}\}$$

$$= [(M - U/N)/I]^{1/(N-1)}$$

for $M \in [U/N, U/N + I]$, which gives the support of $\hat{G}$. With this, we may express the optimality condition for the consumers' reservation utility rule as:

$$\text{(10)}\qquad \underline{W}^r(\underline{M}) = W(P^*(\underline{M}))\hat{G}(\underline{M}) + \int_{\underline{M}}^{U/N+I} W(P^*(M))d\hat{G}(M) - c$$

For given $\underline{M}$, this relationship defines the implied value for $\underline{W}$, under which the reservation utility level is equal to the expected value of an additional search.

c. Equilibrium

At this point, two relationships exist between the critical market share $\underline{M}$ and the consumers' reservation utility level $\underline{W}$. First, from the firms' profit–maximization problem, we have derived (5), which determines $\underline{M}$ for any fixed $\underline{W}$. It is convenient to write this relationship in inverse form as follows:

$$\text{(11)}\qquad \underline{W}^m(\underline{M}) = W(P^*(\underline{M}))$$

Second, as (10) indicates, when uninformed consumers search optimally, the reservation utility level $\underline{W}$ is determines for any fixed critical market share level $\underline{M}$. In equilibrium, the profit–maximization relationship (11) must be consistent with the optimal–search relationship (10).

The profit–maximization and optimal–search problems are jointly solved at the intersection of $\underline{W}^r$ and $\underline{W}^m$. To characterize this solution, we first extablish properties of the two functions $\underline{W}^r$ and $\underline{W}^m$. Straightforward analysis of (10) and (11) reveals:

Lemma 4. (a) $\underline{W}^r$ and $\underline{W}^m$ are strictly increasing.

(b) For $\underline{M} \in (U/N, U/N + I)$, we have:

$$\frac{\partial \underline{W}^r(\underline{M})}{\partial \underline{M}} < \frac{\partial \underline{W}^m(\underline{M})}{\partial \underline{M}}$$

(c) $\underline{W}^r$ is strictly decreasing in c, $\underline{W}^m$ is independent of c, and $\underline{W}^m(U/N + I) - \underline{W}^r(U/N + I) = c$.

Figure 1 illustrates the properties listed in Lemma 4. For low values of c, there is a unique intersection of $\underline{W}^r$ and $\underline{W}^m$, and the values of $\underline{M}$ and $\underline{W}$ at the intersection point allow for joint solution of the profit–maximization and search problems. For a sufficiently high level of c, given by $\bar{c} = \underline{W}^m(U/N) - \underline{W}^r(U/N)$, the intersection point has $\underline{M} = U/N$, and $\underline{M}$ can go no lower as c rises further; the joint solution is then given by $\underline{M} = U/N$ and $\underline{W} = \underline{W}^r(U/N)$. In this case, search costs are so high that firms are not constrained in their price choices by the threat of search. For still higher levels of c, we have $\underline{W}^r(U/N) < 0$, so that consumers would have no desire to take even an initial search in this market; we will assume that search costs are not this high.

With the reservation utility level and corresponding critical market share now uniquely determined, we have established that (6) suffices for the existence of a symmetric equilibrium in which consumers always locate an acceptable price on their first search. The following proposition summarizes our main findings to this point:

Proposition 2. Suppose $N \geq 2$. If (6) is satisfied at the level of $\underline{W}$ determined from $\underline{W}^r$ and $\underline{W}^m$, then there exists a symmetric equilibrium in which:

(a) $\hat{F}(A)$ is defined by (7), and the choices made in conjunction with A are given by:

$$\hat{Z}(A) = (\tilde{P}(M,\underline{W}), \tilde{K}(\bar{P}(M,\underline{W}), M))$$

where $M = U/N + \hat{F}(A)^{N-1}I$;

(b) Uninformed consumers maximize expected utility by using the reservation utility rule with reservation level $\underline{W}$; and

(c) Informed consumers maximize expected utility by using the advertising search rule.

In contrast to the no–advertising benchmark considered in Proposition 1, the equilibrium featured in Proposition 2 exhibits price rivalry among the firms, which may be understood in the following intuitive terms. Firms compete with advertising to capture informed consumers, and so a firm's expected market share is rising with its advertising expenditures. This implies in turn that higher–advertising firms invest more in cost reduction and therefore offer lower prices. In other words, when firms choose cost–reducing investment levels as well as prices and advertising expenditures, price dispersion is a direct consequence of competition in advertising. An important implication of equilibrium price dispersion is that uninformed consumers can credibly threaten to search again, if the visited firm selects a price that is too high. This credible threat from the uninformed consumers means that a firm with a sufficiently low expected market share cannot charge its monopoly price; rather, it must price below its monopoly level, if it is to retain the business of uninformed consumers. Specifically, firms with low advertising levels, determined by $U/N + \hat{F}(A)^{N-1}I < \underline{M}$, are constrained to charge $\underline{P}$, rather than their monopoly price P^*.

It is interesting to consider the implications of the search cost c for equilibrium pricing. As Figure 1 illustrates, when c falls, the critical market share $\underline{M}$ rises, and so the firms are forced to charge the sub–monopoly price $\underline{P}$ for a larger set of expected market shares. Moreover, the price $\underline{P}$ itself drops as c is reduced, reflecting the fact that uninformed consumers are more willing to search again when the search cost is lower.

This effect of a reduction in search costs on the equilibrium pricing rule is illustrated in Figure 2. As the search cost approaches zero, we have that $\underline{P} \to P^*(U/N + I)$; thus, in the zero–search–cost limit, price dispersion disappears, and all firms must offer the same deal as would the largest firm in the market. It follows that price rivalry sharpens as search costs fall, and in the limit we obtain the maximum realization of coordination economies consistent with uniformly–randomizing search decisions by the uninformed.

Industry concentration affects the extent of coordination economies that may be realized in equilibrium. From (9) it may be seen that $\hat{G}$ decreases as N falls, and so it follows easily that the curve $\underline{W}^r$ shifts upward with lower N. Consequently, the equilibrium value of $\underline{W}$ is greater, meaning greater equilibrium expected utility for the uninformed consumers. The informed consumers purchase at the highest of the realized values of M, so their expected utility is determined with reference to the following distribution:

$$\hat{G}(M)^N = ((M - U/N)/I)^{N/(N-1)}$$

It may be seen that $\hat{G}^N$ falls, and thus the informed obtain greater expected utility, when N falls.

We summarize these observations in the following corollary:

Corollary 1. In the equilibrium derived in Proposition 2:

(a) A fall in c or N lead to lower $\underline{P}$, higher $\underline{W}$, and higher expected utility for all consumers; and

(b) $\lim_{c \to 0} \underline{P} = P^*(U/N + I)$.

Thus, prices tend to lower levels as search costs fall and as the market becomes more concentrated.

We now remark on the implications of our findings for the Diamond paradox. Our theory predicts that market prices decline as the level of search costs is reduced, which resolves the paradox in part. The resolution is not complete, however, in the sense that the market price that arises as search costs approach zero differs from that which would occur under Bertrand competition. After demonstrating in the next section that a similar resolution arises when there are few uninformed consumers per firm, we allow for free entry in Section 5, which will yield a more complete "long–run" resolution of the Diamond paradox.

Up to this point, we have assumed that at least two firms operate in the market. We conclude this section by contrasting the advertising equilibrium developed here with the market outcome that would arise if there were a single firm. In the case of monopoly, there is no price rivalry, and so consumers do not receive sub–monopoly prices. On the other hand, the monopolist is able fully to realize the coordination economies present in the market, since it receives the entire market share (i.e., $M = 1$) with probability one. These competing considerations are unambiguously resolved in the model developed here, as consumer welfare and social welfare are *necessarily higher* when the market is fully concentrated, with a single firm selling to all consumers. This is because the full–market monopoly price, $P^*(1)$, lies below the lowest price charged as part of an advertising equilibrium, $P^*(U/N + I)$. Intuitively, while advertising competition gives rise to price rivalry, prices never drop below the monopoly price of the largest possible firm, and so a monopoly market is sure to increase consumer welfare and social surplus. In markets with coordination economies, therefore, more concentrated market structures are unambiguously welfare–increasing given the information imperfections that we consider, no matter how low the search cost.

4. Advertising Equilibria with Few Uninformed Consumers per Firm

Condition (6) may fail if the share of uninformed consumers captured by each firm is small, since if P^* declines steeply, then a relatively small firm catering only to its share of the uninformed might not be able profitably to charge a price near that selected by a large firm. In this case, firms at the lower end of the market–share distribution might find it unprofitable to offer the equilibrium reservation utility level $\underline{W}$. Thus, the mixed strategy constructed in (7) would not give an equilibrium when (6) fails, since a low–advertising firm would prefer to deviate to $A = 0$ and $Z = Z^*(0)$, even though it would then receive zero expected profits by virtue of $W(P^*(0)) < \underline{W}$.

This section extends the previous construction to take account of this possibility. We derive a new equilibrium, in which firms continue to choose a mixed advertising strategy having continuous distribution $\hat{F}(A)$, but there is now a critical advertising level $\underline{A} > 0$ such that firms choosing $A < \underline{A}$ also choose $W(P) < \underline{W}$, while $A > \underline{A}$ will be accompanied by $W(P) \geq \underline{W}$. One interesting feature of this model, therefore, is that an uninformed consumer will search more than once in equilibrium, if he happens initially to visit a low–advertising firm. Further, we continue to find that higher advertising levels are associated with lower equilibrium prices.

To begin, fix a reservation utility level $\underline{W} \in [0,W(P^*(1))]$. We first consider the pricing behavior of firms that choose $A < \underline{A}$. Holding fixed the equilibrium strategies of rival firms and the search rules of consumers, we let $M^O(P)$ denote the expected share of the uninformed consumers that a firm obtains when it chooses P together with $A < \underline{A}$. If the expected share of informed consumers is unaffected by P, being instead determined by A, then the firm's choice of P affects its market share only insofar as it alters the expected share of uninformed consumers to whom it sells. Given that M^O is differentiable, the profit–maximizing price must satisfy:

$$\Pi_P(P,\bar{K}(P,M)\,|\,M) + \Pi_M(P,\bar{K}(P,M)\,|\,M)\,\frac{\partial M^O(P)}{\partial P}\,U = 0 \tag{12}$$

where the firm's total market share is given by $M = M^O(P)U + \hat{F}(A)^{N-1}I$. If a given firm choosing $A < \underline{A}$ captures any uninformed consumers, it will only be after they have searched every firm and found that the given firm offers the lowest price. Thus, in equilibrium we must have that $M^O(P) = \hat{F}(A)^{N-1}$, corresponding to the equilibrium probability that the firm captures all uninformed consumers. As this is also the equilibrium probability that the firm captures all of the informed consumers, we have from $U + I = 1$ that the firm's equilibrium expected market share is $M = M^O(P) = \hat{F}(A)^{N-1}$.

Substituting the equilibrium relation $M = M^O(P)$ into (12) and rearranging, we have:

$$\frac{\partial P^O(M)}{\partial M} = \frac{-\Pi_M(P^O(M),\bar{K}(P^O(M),M)\,|\,M)U}{\Pi_P(P^O(M),\bar{K}(P^O(M),M)\,|\,M)} \tag{13}$$

where $P^O(M)$ gives the inverse of $M^O(P)$. Expression (13) indicates the extent to which price must be reduced in order to achieve an expansion in the firm's share of uninformed consumers, starting at a point at which the firm's price is profit–maximizing and its share of the uninformed consumer population equals its share of the total consumer population. Let us take P^O to be the solution of (13) that has $P^O(0) = P^*(0)$, and also put $\Pi^O(M) = \Pi(P^O(M),\bar{K}(P^O(M),M)\,|\,M)$. Clearly, P^O is strictly decreasing; further, we have that $P^O(M) < P^*(M)$ for $M > 0$, as is evident from the profit–maximization condition (12). Observe also that Π^O is nonnegative and strictly increasing. We will demonstrate below that $P = P^O(\hat{F}(A)^{N-1})$ gives the profit–maximizing price for firms choosing $A < \underline{A}$, while

$\bar{P}$ defined in (5) will determine the equilibrium prices for firms choosing $A \geq \underline{A}$. Figure 3 illustrates the pricing profiles P^O and $\bar{P}$.

Next, consider a firm that chooses $W(P) \geq \underline{W}$. The expected market share obtained by such a firm, for whatever level of A it selects, is given by:

$$(14) \qquad \overline{M}(\underline{F},F) = \Sigma_{i=0}^{N-1} \binom{N-1}{i} \underline{F}^i (1-\underline{F})^{N-1-i} U/(N-i) + F^{N-1} I$$

where $\underline{F} = \hat{F}(\underline{A})$ and $F = \hat{F}(A)$. The first term on the right–hand side of (14) indicates how the firm divides the uninformed consumers with rival firms that choose $A \geq \underline{A}$, which are the firms that offer $W(P) \geq \underline{W}$ in equilibrium. The second term gives the expected market share obtained from rivalry for the informed consumers. Note that $\overline{M}$ is continuous in both arguments, strictly increasing in F, and $\overline{M}(F,F) > F^{N-1}$.

Now define $\underline{F}$ as follows:

$$(15) \qquad \underline{F} = \inf\{F' \mid \Pi^O(F^{N-1}) > \tilde{\Pi}(\overline{M}(F',F),\underline{W}) \text{ for all } F < F'\}$$

Observe that $\underline{F}$ gives the highest value that can be assigned to $\hat{F}(\underline{A})$ subject to the restriction that firms must prefer to choose $W(P) < \underline{W}$ when they select $A < \underline{A}$. Important facts about $\underline{F}$ are given in the following lemma.

Lemma 5. (a) $\underline{F} \in (0,1)$ and $\underline{F}$ is increasing in $\underline{W}$;

(b) $\Pi^O(\underline{F}^{N-1}) = \tilde{\Pi}(\overline{M}(\underline{F},\underline{F}),\underline{W})$;

(c) $P^O(\underline{F}^{N-1}) > \underline{P}$ and $\underline{M} > \overline{M}(\underline{F},\underline{F})$; and

(d) The expected share of the uninformed captured by a firm choosing $A \geq \underline{A}$ is:

$$\underline{U} = \overline{M}(\underline{F},\underline{F}) - \underline{F}^{N-1} I > U/N$$

Proof. Given in the Appendix.

Parts (a) and (b) follow straightforwardly from (14) and (15). Part (c) establishes that prices chosen by firms expecting $M \leq \underline{F}^{N-1}$ indeed lie above the reservation price, and it further verifies the existence of a range of equilibrium market shares on which firms choose $\underline{P}$, as shown in Figure 3. As shown in part (d), by choosing $P \leq \underline{P}$, firms obtain a strictly greater expected share of the uninformed than in the equilibrium of Proposition 2, which reflects the positive probability that rival firms choose $P > \underline{P}$.

The equilibrium advertising distribution may now be constructed, as follows. For small values of A, $\hat{F}(A)$ is determined by:

$$(16) \qquad \Pi^{O}(\hat{F}(A)^{N-1}) - A = 0$$

As A rises, eventially we have $\hat{F}(\underline{A}) = \underline{F}$, and this defines $\underline{A}$. For $A \geq \underline{A}$, $\hat{F}$ is given by by:

$$(17) \qquad \bar{\Pi}(\overline{M}(\underline{F},\hat{F}(A)),\underline{W}) - A = 0$$

The continuity and monotonicity properties of the functions $\hat{\Pi}$, $\bar{\Pi}$ and $\overline{M}$ assure that $\hat{F}$ is continuous and strictly increasing, as defined by (16) and (17). The upper bound of the support is determined by $\hat{F}(\bar{A}) = 1$. The following lemma verifies that the mixed advertising strategy, together with the accompanying price and investment choices, maximizes the firms' expected profits.

Lemma 6. For given $A \in [0,\bar{A}]$, let the price and investment choices be:

$$(18) \qquad \hat{Z}(A) = \begin{cases} (P^{o}(\hat{F}(A)^{N-1}), \tilde{K}(P^{o}(\hat{F}(A)^{N-1}), \hat{F}(A)^{N-1})), & A < \underline{A} \\ (\tilde{P}(\overline{M}, \underline{W}), \tilde{K}(\tilde{P}(\overline{M}, \underline{W}), \overline{M})), & A \geq \underline{A} \end{cases}$$

where $\overline{M}$ is evaluated at $(\underline{F}, \hat{F}(A))$. Then the mixed strategy $\hat{F}(A)$ together with $\hat{Z}(A)$ constitutes a Nash equilibrium for the firms, given the consumers' search rules.

Proof. Given in the Appendix.

Since the equilibrium price is a nondecreasing function of the advertising level, as may be seen in Figure 3, it follows that the advertising search rule is utility–maximizing for the informed consumers, and their search decisions conditional on off–equilibrium–path price observations are easily derived. It remains to verify the existence of an optimal reservation utility rule for the uninformed consumers. Letting $\hat{G}$ denote the probability distribution over M that is induced by the $\hat{F}$ determined in the present section, it follows that $\underline{W}^{r}(\underline{M})$ defined in (10) will continue to express the optimal reservation utility as a function of $\underline{M}$, and we have an equilibrium for $\underline{M}$ and $\underline{W}$ at the intersection of $\underline{W}^{r}$ and $\underline{W}^{m}$, just as before. This completes the proof of:

Proposition 3. Suppose $N \geq 2$. If (6) fails to hold at the level of $\underline{W}$ determined from $\underline{W}^{r}$ and $\underline{W}^{m}$, then there exists a symmetric equilibrium in which:

(a) $\hat{F}(A)$ is defined by (16) and (17), and the choices made in conjunction with A are given by (18):

(b) Uninformed consumers maximize expected utility by using the researvation–utility search rule with reservation level $\underline{W}$; and

(c) Informed consumers maximize expected utility by using the advertising search rule.

Just as in the earlier case, a fall in c or N will lead to lower $\underline{P}$ and higher $\underline{W}$; the consequences for expected utility are unclear in the present case, however, since $\underline{F}$ will also be higher. Thus, the indicated parameter changes will raise utility conditional on locating a firm choosing $P \leq \underline{P}$, but locating a firm choosing $P \leq \underline{P}$ requires a greater number of searches on average, and there is an increased chance that all firms choose $P > \underline{P}$. Here a reduction in search costs will tend to shift weight to *both* tails of the equilibrium price distribution. As $c \to 0$, we have $\underline{P} \to P^{*}(\underline{U} + I)$, so that the reservation price is driven to the lowest level in the market, but also the range of firms choosing $P > \underline{P}$ is at its greatest.

5. Endogeneous Market Structure and the Diamond Paradox

In this section, we introduce an initial free–entry stage that endogenizes the number of firms in the industry. We also contrast our equilibria with the outcomes that arise under perfect price information; this allows us to reassess Diamond's paradoxical result that market prices jump discontinuously from competitive to monopolistic levels at the point of zero search costs.

We now suppose that there is a large number of potential entrant firms that simultaneously choose whether or not to enter at an initial entry stage; let $E \in (0, \Pi^{*}(1))$ denote the sunk entry cost. Following the entry stage, all agents observe the number of entrants, denoted by N, and for each subgame with $N \geq 2$ there arises an advertising equilibrium of the form derived in Propositions 2 and 3. In subgames with $N = 1$, we have $Z = Z^{*}(1)$ and $A = 0$, i.e. the monopoly outcome obtains.

Let N^{A} denote the number of firms that choose to enter in equilibrium. It is clear that (6) must hold at $N = N^{A}$, since the advertising equilibria yield zero expected profits in subgames for which (6) fails, according to Proposition 3. Thus, provided that $N^{A} \geq 2$, it follows that equilibrium advertising and search behavior is characterized as in Proposition 2, corresponding to the case in which each firm obtains a large share of the uninformed. We therefore determine N^A as follows:

$$(19) \qquad \tilde{\Pi}(U/N^A,\underline{W}) \geq E > \tilde{\Pi}(U/(N^A+1),\underline{W})$$

where the dependence of $\underline{W}$ on N has been suppressed.[4] Condition (19) can be satisfied for some $N^A \geq 2$ as long as $\tilde{\Pi}(U/2,\underline{W}) - E \geq 0$; if the latter condition fails, then we have $N^A = 1$.

We may now assess the effect of search costs on equilibrium market structure. For a fixed number of firms, as search costs fall, expected utility rises, according to Corollary 1; this reduces $\tilde{\Pi}(U/N,\underline{W})$ for given N, and strictly so if $\tilde{P}(U/N,\underline{W}) = \underline{P}$. It follows that the equilibrium number of firms, N^A, must fall as the search cost, c, falls. In fact, (6) will be violated when search costs are sufficiently small if even two firms enter, under the following condition:

$$(20) \qquad \Pi(P^*(U/2+I),\tilde{K}(P^*(U/2+I),U/2)|U/2) < E$$

Thus, when (20) holds, the natural monopoly outcome $N^A = 1$ obtains as search costs approach zero. Note that (20) must hold when I is sufficiently close to unity.

Similarly, a rise in I, which reduces U, must also lead to a reduction in N^A. This proves the following proposition.

Proposition 4. (a) In free–entry equilibria, the number of firms is given by (19) if $\tilde{\Pi}(U/2,\underline{W}) - E \geq 0$, and by $N^A = 1$ otherwise;

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[4] In (19) we assume that firms choose to enter if indifference prevails.

(b) N^A is (weakly) increasing in c, with $\lim_{c \to 0} N^A = 1$ if (20) holds, and $\lim_{c \to 0} N^A \geq 2$ otherwise; and

(c) N^A is (weakly) decreasing in I.

Intuitively, lower search costs and a greater proportion of informed consumers enhance the transmission of price and advertising information and thereby lead to more concentrated market structures. We may view improvements in information transmission as complementary to the realization of coordination economies. The natural–monopoly outcome emerges as search costs approach zero, so long as (20) holds, meaning that coordination economies are sufficiently pronounced (e.g., (20) is satisfied when $P^*(U/2 + I)$ lies below $C(K^*(U/2))$). Further, (20) becomes easier to satisfy when E and I are increased: sunk entry costs give another element of the production cost structure contributing to natural monopoly, while high I leads firms to incur high fixed marketing costs, in the form of advertising expenditures, again contributing to natural monopoly.

We may directly compare the free–entry advertising equilibrium with the free–entry equilibrium in the no–advertising case, where equilibria arising in N–entrant subgames are given in Proposition 1. Letting N^N denote the equilibrium number of entrants when there is no advertising, we have:

$$\Pi^*(1/N^N) \geq E > \Pi^*(1/(N^N + 1)) \tag{21}$$

Comparing (19) and (21), we have $\tilde{\Pi}(U/N^A,\underline{W}) \cong \Pi^*(1/N^N) \cong E$ in the $N^A \geq 2$ case, and so $N^A \leq N^N$ follows as a consequence of $U < 1$ together with the constraint $W(P) \geq \underline{W}$. Further, $\Pi^*(U/N^A) \geq \tilde{\Pi}(U/N^A,\underline{W})$ implies $U/N^A \geq 1/N^N$, whence $P^*(U/N^A) \leq P^*(1/N^N)$. Thus, in equilibria with advertising, the market is more concentrated and consumers obtain lower prices with probability one than in equilibria without advertising.

As we have argued previously (Bagwell and Ramey (1994a,b)), these findings provide a consistent interpretation of the empirical results obtained by Benham (1972) and others, in which the ability to advertise is correlated with more concentrated market structures and lower prices, even when the direct price information transmitted by advertising is limited.

Finally, our results supply a "long–run" resolution to the Diamond paradox, so long as coordination economies are sufficiently great. To show this, we must first derive the equilibrium outcome under perfect information, which is carried out in the following proposition:

Proposition 5. Suppose consumers can freely observe firms' price choices. Then there is a unique symmetric equilibrium outcome for each $N \geq 2$, in which firms earn zero expected profits.

Proof. Given in the Appendix.

From Proposition 5, it follows that the natural monopoly outcome must obtain when price information is freely observed, since any level of sunk entry costs will deter a second entrant. Comparing Propositions 4(b) and 5, it follows that the perfect–information outcome is realized as the limit of equilibria with positive search costs when (20) holds, which is the case of significant coordination economies. Thus, the Diamond paradox does not arise here if coordination economies are sufficiently pronounced to generate the natural–monopoly outcome under low search costs. If (20) fails, then a discontinuity continues to exist at the zero–search–cost limit, although our results still mitigate against the Diamond paradox, to the extent that concentration is increased as search costs fall.

6. Loss–Leader Advertising

For multiproduct sellers, advertising may take the form of announced low prices on a subset of the products, which are called *loss leaders*. In this case, advertising provides direct information as to the price of the loss–leader goods as well as indirect information as to the prices of the other goods. In our setting, low prices on part of the product line may be complementary to high levels of cost–reducing investment and low prices on the remainder of the product line, and this can justify consumers' responsiveness to loss–leader advertising.

This point is easily made via a slight extension of our model. Let there now be two products, denoted products 1 and 2, having prices denoted by P_1 and P_2. For simplicity, the utility function is given by the sum over the two goods of $U_i(Q_i) - P_iQ_i$, where Q_i gives the consumption of product i; let Assumption 1 hold for each i. It follows that consumer demand for product i takes the form $D_i(P_i)$, i.e. there are no cross–product interactions on the demand side. Let $W(P)$ denote the maximized utility level as a function of $P = (P_1,P_2)$.

On the production side, we assume that $C_i(K)$, which gives the unit production cost of product i, satisfies Assumption 2 for each i. The profit function is now given by:

$$\Pi(Z\,|\,M) = \Sigma_{i=1}^{2}\,(P_i - C_i(K))MD_i(P_i) - K$$

where $Z = (P_1,P_2,K)$. We take Π to be strictly concave in Z. Let $P_1^*(P_2,M)$ and $K^*(P_2,M)$ give the profit–maximizing choices of P_1 and K for given P_2 and M; it is straightforward to verify that P_1^* is strictly increasing in P_2 and strictly decreasing in M, while the opposite effects hold for K^*. Finally, Assumption 4 must hold separately for the partial derivatives with respect to P_1 and P_2, where $\tilde{K}$ continues to be defined as in

(2).

As before, all consumers must engage in sequential search in order to observe firms' choices of P_1. Informed consumers, however, may directly observe the choices of P_2, and the advertising search rule takes the form of visiting the firm(s) having the lowest level of P_2. The uninformed consumers can only observe P_2 via search. We consider symmetric equilibria in which firms use mixed strategies. We carry out the equilibrium construction in the Appendix, and summarize here with the following proposition.

Proposition 6. In the loss–leader model, if $N \geq 2$, then there exists a symmetric equilibrium in which:

(a) Firms choose a mixed strategy in which lower P_2 is associated with (weakly) greater values of $W(P_1, P_2)$;

(b) Uninformed consumers maximize expected utility by using the reservation utility rule with reservation level $\underline{W}$; and

(c) Informed consumers maximize expected utility by using the advertising search rule.

It follows that coordination economies give an explanation for why loss–leader price reductions may be attractive to consumers: by expanding its market share, a low loss–leader price induces a firm to choose high investment in cost reduction, which implies that consumers will obtain good deals on the other products as well. For sufficiently low levels of search costs, there is an interval of P_2 such that $W(P) = \underline{W}$ for any firm choosing P_2. On this interval, a lower loss–leader price is associated with a *higher* price $\underline{P}_1(P_2)$ for the other good; thus, our equilibrium rationalizes the common observation that loss–leader prices may be accompanied by higher prices on non–advertised products. In this instance, consumer utility remains constant at $\underline{W}$, while for the lowest values of P_2 in the support of the equilibrium strategy, a reduction in the loss–leader price signals a lower level of the other price. Further, for very low values of c, the constraint $W(P) \geq \underline{W}$ may bind for nearly all of the equilibrium P_2 values, so that reductions in P_2 will nearly always be associated with higher levels of P_1, even as consumer utility in the loss–leader equilibria is driven to a high level.

7. Conclusion

We have developed a theory of retail markets that endogenizes the advertising, pricing, technology and entry decisions of all firms as well as the search decisions of consumers subject to two different consumer information channels, advertising and sequential search. Our model delivers four main conclusions. First, our finding that dissipative advertising can direct consumers to the lowest price in the market continues to hold when consumers are allowed to engage in sequential price search. Second, we find that sequential search by advertising–uninformed consumers leads low–advertising firms to face price rivalry that drives them from their monopoly price levels. This gives a new resolution to the Diamond paradox. In particular, we show that our model can approximate the long–run zero–search–cost outcome, which is monopoly, when search costs are positive but low. Third, we argue that greater industry concentration promotes consumer welfare in retail markets that are subject to the kinds of information imperfections that we consider. Finally, we develop a new theory of loss–leader advertising in which consumers are rational in responding to loss leaders, despite the prospect for higher prices on unadvertised goods.

The research presented here might be extended in a variety of directions. One important extension concerns the dynamic interaction between firms and consumers. The dynamic evolution of advertising competition in markets with coordination economies constitutes a topic of particular interest.

Appendix

Proof of Proposition 1. Let $\hat{P}$ denote the symmetric equilibrium price choice of each firm. Symmetry of price choices and search strategies implies that $1/N$ consumers purchase from each firm. If $\hat{P} > P^*(1/N)$, then a firm could reduce its price slightly without losing any purchasers, since the expected gain to a visiting consumer from searching again would be negative. If $\hat{P} < P^*(1/N)$, then the firm could raise its price slightly, so that $W(\hat{P} + \epsilon) > W(\hat{P}) - c$. Again, it would lose no purchasers, since their return from another search would be $W(\hat{P}) - c$. In either of these cases, Firm i can profitably deviate toward $P^*(1/N)$, and so $\hat{P} = P^*(1/N)$ is necessary. Correspondingly, $K^*(1/N)$ gives the equilibrium investment choice. We have an equilibrium since consumers are indifferent as to which firms to visit, while firms make profit–maximizing strategies given their expected number of visitors. *Q.E.D.*

Proof of Lemma 2. Note first that $\bar{K}(P,M)$, defined in (2), is a strictly increasing function of M, as may be seen by direct differentiation. For $M \in (U/N,\underline{M})$, we have:

$$\frac{\partial\bar{\Pi}(M\ ,\ \underline{W})}{\partial M} = (\underline{P} - C(\bar{K}(\underline{P},M)))D(\underline{P}) > (\underline{P} - C(\bar{K}(\underline{P},U/N)))D(\underline{P}) > 0$$

where the first inequality follows from $\bar{K}(\underline{P},M) > \bar{K}(\underline{P},U/N)$, and the second inequality invokes (6). For $M \in [\underline{M},1]$, we have $\bar{\Pi}(M,\underline{W}) = \Pi^*(M)$, which is strictly increasing in M by Lemma 1(a). *Q.E.D.*

Proof of Lemma 3. Consider a given firm whose expected market share is such that the firm's best–deviant price is P^d. Holding P^d fixed, we now consider the incentive to deviate from $\underline{P}$ to P^d as a function of expected market share. We have $\bar{\Pi}(M,\underline{W}) = \Pi(\underline{P},\tilde{K}(\underline{P},M)|M)$, and we may write:

$$\Delta(M) = \Pi(\underline{P},\tilde{K}(\underline{P},M)|M) - \Pi(P^d,\tilde{K}(P^d,M - U/N)|M - U/N)$$

$$= -\int_{\underline{P}}^{P^d} \Pi_P(P,\tilde{K}(P,M)|M)dP + \int_{M-U/N}^{M} \Pi_M(P^d,\tilde{K}(P^d,X)|X)dX$$

Using (6), we have $\Delta(U/N) = \Pi(\underline{P},\tilde{K}(\underline{P},U/N)|U/N) \geq 0$, and:

$$\frac{\partial\Delta(M)}{\partial M} = -\int_{\underline{P}}^{P^d} \frac{d}{dM}\Pi_P(P,\tilde{K}(P,M)|M)dP$$

$$+ [\Pi_M(P^d,\tilde{K}(P^d,M)|M) - \Pi_M(P^d,\tilde{K}(P^d,M - U/N)|M - U/N)] > 0$$

where the integral term is positive by virtue of Assumption 4, while the bracketed term is positive as a consequence of $\tilde{K}(P^d,M) > \tilde{K}(P^d,M - U/N)$ and $\Pi_{MK}(P,\tilde{K}(P,M)|M) = -C'(K)D(P) > 0$. It follows that the given firm cannot gain with a deviation to P^d.

Q.E.D.

Proof of Lemma 5. To see that the set on the right–hand side of (15) is nonempty, note that $\overline{M}(0,0) = U/N$, so $\bar{\Pi}(\overline{M}(0,0),\underline{W}) < 0 = \Pi^o(0)$, using the fact that (6) fails to hold. Further, $\overline{M}(1,1) = 1$ and $\underline{W} \leq W(P^*(1))$ assure that $\bar{\Pi}(\overline{M}(1,1),\underline{W}) = \Pi^*(1) > \Pi^o(1)$, so values of F' near unity cannot be in the set. Thus, $\underline{F}$ exists and satisfies $\underline{F} \in (0,1)$, while the fact that $\underline{F}$ increases in $\underline{W}$ is immediate from (15). This establishes (a), and (b) is immediate from the continuity of the functions $\hat{\Pi}$, $\tilde{\Pi}$ and $\overline{M}$. As for (c), note that we cannot have $P^O(\underline{F}^{N-1}) \leq \underline{P}$, else the following inequalities hold:

$$\tilde{\Pi}(\overline{M}(\underline{F},\underline{F}),\underline{W}) \geq \Pi(P^O(\underline{F}^{N-1}),\tilde{K}(P^O(\underline{F}^{N-1}),\overline{M}(\underline{F},\underline{F}))\,|\,\overline{M}(\underline{F},\underline{F}))$$

$$> \Pi(P^O(\underline{F}^{N-1}),\tilde{K}(P^O(\underline{F}^{N-1}),\underline{F}^{N-1})\,|\,\underline{F}^{N-1}) = \Pi^O(\underline{F}^{N-1})$$

where the second inequality uses $\overline{M}(\underline{F},\underline{F}) > \underline{F}^{N-1}$. Further, $\Pi^*(\overline{M}(\underline{F},\underline{F})) > \Pi^*(\underline{F}^{N-1}) > \Pi^O(\underline{F}^{N-1})$, so we must have $\tilde{P}(\overline{M}(\underline{F},\underline{F}),\underline{W}) < P^*(\overline{M}(\underline{F},\underline{F}))$, whence $\underline{M} > \overline{M}(\underline{F},\underline{F})$. Part (d) follows directly from (14) and $\Sigma_{i=0}^{N-1} \binom{N-1}{i}\underline{F}^i(1-\underline{F})^{N-1-i} = 1$. *Q.E.D.*

Proof of Lemma 6. A firm choosing $A < \underline{A}$ captures the informed consumers with probability $\hat{F}(A)^{N-1}$, and choosing $P \in [P^O(\underline{F}^{N-1}),P^*(0)]$ implies that $M^O(P)$ gives the expected share of the uninformed, since it is the probability that all rival firms choose prices above P, under the hypothesis that they use the strategy $\hat{F}$ together with $\hat{Z}$. As long as the expected share of informed consumers is unaffected by P, the profit–maximizing price satisfies:

$$\Pi_P(P,\tilde{K}(P,M)\,|\,M) + \Pi_M(P,\tilde{K}(P,M)\,|\,M)\,\frac{\partial M^O(P)}{\partial P}\,U = 0$$

where $M = M^O(P)U + \hat{F}(A)^{N-1}I$. By (13), this condition holds at $P = P^O(\hat{F}(A)^{N-1})$, which gives $M^O(P) = \hat{F}(A)^{N-1}$ for this P. To see that second–order conditions are satisfied, suppose a firm choosing the higher advertising level $A' \in (A,\underline{A})$ were to select $P^O(\hat{F}(A)^{N-1})$. The expected market share for this firm would be $\hat{F}(A)^{N-1}U + \hat{F}(A')^{N-1}I > \hat{F}(A)^{N-1}$, which gives a lower value to the right–hand side of (13), holding

$P^O(\hat{F}(A)^{N-1})$ constant; this step invokes Assumption 4. Thus, such a firm actually profits from a small shift along the P^O locus toward lower prices, as shown in Figure 4. Similarly, a firm choosing $A' < A$ would prefer prices slightly above $P^O(\hat{F}(A)^{N-1})$ on the P^O locus. It follows that the choices $P = P^O(\hat{F}(A)^{N-1})$ give profit–maxima for a firm choosing A, among prices $P > \underline{P}$, subject to the assumption that P does not affect the expected share of informed consumers.

In fact, we have obtained profit–maximizing choices even when informed consumers react to P according to their equilibrium strategy. Choosing $P < P^O(\hat{F}(A)^{N-1})$ cannot increase a firm's expected share of informed, since given that it has the highest advertising level, it already sells to all of them. Choosing $P > P^O(\hat{F}(A)^{N-1})$ may reduce the expected share of the informed, if some rival firm should happen to choose a price that is close enough to $P^O(\hat{F}(A)^{N-1})$ to induce the informed to visit it; this only makes such a P less attractive, however, so $P^O(\hat{F}(A)^{N-1})$ is still profit–maximizing. Further, (15) assures that $\Pi^O(\hat{F}(A)^{N-1}) > \tilde{\Pi}(\overline{M}(\underline{F},\hat{F}(A)),\underline{W})$, so prices below $\underline{P}$ cannot give profit–maximizers. Finally, (16) implies that all choices $A \in [0,\underline{A})$ yield zero expected profit.

Consider next a firm choosing $A \geq \underline{A}$ with $\tilde{P}(\overline{M}(\underline{F},\hat{F}(A)),\underline{W}) = \underline{P}$. If the firm instead selects $P \in (\underline{P},P^O(\underline{F}^{N-1})]$, then it obtains expected share $\underline{F}^{N-1}$ of the uninformed, since it will have the lowest price if and only if all the other firms choose advertising levels below $\underline{A}$ and prices above $P^O(\underline{F}^{N-1})$; consequently, the loss of uninformed consumers reduces its expected share by $\overline{M}(\underline{F},\underline{F}) - \underline{F}^{N-1} = M' > 0$. As long as $\overline{P} \geq P^O(\underline{F}^{N-1})$ (where we have defined $\overline{P}$ by $W(\overline{P}) = W(\underline{P}) - c$), the firm can be sure that the informed will not search again if it chooses $P \in (\underline{P},P^O(\underline{F}^{N-1})]$, since any rival firm choosing a lower advertising level will also choose $P \geq \underline{P}$; thus, $P = P^O(\underline{F}^{N-1})$ is profit–maximizing among prices $P \in (\underline{P},P^O(\underline{F}^{N-1})]$. Further, since $F(A)^{N-1} \geq \underline{F}^{N-1}$ gives the firm's expected share of the informed, it follows that no $P > P^O(\underline{F}^{N-1})$ can give greater expected profits, based on the argument used earlier in the proof, and we have in this case that $P^d = \min\{P^o(\underline{F}^{N-1}), P^*(\underline{F}^{N-1}U + F(A)^{N-1}I)\}$ is profit–maximizing among prices $P > \underline{P}$.

Now, along the lines of Lemma 3, we have:

$$\Delta(M) = \Pi(\underline{P},\tilde{K}(\underline{P},M)\,|\,M) - \Pi(P^d,\tilde{K}(P^d,M-M')\,|\,M-M')$$

$$= -\int_{\underline{P}}^{P^d} \Pi_P(P,\tilde{K}(P,M)\,|\,M)dP + \int_{M-M'}^{M} \Pi_M(P^d,\tilde{K}(P^d,X)\,|\,X)dX$$

Note that $\tilde{\Pi}(\overline{M}(\underline{F},\underline{F}),\underline{W}) = \Pi^o(\underline{F}^{N-1})$, from Lemma 4(b), implies $\Delta(\overline{M}(\underline{F},\underline{F})) = 0$, while we have $\partial\Delta/\partial M > 0$, just as in the proof of Lemma 3, again invoking Assumption 4. Using $\overline{M}(\underline{F},\hat{F}(A)) > \overline{M}(\underline{F},\underline{F})$ and $\overline{M}(\underline{F},\hat{F}(A)) - M' = \underline{F}^{N-1}U + \hat{F}(A)^{N-1}I$, we have:

$$\Pi(\underline{P},\tilde{K}(\underline{P},\overline{M}(\underline{F},\hat{F}(A)))\,|\,\overline{M}(\underline{F},\hat{F}(A)))$$

$$> \Pi(P^d,\tilde{K}(P^d,\underline{F}^{N-1}U + \hat{F}(A)^{N-1}I)\,|\,\underline{F}^{N-1}U + \hat{F}(A)^{N-1}I)$$

Thus, $\underline{P}$ is preferred to P^d when a firm chooses $A \geq \underline{A}$. If $\overline{P} < P^o(\underline{F}^{N-1})$, then the possibility arises that the informed consumers will search again should P^d be chosen, which can only make deviations to $P > \underline{P}$ less profitable. Clearly, no $P > \underline{P}$ will be attractive to a firm choosing $A \geq \underline{A}$ and $\tilde{P}(\overline{M}(\underline{F},\hat{F}(A)),\underline{W}) = P^*(\overline{M}(\underline{F},\hat{F}(A)))$. Subject to the profit–maximizing choices of price and investment, (17) implies that all selections $A \in [\underline{A},\overline{A}]$ yield zero profits. Since $M = \overline{M}(\underline{F},1)$ for all $A > \overline{A}$, it follows that any such A gives strictly negative profit. This proves that the indicated mixed strategy is profit–maximizing for a given firm, when all the other firms use the strategy and consumers use the indicated search rules. *Q.E.D.*

Proof of Proposition 5. Let us first consider the equilibria in N–entrant subgames. Since price information is perfect, consumers will ignore advertising in making their search decisions, and firms choose $A = 0$. For $N \geq 2$, let $\hat{F}(P)$ denote the probability distribution over prices that serves as the firms' mixed pricing strategy, and suppose that $\hat{F}$ is continuous. In equilibrium, we have $M = (1 - \hat{F}(P))^{N-1}$, and the mixed pricing strategy is determined by:

$$\Pi(P,\bar{K}(P,(1-\hat{F}(P))^{N-1})\,|\,(1-\hat{F}(P))^{N-1})) = 0$$

It is easily checked that this defines $\hat{F}$ as a strictly increasing function for $P < P^*(0)$, with $\hat{F}(P^*(0)) = 1$. The lower bound of the support of $\hat{F}$, written $\underline{P}$, is determined by:

$$\Pi(\underline{P},\tilde{K}(\underline{P},1)\,|\,1) = 0$$

Thus, $\hat{F}$ gives a Nash equilibrium in prices, and further it can be shown using standard techniques (see Varian (1980), for example) that $\hat{F}$ gives the unique symmetric equilibrium of the subgame. *Q.E.D.*

Proof of Proposition 6. First consider the firms' profit–maximizing price choices. When pricing below the reservation level, firms solve the following constrained–maximization problem:

$$\text{(A1)} \qquad \tilde{P}_1(P_2,M,\underline{W}) = \arg\max_{P_1} \Pi(P,\bar{K}(P,M)\,|\,M) \;\; \text{subject to} \;\; W(P) \geq \underline{W}$$

For sufficiently low P_2 and high M, the constraint will not bind and we have $\tilde{P}_1(P_2,M,\underline{W}) = P_1^*(P_2,M)$. For high P_2 and low M, in contrast, the solution is $\tilde{P}(P_2,M,\underline{W}) = \underline{P}_1(P_2)$, where $W(\underline{P}_1(P_2),P_2) = \underline{W}$. Let $\bar{\Pi}(P_2,M,\underline{W})$ give the maximized profit level corresponding to (A1).

We now obtain profit–maximizing prices above the reservation level. For $M > 0$ and P_1 sufficiently close to $P_1^*(M)$, we define $P_2^z(P_1,M)$ by:

$$\text{(A2)}\quad \Pi(P_1,P_2^z,\tilde{K}(P_1,P_2^z,M)\,|\,M) = 0 \;\text{ and }\; P_2^z < \arg\max_{P_2} \Pi(P_1,P_2,\tilde{K}(P_1,P_2^z,M)\,|\,M)$$

Put $P_2^z(P_1,0) = \lim_{M\to 0} P_2^z(P_1,M)$. Clearly, we have $\partial P_2^z/\partial M < 0$.

Letting $M^o(P_1)$ give the expected share of the uninformed obtained when the firm prices above the reservation level, it follows that the profit–maximizing choice of P_1, holding P_2^z constant, must satisfy:

$$\text{(A3)}\qquad \Pi_{P_1}(P,\tilde{K}(P,M)\,|\,M) + \Pi_M(P,\tilde{K}(P,M)\,|\,M)\,\frac{\partial M^o(P_1)}{\partial P_1}\,U = 0$$

where $P = (P_1,P_2^z(P_1,M))$. As before, we have $M = M^o(P_1)$ in equilibrium, and inverting M^o and rearranging gives:

$$\text{(A4)}\qquad \frac{\partial P_1^o(M)}{\partial M} = \frac{-\Pi_M(P_1^o(M),P_2^z,\tilde{K}(P_1^o(M),P_2^z,M)\,|\,M)\,U}{\Pi_{P_1}(P_1^o(M),P_2^z,\tilde{K}(P_1^o(M),P_2^z,M)\,|\,M)}$$

where P_2^z is evaluated at $(P_1^o(M),M)$. We take P_1^o to be the solution of (A4) with $P_1^o(0)$

$= P_1^*(0)$; clearly, $P_1^o(M)$ is strictly decreasing and $P_1^o(M) < P_1^*(P_2^z(P_1^o(M),M),M)$. Further, we have that $P_2^z(P_1^o(M),M)$ is strictly decreasing in M.

Defining $\overline{M}(\underline{F},F)$ as in (14), let $\underline{F}(P_2)$ be given by:

$$\underline{F}(P_2) = \inf\{F' \mid \tilde{\Pi}(P_2,\overline{M}(F',F),\underline{W}) < 0 \text{ for all } F < F'\}$$

Clearly, $\tilde{\Pi}(P_2,\overline{M}(\underline{F}(P_2),\underline{F}(P_2)),\underline{W}) = 0$.

We now construct the mixed strategy $\hat{H}(P_2)$, as follows. First, let highest price in the support of $\hat{H}$, denoted by $\overline{P}_2$, be given by:

$$\overline{P}_2 = \arg\max_{P_2} \tilde{\Pi}(P_2,U/N,\underline{W})$$

We obtain an equilibrium in which $W(P) \geq \underline{W}$ with probability one as long as the following condition holds:

$$\text{(A5)} \qquad \tilde{\Pi}(\overline{P}_2,U/N,\underline{W}) \geq 0$$

In this case, for $P_2 < \overline{P}_2$, define $\hat{H}$ by:

$$\text{(A6)} \qquad \tilde{\Pi}(P_2,U/N + (1 - \hat{H}(P_2))^{N-1}I,\underline{W}) = \tilde{\Pi}(\overline{P}_2,U/N,\underline{W})$$

Clearly, $\hat{H}$ is continuous, and using (A5) it may be shown that $\hat{H}$ must be strictly increasing in P_2. If (A5) does not hold, then we instead put $\overline{P}_2 = P_2^*(0)$. Letting $M^z(P_2)$ give the inverse of $P_2^z(P_1^o(M),M)$, $\hat{H}$ is given by:

$$\text{(A7)} \qquad (1 - \hat{H}(P_2))^{N-1} = M^z(P_2)$$

Eventually we must have $1 - \hat{H}(\underline{P}_2) = \underline{F}(\underline{P}_2)$, which defines $\underline{P}_2$. For smaller P_2, $\hat{H}$ is given by:

$$\tilde{\Pi}(P_2, \overline{M}(\underline{F}(P_2), 1 - \hat{H}(P_2)), \underline{W}) = 0 \tag{A8}$$

In this case also, $\hat{H}$ is clearly continuous and strictly increasing. Further, we have $W(P_1^o(\underline{F}(\underline{P}_2)^{N-1}), \underline{P}_2) < \underline{W}$, for otherwise we could write:

$$\tilde{\Pi}(\underline{P}_2, \overline{M}(\underline{F}, \underline{F}), \underline{W}) \geq \Pi(P_1^o(\underline{F}(\underline{P}_2)^{N-1}), \underline{P}_2, \tilde{K} \mid \overline{M}(\underline{F}, \underline{F}))$$

$$> \Pi(P_1^o(\underline{F}(\underline{P}_2)^{N-1}), \underline{P}_2, \tilde{K} \mid \underline{F}(\underline{P}_2)^{N-1}) = 0$$

Let us now check that the choices of P_1 corresponding to P_2 are profit–maximizing. In the case that (A5) holds, a firm choosing $P_1 = \underline{P}_1(P_2)$ can deviate to $\overline{P}_1(P_2) > \underline{P}_1(P_2)$, where $W(\overline{P}_1(P_2), P_2) = \underline{W} - c$; thus, $P_1^d(P_2) = \min\{\overline{P}_1(P_2), P_1^*(P_2; (1 - \hat{H}(P_2))^{N-1}I)\}$ gives the best deviation. We may define $\Delta(P_2, M)$ as in the proof of Lemma 3, and we have:

$$\frac{d}{dM} \Delta(\hat{P}_2(M), M) = \{ -\Pi_{P_1}(P_1^d, \hat{P}_2, \tilde{K} \mid M) \frac{\partial \overline{P}_1}{\partial P_2} + \Pi_{P_1}(\underline{P}_1, \hat{P}_2, \tilde{K} \mid M) \frac{\partial \underline{P}_1}{\partial P_2}$$

$$- \int_{\underline{P}_1}^{P_1^d} \frac{\partial}{\partial P_2} \Pi_{P_1}(P, \hat{P}_2, \tilde{K} \mid M) dP + \int_{M-U/N}^{M} \frac{\partial}{\partial P_2} \Pi_M(P_1^d, \hat{P}_2, \tilde{K} \mid X) dX \} \frac{\partial \hat{P}_2}{\partial M}$$

$$-\int_{\underline{P}_1}^{P_1^d} \frac{\partial}{\partial M} \Pi_{P_1}(P,\hat{P}_2,\tilde{K}\,|\,M)dP$$

$$+ [\Pi_M(P_1^d,\tilde{K}(P_1^d,M)\,|\,M) - \Pi_M(P_1^d,\tilde{K}(P_1^d,M - U/N)\,|\,M - U/N)]$$

where $\hat{P}_2(M)$ is the inverse of $M = U/N + (1 - \hat{H}(P_2))^{N-1}I$. To see that the term in braces is strictly negative, note first that $0 \leq \Pi_{P_1}(P_1^d,\hat{P}_2,\tilde{K}\,|\,M) < \Pi_{P_1}(\underline{P}_1,\hat{P}_2,\tilde{K}\,|\,M)$, and further, $0 > \partial\overline{P}_1/\partial P_2 > \partial\underline{P}_1/\partial P_2$, due to the convexity of $W(P)$ in P_1; thus, the sum of the first two terms within the braces is strictly negative. For the third and fourth terms, we have:

$$-\frac{\partial}{\partial P_2} \Pi_{P_1}(P_1,\hat{P}_2,\tilde{K}\,|\,M) = -\Pi_{KP_1} \frac{\partial \tilde{K}}{\partial P_2} < 0$$

$$\frac{\partial}{\partial P_2} \Pi_M(P_1^d,\hat{P}_2,\tilde{K}\,|\,M) = \frac{1}{M} \Pi_{P_1}(P_1^d,\hat{P}_2,\tilde{K}\,|\,M) \frac{\partial \overline{P}_1}{\partial P_2} + \frac{\partial}{\partial M} \Pi_{P_2}(P_1^d,\hat{P}_2,\tilde{K}\,|\,M) < 0$$

where the second inequality invokes Assumption 4. Since $\partial\hat{P}_2/\partial M < 0$, while the remaining terms are positive as in the proof of Lemma 2, we have that $d\Delta(\hat{P}_2(M),M)/dM > 0$; since $\Delta(\hat{P}_2(U/N),U/N) \geq 0$, using (A5), we have that the firm will not prefer to deviate to P_1^d.

As for the case in which (A5) fails, for $P_1 < P_1^*(\hat{P}_2(M),M)$, we have:

$$\frac{d}{dM} \frac{-\Pi_M(P_1,\hat{P}_2,\tilde{K}\,|\,M)\,U}{\Pi_{P_1}(P_1,\hat{P}_2,\tilde{K}\,|\,M)} = -\{\, \Pi_{P_1} [\frac{\partial}{\partial M} \Pi_{P_2}(P_1,\hat{P}_2,\tilde{K}\,|\,M) \frac{\partial \hat{P}_2}{\partial M} + \Pi_{MK} \frac{\partial \tilde{K}}{\partial M}]$$

$$- \Pi_M[\Pi_{P_1K} \frac{\partial \bar{K}}{\partial P_2} \frac{\partial \hat{P}_2}{\partial M} + \frac{\partial}{\partial M} \Pi_{P_1}(P_1,\hat{P}_2,\tilde{K}|M)] \}/\Pi^2_{P_1} < 0$$

using Assumption 4. Thus, the same argument as in the proof of Lemma 6 implies that $P_1^o(M)$ defines profit–maximizing prices for firms choosing $P_2 > \underline{P}_2$, and similarly we can show that the indicated selections of P_1 are profit–maximizing for $P_2 \leq \underline{P}_2$.

Whether or not (A5) holds, as P_2 falls, the equilibrium expected market share $\hat{M}(P_2) = U/N + (1 - \hat{H}(P_2))^{N-1}I$ rises; thus, we may define $\underline{M}^m(\underline{W})$ by:

$$\underline{M}^m(\underline{W}) = \inf\{M \,|\, \bar{P}_1(P_2,\hat{M}(P_2),\underline{W}) = P_1^*(P_2,\hat{M}(P_2)) \text{ for } M = \hat{M}(P_2)\}$$

and we have that a firm chooses $W(P) > \underline{W}$ if and only if its expected market share satisfies $M > \underline{M}^m(\underline{W})$. Thus, we may express the search problem of the uninformed in terms of the distribution of expected market shares, where the distribution $\hat{G}$ is exactly as given in (9). We have:

$$\text{(A9)}\quad \underline{W}^r(\underline{W}) = \underline{W}\hat{G}(\underline{M}^m(\underline{W})) + \int_{\underline{M}^m(\underline{W})}^{1} W(P_1^*(\hat{P}_2(M,\underline{W}),M),\hat{P}_2(M,\underline{W}))d\hat{G}(M) - c$$

where $\hat{P}_2(M,\underline{W})$ gives the inverse of $\hat{M}(P_2)$, making explicit the dependence on $\underline{W}$. We have an equilibrium at $\underline{W} = \underline{W}^r(\underline{W})$, where existence of the fixed point follows from boundedness of the price choices along with continuity of the relevant functions on the right–hand side of (A9). *Q.E.D.*

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Figure 1

Determination of Equilibrium

A - equilibrium for $c < \bar{c}$

B - equilibrium for $c = \bar{c}$

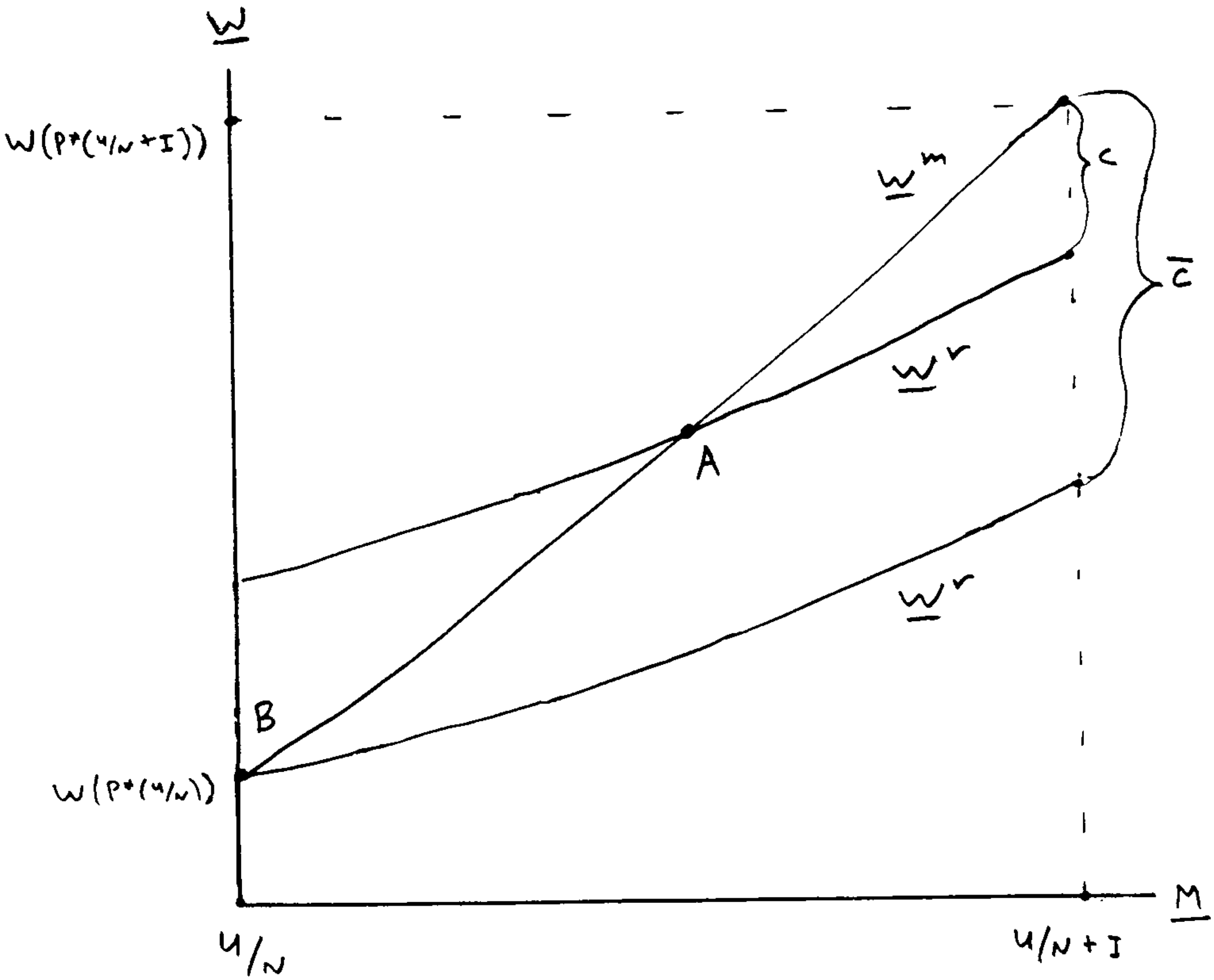

Figure 2

Effect of Decrease in c
on Equilibrium Pricing

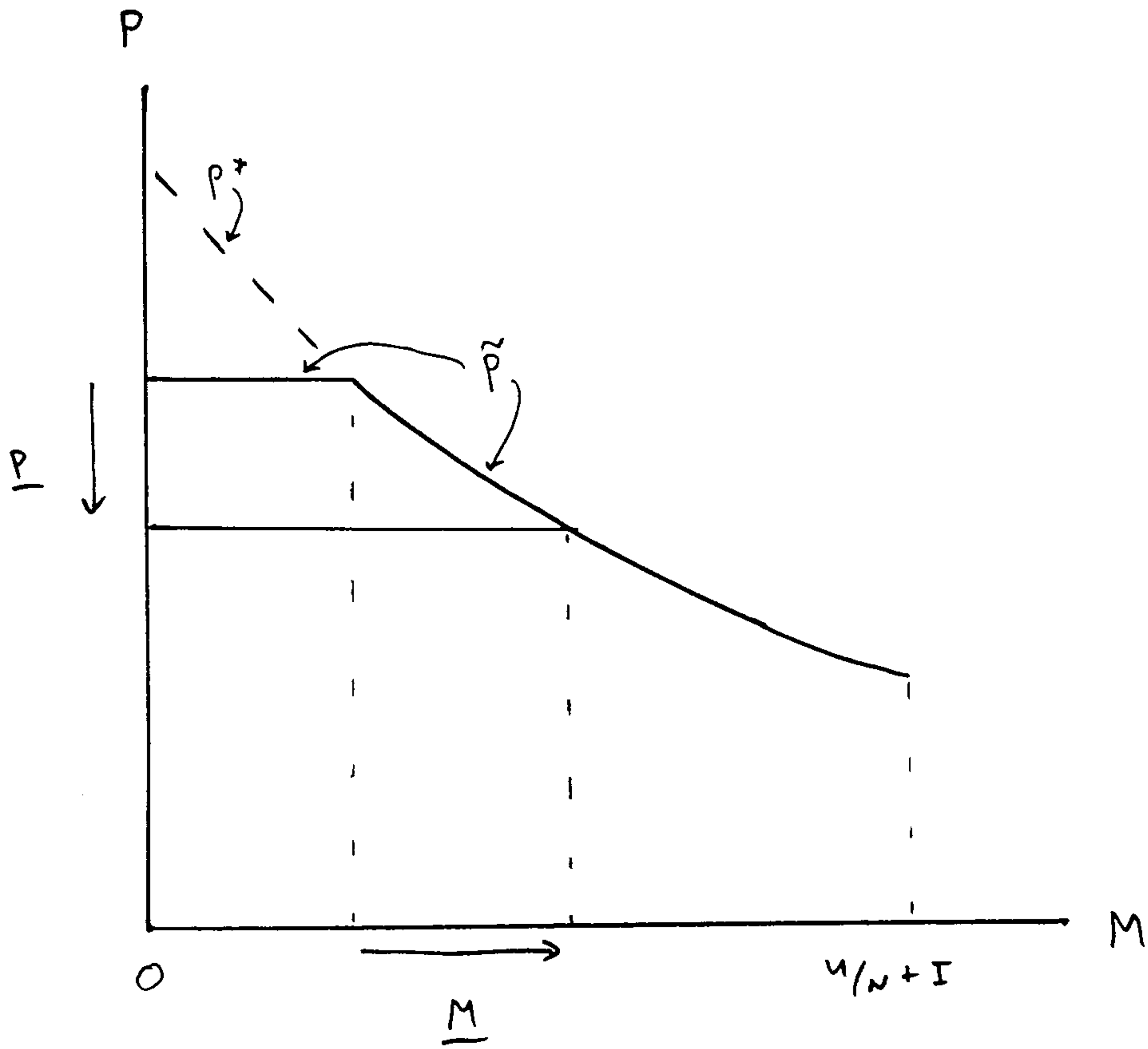

Figure 3

Equilibrium Pricing

in Small u/N Case

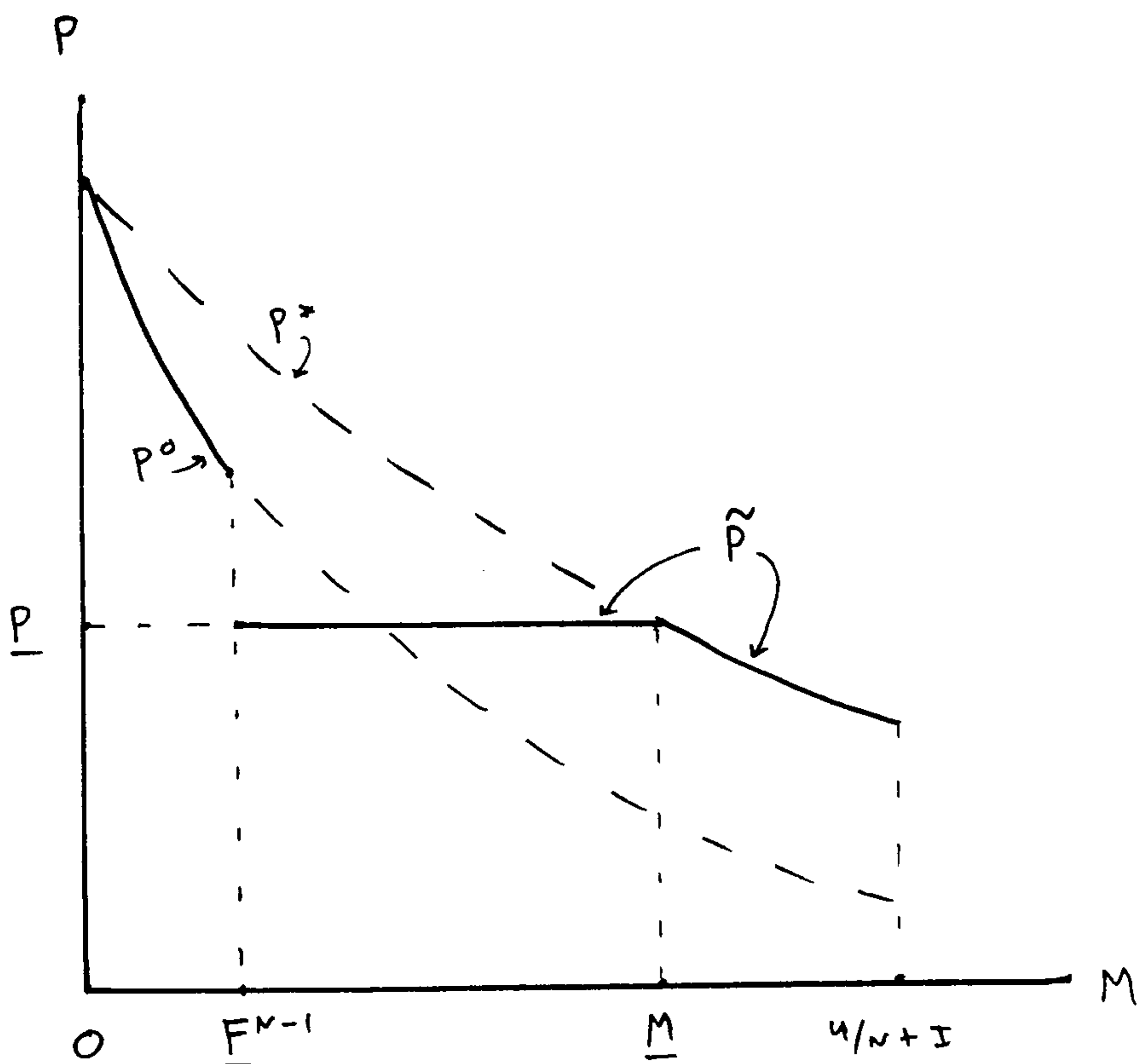

Figure 4

Verification of Optimality of P^o