Discussion Paper No. 1145

SCREENING CONSUMERS THROUGH ALTERNATIVE PRICING MECHANISMS

by

Eugenio J. Miravete
Department of Economics
Northwestern University

First version: May 10th, 1995
This version: December 19th, 1995

Screening Consumers Through Alternative Pricing Mechanisms

Eugenio J. Miravete † *

Department of Economics
Northwestern University

First version: May 10th, 1995
This version: December 19th, 1995

Abstract

This paper addresses the optimal design of optional nonlinear tariffs. Two particular solutions commonly used in telecommunications and other industries are fully characterized. These optimal outlay schedules illustrate how the tariff design is altered when there exists a time lag between tariff choice and consumption. In this model consumers' uncertainty is resolved in the *interim*, between the tariff choice and the usage decision, through changes in their types. The paper studies whether the monopolist may profit from screening consumers according to different information sets, and it shows that expected profits are higher under an ex–post tariff if the variance of the ex–ante type distribution is large enough. The paper also shows that no results regarding social efficiency may be obtained in general. Welfare comparison of optional tariffs will be very sensitive to type distributions, how types enter demand specifications, and the relative variance of the type components. JEL: D42, D82, L96.

---

† Department of Economics. Northwestern University. 2003 Sheridan Road. Evanston, IL 60208-2600, U.S.A. Internet: *miravete@merle.acns.nwu.edu*

* I wish to thank John Panzar for his guidance on the present research and, for his many suggestions over several endless discussions on this paper. I also thank Kyle Bagwell, Robert Porter and Daniel Spulber for helpful comments and suggestions. Financial support from the Ministerio de Educación y Ciencia, Spain is gratefully acknowledged.

:/.../eugenio/thesis/thesis01.tex

1 Introduction

Telecommunications, and in particular the telephone industry, has given rise to several interesting pricing issues for over a century. Many of these pricing practices have been applied to other industries. A common practice in telecommunications pricing is optional tariffs. Even though most residential telephone customers in the United States pay a flat fee for local telephone service or a flat charge per untimed call, local exchange carriers (LECs) also offer them optional local measured service.

Local measured service includes a fixed fee for the provision of a dial tone, and a variable usage unit charge. In general, the usage charge is a multidimensional tariff that distinguishes calls by time–of–day or distance, but also includes quantity discounts. Many LECs provide a calling allowance with the fixed fee, offer reduced rates for frequently called numbers or bulk rates for business customers. In general, measured service is a class of taper, *i.e.*, a particular declining block tariff for which the effective marginal tariff is determined at the end of the billing period, when consumption is realized. Any declining block tariff is a piecewise linear and concave outlay schedule and, according to Faulhaber and Panzar (1977), it is equivalent to the set of self–selecting two–part tariffs which forms the lower envelope of the schedule provided that consumers' demands are non–stochastic. Since the effective price is determined according to actual consumption, once any kind of consumer uncertainty is resolved, tapers constitute a "pay-as-you-go" billing method. I will call tapers *"ex-post pricing"* and denote by $\{\hat{p}(\theta), \hat{A}(\theta)\}$ the associated ex–post two–part nonlinear price schedule, where θ is the ex–post consumer type.

Flat rate service consists of a fixed fee for local telephone service which includes the provision of a dial tone and unlimited calling to a local area. Therefore, flat rate service may be considered one of the options of the monopolist's optional calling plans (OCPs), *i.e.*, a set of two–part tariffs, each consisting of a subscription fee and an usage charge that is chosen by consumers before their consumption is realized. Flat rate service consists of a unique two–part tariff OCP with a fixed fee and zero marginal charge. In contrast to the features of the ex–post tariff, the choice of an OCP is made at the beginning of the billing period when consumers are uncertain about circumstances that will determine their usage. I will call OCPs *"ex-ante pricing"* and denote by $\{\tilde{p}(\theta_1), \tilde{A}(\theta_1)\}$ the set of ex–ante two–part nonlinear tariffs, where θ_1 denotes the ex–ante consumer type.

Ex–ante tariff choice and its implications for the optimal tariff design have not been addressed very often, even though their use is quite common [Wilson (1993, §2)]. Several empirical studies rely on the underlying consumer maximization problem, but none address the issue of an ex–ante tariff choice that may depend on variables different from those that condition the usage decision. MacKie–Mason and Lawson (1993), assume that usage and tariff choice are simultaneous decisions and therefore explained by the same exogenous variables. It may be unavoidable to assume simultaneous choice for empirical purposes because of the lack of detailed information, even when the timing of the problem still matters for the specification of the stochastic structure of the empirical model, but the theoretical framework must deal with this aspect of the problem.

The aim of this paper is to study the optimal design of $\{\hat{p}(\theta), \hat{A}(\theta)\}$ and $\{\tilde{p}(\theta_1), \tilde{A}(\theta_1)\}$ by a monopolist. I study the case in which these tariffs are offered in isolation. This procedure allows me to evaluate whether the ex–ante or the ex–post tariff has a greater power to screen consumers and provide the monopolist with higher profits. The approach will also be helpful to evaluate whether it is socially efficient for the regulatory agency to allow the existence of optional calling plans, or to restrict telephone pricing schedules to measured service.

The present approach considers the existence of stochastic features for consumers demands through changes in consumers' types between the choice of tariff and the usage decision. The idea is that the tariff choice depends on conjectures about the state of the world, while consumption depends on the realized state. I capture this increase in the information set of customers by assuming a one–dimensional ex–post consumer type $\theta = \theta(\theta_1, \theta_2)$ which depends on two ex–ante dimensions. When consumers choose between tariffs, they only know one dimension of their type, θ_1, but when they decide how much to consume, they know both dimensions. By this construction, the lag between tariff choice and usage level involves time so that the revelation of the state defines consumers' ex–post type. The ex–ante known dimension θ_1 of the ex–post two–dimensional type constitutes the ex–ante type, which captures the expected state of the world that will condition consumers' decisions on usage level. I will call the other component θ_2 of the ex–post type, the *shock*. The shock defines the ex–post type "around" the ex–ante type, and it embodies the increase in the consumer information set from the tariff choice to the usage level decision. As a consequence, the optimal ex–ante tariff will equal the ex–post tariff in absence of these stochastic elements, provided that the optimal outlays are concave.

There are few works that deal explicitly with the information structure of an asymmetric information problem. As in Lewis and Sappington (1994), the present model gives the monopolist the ability to decide which information structure will consumers use to choose among tariffs but not to decide on consumption because in the present model learning occurs independently of the monopolist's decision, and optimal consumption is determined by the ex–post information structure given the previous tariff choice. Therefore, the monopolist decision on the information structure will affect the optimal purchase only through the choice of tariff.

The work of Clay, Sibley and Srinagesh (1992) is the most complete existing approach to the optional tariff problem[1]. They assume that OCPs and tapers can be

[1] Besides this work, only Dansby (1983) addresses welfare comparison of ex–ante and ex–post based nonlinear tariffs. It is shown that for many demand distributions welfare is lower with flat rate service (ex–post tariff).

implemented by a menu of two–part tariffs which maximize the corresponding expected profit function. The analysis is carried out assuming a particular demand specification and a discrete symmetric distribution of the ex–post type around a finite number of ex–ante mean types. The range of the shock is restricted to be small enough in order to collapse the two dimensional ex–ante consumer type into a one dimensional ex–post characterization. All these assumptions lead to expected demand and expected consumer surplus that are independent of the support and distribution of the ex–post type. Their main results are that for small levels of uncertainty regarding customer demand, expected profit to the firm is higher with the OCPs than with a taper and, second, total surplus and consumer surplus (under very restrictive conditions) are higher on the profit maximizing set of optional two–part tariffs than on the profit maximizing taper.

The major criticism of Clay, Sibley, and Srinagesh's (1992) paper is that they infer general results from a very particular formulation of the problem. By contrast with their work, the present one characterizes the optimal tariffs applying the Revelation Principle [Myerson (1979)]. The existence of sets of self–selecting tariffs that implement the corresponding outlay schedules can be therefore ensured by the choice of a distribution of consumers types with the increasing hazard rate property. The present model employs a continuum instead of a discrete number of types. This procedure simplifies the analysis by avoiding any possibility of pooling of the ex–post types around the corresponding ex–ante types. Using this approach, I present an example that provides different results than those of Clay, Sibley, and Srinagesh (1992).

This example shows that welfare results critically depends on the relative variance of type components. The monopolist will offer an ex–post outlay if consumers are heterogeneous enough ex–post. Large ex–post heterogeneity is determined either by a large enough ex–ante heterogeneity, or by a large variance of the shock that adds to a relatively low ex–ante heterogeneity. By contrast, consumers only prefer the ex–post schedule if the shock has a strong effect on the definition of the ex–post type.

The main result of this paper is to show how the relative variance of type components affects the expected social efficiency of the tariffs. The example shows that results on the private and social desirability of ex–ante *v.s.* ex–post tariffs cannot be established in general if stochastic elements are present in consumers demands. Furthermore, this result is established assuming that consumers are risk neutral.

The work is organized as follows. In section 2, I review several examples of optional tariffs. This section analyzes pricing and marketing practices which are relevant beyond the limits of telecommunications. In section 3, the optimal taper and optimal OCPs are fully characterized. Section 4 presents the expressions which characterize the efficient pricing for each case. Section 5 compares the ex–ante and ex–post tariffs, and develops an example to evaluate their relative dominance. Finally, section 6 summarizes the main conclusions of the paper.

2 Examples of Optional Tariffs

Over the last decade, optional tariffs have grown in use in US telecommunications. They are not limited to local calls: interexchange carriers (IXCs) –AT&T, MCI, and Sprint– compete by offering optional calling plans with different features for long distance calls. AT&T introduced the *"Reach-Out of America"* plan in June 1984, after approval by the Federal Communication Commission (FCC). Initially the plan included three billing methods, all of them measured services that required the payment of different monthly fees. The *Two-Part Plan* offered a uniform percentage discount in off–peak periods. The *Tapered Plan* reduced the marginal usage charge after the first hour of usage in off–peak periods. Finally, the *Block–of–Time Plan* offered the allowance of one hour of usage in off–peak periods, and a lower marginal tariff for additional usage. This OCP tried to divide customers into three separate groups depending whether they were low, medium, or high usage. Since then, the *"Reach-Out of America"* plan has evolved, developing a broader variety of pricing structures and lowering tariffs as the LECs reduced their access charges. Tariffs were also reduced to account for imputed productivity gains since 1989, as the OCPs have been subject to the overall price–cap regulation on AT&T's residential and small business services. MCI and Sprint also offer OCPs for long distance calls in off–peak periods[2]. Business customers may also choose among optional wide area telecommunications service (WATS) tariffs. AT&T offers a declining block tariff with quantity discounts, *"PRO WATS"*, or the optional *"Megacom WATS"* that ensures the lowest marginal usage charge if consumption exceeds 500–600 hours per month, according to Mitchell and Vogelsang (1991, §9).

In addition, OCPs have been used as marketing strategies with interesting economic insights, as for example MCI's 1991 *"Friends and Family Circle"* plan. This plan offered a 20% discount on calls to twelve pre–selected numbers whenever the called customers also used MCI as their long distance carrier. This plan involves aspects of the ex–ante plan choice *vs.* ex–post usage, together with consumers' switching costs through induced network externalities as a way of expanding the carrier's activity.

Of particular interest is the comparison between MCI's *"Friends Around the World Anytime"* and Sprint's *"The Most"* plans. Both are measured services that require monthly payments. MCI's plan offers low rates in off–peak periods for international calls plus an additional 20% discount to some pre–selected international numbers. Sprint's plan is similar but instead of forcing customers to choose ex–ante which numbers to apply the additional 20%, a 50% discount applies to the most often called number in each billing period. Therefore, while MCI's plan is mainly addressed to customers with quite stable calling patterns, Sprint's plan is better designed for customers with high pattern variability in their calling, so that they receive the discounts according to the ex–post decision on telephone usage. The point here is that IXCs specialize into ex–ante and ex–post discounts

---

[2] See Mitchell and Vogelsang (1991, §8) for a description of these plans by mid–1980.

when they compete through tariff design. This may be an interesting aspect of competition in nonlinear tariffs for almost perfect substitutive goods when consumers face demand uncertainty. Another interesting aspect of this case is that competitive carriers may design tariffs to product differentiate when services are perfect substitutes. This framework may provide a natural extension of the basic model of optional tariff design[3].

Cellular phones companies also offer several optional tariff plans to business and non–business customers. Cellular One Group offers ex–ante and ex–post based OCPs to business customers but only ex–ante based OCPs to non–business customers in the Chicago area. Calling plans consist of a set of two–part tariffs plus a calling allowance. The marginal tariff distinguishes between peak and off–peak demand periods. There are some differences between *Business* and *Non–Business* plans. Under any *Business* plan, customers are free to use their allowance either at *Prime Time* or at *Non–Prime Time*. However, all *Non–Business* plans distinguish between *Prime Time* and *Non–Prime Time* allowances. Moreover, *Non–Prime Time* is four hours longer than for any *Business* plan (7p.m.–7a.m. *vs.* 10p.m.–7a.m.). Finally, only *Business* offers the possibility of an ex–post based tariff, the *Security* plan.

Optional tariffs in the sense described above *i.e.*, ex–ante choice between a particular ex–ante based plan and the ex–post tariff, are not unique to the telecommunications industry. Some travellers may choose between buying a full fare airline ticket in order to get a frequent flyer reward or a less expensive industry discount ticket without a mileage premium. Users of public transport systems have to decide between paying each trip or buying a weekly card that allows free access to transport services within the chosen concentric band areas of an integrated tariff transportation system (*e.g.*, Paris).

Car rentals also offer optional tariffs in at least two dimensions of their pricing procedure: mileage and fuel. Consumers are usually offered the option between *"free mileage"* (flat rate service) or some mileage allowance plus some charge for additional miles (measured service) plus a lower fixed charge than the *"free mileage"* option. Car rental customers are also offered the possibility of buying a *"full tank option"*: the consumer may pay a fixed charge and forget about how full the tank of the rented car is when it is returned, or pay a lower fixed charge plus the value of the fuel needed to fill the tank (generally priced above the market standard).

Electricité de France (EDF) offers its customers the choice among several (multi-dimensional) tariffs. Customers choose ex–ante which plan they want and they are billed ex–post, according to their usage and the subscribed options. In this case, customers of

---

[3] Similar to this case but related to "quality" differentiation are AT&T's optional plans. Customers may either choose to pay a fee and obtain tariff discounts in some restricted hours or, without any payment, receive tariff discounts for calls directed to a chosen foreign country without any time constraint. The choice will depend on the time/geography patterns of customers calls.

EDF do not commit to a specific plan from a set of self–selecting two–part tariffs, but instead to a nonlinear plan from a set of nonlinear optional tariffs. Tariffs are divided by range of monthly power loads: *Tariff Bleu* up to 36 kVA, *Tariff Jaune* 36–250 kVA for residents and farmers, and *Tariff Vert A* up to 10000 kW, *Tariff Vert B* 10000–40000 kW,... for industrial and commercial customers. The total payment depends on the pre–selected options, *i.e.*, peak or off–peak consumption periods against the basic option or load duration. If the usage or time of consumption of customers get close to the limits established in his chosen tariff, then they are disproportionately surcharged[4].

3 Tariff Design

General treatments of the theory of nonlinear pricing can be found in Goldman, Leland, and Sibley (1984), Maskin and Riley (1984), Tirole (1989, §3), and Wilson (1993, §6). All these works address a static game of incomplete information where the monopolist only knows the distribution of consumers' types. According to this informational constraint, the monopolist designs an optimal fully nonlinear schedule that can be implemented by a set of two–part tariffs under reasonable assumptions on the distribution of consumers's type. The basic extensions of this model include the design of multidimensional and multiproduct tariffs[5], the design of state contingent nonlinear schedules [Spulber (1992a) and Spulber (1992b)]. The present model differs from the state contigent formulation not only because the shock may be different for each consumer but also because it remains private information for them so that the monopolist only knows its distribution even when the state of nature has been revealed (to consumers only).

In this section I will obtain the expressions for the optimal ex–ante and ex–post tariffs. In order to do that, suppose the following timing for the game. A monopolist without any capacity constraint may offer his customers either the choice among ex–ante tariff plans or among ex–post tariff plans. In the case of the ex–ante tariff a representative risk neutral consumer chooses the tariff plan before she knows her ex–post type, and the monopolist commits to bill her according to the plan chosen. When the state of nature is revealed and the consumer (only) learns her ex–post type, usage level is determined by maximizing her utility subject to the billing system that she chose. In the case of the ex–post tariff, the consumer chooses simultaneously the usage and the tariff plan when she knows her ex–post type. The ex–ante type θ_1 is private information for the consumer,

[4] For example, the *"critical–times"* option of the blue tariff offered in 1987 a 36% lower energy charge than the basic option, but imposes an 800% surcharge for the energy consumed after the utility broadcasts a "power in scarce supply" announcement. For a more detailed review of the electric industry see Wilson (1993; §2.2, §2.3).

[5] On multidimensional tariffs see Oren, Smith, and Wilson (1985), Panzar and Sibley (1978), and Wilson (1993, §9–11). On multiproduct tariffs see Calem and Spulber (1984), Oren, Smith, and Wilson (1982), Sibley and Srinagesh (1995), Spulber (1989), and Wilson (1993, §12–14).

and the monopolist only knows its distribution. However, the monopolist and consumers have the same prior on the distribution of the shock θ_2. Since the monopolist knows the distribution of both dimensions, he can compute the optimal nonlinear schedules using either the distribution of the ex–ante type θ_1 or the distribution of the ex–post type $\theta = \theta(\theta_1, \theta_2)$. Before obtaining the optimal tariff, I will present the technical requirements of the model.

Individuals are described by their type. There exists a multidimensional characterization of consumers' ex–ante type. The analysis will be carried out within a two–dimensional space ($l = 2$), $(\theta_1, \theta_2) \in R^2$. The definition of the ex–post type $\theta \in R$ is normalized such that $\frac{\partial \theta}{\partial \theta_i} > 0$, for $i = 1, 2$. Type parameters are distributed according to a distribution function $(\theta_1, \theta_2) \sim F(\theta_1, \theta_2)$ that is assumed to have a continuously differentiable density function $f(\theta_1, \theta_2)$ defined on a convex support $D_1 \times D_2 = [\underline{\theta}_1, \overline{\theta}_1] \times [\underline{\theta}_2, \overline{\theta}_2] \subseteq R^2$. Since I assume that the effect of the shock is to modify the ex–ante type through some functional specification, I also normalize the support of the shock in order to obtain an expected ex–post type centered around the corresponding ex–ante type:

$$\int_{D_2} \theta(\theta_1, \theta_2) f(\theta_2 \mid \theta_1) d\theta_2 = \theta_1 \qquad : \qquad \forall \theta_1 \tag{1}$$

where $f(\theta_2 \mid \theta_1)$ denotes the density function of the shock conditional on the consumer's ex–ante type θ_1. From here, through the definition of the ex–post type $\theta(\theta_1, \theta_2)$, the joint distribution $F(\cdot)$ gives the distribution of the transformed variable $G(\theta)$ and its support D.

The one–dimensional characterization of the ex–post type through the real valued function $\theta = \theta(\theta_1, \theta_2)$, enables us to avoid the complex analysis of a proper multidimensional ex–post type directly related to the multidimensional ex–ante type formulation of the model. This is because if ex–post types are ordered along this one–dimensional curve there is no qualitative difference between the multidimensional and the one–dimensional type formulation [Wilson (1993, §8.4)]. There are further reasons to avoid general multidimensional formulations[6]. In particular, Wilson (1993; §12, §14) and Wilson (1995) have recently addressed the computational difficulties of solving general models with multidimensional consumer's type. His results (drawn from particular examples) show qualitative differences with the one–dimensional type model. Prices are no longer monotone functions of the quantities purchased and, in the case of a two–product monopolist, consumers purchase either both commodities or neither.

In the present model, there is one ($n = 1$) good x (local telephone service) with price p (marginal tariff), while income y is taken as the numeraire. Therefore, $(p, 1) \in R^2$. In addition, and for simplicity, I assume that there is no income effect. The small share

---

[6] A two–dimensional type model of price discrimination was first solved by Laffont, Maskin, and Rochet (1987).

of local phone bills in consumers' incomes justifies the constant marginal utility of income assumption. The assumed net indirect utility function is:

$$V(p,\theta,A) = v(p,\theta) - A = \int_p^{\infty} x(z,\theta)dz - A \tag{2}$$

where A is the fixed charge associated to the chosen tariff plan and $x(p,\theta)$ is consumer's demand function. Each nonlinear price schedule is implemented through a continuum of two–part tariffs. The subutility function $v(p,\theta)$ is assumed to be at least three times continuously differentiable over $[0,\infty) \times D$. In addition I also assume:

$$v(\infty,\theta) = 0 \quad \forall\theta \in D \tag{3.a}$$

$$\nabla_\theta[v(p,\theta)] > 0 \quad \Longleftrightarrow \quad (v_{\theta_1}(p,\theta), v_{\theta_2}(p,\theta)) > (0,0) \tag{3.b}$$

Because of the no–income effect assumption Roy's Law ensures that:

$$V_p(p,\theta,A) = v_p(p,\theta) = -x(p,\theta) \tag{4}$$

The indirect utility function is convex in price and the single crossing property (SCP) relative to θ is assumed to hold. Therefore, since by normalization $\frac{\partial\theta}{\partial\theta_i} > 0 \ \forall i$, demand derivatives have the following signs:

$$x_p(p,\theta) = -v_{pp}(p,\theta) < 0 \tag{5.a}$$

$$x_{\theta_1}(p,\theta) = -v_{p\theta_1}(p,\theta) > 0 \tag{5.b}$$

$$x_{\theta_2}(p,\theta) = -v_{p\theta_2}(p,\theta) > 0 \tag{5.c}$$

For simplicity, I consider that the monopolist's cost function includes a constant marginal cost c, and a fixed cost k. Regarding pricing, the monopolist has two options. He can offer either an ex–ante or an ex–post tariff on a stand alone basis. Here, as in Clay, Sibley and Srinagesh (1992), I study whether the presence of customer–level demand uncertainty makes the monopolist prefers to have consumer choice take place at the start of the billing period, when demand uncertainty is present, or at the end of the billing period, when it is absent.

3.1 Characterization of the Ex–Post Tariff

With an ex–post tariff, the consumer pays according to her consumption at the end of the billing period. Consumers optimally choose the usage level given their (known) preferences and the monopolist's tariff. Provided that this outlay schedule is concave, it is equivalent to address the dual problem of choosing the optimal two–part tariff that will apply to consumers' optimal purchases.

As is now standard in the literature, the monopolist's optimal design of the ex–post tariff may be solved using the Revelation Principle. The monopolist (principal) designs an optimal direct mechanism $\{\hat{p}(\theta), \hat{A}(\theta)\}$ which induce consumers (agents) to truthfully reveal their ex–post, private information. For this mechanism to be incentive compatible (IC) consumers must find it optimal to reveal their true ex–post type to the monopolist, *i.e.*:

$$\theta \in \arg\max_{\theta'}[v(p(\theta'), \theta) - A(\theta')]$$

so that a consumer of type θ will be billed according to the two-part tariff defined by the mechanism $\{\hat{p}(\theta'), \hat{A}(\theta')\}$ if in the communication game she reveals θ' as her type. Solving this problem we obtain the IC condition:

$$A'(\theta) = -x(p(\theta), \theta)p'(\theta) \qquad \forall \theta \in D \tag{6}$$

The sufficient condition for this problem to be concave is that $p'(\theta) \leq 0$, *i.e.*, that consumers with higher valuations pay lower marginal tariffs[7]. Let us now denote by $V(\theta) = v(p(\theta), \theta) - A(\theta)$ the consumer's surplus when she subscribes an ex–post tariff plan. By the Envelope Theorem, using equation (6) we get the following condition:

$$V'(\theta) = v_\theta(p(\theta), \theta) \tag{7}$$

Given the consumer's optimal tariff plan choice, the monopolist offers a continuum of two–part tariffs which solves the following problem:

$$\max_{p(\theta), A(\theta)} \int_{\theta^*}^{\overline{\theta}} [A(\theta) + (p(\theta) - c)x(p(\theta), \theta)]\, g(\theta) d\theta - k$$

$$\text{s.t.} \quad V(\theta) = v(p(\theta), \theta) - A(\theta)$$

$$V'(\theta) = v_\theta(p(\theta), \theta)$$

$$V(\theta^*) \geq 0$$

As it is shown in the appendix, the solution to this problem characterizes the optimal ex–post tariff. The outcome functions of the ex–post mechanism are:

$$\hat{p}(\theta) = c - \frac{1 - G(\theta)}{g(\theta)} \frac{x_\theta(p(\theta), \theta)}{x_p(p(\theta), \theta)} \tag{8}$$

$$\hat{A}(\theta) = v(\hat{p}(\theta), \theta) - V(\theta^*) - \int_{\theta^*}^{\theta} v_\theta(\hat{p}(z), z) dz \tag{9}$$

---

[7] The necessary condition is that $V_{\theta'}[p(\theta), \theta, A(\theta)] = 0$. Totally differentiating this condition leads to $V_{\theta'\theta'}[p(\theta), \theta, A(\theta)] = -V_{\theta\theta'}[p(\theta), \theta, A(\theta)]$. Therefore, the local sufficient condition is $V_{\theta\theta'}[p(\theta), \theta, A(\theta)] = -x_\theta(p(\theta), \theta)p'(\theta) \geq 0$. Since $x_\theta(\cdot) > 0$ by equation (5), the fulfillment of the second order condition requires that $p'(\theta) \leq 0$.

In fact for most commonly used indirect utility functions (with SCP non increasing in p), these equations suffice to characterize the optimal solution if the individual net revenue function is concave in p. This result is stated in the following proposition. All results are proved in the appendix.

PROPOSITION 1: *Equations* $(8)-(9)$ *suffice to characterize the monopolist's maximum expected profit if net revenue is concave in* p, *and* $v_{pp\theta}(p,\theta) \leq 0$.

The ex–post pricing problem becomes a standard nonlinear pricing problem because of the assumed single dimension of consumers ex–post types. Therefore, provided that the single crossing property holds, and the hazard rate of the distribution of ex–post types is increasing, the monopolist can discriminate among consumers (no bunching) by offering a continuum of self–selecting two–part tariffs that implement the optimal nonlinear outlay schedule. It also follows that consumers with higher valuation buy larger quantities of the monopolized good, and that only the highest ex–post consumer type is efficiently priced. All these results are formally presented in the following theorem.

THEOREM 1: *Assume that the SCP holds and that the ex–post distribution function* $F(\theta)$ *has an increasing hazard rate. Then, the solution to the ex–post problem has the following properties:*

a) If $\{\hat{p}(\theta), \hat{A}(\theta)\}$ *is an IC mechanism, it is also almost everywhere differentiable. Consumers with higher valuations pay lower marginal tariffs but higher fixed fees.*

b) Consumers with higher valuations purchase larger quantities.

c) The marginal willingness to pay of any consumer exceeds the marginal cost, except for the highest ex–post consumer type.

Under the conditions stated in this theorem, the monopolist may screen consumers by offering them a continuum of self–selecting two–part tariffs, since each consumer type finds that one and only one of these tariff plans maximize her utility. Each two–part tariff is the optimal solution for only one ex–post consumer type, and therefore the equilibrium is ensured to be completely separating. A sufficient condition for a continuum of two–part tariffs to be self–selecting is that its lower envelope be concave in consumption [Faulhaber and Panzar (1977)]. Since part *a)* of Theorem 1 ensures that $\hat{p}'(\theta) < 0$, and part *b)* that $\hat{X}(\theta) > 0$, it follows that $\hat{p}'(X) < 0$ so that the outlay schedule is concave.

3.2 Characterization of the Ex–Ante Tariff

In this case, at the end of the billing period each consumer pays according to her consumption and to the tariff plan that she chose at the begining of the billing period. The choice of tariff and consumption are no longer dual problems because of the existence of stochastic elements in consumers' demands. Now, the consumer optimally chooses her tariff plan given her expected ex–post type. Later, the same consumer will optimally choose her usage level given her ex–post preferences and her previously chosen tariff.

Two issues are worth mentioning at this point. First, tariff plans do not need to be linear. I work with two–part tariffs because they are widely used as tariff plans. However, more complicated tariff plans are possible, and they can be addressed with few modifications of the present model. Second, in the case of ex–ante tariffs consumers first choose the tariff plan, and later they choose consumption, once they learn their ex–post type. The sequential feature of the choice process distinguishes two IC constraints; one for tariff choice, and other for the usage decision.

For the optimal design of the ex–ante tariff the monopolist also designs an optimal direct mechanism $\{\tilde{p}(\theta_1), \tilde{A}(\theta_1)\}$ which induces consumers to truthfully reveal their private information at the begining of the billing period. Now, for this mechanism to be incentive compatible (IC) consumers must find it optimal to reveal their true ex–ante type to the monopolist at the begining of the billing period, *i.e.:*

$$\theta_1 \in \arg\max_{\theta_1'} \int_{\underline{\theta}_2}^{\overline{\theta}_2} [v(p(\theta_1'), \theta) - A(\theta_1')] f(\theta_2 \mid \theta_1) d\theta_2$$

Solving this problem we obtain the incentive compatibility condition:

$$A'(\theta_1) = -E_{\theta_2|\theta_1}[x(p(\theta_1), \theta)]p'(\theta_1) \qquad \forall \theta_1 \in D_1 \tag{10}$$

where $E_{\theta_2|\theta_1}[\cdot]$ denotes the expectation of demand conditional on consumer's ex–ante type. The sufficient condition for this problem to be concave is that $p'(\theta_1) < 0$, *i.e.*, that consumers with higher ex–ante valuations choose tariff plans with lower marginal charges. Next, denote by $\vartheta(\theta_1)$ the consumer's conditional expectation on her surplus when she subscribes to a particular ex–ante tariff plan, $E_{\theta_2|\theta_1}[v(p(\theta_1), \theta) - A(\theta_1)]$. By the Envelope Theorem, using equation (10), we get the following condition to be used in the monopolist's ex–ante pricing problem:

$$\vartheta'(\theta_1) = E_{\theta_2|\theta_1}\left[v_\theta(p(\theta), \theta)\frac{\partial\theta}{\partial\theta_1}\right] \tag{11}$$

If the monopolist decides to screen consumers according to their ex–ante type, he offers a continuum of two–part tariffs which solves the following problem:

$$\max_{p(\theta_1), A(\theta_1)} \int_{\theta_1^*}^{\overline{\theta}_1} \left[A(\theta_1) + (p(\theta_1) - c)E_{\theta_2|\theta_1}[x(p(\theta_1), \theta)]\right] f_1(\theta_1) d\theta_1 - k$$

$$\text{s.t.} \quad \vartheta(\theta_1) = E_{\theta_2|\theta_1}[v(p(\theta_1), \theta)] - A(\theta_1)$$

$$\vartheta'(\theta_1) = E_{\theta_2|\theta_1}\left[v_\theta(p(\theta), \theta)\frac{\partial\theta}{\partial\theta_1}\right]$$

$$\vartheta(\theta_1^*) \geq 0$$

where $f_1(\theta_1)$ is the marginal density function of the ex–ante type. The solution to this problem is similar to the ex–post tariff, and its derivation will be omitted. The outcome functions of the ex–ante mechanism are:

$$\tilde{p}(\theta_1) = c - \frac{1 - F_1(\theta_1)}{f_1(\theta_1)} \frac{E_{\theta_2|\theta_1}\left[x_\theta(p(\theta_1), \theta)\frac{\partial\theta}{\partial\theta_1}\right]}{E_{\theta_2|\theta_1}[x_p(p(\theta_1), \theta)]} \tag{12}$$

$$\tilde{A}(\theta_1) = E_{\theta_2|\theta_1}\left[v(\tilde{p}(\theta_1), \theta) - \vartheta(\theta_1^*) - \int_{\theta_1^*}^{\theta_1} v_\theta(\tilde{p}(z), \theta(z, \theta_2))\frac{\partial\theta}{\partial z}dz\right] \tag{13}$$

PROPOSITION 2: *Equations* (12)−(13) *are sufficient conditions for the monopolist's to maximize expected profit if consumer's net revenue is concave in p, the indirect utility function is such that* $v_{pp\theta}(p, \theta) \leq 0$ *and the ex–post and ex–ante types are positively related.*

The ex–ante pricing problem also becomes a standard nonlinear pricing problem since consumer's ex–ante type is one–dimensional, but with quite different implications. The following theorem shows that within this framework the monopolist can also discriminate among consumers' ex–ante types by offering a continuum of self–selecting two–part tariffs, provided that the single crossing property holds, and the hazard rate of the distribution of ex–ante types is increasing.

THEOREM 2: *Assume that the SCP holds, the ex–ante distribution function* $F_1(\theta_1)$ *of the ex–ante type has an increasing hazard rate, and the definition of the ex–post type is normalized such that* $\frac{\partial\theta}{\partial\theta_i} > 0$, *for* $i = 1, 2$. *Then, the solution to the ex–ante problem has the following properties:*

a) *If* $\{\tilde{p}(\theta_1), \tilde{A}(\theta_1)\}$ *is an IC mechanism, it is also almost everywhere differentiable. Consumers with ex–ante higher valuations choose plans with lower marginal tariffs but higher fixed fees.*

b) *A higher ex–ante valuation induces larger purchases of the good on average.*

c) *The marginal willingness to pay of any consumer exceeds the marginal cost, except for the highest ex–ante consumer type.*

Two points of this theorem should be clarified. First, even when the mathematical lower envelope of the ex–ante tariff exists, its meaning is quite different from the usual. It says that the monopolist can design a continuum of two–part tariffs from which consumers will self–select according to their ex–ante type. But this lower envelope does not represent the optimal combinations of tariff–purchases for each ex–post type. The explanation is that each consumer commits to a particular tariff plan at the beginning of the billing period, and when the shock is realized, its effect is to move the consumer's combination of total payments and purchases along the chosen linear tariff instead of the lower envelope. This means that the monopolist can discriminate among consumers when they are uncertain about their future type by offering a set of optional tariff plans. Furthermore, since by definition both the ex-ante type and the shock are directly related to the ex-post type, countervailing incentives [Lewis and Sappington (1989)] are not present and pooling is not an equilibrium feature. Second, it is proved that on average consumers with higher ex-ante valuation tend to buy larger quantities of the monopolized good, but the final result is ambiguous because shocks move consumers along their chosen plan and not along the lower envelope. Hence, there is no longer a one-to-one relationship between optimal purchase, marginal tariff and fixed fee. With an ex-ante nonlinear tariff schedule it is possible that the same quantity be billed at different prices since consumers have chosen different tariff plans. Therefore, consumers who chose different plans because their ex-ante type were different, may end up purchasing the same amount if they receive the appropiate opposite shocks.

4 Efficient Tariffs

In many cases, optional tariffs are offered by regulated monopolists. In particular, this is the case for local telephone services. Monopolists are allowed revenues only sufficient to cover their total costs. This is a common situation when first best marginal cost pricing cannot be achieved without negative profits because the existence of increasing returns. In this section, for the sake of completeness, I briefly describe the problem of efficient pricing with optional tariffs and obtain the ex-ante and ex-post Ramsey tariffs.

A regulated monopolist that offers optional tariffs has to design them in order to maximize the expected total surplus subject to the condition that expected revenues recover total costs. For the ex-post and ex-ante problem, this break-even budget constraint are repectively:

$$\int_{\theta^*}^{\overline{\theta}} \left[A(\theta) + (p(\theta) - c)x(p(\theta), \theta)\right] g(\theta)d\theta - k \geq 0$$

$$\int_{\theta_1^*}^{\overline{\theta}_1} \left[A(\theta_1) + (p(\theta_1) - c)E_{\theta_2|\theta_1}[x(p(\theta_1), \theta)]\right] f_1(\theta_1)d\theta_1 - k \geq 0$$

Let δ denote the corresponding Lagrange multiplier for these budget constraints. After simple manipulations, the regulated monopolist's problems may be written as follows:

Ex–Post Ramsey problem:

$$\max_{p(\theta),V(\theta)} \int_{\theta^*}^{\overline{\theta}} \left[v(p(\theta),\theta) + (p(\theta)-c)x(p(\theta),\theta) - \frac{\delta}{1+\delta} V(\theta) \right] g(\theta) d\theta - k$$

$$V'(\theta) = v_\theta(p(\theta),\theta)$$

$$V(\theta^*) \geq 0$$

Ex–Ante Ramsey problem:

$$\max_{p(\theta_1),\vartheta(\theta_1)} \int_{\theta_1^*}^{\overline{\theta}_1} \left[E_{\theta_2|\theta_1}[v(p(\theta_1),\theta)] + (p(\theta_1)-c)E_{\theta_2|\theta_1}[x(p(\theta_1),\theta)] - \frac{\delta}{1+\delta}\vartheta(\theta) \right] f_1(\theta_1) d\theta_1 - k$$

$$\vartheta'(\theta_1) = E_{\theta_2|\theta_1}\left[v_\theta(p(\theta),\theta)\frac{\partial\theta}{\partial\theta_1} \right]$$

$$\vartheta(\theta_1^*) \geq 0$$

The solutions for the marginal Ramsey tariffs are[8]:

$$\hat{p}^*(\theta) = c - \frac{\delta}{1+\delta}\frac{1-G(\theta)}{g(\theta)}\frac{x_\theta(p(\theta),\theta)}{x_p(p(\theta),\theta)} \tag{14}$$

$$\tilde{p}^*(\theta_1) = c - \frac{\delta}{1+\delta}\frac{1-F_1(\theta_1)}{f_1(\theta_1)}\frac{E_{\theta_2|\theta_1}\left[x_\theta(p(\theta_1),\theta)\frac{\partial\theta}{\partial\theta_1} \right]}{E_{\theta_2|\theta_1}[x_p(p(\theta_1),\theta)]} \tag{15}$$

The Lagrange multiplier δ may be interpreted as the shadow cost of public funds [Laffont and Tirole (1993, §1.2)]. As it is more costly to obtain public funds to cover the total costs of the monopolist $\delta \to \infty$, the Ramsey price approaches the optimal monopolist tariffs. This is the case when total costs are so large that profit maximization is required. On the contrary, when taxation to obtain public funds does not generate any distortions $\delta \to 0$, Ramsey prices involve marginal cost pricing because public transfers cover negative profits. Observe that the effect of a positive δ is just to reduce the cost of consumers' surplus in the regulated monopolist's objective function.

---

[8] The corresponding fixed fees are computed as in equations (9) and (13) using (14) and (15) instead of (8) and (12).

5 Ex–ante vs. Ex–post Tariffs

The monopolist may choose to screen consumers according to their ex–ante type at the begining of the billing period if this pricing strategy takes advantage of consumers uncertainty and generates higher expected profits than the ex–post tariff. The monopolist's decision will depend on the relative profitability of each tariff. In this section, I will address the relative magnitude of the expected consumer surplus, profits, and welfare under each tariff scheme.

In the case of an ex–ante tariff, the optimal usage is $x(\theta_1, \theta_2)$, where θ_1 was revealed to the monopolist at the choice of tariff stage, and θ_2 is truthfully revealed to the monopolist at the purchase stage. Therefore, θ_2 is the solution of the following communication game:

$$\theta_2 \in \arg\max_{\theta_2'} [u(x(\theta_1, \theta_2'), \theta(\theta_1, \theta_2)) + y - A(\theta_1) - p(\theta_1)x(\theta_1, \theta_2')]$$

where $u(\cdot)$ is the corresponding consumer's direct utility. Therefore, the consumer buys the amount of good that equals her marginal utility with the marginal tariff of her tariff plan which was chosen optimally at the beginning of the billing period. The solution to this problem provides the IC constraint for usage choice. In the case of the ex–post tariff, this IC constraint is the same as the one that characterizes the choice of tariff.

We might expect that ex–post "mistakes" in the tariff choice are balanced in the sense that there should be as many consumers who choose tariff plans, which are ex–post efficient for larger usage levels than their realized usage, as consumers who choose ex–post efficient plans for lower usage levels. But several empirical studies report that consumers' tariff choices are generally biased towards low marginal tariff plans, so that they usually end up paying in excess because their consumption is not large enough for the chosen tariff plan. For instance, Kridel, Lehman and Weisman (1993) examined data for Missouri and Arkansas in 1985 and found that 55% of the customers on the flat rate service would have been better off choosing measured service because their usage was too low, while only 10% of those on measured service had a usage too high to justify this choice. They would have been better off choosing the flat rate service. Using data from local usage and tariff choice of electricity by residential customers in Delaware from September through November 1985, Train, Ben–Akiva, and Atherton (1989) also suggest that many consumers do not correctly anticipate their individual demands from a cost minimization point of view. Hobson and Spady (1988) find the same bias for telephone usage data from the Subscriber Line Usage Survey (SLUS). Finally, MacKie–Mason and Lawson (1993) find that when telephone customers in Kentucky were given the choice, over 33% who chose flat rate service would save at least 25% by switching to the measured service.

The usual argument to explain this observed behavior is to assume that customers exhibit substantial risk aversion when faced with bill uncertainty and therefore customers with average usage level below the critical level will choose the (ex–ante based) flat rate service to ensure low monthly bill variation. However, the risk aversion argument cannot be sustained easily because telephone bills are small relative to customers' income. The risk involved is too small to warrant insurance and the marginal utility of income must be constant over the usual range of expenditure. Clay, Sibley and Srinagesh (1992), Mitchell and Vogelsang (1991, §8.1.1), and Srinagesh (1992) reject the risk aversion argument on this basis, as do I[9].

Only if the shock takes a particular value, *e.g.*, $\theta(\theta_1, \theta_2) = \theta_1$, the ex–ante choice of tariff plan turns out to be ex–post efficient. In any other case, consumers would have been better off ex–post if they had chosen a different tariff plan ex–ante. Clay, Sibley, and Srinagesh (1992) argue that for low levels of uncertainty, the ex–ante tariff may generate higher expected profits because it induces consumers to choose ex–ante tariff plans that are not IC ex–post. However, consumers do not violate any IC constraint. At the time of the choice of tariff plan, they choose the best option according to their expectation about their future consumption. Later, when the schock is realized, each consumer chooses her optimal usage level constrained to the chosen tariff plan. Clearly, consumers would have been better off if they had chosen ex–ante their ex–post efficient plan, but it is not obvious whether the establishment of ex–ante tariffs is socially efficient.

Observe that ex–post, consumers could always improve their utility by choosing the tariff plan that corresponds to the tariff lower envelope. If the mathematical lower envelope of the ex–ante tariff coincides with the optimal ex–post tariff, consumers who are offered an ex–ante tariff would have always been in expectation at least as well as if they were offered an ex–post tariff. However, this is only a limiting case that holds when demand uncertainty is not significant. Suppose that the shock is the same for every consumer. In this case, the marginal distribution of the shock becomes degenerate at the shock's mean value. Because of the normalization of the support of the distribution of the shock given by (1), equations (8) – (9) and (12) – (13) are identical so that the ex–ante and ex–post tariffs are the same. This result is now stated as a theorem and proved in the appendix.

THEOREM 3: *If the distribution of the shock becomes degenerate the ex–ante and ex–post tariffs are the same.*

When the variance of the shock is zero, all consumer differences are captured by the distribution of the ex–ante type, and the choice of tariff plan becomes dual to the choice of consumption. In this limiting case all consumers will be indifferent between an ex–ante or ex–post tariff. Similarly, the monopolist obtains the same expected profits with both tariffs when the distribution of the shock is degenerate.

We can also evaluate the opposite polar case. Suppose now that all consumers have the same ex–ante type. The optimal ex–ante tariff is then a two–part tariff, and if

---

[9] In fact Srinagesh (1992) shows that changes in consumer types may explain the biased choice even for risk neutral consumers. The bias is numerically computed using a compound exponential distribution with the appropiate skewness.

the monopolist decides to offer it, he does because screening consumers according to their ex–post type is not profitable. Observe that when the distribution of the ex–ante type collapses into a point distribution, the monopolist's choice between offering an ex–ante or ex–post tariff is formally equivalent to the choice between a two–part tariff or a fully nonlinear tariff when consumers types are one–dimensional. As it is already well known in the literature [Faulhaber and Panzar (1977, §4), Wilson (1993, §8.3)], welfare is increasing with the number of self–selecting two–part tariffs when consumers's taste differs. As in our case, consumers' types are represented by a continuous variable θ, a fully nonlinear ex–post tariff will Pareto dominate the optimal ex–ante two–part tariff.

The comparison of welfare associated to each tariff depends on how θ enters the demand, how θ is defined in terms of θ_1 and θ_2, and how the convolution distribution of θ relates to the distribution of the ex–ante type and the distribution of the shock. However, at this level of generality it is possible to affirm that desirability of ex–ante over ex–post tariffs seems to depend on the relative variance of type components. We have shown that welfare is higher under the ex–post tariff when there is little difference in consumers' ex–ante types. On the contrary, consumers are indifferent between ex–ante and ex–post types when the distribution of the shock is degenerate. But since the monopolist has the ability to screen consumers according to their ex–ante type by offering optional tariff plans, profits may be weakly higher with an ex–ante tariff because consumers commit to particular tariff plans that make their payments to be slightly above the lower envelope of the ex–post tariff. Therefore, the ex–ante tariff might be socially efficient under this circumstances.

The following example shows that, regardless of the parameterization of the model, expected profits may be higher with an ex–post tariff. This is always the result when the distribution of the shock is almost degenerate, which contradicts the result of Clay, Sibley, and Srinagesh (1992). The result on expected consumer surplus is that an ex–post tariff dominates from consumers' perspective if the distribution of the ex–ante type is not very spread and the variance of the shock is high enough.

5.1 Example: Beta Distribution and Linear Demands

In this section I will address the case of linear demands with unknown intercepts and beta distibution of types. This case is interesting for two kind of reasons. First, it produces results that are common to other distributions of the exponential family. Second, under one particular parameterization, the beta distribution together with linear demands allows to solve the model in closed form [Ivaldi and Martimort (1994)]. Since the beta is a very flexible distribution of the exponential family, this closed form solution is especially suitable for the empirical estimation of the model . In addition, this distribution is defined on a compact support, so that the variability of the intercept of demand may be restricted to positive values. For this example, assume that consumers direct utility function is the following:

$$U(x,\theta,A) = \theta x - \frac{b}{2}x^2 - A - px$$

which implies that demands are linear and that consumer types are indexed by the intercept of those demands. The corresponding indirect utility function is:

$$V(p,\theta,A) = \frac{(\theta-p)^2}{2b} - A \tag{16}$$

which in addition fulfill all demand requirements of Theorem 1 and Theorem 2 for the tariffs' lower envelopes to be concave[10].

To complete the characterization of the problem, the ex–post type is defined as $\theta = \theta_1\theta_2$. The ex–ante type $\theta_1 \in [0,1]$ is assumed to be distributed according to a beta distribution $\beta[1,\frac{1}{\lambda_1}]$. The shock $\varepsilon \in [0,1]$ is assumed to be independently distributed according to the beta distribution $\beta[1+\frac{1}{\lambda_1},\frac{1}{\lambda}-\frac{1}{\lambda_1}]$. In order to ensure that the shock moves the ex–post type around the corresponding ex–ante type, I define the normalized shock as $\theta_2 = 1+\varepsilon-\mu_\varepsilon$ where μ_ε accounts for the mean of ε. Then, for simplicity, while the support of the ex–ante type and ε is $D_1 = [0,1]$ the support of the distribution of the normalized shock is $D_2 = [\underline{\theta}_2,\overline{\theta}_2] = [1-\mu_\varepsilon, 2-\mu_\varepsilon]$. All these assumptions ensure [Jambunathan (1954), Kotlarski (1962)] that the ex–post type is distributed according to a beta distribution $\beta[1,\frac{1}{\lambda}]$ over $D = [0,2-\mu_\varepsilon]$. Normalizing again $V(\theta^\star) = \vartheta(\theta_1^\star) = 0$, the solutions for both problems are:

<u>Ex–post problem:</u>

$$\hat{p}(\theta) = c + \lambda\left[2-\mu_\varepsilon-\theta\right]$$

$$\hat{A}(\theta) = \frac{1}{2b}\left[\theta^2\lambda(1+\lambda) - 2(c+\lambda(2-\mu_\varepsilon))\lambda\theta + (c+\lambda(2-\mu_\varepsilon))^2\right]$$

$$\hat{x}(\theta) = \frac{1}{b}\left[\theta(1+\lambda) - c - \lambda(2-\mu_\varepsilon)\right]$$

$$\hat{V}(\theta) = \frac{1}{2b}\left[\theta^2(1+\lambda) - 2\theta(c+\lambda(2-\mu_\varepsilon))\right]$$

$$\hat{\pi}(\theta) = \frac{1}{2b}\left[\lambda(1+\lambda)\left(2(2-\mu_\varepsilon)\theta-\theta^2\right) - 2\left(c+\lambda(2-\mu_\varepsilon)\right)\lambda(2-\mu_\varepsilon) + \left(c+\lambda(2-\mu_\varepsilon)\right)^2\right] - k$$

---

[10] The assumptions made in the previous section ensure that the generalized SCP of McAfee and McMillan (1988) hold for the present case. In particular, there are fewer goods ($l = 1$) than type dimensions ($k = 2$), it is required that the matrix of partial derivatives of demand with respect to type dimensions has full rank ($n = 1$): $\text{rank}(\nabla_\theta[x(p,\theta)]) = \text{rank}(-v_{p\theta_1}(p,\theta), -v_{p\theta_2}(p,\theta))$ $= 1$. This rank condition is already ensured by demand derivatives assumptions. McAfee and McMillan show that the generalized SCP ensures that isoprice surfaces in type space $P(x,\theta) = \overline{P}$ are hyperplanes, so that optimal pricing may be addressed with a single dimensional sufficient statistic. In addition, Rochet (1985) shows that a sufficient condition for the SCP to hold is that $P(x,\theta)$ be linear in θ as in the present example.

Ex–ante problem:

$$\begin{aligned}
\tilde{p}(\theta_1) &= c + \lambda_1\left[1-\theta_1\right] \\
\tilde{A}(\theta_1) &= \frac{1}{2b}\left[\theta_1^2\lambda_1(1+\lambda_1) - 2(c+\lambda_1)\lambda_1\theta_1 + (c+\lambda_1)^2\right] \\
\tilde{x}(\theta) &= \frac{1}{b}\left[\theta_1(\theta_2+\lambda_1) - c - \lambda_1\right] \\
\tilde{V}(\theta) &= \frac{1}{2b}\left[\theta_1^2\left(\theta_2 + (2\theta_2-1)\lambda_1\right) - 2\theta_1\theta_2(c+\lambda_1)^2\right] \\
\tilde{\pi}(\theta) &= \frac{1}{2b}\left[\lambda_1\left(2(\theta_2+\lambda_1)\theta_1 - (2\theta_2 - 1 + \lambda_1)\theta_1^2\right) - 2(c+\lambda_1)\lambda_1 + (c+\lambda_1)^2\right] - k
\end{aligned}$$

It is straighforward to show that Theorems 1 and 2 hold, and in particular that the outlays are concave as long as $\lambda > 0$ and $\lambda_1 > 0$. These conditions also ensure that the optimal purchases are increasing in θ and θ_1 respectively, because as long as $\lambda > 0$ and $\lambda_1 > 0$ the distributions of the ex–post and ex–ante type have the incresing hazard rate property. Observe that the distribution of the shock, ε, implies that its mean and variance are:

$$\mu_\varepsilon = \frac{\dfrac{\lambda}{1+\lambda}}{\dfrac{\lambda_1}{1+\lambda_1}} \qquad ; \qquad \sigma_\varepsilon^2 = \frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_1}\right)\left(1+\frac{1}{\lambda_1}\right)}{\left(1+\dfrac{1}{\lambda}\right)^2\left(2+\dfrac{1}{\lambda}\right)}$$

Therefore the evaluation of the solutions in the limiting case of a degenerate point distribution for the shock as $\sigma_\varepsilon \to 0$ it is equivalent to evaluate these expresions as λ_1 approaches λ. It happens that as $\lambda_1 \to \lambda$, $\mu_\varepsilon \to 1$, $\mu \to \frac{\lambda}{1+\lambda}$, $\theta_2 \to 1$, and $\theta \to \theta_1$. These relationships suffice to prove that the ex–ante and ex–post solutions given before are identical if the shock has a degenerate point distribution.

This closed form solution however provides us with additional results. In particular the solution ensures that the same ex–post consumer type will always consume a larger amount of good if she is offered an ex–post tariff than if she is offered an ex–ante tariff $\hat{X}(\theta) > \tilde{X}(\theta)$ because:

$$\lim_{\lambda_1 \to \lambda} \tilde{X}(\theta) - \hat{X}(\theta) = 0$$

and,

$$\frac{\partial[\tilde{X}(\theta) - \hat{X}(\theta)]}{\partial\lambda_1} = \frac{1}{b}\left[(\theta_1 - 1) + \lambda\frac{\partial\mu_\varepsilon}{\partial\lambda_1}\right] < 0$$

Hence, when the consumer receives a positive (negative) shock, her consumption increases more (decreases less) with the ex–post tariff than with the ex–ante tariff. This result may be explained by the different marginal tariff profile that the consumer faces in each case. If the monopolist offers an ex–ante tariff the consumer faces a constant marginal tariff that in equilibrium has to be equated to her marginal utility. On the contrary, when the monopolist offers an ex–post tariff, the consumer faces decreasing marginal tariffs if she consumes larger amounts of the good after receiving positive shocks, but increasing marginal tariffs if she receives negative shocks. This result does not implies that consumers' utility is always higher with an ex–post tariff because the marginal tariff can be higher or lower for the same ex–post type.

The example provides a reasonably easy comparison of expected consumer surplus and profits. Given the distribution of the normalized shock we have that $\mu_2 = 1$, $\sigma_2^2 = \sigma_\varepsilon^2$, and the second order moment $\alpha_2 = 1+\sigma_\varepsilon^2$. It follows that the expected values for consumer surplus and profits are:

$$E[\hat{V}(\theta)] = \frac{2-\mu_\varepsilon}{2b}\left[\left(\frac{\lambda^2(1+\lambda)}{1+2\lambda} - \frac{2\lambda^2}{1+\lambda}\right)(2-\mu_\varepsilon) - 2\frac{\lambda c}{1+\lambda}\right] \tag{17}$$

$$E[\tilde{V}(\theta)] = \frac{1}{2b}\left[\frac{\lambda^2}{1+2\lambda}(2-\mu_\varepsilon)^2 + \frac{\lambda^3}{1+2\lambda_1} - 2\frac{\lambda}{1+\lambda}(2-\mu_\varepsilon)(c+\lambda_1)\right] \tag{18}$$

$$E[\hat{\pi}(\theta)] = \frac{1}{2b}\left[c^2 + \lambda^2(2-\mu_\varepsilon)^2\frac{1+\lambda-\lambda^2}{1+2\lambda}\right] - k \tag{19}$$

$$E[\tilde{\pi}(\theta)] = \frac{1}{2b}\left[c^2 + \lambda_1^2\frac{1+\lambda_1-\lambda_1^2}{1+2\lambda_1}\right] - k \tag{20}$$

It is now easy to show whether the monopolist profits from screening consumers either according to their ex–ante or ex–post information set. Since for the variance of the shock to be nonnegative it is necessary that $\lambda < \lambda_1$, let define $\lambda = \gamma\lambda_1$ for $0 \leq \gamma \leq 1$. Then, from equations $(19) - (20)$ the expected gain of introducing an ex–ante tariff is:

$$E[\tilde{\pi}(\theta) - \hat{\pi}(\theta)] = \frac{\lambda_1^2}{2b}\left[\frac{1+\lambda_1-\lambda_1^2}{1+2\lambda_1} - \gamma^2\left(\frac{2-\gamma+\gamma\lambda_1}{1+\gamma\lambda_1}\right)^2\frac{1+\gamma\lambda_1-\gamma^2\lambda_1^2}{1+2\gamma\lambda_1}\right]$$

The sign is ambiguous, but there are two interesting cases that will help to characterize tariffs difference of profitability. First, if the distribution of the shock is degenerate ($\gamma \to 1$), the limit of the expected difference of profits vanishes. In this case all differences in consumers' taste are captured by the ex–ante type and the monopolist is indifferent between offering an ex–ante or an ex–post tariff since they are equal in the limit. Second, we could address the extreme case of heterogeneous ex–ante types but a unique ex–post consumer type. In the case of the present beta distributions, λ and λ_1 are directly related to the spread of the type distributions. Therefore higher values of these parameters imply that the monopolist is more uncertain about consumers types. If $\lambda = 0$ ($\gamma \to 0$) the distribution of the ex–post type becomes degenerate at $\theta = 2$. This is the limiting case of a monopolist that is more certain about the distribution of the ex–post than of the ex–ante type. Under these circumstances the limit of the difference of profits is:

$$\lim_{\gamma \to 0} E[\tilde{\pi}(\theta) - \hat{\pi}(\theta)] = \frac{\lambda_1^2}{2b} \frac{1 + \lambda_1 - \lambda_1^2}{1 + 2\lambda_1}$$

which is positive only if λ_1 is smaller than the golden ratio $\lambda_1^* = (1 + \sqrt{5})/2$.

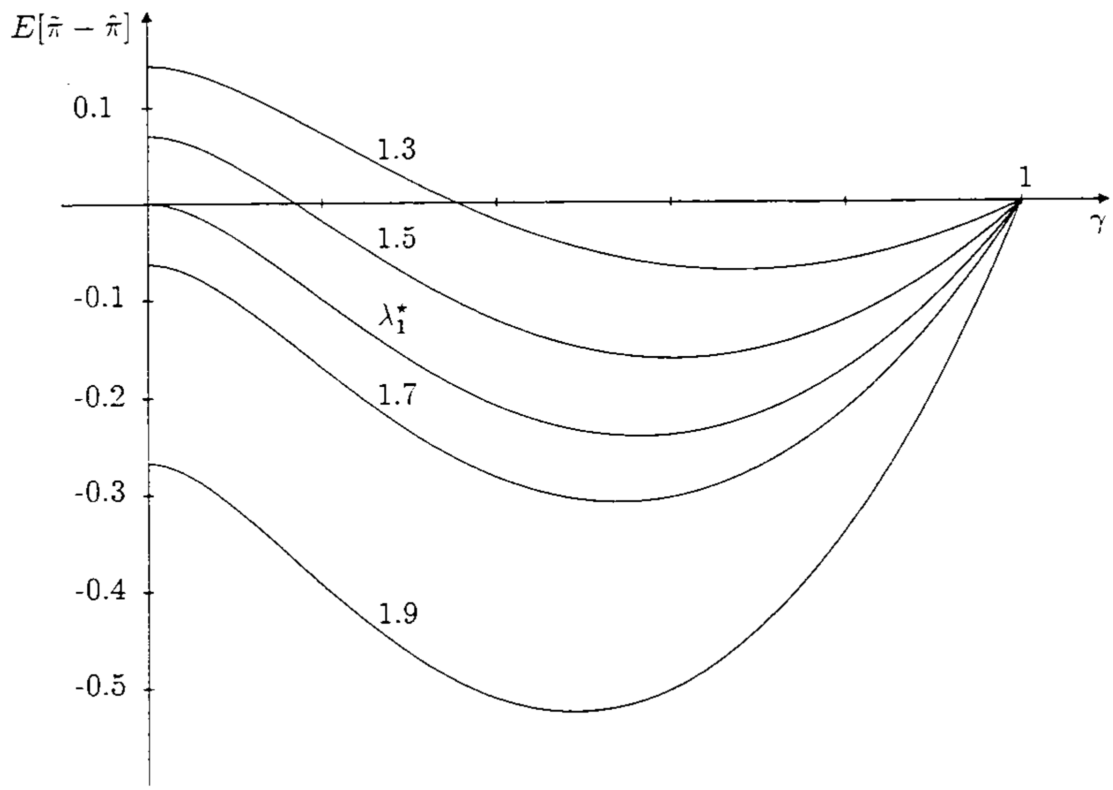

Figure 1

Figure 1 represents the difference of monopolist's expected profits for $0 \leq \gamma \leq 1$ evaluated at $b = 1$. Observe that if $\lambda_1 > \lambda_1^*$ the expected profit under the ex–post tariff always exceeds that of the ex–ante tariff. However, if $\lambda_1 < \lambda_1^*$ the expected profits of the ex–ante tariff exceeds that of the ex–post tariff for a given range of γ. The monopolist profits from the introduction of an ex–ante schedule only if the distribution of the ex–ante types is not very dispersed ($\lambda_1 < \lambda_1^*$) and the distribution of the shock has a low variance (γ is low)[11]. Within this range ($\lambda_1 < \lambda_1^*$), the more spread is the distribution of ex–ante types (larger λ_1), the lower is variance of the shock needed for the optimal ex–ante schedule

[11] The variance of the shock is a concave function of γ. The value of γ for which σ_ϵ^2 reaches the maximum value is decreasing in λ_1. See Appendix.

to be more profitable in expectation than the optimal ex–post schedule. If the variance of the shock is too high, there is a large ex–post heterogeneity of consumer types and the monopolist profits more from screening consumers according to their ex–post types.

Working as before, the expected incresase in consumer surplus from the introduccion of ex–ante tariffs is:

$$E[\tilde{V}(\theta) - \hat{V}(\theta)] = \frac{1}{2b}\left[\frac{\gamma^2\lambda_1^2(2+3\gamma\lambda_1-\gamma^2\lambda_1^2)}{(1+\gamma\lambda_1)(1+2\gamma\lambda_1)}\left(\frac{2-\gamma+\gamma\lambda_1}{1+\gamma\lambda_1}\right)^2 + \frac{\lambda_1^3}{1+2\lambda_1} - \frac{2\gamma\lambda_1^2}{1+\gamma\lambda_1}\left(\frac{2-\gamma+\gamma\lambda_1}{1+\gamma\lambda_1}\right)\right]$$

which also has an undefined sign. As before, when the distribution of the shock is degenerate, consumers are indiferent between an the ex–ante and ex–post tariff because their expected rent is the same. However, if the distribution of the ex–post type is much less spread than the distribution of the ex–ante type, consumers may prefer the ex–ante tariff since it accounts for consumers' ex–ante heterogeneity. In the limit:

$$\lim_{\gamma\to 0} E[\tilde{V}(\theta) - \hat{V}(\theta)] = \frac{1}{2b}\frac{\lambda_1^3}{1+2\lambda_1} > 0$$

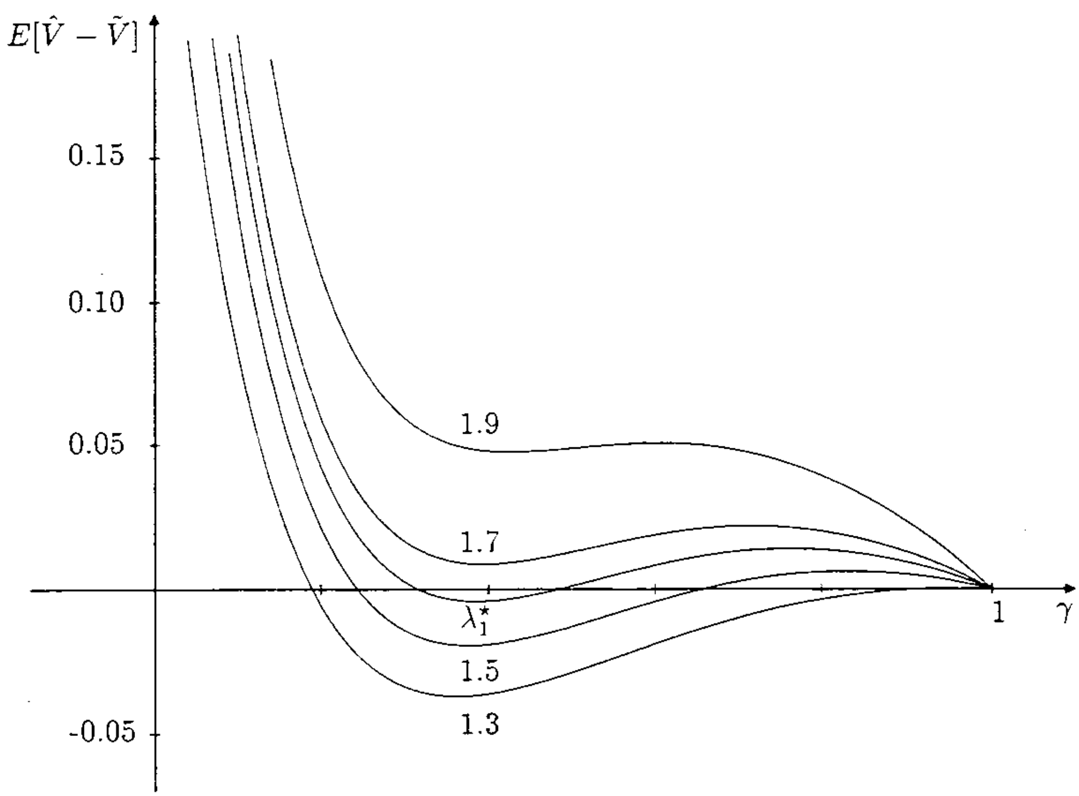

Figure 2

Figure 2 represents the difference of expected consumer surplus for intermediate values of γ. Expected consumer surplus with an ex–ante tariff is always positive when the distribution of λ_1 is spread enough. Consumers prefer the ex–post outlay when they are not too heterogeneous ex–ante (low values of λ_1) but the variance of the shock is large (intermediate values of γ).

This example shows that welfare results critically depends on the relative variance of type components. For the beta distribution case with linear demand described in this section, the monopolist will offer an ex–post outlay if consumers are heterogeneous enough ex–post. High ex–post heterogeneity is determined either by a high enough ex–ante heterogeneity ($\lambda_1 > \lambda_1^*$), or by a large variance of the shock that adds to a relatively low ex–ante heterogeneity. By contrast, consumers only prefer the ex–post schedule if the shock has a strong effect on the definition of the ex–post type, *i.e.*, ex–ante relatively homogeneous consumers but with a high variance for their individual shocks. As a result, the relative social valuation of these tariffs is also ambiguous. Figure 3 shows that in general expected welfare from the ex–post tariff dominates unless the variance of the shock is very low and the distribution of ex–post types becomes much less spread than the distribution of ex–ante types.

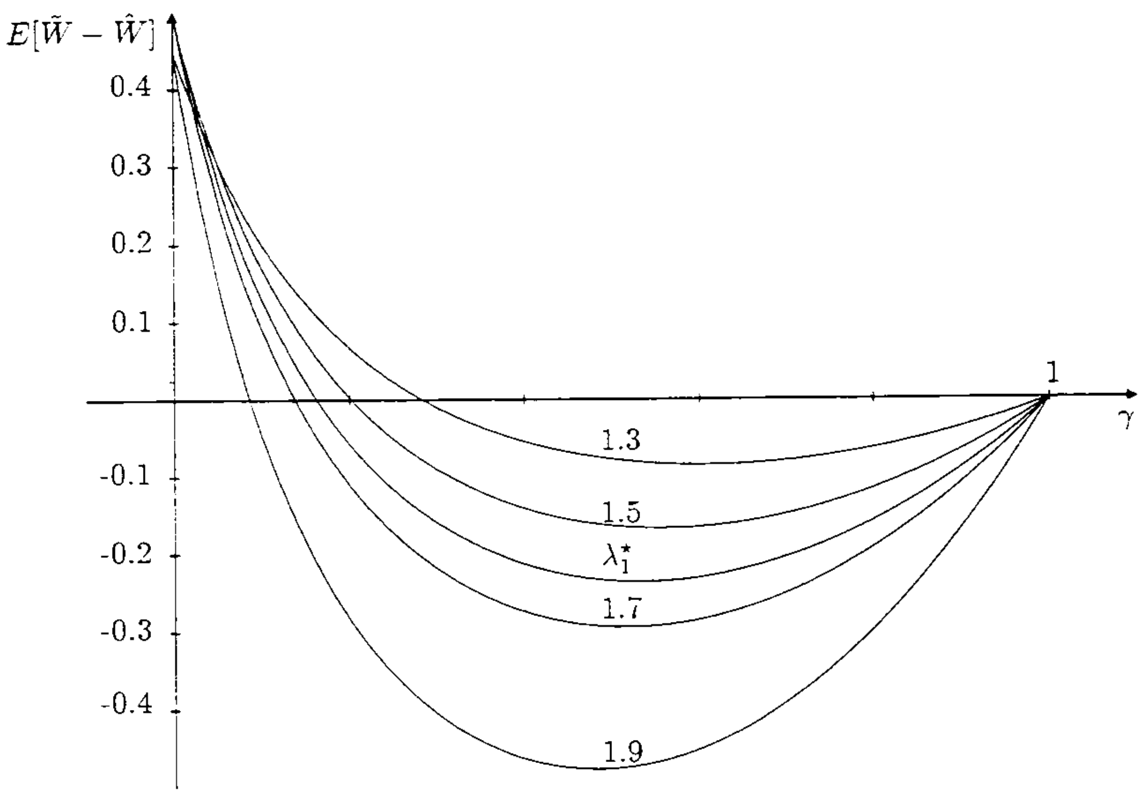

Figure 3

6 Concluding Remarks

The present work offers a characterization of the optimal ex–ante and ex–post nonlinear tariffs when a single dimensional ex–post consumer type includes the effect of shocks on consumer tastes between the time of choice of tariff and usage decision. The paper addresses the relative social efficiency of both tariff schemes. It has been shown that ex–ante and ex–post efficiency will crucially depend on the size of the variance of the shock and the dispersion of the ex–ante type. When there is not ex–ante type dispersion, a continuum of ex–post nonlinear tariff plans clearly dominates. However, and in contrast with previous results, it has been shown for a particular formulation that when there are significative ex–ante differences in tastes, a continuum of ex–post tariffs plans may dominate over a continuum of ex–ante tariff plans. Therefore, there are not general results regarding tariffs' social profitability.

The model has been solved for a continuum of types assuming that consumers are risk neutral. The results of the previous section shows that risk aversion arguments are not necessary to explain why consumers generally prefer ex–ante type based nonlinear tariffs. For the supply side, it has been shown that the monopolist may profits from forcing the consumers to commit to a particular tariff plan ex–ante under some circumstances, but he will offer an ex–post tariff plan if consumers' ex–post heterogeneity is high enough.

These results have been illustrated with a simple case with linear demand and beta distributions of consumers' types (demand intercepts). The features of this example are quite general, and they may be extended for other definitions of the ex–post type and distributions of its components using the reproductive property of the gamma distribution [Johnson and Kotz (1970, §17.2)]. For example, in addition to many cases that can be solved numerically, there are at least two other examples involving distributions of the exponential family that may also be solved in closed form for the linear demand specification and ex–post type defined as $\theta = \theta_1 + \theta_2$. One is the case when the ex–ante type, θ_1, and the shock are independently distributed according to a beta distribution $\beta(1, \frac{1}{\lambda})$ with support on $D_1 = [\underline{\theta}_1, \overline{\theta}_1]$ and $D_2 = [1 - \frac{\lambda}{1+\lambda}, 2 - \frac{\lambda}{1+\lambda}]$ respectively, so that the ex–post type is distributed according to a $\beta(2, \frac{1}{\lambda} - 1)$ on $D = [\underline{\theta}_1 - 1 - \frac{\lambda}{1+\lambda}, \overline{\theta}_1 + 2 - \frac{\lambda}{1+\lambda}]$. In the second case the ex–ante type and the shock are independently distributed according to an exponential distribution $\Gamma(1, \lambda)$ with support on $D_1 = [\underline{\theta} + \lambda, \infty)$ and $D_2 = [-\theta, \infty)$, so that the ex–post type is distributed according to a gamma distribution of the type $\Gamma(2, \lambda)$ on $D = [\underline{\theta}, \infty)$.

Appendix

• *Derivation of the ex–post tariff*

The corresponding Hamiltonian for the monopolist's ex–post problem is:

$$H[V,p,\theta] = [v(p(\theta),\theta) - V(\theta) + (p(\theta) - c)x(p,\theta) - k]\, g(\theta) + \lambda(\theta) v_\theta(p(\theta),\theta)$$

Using (4), the first order necessary conditions become:

$$H_p : (p(\theta) - c)x_p(p,\theta)g(\theta) + \lambda(\theta)v_{p\theta}(p(\theta),\theta) = 0$$
$$H_V : -g(\theta) = -\lambda'(\theta) \qquad ; \qquad \lambda(\overline{\theta}) = 0$$

There is not transversality condition at $\overline{\theta}$ since $V'(\theta) > 0\ \forall\theta$, so that the participation constraint is only binding at $\theta^* < \overline{\theta}$ [Kamien and Schwartz (1991, §II.7)]. Therefore:

$$\lambda(\theta) = \int_{\overline{\theta}}^{\theta} g(z)dz = G(\theta) - 1$$

Equations (8) – (9) follow from substituting this expression and equation (5.b) into the first order necessary conditions.

• *Proof of Proposition 1*

For the monopolist's ex–post problem necessary conditions to be sufficient, it must be the case that the objective and constraint functions be concave in p and V, given that $V(\theta)$ and $\lambda(\theta)$ are continuous and $\lambda(\theta) \geq 0$ [Kamien and Schwartz (1991, §II.3)]. It is straightforward to show that the Hessians of these functions are always singular matrices. Therefore, it suffices that the second derivative of these functions with respect to p be negative. The concavity of the constraint requires $v_{pp\theta}(p,\theta) \leq 0$. Finally, the concavity of the objective function requires $x_p(p,\theta) + (p - c)x_{pp}(p,\theta) \leq 0$. ∎

• *Proof of Theorem 1*

a) Let $\theta > \theta'$. Incentive compatibility implies:

$$v(\hat{p}(\theta),\theta) - \hat{A}(\theta) \geq v(\hat{p}(\theta'),\theta) - \hat{A}(\theta')$$
$$v(\hat{p}(\theta'),\theta') - \hat{A}(\theta') \geq v(\hat{p}(\theta),\theta') - \hat{A}(\theta)$$

Adding these two inequalities yields:

$$\int_{\theta'}^{\theta} v_\theta(\hat{p}(\theta), z)dz = v(\hat{p}(\theta), \theta) - v(\hat{p}(\theta), \theta') \geq [v(\hat{p}(\theta), \theta) - \hat{A}(\theta)] - [v(\hat{p}(\theta'), \theta') - \hat{A}(\theta')]$$

$$\geq v(\hat{p}(\theta'), \theta) - v(\hat{p}(\theta'), \theta') = \int_{\theta'}^{\theta} v_\theta(\hat{p}(\theta'), z)dz$$

This inequality together with the SCP, $v_{p\theta}(\hat{p}(\theta), \theta) < 0$, implies that $\hat{p}(\theta) \leq \hat{p}(\theta')$. Therefore, since $\hat{p}(\theta)$ is monotone, it is almost everywhere continuous, and also almost everywhere differentiable. Observe that from here $\hat{p}'(\theta) < 0$, *i.e.*, higher consumer types pay lower marginal tariffs. This result holds globally because of the SCP, and ensures that local maximum of the consumer tariff choice is also a global maximum.

For the mechanism to be almost everywhere differentiable, it remains to prove that the other outcome function, $\hat{A}(\theta)$ is also almost everywhere differentiable. Observe that IC also implies:

$$v(\hat{p}(\theta), \theta) - v(\hat{p}(\theta'), \theta) \geq \hat{A}(\theta) - \hat{A}(\theta') \geq v(\hat{p}(\theta), \theta') - v(\hat{p}(\theta'), \theta')$$

Then, taking limits and using equation (4), it follows:

$$\lim_{\theta' \to \theta} \frac{v(\hat{p}(\theta), \theta) - v(\hat{p}(\theta'), \theta)}{\theta - \theta'} = \lim_{\theta' \to \theta} \frac{v(\hat{p}(\theta), \theta') - v(\hat{p}(\theta'), \theta')}{\theta - \theta'} =$$

$$\lim_{\theta' \to \theta} \frac{v(\hat{p}(\theta), \theta) - v(\hat{p}(\theta'), \theta)}{\hat{p}(\theta) - \hat{p}(\theta')} \frac{\hat{p}(\theta) - \hat{p}(\theta')}{\theta - \theta'} = v_p(\hat{p}(\theta), \theta)\hat{p}'(\theta) = -x(\hat{p}(\theta), \theta)\hat{p}'(\theta) = \hat{A}'(\theta)$$

Therefore higher consumer types pay higher fixed fees.

b) For an ex–post consumer type θ, the optimal purchase is $\hat{x}(\theta) = x(\hat{p}(\theta), \theta)$. If consumers with higher valuations purchase larger amounts of good, it is necessary that $\hat{x}'(\theta) = x_p(\hat{p}(\theta), \theta)\hat{p}'(\theta) + x_\theta(\hat{p}(\theta), \theta) > 0$. Partial derivatives given by equation (5) and $\hat{p}'(\theta) < 0$ ensure that $\hat{x}'(\theta) > 0$.

c) This proof is straightforward. At $\theta = \overline{\theta}$, $G(\overline{\theta}) = 1$ and the second term of equation (8) cancels, so that $\hat{p}(\overline{\theta}) = c$. ∎

• *Proof of Proposition 2*

The monopolist's Hamiltonian for the ex–ante problem is:

$$H[\vartheta, p, \theta_1] = \Big[E_{\theta_2|\theta_1}[v(p(\theta_1), \theta)] - \vartheta(\theta_1) + (p(\theta_1) - c)E_{\theta_2|\theta_1}[x(p, \theta)] - k\Big] f_1(\theta_1)$$

$$+\lambda_1(\theta_1)E_{\theta_2|\theta_1}\left[v_\theta(p(\theta_1), \theta)\frac{\partial \theta}{\partial \theta_1}\right]$$

As in the proof of Proposition 1, (12)–(13) are sufficient conditions if the second derivatives of the objective and constraint functions with respect to p are negative. It is straightforward to show that these derivatives are negative under the same conditions that those of Proposition 1, but in addition, it is necessary that the definition of the ex–post type be such that $\frac{\partial \theta}{\partial \theta_1} > 0$ for the constraint to be concave. ■

• *Proof of Theorem 2*

Part *c)* is directly proved as in Theorem 1 by substituting θ for θ_1 and defining the corresponding inverse hazard rate of the distribution of ex–ante types $h_1(\theta_1)$. Part *a)* is similarly proved by substituting $v(\hat{p}(\theta'),\theta) - \hat{A}(\theta')$ for $E_{\theta_2|\theta_1}[v(\tilde{p}(\theta'),\theta) - \tilde{A}(\theta')]$ in order to show that $\tilde{p}'(\theta_1) < 0$ and $\tilde{A}'(\theta_1) > 0$. Finally, to prove part *b)*, let define $\tilde{x}(\theta,\theta_1) = x(\tilde{p}(\theta_1),\theta)$ as the optimal purchase of an ex–post type θ who had ex–ante type θ_1 and who chose her optimal ex–ante plan $\{\tilde{p}(\theta_1), \tilde{A}(\theta_1)\}$. Consumers with higher ex–ante valuations are expected to purchase larger amounts of good on average if $\tilde{x}_{\theta_1}(\theta,\theta_1) = x_p(\tilde{p}(\theta_1),\theta)\tilde{p}'(\theta_1) + x_\theta(\tilde{p}(\theta_1),\theta)\frac{\partial \theta}{\partial \theta_1} > 0$. Signs of partial derivatives are given by equation (5), $\tilde{p}'(\theta_1) < 0$, and the normalization in the definition of the ex–post type ensure that $\tilde{x}_{\theta_1}(\theta,\theta_1) > 0$. ■

• *Proof of Theorem 3*

When the variance of the shock is zero the distribution function of the ex–post type is the same that the distribution function of the ex–ante type because the bivariate distribution $F(\theta_1,\theta_2)$ collapses into $F_1(\theta_1)$. Since by equation (1), $\theta(\theta_1,\mu_{2|1}) = \theta_1$, because the distribution of the shock is degenerate, this implies that $F_1(\theta_1) = F_1(\theta) = G(\theta)$ because of the definition of $G(\cdot)$ as the distribution of the transformed variable θ. Therefore $h(\theta) = h_1(\theta_1)$. Next, since $\theta = \theta_1$ when $Var(\theta_2) = 0$, and then $\frac{\partial \theta}{\partial \theta_1} = 1$. The point distribution of the shock also suffices to ensure that $E_{\theta_2|\theta_1}[x_i(p(\theta_1),\theta)] = x_i(p(\theta_1),\theta)$ for $i = \{p,\theta\}$. Therefore equations (8) and (12) are identical when $Var(\theta_2) = 0$. The same argument suffices to show that equations (9) and (13) are also identical under this circumstance. ■

• *Variance of the shock*

The variance of the shock may be rewritten in terms of γ and λ_1 as follows:

$$\sigma_\epsilon^2 = \frac{(\gamma^{-1} - 1)(1 + \lambda_1^{-1})}{\lambda_1[1 + (\gamma\lambda_1)^{-1}]^2[2 + (\gamma\lambda_1)^{-1}]}$$

which is represented in Figure 4 for different values of λ_1. Let denote by $\hat{\gamma}(\lambda_1)$ the positive solution to $\frac{\partial \sigma_\epsilon^2}{\partial \gamma} = 0$. It is then straightforward to show that $\hat{\gamma}(\lambda_1)' < 0$.

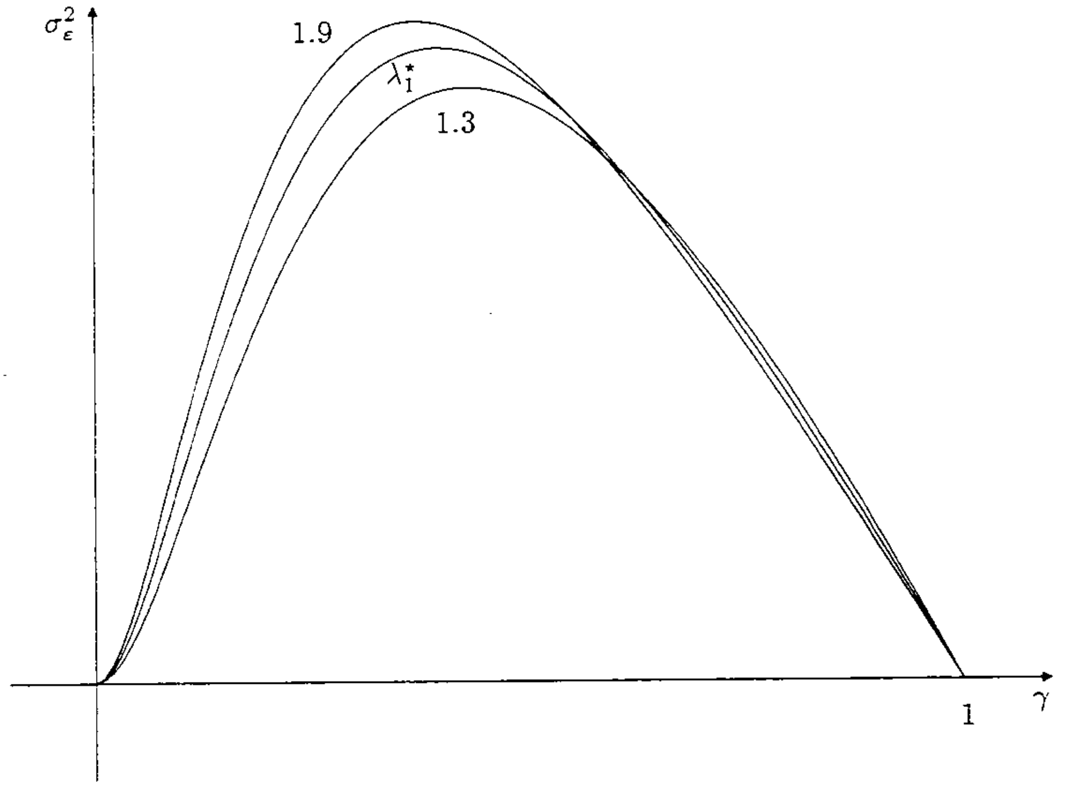

Figure 4

References

CALEM, P.S., AND SPULBER, D.F. (1984): "Multiproduct Two Part Tariffs." *International Journal of Industrial Organization*, 2, 105-115.

CLAY, K., D.S. SIBLEY, AND P. SRINAGESH (1992): "Ex Post *vs.* Ex Ante Pricing: Optional Calling Plans and Tapered Tariff." *Journal of Regulatory Economics*, 4, 115–138.

DANSBY, R.E. (1983): "Economic Analysis of a Measured Service Option," in L. Courville, A. de Fontenay, and R. Dobell (eds.): *Economic Analysis of Telecommunications: Theory and Applications*. North–Holland.

FAULHABER, G.R. AND J.C. PANZAR (1977): "Optimal Two–Part Tariffs with Self–Selection." Bell Laboratories Economic Discussion Paper No. 74.

GOLDMAN, M.B., H.E. LELAND, AND D.S. SIBLEY (1984): "Optimal Nonuniform Prices." *Review of Economic Studies*. 51, 305–319.

HOBSON, M. AND R.H. SPADY (1988): "The Demand for Local Telephone Service Under Optional Local Measured Service." Bellcore Economics Discussion Paper No. 50.

IVALDI, M. AND D. MARTIMORT (1994): "Competition under Nonlinear Pricing." *Annales d'Economie et de Statistique*, 34, 71–114.

JAMBUNATHAN, M.V. (1954): "Some Properties of Beta and Gamma Distributions." *The Annals of Mathematical Statistics*, 25, 401–405.

JOHNSON, N.L. AND S. KOTZ (1970): *Continuous Univariate Distributions*. Houghton Mifflin Co.

KAMIEN, M.I. AND N.L. SCHWARTZ (1991): *Dynamic Optimization*, 2nd edition. North–Holland.

KOTLARSKI, I. (1962): "On Groups of n Independent Random Variables Whose Product Follows the Beta Distribution." *Colloquium Mathematicum*, 9, 325–332.

KRIDEL, D., D. LEHMAN, AND D. WEISMAN (1993): "Option Value, Telecommunications Demand, and Policy." *Information Economics and Policy*, 17, 69–75.

LAFFONT, J.J., E. MASKIN, AND J.C. ROCHET (1987): "Optimal Nonlinear Pricing with Two–Dimensional Characteristics," in T. Groves, R. Radner, and S. Reiter (eds.): *Information, Incentives, and Economic Mechanisms*. University of Minnesota Press.

LAFFONT, J.J., AND J. TIROLE (1993): *A Theory of Incentives in Procurement and Regulation*. MIT Press.

LEWIS, T.R. AND D.E.M. SAPPINGTON (1989): "Countervailing Incentives in Agency Problems." *Journal of Economic Theory*, 49, 294–313.

LEWIS, T.R. AND D.E.M. SAPPINGTON (1994): "Supplying Information to Facilitate Price Discrimination." *International Economic Review*, 35, 309–327.

MACKIE–MASON, J.K. AND D. LAWSON (1993): "Local Telephone Calling Demand when Customers Face Optimal and Nonlinear Price Schedules." Working Paper. Department of Economics. University of Michigan.

MASKIN, E. AND J. RILEY (1984): "Monopoly with incomplete information." *Rand Journal of Economics*, 15, 171–196.

McAfee, R.P. and J. McMillan (1988): "Multidimensional Incentive Compatibility and Mechanism Design." *Journal of Economic Theory*, 46, 335–354.

Mitchell, B.M. and I. Vogelsang (1991): *Telecommunications Pricing. Theory and Practice.* Cambridge University Press.

Myerson, R. (1979): "Incentive Compatibility and the Bargaining Problem." *Econometrica*, 47, 61–73.

Oren, S., S. Smith, and R. Wilson (1982): "Competitive Nonlinear Tariff." *Journal of Economic Theory*, 29, 49–71.

Oren, S., S. Smith, and R. Wilson (1985): "Capacity Pricing." *Econometrica*, 53, 545–566.

Panzar, J.C. and D.S. Sibley (1978): "Public Utility Pricing under Risk: The Case of Self-Rationing." *American Economic Review*, 68, 887–895.

Rochet, J.C. (1985): "The Taxation Principle and Multi-Time Hamilton–Jacobi Equations." *Journal of Mathematical Economics*, 14, 113–128.

Sibley, D. and P. Srinagesh (1995): "Multiproduct Nonlinear Pricing with Multiple Taste Characteristics." Mimeo.

Spulber, D.F. (1989): "Product Variety and Competitive Discounts." *Journal of Economic Theory*, 48, 510–525.

Spulber, D.F. (1992a): "Capacity–Contingent Nonlinear Pricing by Regulated Firms." *Journal of Regulatory Economics*, 4, 299–319.

Spulber, D.F. (1992b): "Optimal Nonlinear Pricing and Contingent Contracts." *International Economic Review*, 33, 747–772.

Srinagesh, P. (1992): "A Dynamic Stochastic Model of Choice." Bellcore Economics Discussion Paper No. 78.

Tirole, J. (1989): *The Theory of Industrial Organization.* MIT Press.

Train, K.E., M. Ben–Akiva, and T. Atherton (1989): "Consumption Patterns and Self-Selecting Tariffs." *The Review of Economics and Statistics*, 50, 62–73.

Wilson, R.B. (1993): *Nonlinear Pricing.* Oxford University Press.

Wilson, R.B. (1995): "Nonlinear Pricing and Mechanism Design," in H. Amman, D. Kendrick, and J. Rust (eds.): *Handbook of Computational Economics*, Vol. I. North–Holland.