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GAMES, COMPUTERS, AND O.R.

by

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The following is an extended abstract of a lecture prepared for the Seventh Annual ACM-SIAM Symposium on Discrete Algorithms.

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The scientific interaction of game theory with computer science and operations research is broad and fundamental. It covers a large variety of applications in all three fields, and it transfers concepts, language, and results across fields. Listing all the areas of overlap will be long and tedious. I will therefore restrict myself to discuss a few, easy to present examples, where the cross-fertilization of ideas was successful, and point to a need for further research.

Graphs in Games

In coalitional games it is most often assumed that the grand coalition, consisting of all the players, will form. But the process of coalition formation is often complex, only partial, and seems difficult to describe mathematically. To model this process rigorously, Myerson (1977) introduced the notion of a communication graph, where each node represents a player and a link represents the ability of the two linked players to communicate. He then extended the Shapley value of the game to reflect this partial communication ability. Kalai, Postlewaite, and Roberts (1979), extending the notion of the Core of a game to incorporate communication graphs, were able to study and compare different trading structures. For example: How does complete free trade, described by a complete comunication graph, compare with centralized trade, where each player is only connected to one fixed middleman?

Multi-Person Operations Research.

Typical operations research problems, where a single objective function is maximized, model one person optimization. Often, however, the real-life problem involves different maximizers with different objectives and different resources. Assignment games, studied by Shapley and Shubik (1972), linear production games, studied by Owen (1975), and spanning tree games, studied by Bird (1976), are all examples of multi-person extensions of well known operations research problems. I will use flow games, studied in Kalai and Zemel (1982), to illustrate the synergy, of operations research optimization techniques with game theoretic solution concepts, obtained in such problems.

Consider a standard max-flow problem with a source **s** and a sink **t** and with a non-negative capacity associated with each of its undirected arcs. Assuming that each unit of flow from **s** to **t** generates \$1 profit, the standard problem is to find the profit maximizing flow pattern. If, however, different arcs are owned by different individuals, or players, then we deal simultaneously with two questions: (1) what is the optimal flow pattern, and (2), how should the profit from the optimal flow be distributed among the players?

Modeling this problem as a coalitional game we assign to every coalition (a subset of players) S a value V(S) describing the maximum profit (**s** to **t** flow) S can generate by using only its own arcs. The coalitional form game described by the collection of these values, V = (V(S)), is the resulting flow game. A vector of individual payoffs is in the Core of the game if the total payoff allocated to all players is feasible, i.e., it sums to no more then the value of the grand coalition, and is "coalitionally stable," i.e., the total payoffs allocated to the members of every coalition S is at least as great as their own value V(S).

Using the standard max-flow min-cut result, it is easy to see that any flow game must be "totally balanced," which means that its Core and the Cores of all its subgames (restricting the game to subsets of players) are non-empty. Conversely, starting with any totally balanced game V, we can find a flow problem describing it. Thus the class of totally balanced games coincides with the class of flow games. Moreover, the intuition obtained shows that every such game can be written as the minimum of additive games. This simple decomposition is useful for understanding this important class of games (an older result of Shapley and Shubik (1969) showed that the class of market games also coincides with the class of totally balanced games) and the strategic structure of multi-person flow problems.

The Complexity of Playing a Game

A traditional assumption in non-cooperative game theory is that players are fully rational and have unlimited computational ability. Made for modeling convenience, this assumption is especially disturbing when we deal with complex games. First rigorous studies (see, for example, Ben-Porath (1986) Neyman (1985) and Rubinstein (1986)), showed that concepts from computer science are useful for modeling bounded computational ability. But we must be precise in classifying the types of complexities that can arise. We first discuss the notion of *strategic complexity*, through the example of a 2-person repeated prisoners' dilemma game.

In successive discrete periods, each of the two players must choose one of the actions: "cooperate" (C), or "fight" (F). However, before making his t-period choice, a player is told the full history of play, consisting of the t-1 pairs of earlier choices. Thus, formally, a player's strategy is a function choosing C or F for every history, i.e., a finite length string consisting of pairs of actions. Clearly this definition is too broad, and it even includes strategies that cannot be described in any finite time.

Aumann (1981) proposed using automata to measure strategic complexity. Consider automata, Moore machines, whose inputs are history strings and outputs are recommended actions. Then for every strategy let its complexity be measured by the number of states of the smallest automaton describing it. If we allow countably many states then every strategy can be described by such an automaton, and paralleling the Myhill-Nerode Theorem, Kalai and Stanford (1988) show that the complexity of a strategy equals the number of distinct strategies it induces (a strategy f induces the strategy g, if a player using f will be led to use g in the "new" infinitely repeated game that starts after some finite history).

Using the above measure of complexity, a resolution of the finitely repeated Prisoners' Dilemma Paradox was obtained. In such a game the (rational) equilibrium strategies prescribe fighting throughout the game, yet most people consider this unreasonable. Neyman (1985) showed that in the game where the players are restricted to use bounded complexity strategies there are highly cooperative equilibria. This holds true even if the exogenously given number, describing the upper bound on the complexity of strategies that can be used, is large.

Rubinstein (1986) and Abreu and Rubinstein (1988) showed that the high degree of outcome indeterminacy (the Folk Theorem), exhibited in infinitely repeated games with patient players, can be significantly reduced. If we impose minimal additional costs for using more complex strategies, the set of equilibrium payoffs of the infinitely repeated prisoners' dilemma game shrinks from a two dimensional set to a single line segment.

Ben-Porath (1986) studied the advantage of using a bigger automaton in a repeated 2-person 0-sum games with patient players. Having a bigger automaton than the opponent's can be highly advantageous, but no advantage is materialized unless the automaton is exponentially bigger.

In unmodified infinitely repeated games, with no exogenously imposed complexity bounds or added complexity costs, interpersonal complexity bounds exist. For example, players must be using equally complex strategies at generic equilibria of 2-person games. Moreover, the set of equilibrium payoffs is uniformly approximated by the equilibria where players choose to use strategies of a given level of high, but finite, complexity (see Kalai and Stanford (1988)).

Restricted strategic complexity, through the use of automata or Turing Machines, has been the topic of many papers. The reference list contains some additional examples.

The Complexity of Solving a Game

A different issue is the difficulty of determining optimal strategies (without complexity or other restrictions) for playing a game. This turns out to be more of a computational problem and, not surprisingly, measures of algorithmic complexity became useful.

Gilboa (1988) studied the difficulty of computing an optimal automaton in a repeated game. If the number of opponents is known, then this turns out to be only of polynomial difficulty. However, with an unknown number of opponents the problem is NP complete. Gilboa and Zemel (1989) studied the difficulty of computing equilibria for one shot normal form games with large number of strategies. Questions regarding Nash equilibria tend to be difficult, often NP hard, while questions about correlated equilibria tend to be easy. This suggests that Nash equilibrium, the most established solution of non-cooperative game theory, may be the wrong concept for modeling the behavior of players in large games. Our references contain additional examples of such studies.

Modeling Boundedly-Rational Players

Combining restrictions to simple strategies with restrictions to computable solution concepts leads to new problems, not identified when these two restrictions are dealt with separately. Papadimitriou (1992) shows a model where computing an optimal automaton with no restrictions is easy, yet computing an optimal automaton subject to restriction on the number of states is hard. Similarly, incorporating complexity costs into the objective function makes the optimization problem significantly harder.

Papadimitriou's results suggest a modeling difficulty. As players become bounded their computational requirements increase since their optimization problems are objectively more difficult. It seems that this type of difficulty is even more severe.

One of game theory's basic premises is that a rational player, in order to guess opponents' actions, puts himself in the opponents' shoes, and simultaneously with his own he also solves the opponents' optimization problems. In a fully rational world this assumption leads to a Nash equilibrium where n-players simultaneously solve n interrelated problems, and each solution is optimal relative to the other n-1 solutions.

In moving to a boundedly rational world it is still reasonable to assume that players put themselves in their opponents' shoes, at least to a limited degree. This means that the bounded player has to make assumptions about the way that the bounded opponents behave (he may even have to make assumptions about the assumptions that they make about his own bounded behavior, and so on). While there is only one way to be rational (I assume maximization of expected utility and ignore the issue of multiple solutions) there are many behavior modes possible under bounded rationality. So we may have him assign probabilities to all the bounded modes of behavior that they may follow, and following a Bayesian game approach a la Harsanyi (1967), have him select an optimal response. But this modeling approach is faulty since it solves the bounded rationality problem by assuming that players are extremely rational (the full treatment, with assumptions about assumptions, will require putting probability distributions on probability distributions, etc.).

As game theory attempts to model larger (in the number of players, strategies, and periods of play) games realistically , the need for a good model of boundedly rational players becomes urgent. Creative conceptual input from other fields, computer science, artificial intelligence, mathematical psychology and alike, may help us overcome modeling difficulties described above.

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