DYNAMIC RETAIL PRICE AND INVESTMENT COMPETITION*

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Abstract: We develop a simple model of retail competition in which retailers select prices and investments in cost reduction. Unable to observe firms' current prices prior to costly search, consumers monitor firms' historic pricing behavior. An equilibrium is constructed in which several identical firms enter and then engage in a phase of vigorous price competition, corresponding to a battle for low-price reputations. This phase is concluded with a "shakeout," as a low-price, low-cost firm comes to dominate the market while other firms lose market share. A central feature of the equilibrium is that low prices are complementary to large investments in cost reduction. Even though the dominant firm's price rises through time, and initially may be below marginal cost, we argue that an interpretation of predatory pricing may be inappropriate, since the dominant firm is also the most-efficient (lowest-cost) firm in the market.

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I. Introduction

In many retail markets, a few dominant retailers emerge from a pack of entrants. Examples include Wal-Mart and Kmart in the discount retail market, and Toys "R" Us, Circuit City and Home Depot in the retail toy, electric appliance and hardware markets, respectively. While the dominant firms differ in many respects, they also appear to share a cluster of attributes, including low prices, huge sales volumes, and large investments in sophisticated sales and distribution technologies. Consistent with these observations, this paper presents a dynamic model of retail competition in which several identical firms initially enter, and then, after a phase of vigorous price competition, a high-tech, low-price retailer eventually dominates, wresting the market from its competitors.[1]

We develop our ideas in a simple three-stage model of retail competition that gives particular emphasis to firms' pricing and investment strategies. The game begins with an initial *entry phase* of the market, during which each of several identical potential entrants decides whether to participate in the industry. Next, in the market's *competitive phase*, firms select prices, with each recognizing that a low price in this stage may inspire a large future market share. Firms also make investments in cost-reducing capital during this phase. Finally, in the *mature phase* of the market, firms set prices and realize their respective final market shares.

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[1]Empirical research on the dynamics of retail markets is scant, but retail commentators seem to agree with the basic trends of retail markets as described here. For example, Bendetti and Zellner (1992, p. 66) observe that:

> *"In category after category, giant 'power retailers' are using sophisticated inventory management, finely tuned selections, and, above all, competitive pricing to crowd out weaker players. Consultants Management Horizons predicts that retailers now accounting for half of all sales will disappear by the year 2000 through bankruptcy, mergers and other reorganizations. Triumphing over them are superpowers including Wal-Mart, Kmart, Toys 'R' Us, Home Depot, Circuit City Stores, Dillard Department Stores, Target Stores and Costco, among others."*

In this article and many others (Chakravarty, 1991; Chanil, 1991, 1992; Power and Dunkin, 1990; Sapporito, 1991; Zinn and Power, 1990), the same themes arise repeatedly: a shakeout or consolidation is underway, as many retailers exit or diminish in size, leaving the market dominated by a few large retailers with low costs and prices.

A key ingredient in our model is the assumption that consumers can observe a firm's contemporaneous price only after incurring a search cost. In such a situation, consumers are motivated to use all available information so as to better forecast the prices available at the various firms. For example, consumers may communicate with one another and share past price experiences, if it is expected that a firm's current and past prices are in some way related. In order to allow for this possibility, we suppose that word-of-mouth communication is extensive, and we assume that consumers learn all competitive-phase prices prior to entering the mature phase. We then consider whether a rational consumer would ever grant a firm a "low-price reputation," whereby the consumer concludes that a low competitive-phase price portends a low mature-phase price as well.

Specifically, we propose a simple *comparison strategy* for consumers, in which (i). consumers divide up evenly over all entering firms in the competitive phase, and then (ii). in the mature-phase, those consumers that do not face prohibitive switching costs elect to visit the firm that is the low-price leader, i.e., the firm whose competitive-phase price was lowest. This is a simple consumer search rule, and it has two important implications for firm behavior. First, firms battle in the competitive phase for the reputation of being the low-price leader. Second, a consolidation or *shakeout* is induced: a number of firms enter, with each hoping to emerge as the victor, but in the end only one firm dominates.

The consumer search rule of purchasing from the historic low-price leader is a simple rule, but it is also a rational rule. This is because a firm's competitive-phase price serves as a credible signal of its investment in cost reduction and thus its mature-phase price. Intuitively, a firm that prices low in the competitive phase has high contemporaneous sales, and thus recognizes substantial benefits from investment that reduces marginal cost. This permanent reduction in marginal cost in turn creates an incentive for the firm also to set a low price in the future. A similar effect derives from the relationship between the firm's current price and its expected future sales. Given the consumers' comparison search rule, a firm with a low current price is rewarded with greater expected future patronage. This provides an additional channel through which a low current price sparks the firm's incentive to invest in marginal-cost reduction, thereby again lowering the firm's optimal future price.

Building on these basic insights, we construct a free-entry equilibrium that conforms well with the features observed in retail markets. In this equilibrium, a shakeout occurs, as several ex ante identical firms enter the market and yet a single firm comes to dominate the market in its mature phase. This firm offers the lowest price, selects the greatest investment level, and obtains the highest sales volume in the competitive phase. Given its low-cost structure and its reputation for low prices, the dominant firm also has the lowest price and achieves an even higher volume in the mature phase.

In view of the aggressive pricing that transpires during the competitive phase, and the ensuing shakeout that concentrates market share in the hands of the low-price leader, it might seem natural to interpret the leader's pricing behavior as predatory. Further, we show that the leader's mature-phase price, and indeed the mature-phase prices of all firms, must be higher than the competitive-phase level; this makes the predatory interpretation of the leader's conduct all the more compelling. We argue that this interpretation is inappropriate, however, since the leader's price cut simply signals that it is the most efficient firm, and its gain in market share represents consumers' natural response to this information. Major legal tests for judging for predation may nonetheless incorrectly condemn the leader: the standard of Baumol (1979) condemns the leader as a consequence of its post-shakeout price increase, while the Areeda-Turner (1975) rule has the perverse effect of finding predation if and only if rivalry generates the sharpest price cuts and greatest consumer benefits.

The role played in the model by marginal-cost-reducing investment is crucial, and the relevance of such investments for retail markets thus warrants some discussion. In fact, investments of this nature are especially relevant for retail firms, and correspond naturally to expenditures on computers, communication systems, warehouses, and trucks that lower the marginal cost of information processing, inventory management, and product distribution. For example, Wal-Mart has reduced its variable selling costs with heavy investments in state-of-theart information and distribution systems, including a comprehensive electronic scanner checkout system and privately-owned satellites and warehouses (Stone, 1991; Sawaya, 1992; Huey, 1989). These investments reduce marginal selling costs, by leading to more efficient inventory management and sharply-reduced telephone costs, and by cutting out of the "middleman" and the corresponding doubling of margins.[2] Analogous investment strategies have been adopted by other giant retailers, including Kmart (Stone, 1991; Chakravarty, 1991), J.C. Penney (*Chain Store Executive Age*, 1988), and Toys "R" Us (Stern and Schoenhaus, 1990, p. 276).

Our research relates to several strands of previous work. For example, low-price reputations have been previously considered in the context of signaling models, where a low price today signals a low-cost type and thereby a low price tomorrow (Bagwell, 1987).[3] By contrast, the "signaling" process in the present research does not involve exogenous cost types; rather, a low price today signals a *choice* to invest a large amount in cost reduction, leading in turn to the expectation of a low price tomorrow. We also go further than the previous work in examining the dynamics of market structure and the issue of predatory pricing.

Our work is also related to the learning-by-doing model of Cabral and Riordan (1994). They consider a dynamic duopoly model in which learning-by-doing stimulates aggressive pricing in the short-run, followed by a long-run consolidation of market share at the low-cost firm and potential exit by the high-cost rival. Cabral and Riordan point out as well that predatory pricing tests can have perverse consequences in some equilibria. While we share with Cabral and Riordan an emphasis on endogenous cost structures and battles for dominance, we offer a model that stresses important aspects of the retail industry, such as cost-reducing investments, price reputations and consumer search behavior.

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[2]The reductions in marginal costs are apparently quite drastic, and in fact Wal-Mart's retail prices often lie below the wholesale prices of competing merchants (Sidey, 1992).

[3]See also the loss-leader model of Simester (1992) and the repeated-game theory of low-price reputations developed by Bagwell and Ramey (1992). The latter paper emphasizes how consumers can discipline firms' pricing by threatening to direct future search away from firms that raise current prices.

The characteristics of large retail firms are studied by Bagwell and Ramey (1994a,b), who show that high volume, low prices, advanced selling technologies and extensive product selection are complementary to one another. They then use this finding to argue that consumers can benefit by coordinating their purchase behavior and propose a theory of advertising that is consistent with rational consumer behavior. The present research emphasizes firms' pricing and investment strategies, rather than advertising and consumer coordination, and constructs a *dynamic* model of retail competition that offers new insights regarding low-price reputations, shakeout and predation.[4]

Finally, the findings developed here contrast strikingly with those developed in the switching-cost literature, as surveyed by Klemperer (1992). In that literature, consumers incur switching costs when visiting new firms, and consumers are perfectly informed of current prices. The presence of switching costs leads firms to price aggressively in the competitive phase, as they compete for the right to monopolize "locked-in" consumers at a later date. In our model, by contrast, consumers are imperfectly informed of current prices, and they examine historic pricing patterns in forming current search strategies. The incentive to price aggressively in the competitive phase now may be linked to the *absence* of switching costs, since a firm prices lower in the competitive phase in order to induce consumers to switch to it in the future.

The paper is organized as follows. The basic model is presented in Section II. Next, the mature-phase strategies are described in Section III, and the competitive-phase strategies and the market equilibrium are found in Section IV. Entry and shakeout are discussed in Section V, while Section VI features equilibrium price dynamics and predatory pricing, respectively. Section VII concludes.

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[4]Portions of our formal analysis draw heavily on that developed by Bagwell (1992) for a limit pricing game with multiple incumbents. He constructs a mixed-strategy equilibrium, in which the entrant uses a comparison strategy and enters against the incumbent with the highest preentry price. This comparison strategy is rational since a low preenty price is complementary to a heavy investment in marginal-cost reduction. More broadly, our techniques build on those developed in the mixed-strategy price-dispersion literature (Bagwell and Ramey, 1994b; Rosenthal, 1980; Varian, 1980).

II. The Model

In this section, we describe formally the competitive and mature phases of our model in terms of a two-stage game. The central demand, cost and profit assumptions that underlie our analysis are also developed in this section. In subsequent sections, we characterize the game's equilibria, and consider a larger game that also includes an initial entry phase.

The two-stage game has $N \geq 2$ symmetric firms and the following sequence of actions:

Stage 1: Each firm simultaneously picks a Stage-1 price, $P \in \Re_+$, and a level of investment, $I \in \Re_+$. Consumers observe neither of these choices and simultaneously select firms from which to buy.

Stage 2: Both consumers and firms learn all of the prices of the previous stage, although each firm's investment level remains its private knowledge. Firms then pick their Stage-2 price, $R \in \Re_+$, at the same time that consumers decide which firm to visit.

We refer to Stage 1 as the market's *competitive phase* and Stage 2 as the market's *mature phase*.

In retail markets, consumers acquire price information through direct search experience and also through word-of-mouth communication. These possibilities are captured in our game with the assumptions that current prices can be observed only through search while past price information can be learned through communication. Formally, our model may be understood as one in which: (i). consumers face a search cost of zero for the first firm visited per stage, but the cost of visiting a second firm within a given stage is prohibitive, and (ii). consumers share all information regarding past price experiences. We therefore have a simple structure with which to explore the idea that consumers may communicate and learn about past pricing in order to better forecast current prices and search more efficiently.

We further enrich the model by assuming that a fraction 1-s of consumers have high switching costs (Klemperer, 1992) and thus remain in Stage 2 with the firm with which they began in Stage 1. The remaining fraction $s \in (0,1)$ of consumers are price-sensitive relative to their switching costs and are therefore able to change firms. Our model includes the situation in which all consumers are able to switch as a limiting case. As will become clear, the presence of switch-capable consumers plays an important role in determining the responsiveness of a firm's current market share to its past price.

The model admits an abundance of sequential equilibria (Kreps and Wilson, 1982). To deal with the multiplicity issue, we follow Bagwell and Ramey (1994b) and Bagwell (1992) and select among the set of equilibria by looking at those in which consumers follow plausible - but still fully rational - rules of thumb. In particular, we restrict attention to sequential equilibria in which consumers employ a simple *comparison strategy*, whereby (i). consumers divide up evenly over entering firms in Stage 1, and (ii). consumers who are capable of switching firms divide up evenly in Stage 2 over those firms with the lowest Stage-1 price. In other words, consumers compare historic prices and reward low-price leaders from the competitive phase with greater market share in the mature phase.

An *equilibrium* is now defined as a sequential equilibrium in which consumers use the comparison strategy. Our general approach is first to assume that consumers adopt the comparison strategy, then to determine the consequent optimal firm behavior, and finally to verify that the consumer comparison strategy is in fact optimal among all possible search strategies.

Before turning to a derivation of the set of equilibria, however, we first develop the assumptions on demand, cost and profit functions that underlie our analysis. Aggregate consumer demand is assumed stationary over the two stages. Specifically, there is a unit mass of consumers, and each consumer possesses a demand function, D(P), which is positive, twice differentiable, and downward-sloping for all P.[5] In Stage 1, consumers are divided up evenly among firms, and so each firm's demand is D(P)/N. In Stage 2, a firm's demand will depend

[5]Our analysis is easily extended to allow for demand to be zero for P above some "choke price," so long as the choke price is not too small.

upon whether or not it charged the lowest price in the preceding competitive phase. If a firm's Stage-1 price is not among the lowest, then its Stage-2 demand is $[(1-s)/N]D(P)$. If instead its price is among the lowest, then the firm's demand is $[(1-s)/N + s/M]D(P)$, where M is the number of low-priced firms.

The firm's cost function is also taken to be stationary through time, and, letting Q denote the output sold by a firm, we represent this cost function as follows:

Assumption 1: A firm's cost function takes the form $c(I)Q$, where $c(I)$ is twice differentiable, $c'(I) < 0$ and $\lim_{I \to \infty} c(I) > 0$.

Thus, each firm has a constant marginal cost of production that decreases with investment. As discussed in the Introduction, the assumption that investment lowers marginal costs is especially appropriate for the retail firm, since investment can be interpreted in terms of electronic scanner checkout systems and privately-owned satellites and warehouses. Note that $c(I)Q$ represents the operating costs of a firm and does not include the fixed cost of entry or the expenses associated with investment. These latter costs are introduced below.

Let $F \in (0,1]$ be the fraction of consumers obtained by the firm, so that operating profits in each stage have the form:

(1). $\Pi(P,I,F) = [P - c(I)]D(P)F.$

The following assumption guarantees that the firm's single-stage profit function has a well-defined optimal price:

Assumption 2: For all $P, I \in \Re_+$ and $F \in (0,1]$, the profit function $\Pi(P,I,F)$ is concave in (P,I). Further, there is a unique profit-maximizing price, $P = R^*(I)$, at which $\Pi_P(P,I,F) = 0$.

The concavity of operating profit becomes useful below for purposes of comparative statics. Observe further that R* is independent of F, since the cost function exhibits constant returns to scale when investment is held constant.

Given the existence of the profit-maximizing price, it is convenient also to define the corresponding level of maximized profits:

(2). $\Pi^*(I,F) \equiv \Pi(R^*(I),I,F)$.

It is easy to show that $\Pi^* > 0$, since $R^* > c(I)$ follows from the first order condition.[6] R* and Π^* will play important roles in the subsequent section, where we discuss the nature of firm behavior in the market's mature phase.

III. Mature-Phase Prices and Profits

Define a *low-priced firm* as any firm with the lowest price in Stage 1, and a *high-priced firm* as any firm that did not charge the lowest price in Stage 1. This section provides a characterization of the Stage-2 equilibrium prices and profits of low- and high-priced firms.

We begin with the following lemma:

Lemma 1: In any equilibrium of the two-stage game, if the number of low-priced firms entering the market's mature phase is M, then:

(i). A low-priced firm with previous investment level I selects the mature-phase price $R = R^*(I)$ and earns the mature-phase profit $\Pi^*(I,(1-s)/N + s/M)$.

(ii). A high-priced firm with previous investment level I selects the mature-phase price $R = R^*(I)$ and earns the mature-phase profit $\Pi^*(I,(1-s)/N)$.

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[6]Given our assumption that $D(P) > 0$ for all P, maximized profits must be positive for every I. If instead a choke price P' exists, then we must add the assumption that $c(0) < P'$.

Observe that investment provides an important link between the two stages, as the mature-phase equilibrium price is completely determined by the previous investment choice. The proof of the lemma is immediate.

We consider next how the firm's mature-phase equilibrium price depends upon its prior investment activity. Simple calculations reveal that:

(3). $\text{sign } R^{*\prime}(I) = \text{sign } c'(I) < 0.$

Intuitively, firms that invest more have lower marginal costs and thus find lower prices more desirable. In this sense, a high investment level in the competitive phase "commits" a firm to a lower price in the subsequent mature phase.

Consider next the mature-phase equilibrium profits of firms. Observe first that firms that invest more earn greater future profits:

(4). $\text{sign } \Pi_I^*(I,F) = - \text{ sign } c'(I) > 0.$

This property follows easily from (1), (2) and the envelope theorem, and it simply captures the notion that a firm's profits must increase when its total costs decline. A second observation is that a profit-maximizing firm benefits from greater market share:

(5). $\text{sign } \Pi_F^*(I,F) = \text{sign } \Pi^*(I,F)/F > 0.$

This property follows because the optimal price must exceed marginal cost, given that this price satisfies the first-order condition, $\Pi_P = 0$. Thus, greater market share must increase profits.

Taken together, these findings indicate that a low-priced firm offers a better deal to consumers and makes greater profit in the mature phase, *if a low-priced firm also invests a* *greater amount in cost reduction.* The next step is therefore to examine firms' price and investment strategies in the competitive phase.

IV. Competitive-Phase Prices and Profits

This section derives the competitive-phase pricing and investment strategies of the firm. We proceed by first defining the optimal investment and price for a firm, when the likelihood of being the low-priced firm is taken to be exogenous. Then, using the comparative-statics results derived through this exercise, necessary characteristics of equilibrium behavior are derived, and an equilibrium is constructed.

A. Comparative-Statics Results

We begin by modeling the firm as if it has a fixed probability $\rho \in [0,1]$ of being the low-priced firm and an associated probability $1-\rho$ of being a high-priced firm.[7] In this event, the expected discounted profit to the firm as viewed from Stage 1 is given by:

$$(6).\qquad V(P,I,\rho) \equiv \Pi(P,I,1/N) - kI + \delta[\rho\Pi^*(I,(1-s)/N + s) + (1-\rho)\Pi^*(I,(1-s)/N)],$$

where $\delta \in \Re_+$ is the discount factor and $k > 0$ is the per unit cost of investment. Note that we allow $\delta > 1$ in order to introduce the possibility that the mature phase is long relative to the competitive phase.

We now place some mild restrictions on this payoff function:

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[7]Observe that this exercise embodies two assumptions. First, the probability of being the low-priced firm is taken to be independent of the firm's price. As explained, this assumption will be relaxed subsequently and enables us to define a function that will be useful in characterizing the equilibrium. Second, a firm is either the low-priced firm or a high-priced firm; in other words, situations in which the firm ties and is one of the low-priced firms are not captured when a single parameter ρ is employed. We show later that ties never occur in equilibrium.

Assumption 3: For all P, I $\in \Re_+$ and $\rho \in [0,1]$,

(i). $V(P,I,\rho)$ is maximized over P and I by a unique pair $(P^*(\rho),I^*(\rho)) > 0$ at which $V_P(P,I,\rho) = V_I(P,I,\rho) = 0$.

(ii). For a fixed P, $V(P,I,\rho)$ is maximized over I by a unique selection, $\tilde{I}(P,\rho) > 0$, at which $V_I(P,I,\rho) = 0$.

(iii). $V(0,\tilde{I}(0,\rho),\rho) \leq 0$.

Assumptions 3 (i) and (ii) are standard and simply amount to interiority conditions, since we show below that V is concave. We may understand $(P^*(\rho),I^*(\rho))$ to be the *monopoly price-investment pair*, given some probability ρ of being the low-priced firm. Similarly, $\tilde{I}(P,\rho)$ is the *optimal investment level*, holding fixed the Stage-1 price and the probability of being the low-priced firm. Assumption 3 (iii) holds if D(0) is large.

The comparative-statics properties of the functions P^*, I^* and $\tilde{I}$ will be important in the sequel. To address these properties, we first establish that:

Lemma 2: For all $\rho \in [0,1]$, $V(P,I,\rho)$ is concave in (P,I).

The proof of this lemma is found in the Appendix. It also is useful to observe that $P^*(\rho) = R^*(I^*(\rho))$ and $I^*(\rho) = \tilde{I}(P^*(\rho),\rho)$. Finally, we may define the monopoly two-stage payoff function $V^*(\rho) \equiv V(P^*(\rho),I^*(\rho),\rho)$, and it follows directly that $V^*(\rho) > 0$.

With the relevant definitions and assumptions now in place, we are prepared to investigate the relationships that exist through the payoff function between price, investment and the probability of entry. The following results are confirmed in the Appendix:

Lemma 3:

(i). $V_{PI}(P,I,\rho) < 0$.

(ii). $V_{P\rho}(P,I,\rho) = 0$.

(iii). $V_{I\rho}(P,I,\rho) > 0$.

(iv). $V_{\rho}(P,I,\rho) > 0$.

Part (i) indicates a complementarity between low prices and high investment; this follows since a low price stimulates sales and thus raises the immediate benefits from a lower marginal cost. A similar logic underlies part (iii): when the probability of being the low-priced firm rises, the expected number of future customers is increased, and so the benefit of investment that lowers future marginal costs also increases. Observe from part (ii) that no direct relationship exists between prices and the probability of being the low-priced firm. Finally, part (iv) establishes that expected discounted profit rises when the odds of being the low-priced firm improve.

Given Lemmas 2 and 3, the following comparative-statics results are direct:

Proposition 1:

(i). $\tilde{I}_P(P,\rho) < 0$ and $\tilde{I}_\rho(P,\rho) > 0$.

(ii). $I^{*\prime}(\rho) > 0 > P^{*\prime}(\rho)$.

(iii). $V^{*\prime}(\rho) > 0$.

Intuitively, the optimal investment level rises whenever current sales rise (due to a lower price) or expected future sales rise (due to a greater chance of being the low-priced firm), since a reduction in marginal cost is then especially valuable. Even though there is no direct complementarity between a low price and the probability that the firm will have the lowest price, a greater likelihood of being the low-priced firm induces the firm to invest more, and this in turn implies that the firm's monopoly price declines as the probability of being the low-price leader increases. Finally, using Lemma 3 and the envelope theorem, it follows easily that the monopoly payoff function increases with the probability of being the low-priced firm.

B. Competitive-Phase Equilibrium Strategies: Necessary Properties

With the basic properties of the key competitive-phase variables now recorded, we are prepared to endogenize the probability that a firm becomes the low-priced firm and thereby to characterize the necessary features of firms' competitive-phase equilibrium strategies. For expositional purposes, we give here only the main ideas and defer details to the Appendix.

Given the discontinuous nature of the consumer comparison strategy, whereby the lowest-priced firm wins all switch-capable consumers, it is not surprising that pure-strategy equilibria fail to exist. We thus focus on *symmetric equilibria* in mixed strategies, defined as equilibria in which firms all use the same randomized price and investment strategies. Let $\hat{G}(P)$ represent the symmetric equilibrium distribution function for firm's competitive-phase prices. It is straightforward to establish that $\hat{G}(P)$ must be continuous; i.e., there can be no point masses in a firm's equilibrium probability distribution over prices. This in turn implies that ties occur with probability zero: a firm will either be *the* low-priced firm or a high-priced firm. These results are presented as Propositions A and B in the Appendix.

With ties ruled out, it follows that a single parameter, ρ, describes the probability distribution facing a firm with respect to the number of the firm's future consumers. The results derived in the previous subsection may therefore be applied. In particular, the firm randomizes over price-investment pairs in equilibrium, and these pairs must take the form $(P, \tilde{I}(P,\hat{\rho}(P))$, where $\hat{\rho}(P) \equiv [1-\hat{G}(P)]^{N-1}$ is the equilibrium probability that a firm selecting the price P will be the low-priced firm. This follows because investment is unobservable and a firm thus chooses investment to maximize expected discounted profit, given the price it charges in the competitive phase and the implied probability that the firm will be dominant in the mature phase.

Let the support of $\hat{G}(P)$ be given by $[\underline{P},\bar{P}]$; i.e., $\underline{P}$ is the largest price for which $\hat{G}(P) = 0$, while $\bar{P}$ is the smallest price satisfying $\hat{G}(P) = 1$. In fact, as we show formally in the Appendix as Proposition C, $\bar{P}$ is the monopoly price for a firm with zero likelihood of being the low-priced firm, i.e., $\bar{P} = P^*(0)$. Intuitively, since there are no point masses, a firm charging $\bar{P}$ is sure not to be the low-priced firm; consequently, there would no obstacle to a deviation to the price $P^*(0)$ were $\bar{P} \neq P^*(0)$.

To characterize the remaining features of $\hat{G}(P)$, it is useful to define the expected discounted profit for any given price and probability when investment is selected optimally:

(7). $\tilde{V}(P,\rho) \equiv V(P,\tilde{I}(P,\rho),\rho)$.

This function has a number of useful and important properties:

(8). $\tilde{V}_P(P,\rho) = V_P(P,\tilde{I}(P,\rho),\rho)$

(9). $\tilde{V}_{PP}(P,\rho) = V_{PP}(P,\tilde{I}(P,\rho),\rho) + V_{PI}(P,\tilde{I}(P,\rho),\rho)\tilde{I}_P(P,\rho) < 0$

(10). $\tilde{V}(0,\rho) \leq 0$

(11). $\tilde{V}_\rho(P,\rho) = V_\rho(P,\tilde{I}(P,\rho),\rho) > 0$,

where these properties follow easily from the assumptions and lemmas presented above. Recalling that $\tilde{I}(P^*(\rho),\rho) = I^*(\rho)$ and using (8), it is evident that $\tilde{V}_P(P,\rho) = 0$ at $P = P^*(\rho)$ and thus that $\tilde{V}$ is maximized at $P^*(\rho)$. These properties are illustrated in Figure 1.

[Insert Figure 1 about here.]

We are now prepared to characterize the lowest possible equilibrium price, $\underline{P}$. To this end, observe first that the expected discounted profit earned in the mixed-strategy equilibrium must be $V^*(0)$, as this is the expected discounted profit earned by a firm choosing the price $\bar{P}$.

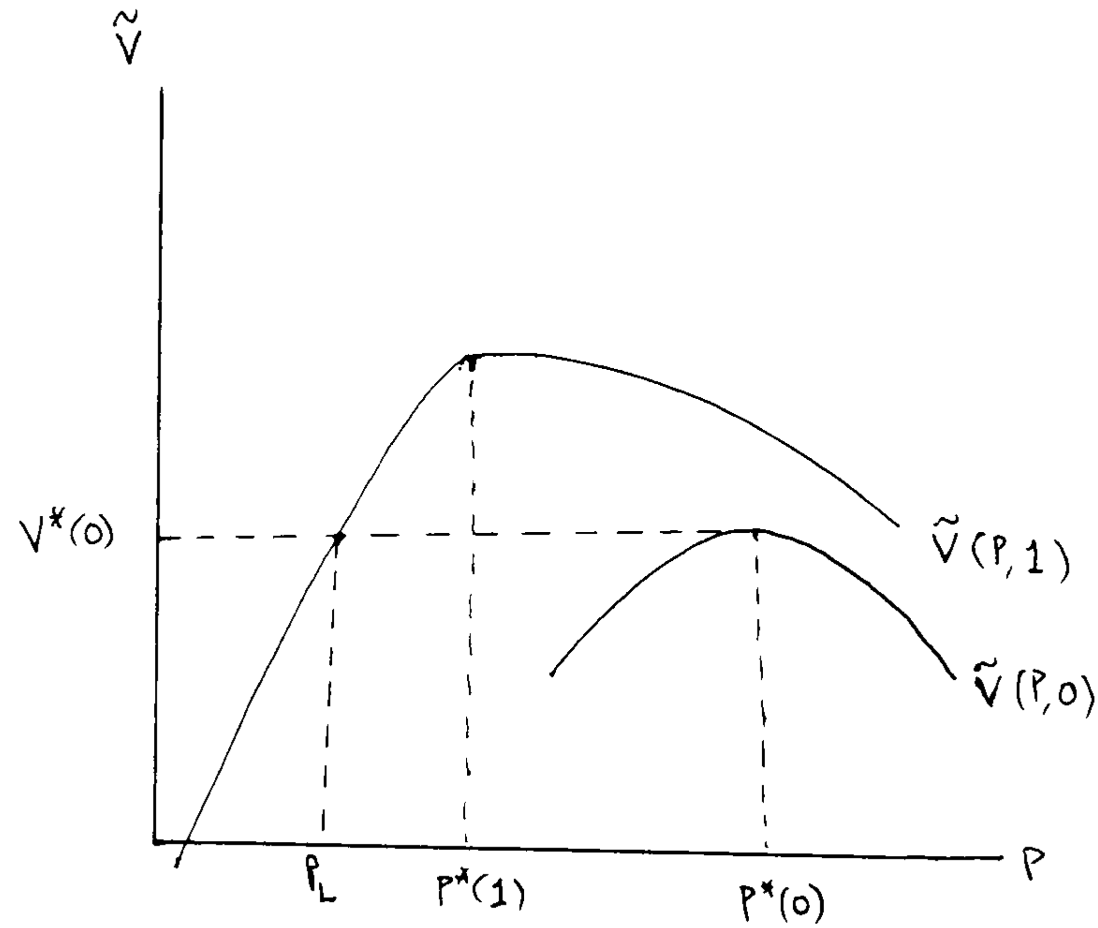

Figure 1

Next, the absence of point masses ensures that a firm charging $\underline{P}$ will be the lowest-priced firm with probability one, and thus will earn $\tilde{V}(\underline{P},1)$. Finally, referring to Figure 1, observe that a unique price $P_L < P^*(1)$ can be found such that $\tilde{V}(P_L,1) = V^*(0)$. Since expected discounted profit must be constant along the support of the mixed strategy, it follows that P_L is the lowest price that could ever be charged in any symmetric equilibrium, i.e., $\underline{P} = P_L$.

A final finding is that gaps cannot exist in the support of the mixed strategy distribution; this result is proved in the Appendix as Proposition D. With gaps ruled out, it must be that all prices in the interval $[P_L,P^*(0)]$ yield the same expected discounted profit, $V^*(0)$. Letting $\hat{I}(P)$ denote the equilibrium investment choice paired with a particular price selection, we conclude that:

Proposition 2: In any symmetric equilibrium of the two-stage game, for any $P \in [P_L,P^*(0)]$, the equilibrium price distribution function, $\hat{G}(P)$, and the equilibrium investment function, $\hat{I}(P)$, must satisfy:

(12). $\tilde{V}(P,\hat{\rho}(P)) = V^*(0)$

(13). $\hat{I}(P) = \tilde{I}(P,\hat{\rho}(P))$

where $\hat{\rho}(P) \equiv [1 - \hat{G}(P)]^{N-1}$.

Equation (12) simply indicates that expected discounted profit must be constant over the support, while (13) defines the optimal investment level, given a price and the associated probability of becoming the low-priced firm.

Figure 2 illustrates the tradeoff that confronts firms as represented in (12): by pricing lower, a firm sacrifices some profit in the competitive phase, but it also increases the likelihood that it will become the low-price leader once the market's mature phase is reached. Corollary 1 further sharpens this tradeoff, by capturing a specific sense in which firms compete in the competitive phase to become the low-price leader:

Corollary 1: In any symmetric equilibrium of the two-stage game, for any $P \in [P_L, P^*(0))$, $P < P^*(\hat{\rho}(P))$ and $\hat{I}(P) > I^*(\hat{\rho}(P))$, where $\hat{\rho}(P) \equiv [1 - \hat{G}(P)]^{N-1}$.

Proof: Fix $P_1 \in [P_L, P^*(0))$ so that $\hat{\rho}(P_1) \in (0,1]$. From Figure 2, it is clear that $P_1 < P^*(\hat{\rho}(P_1))$. Also, $\hat{I}(P_1) = \tilde{I}(P_1, \hat{\rho}(P_1)) > \tilde{I}(P^*(\hat{\rho}(P_1)), \hat{\rho}(P_1)) = I^*(\hat{\rho}(P_1))$. Q.E.D.

Thus, for any given price P, a probability of being the low-priced firm is implied, and the firm would prefer to raise its price (toward $P^*(\hat{\rho}(P))$ if it could do so without reducing the probability that it would be the low-price leader. In other words, low introductory pricing is motivated by a desire to increase future market share, since a firm would wish to raise its price and sell less if this did not lower the likelihood that it would become dominant in the future.[8] Note that downward distortions in price "spill over" into upward distortions in investment, because the associated expansion in competitive-phase sales enhances the incentive to invest.

[Insert Figure 2 about here.]

C. Existence of Equilibrium

Thus far, we have argued that if consumers use a comparison strategy, then in any symmetric equilibrium entering firms must randomize their price and investment selections, with the randomization simply reflecting the tradeoff between pricing low/investing high and improving the odds of becoming the dominant retailer versus pricing high/investing low and

[8]Some previous research (Baumol, 1959, Williamson, 1964; Farrell and Saloner, 1986, Katz and Shapiro, 1986; Fershtman and Judd, 1987; Klemperer, 1992) also suggests that even profit-maximizing firms may have a market-share-maximization objective. In the present research, however, managers do not care directly about market share, direct network externalities are not imposed, strategic managerial payment schemes are not considered, and the *absence* of switching costs is important.

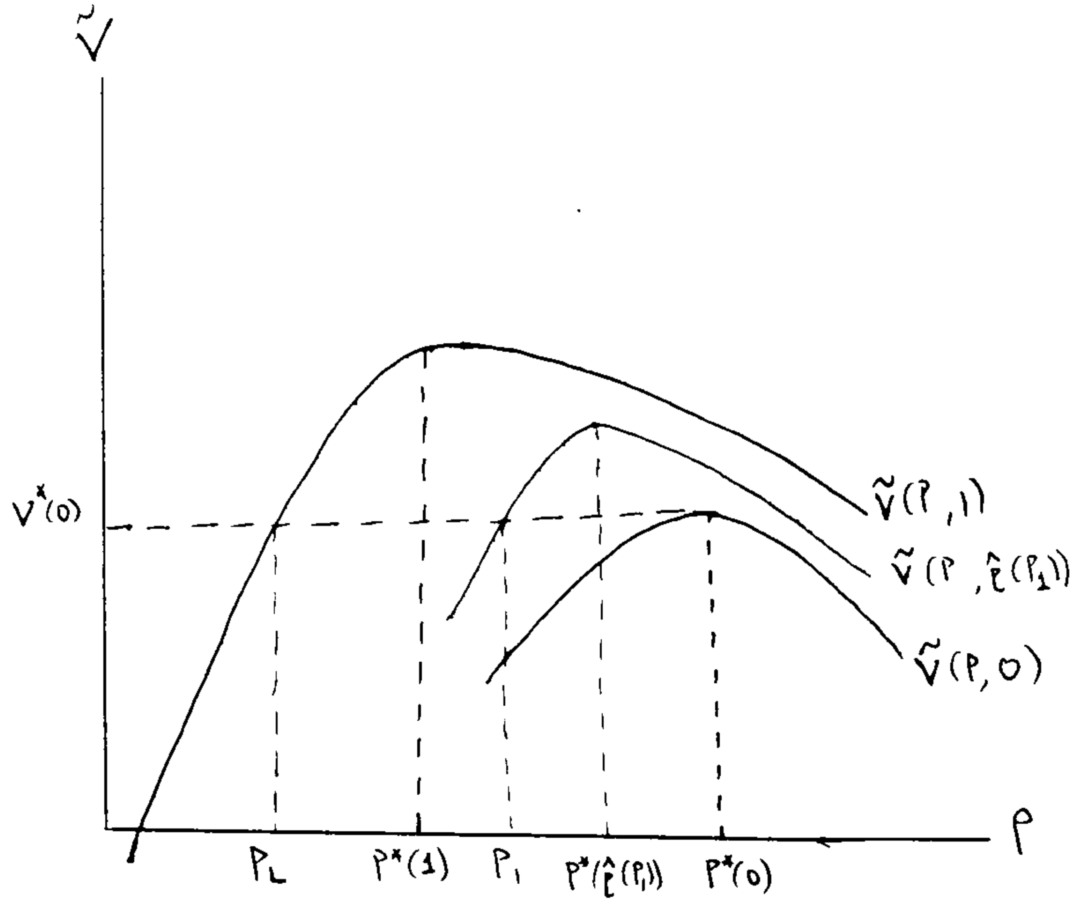
$\tilde{V}$
$V^*(0)$
$\tilde{V}(P,1)$
$\tilde{V}(P,\hat{\ell}(P_1))$
$\tilde{V}(P,0)$
P
P_L
$P^*(1)$
P_1
$P^*(\hat{\ell}(P_1))$
$P^*(0)$

Figure 2

earning greater profit in the short term. Our goal in this subsection is to show that a symmetric equilibrium exists.

To this end, we first verify that the firms' pricing strategy, $\hat{G}(P)$, as defined implicitly in (12) is indeed a well-defined distribution function. This follows because $\tilde{V}$ is increasing in ρ, as (11) indicates, an so $\hat{G}(P)$ may be defined via the implicit function theorem as applied to (12). Intuitively, and as Figure 2 illustrates, as P drops, the probability of being the low-priced firm must rise, in order to preserve indifference. Thus, $\hat{\rho}(P)$ must be decreasing in P, at least for $P < P^*(0)$. With $\hat{\rho}(P) \equiv [1 - \hat{G}(P)]^{N-1}$, it then follows that $\hat{G}(P)$ is increasing in P. One can also verify from (12) that $\hat{G}(P_L) = 0 < 1 = \hat{G}(P^*(0))$.

Next, we must confirm that the firms' strategies as defined in (12) and (13) are in fact profit maximizing. Observe that no firm can gain by altering the probability with which it selects some $P \in [P_L, P^*(0)]$. Further, $P < P_L$ is less profitable in expectation than is P_L, and $P > P^*(0)$ is similarly inferior to $P^*(0)$, taking as given other players' strategies.

The final and most interesting step is to verify that the imposed consumer comparison strategy is in fact a best response among all possible search strategies to the induced behavior of firms. Note that it is optimal for consumers to pick firms randomly in the competitive phase, since firms employ symmetric strategies. Furthermore, a switch-capable consumer does best in the mature phase by visiting the firm with the lowest competitive-phase price. For suppose that $P_a < P_b$ are both observed in the competitive phase, and that P_a is the price of the lowest-priced firm. Then using (3) the mature-phase prices of firms a and b satisfy:

$$
\begin{aligned}
(14). \qquad R_a &= R^*(\tilde{I}(P_a, \hat{\rho}(P_a))) \\
&< R^*(\tilde{I}(P_b, \hat{\rho}(P_a))) \\
&< R^*(\tilde{I}(P_b, \hat{\rho}(P_b))) \\
&= R_b,
\end{aligned}
$$

and so firm a, the low-price firm, charges a lower price in the mature phase as well.

Intuitively, when a firm charges a lower price in the competitive phase, it realizes that it will sell a greater volume of output in that phase (since $D' < 0$), and it also expects to sell a larger volume in the future, because it now has a better chance of emerging as the low-price leader (since $\hat{\rho}'(P) < 0$ in equilibrium). Thus, as a firm lowers its competitive-phase price, it raises its expected current and future sales volumes, and it thereby commits itself to the associated higher investment level and the consequent lower mature-phase price. Switch-capable consumers are therefore entirely rational in employing the comparison strategy, since the firm that offered the lowest price in the competitive phase also chose the highest investment level, implying that this firm will select the lowest mature-phase price as well. In other words, consumers are rational in using a simple rule-of-thumb that grants a "low-price reputation" to the firm with the lowest historic price.[9]

We now summarize the preceding discussion with the following proposition:

Proposition 3: For the two-stage game, there exists a symmetric equilibrium in which:

(i). The competitive-phase firm strategies, $\hat{G}(P)$ and $\hat{I}(P)$, satisfy (12) and (13).

(ii). The mature-phase firm strategy, R, is as described in Lemma 1.

V. Entry and Shakeout

We now append to the model an earlier Stage 0, in which the number of firms is endogenously determined as a function of market structure parameters. With the number of firms determined in this manner, we then characterize the level of consolidation or *shakeout* that occurs as the market passes from its competitive to its mature phase.[10]

---

[9]Consumers' belief as to a firm's investment level is correct for $P \in [P_L, P^*(0)]$. For $P \notin [P_L, P^*(0)]$, the consumer search rule remains optimal if the consumer belief function is completed in the natural manner, with the belief that $I = \tilde{I}(P,1)$ for $P < P_L$ and $I = \tilde{I}(P,0)$ for $P > P^*(0)$.

[10]Alternative theories of shakeout and exit have been previously developed by Clarida (1993), Fudenberg and Tirole (1986), Ghemawat and Nalebluff (1985), and Whinston (1988).

Specifically, the two-stage game presented above is amended to include an earlier stage:

Stage 0: Firms simultaneously decide whether or not to enter, where the one-time sunk cost associated with entry is E.

While it is in no way essential, we assume that firms use nonrandom strategies in this stage. A *symmetric equilibrium* for this three-stage game is then a sequential equilibrium in which firms use nonrandom entry strategies in Stage 0 and the behavior of firms and consumers in Stages 1 and 2 is as described in Proposition 3.[11]

We turn first to a determination of the number of entrants. Given the symmetry of firm equilibrium strategies in Stages 1 and 2, it follows that all entrants earn the same expected discounted profit. Furthermore, since expected discounted profits must be constant along the support of the equilibrium strategy, each entrant earns:

$$
\begin{aligned}
(15).\quad V^*(0) &= V(P^*(0),I^*(0),0) \\
&= \Pi(P^*(0),I^*(0),1/N) - kI^*(0) + \delta\Pi^*(I^*(0),(1-s)/N) \\
&= \Pi^*(I^*(0),1/N) - kI^*(0) + \delta\Pi^*(I^*(0),(1-s)/N),
\end{aligned}
$$

where the last equality follows since $P^*(0) = R^*(I^*(0))$.

The firms' cost function as described in Assumption 1 admits no fixed costs, and so we introduce a cost of entry in order to bound the number of entrants. We assume that the one-time sunk cost of entry, E, satisfies $E > 0$ and $E \leq V^*(0)$ when $N = 2$, where the latter inequality ensures that duopoly is viable. As is clear from (15), for sufficiently large N, $V^*(0)$ is sure to

[11]A full specification of equilibrium requires also that behavior be defined when $N = 1$. In the event of only one entrant, we select the equilibrium for the two-stage game in which all consumers visit this firm in each stage and the firm selects $(P^*(1),I^*(1))$ and $R^*(I^*(1))$ in the competitive and mature phases, respectively. We make assumptions below that ensure that at least two firms enter in the symmetric equilibria of the three-stage game.

lie below E; thus, there exists some $N^* \geq 2$ at which $V^*(0) = E$. Ignoring integer constraints, it follows that an equilibrium exists for the three-stage game in which N^* entrants enter the market; moreover, N^* is easily shown to be unique:

Proposition 4: There is a unique symmetric equilibrium number of entrants, $N^* \geq 2$, for the three-stage game, and this number is determined by $V^*(0) = E$.

Thus, the symmetric equilibrium of the three-stage game is essentially unique, with multiple equilibria arising only in that the model does not pin down the identity of the N^* entrants.[12]

It is now apparent that a consolidation or shakeout occurs in symmetric equilibria of the model. Many firms enter and engage in price rivalry, but in the end only one firm becomes dominant and collects all switch-capable consumers. All non-dominant firms are shaken out, in the sense that their sales volumes dwindle as the industry moves from the competitive to the mature phase. In the limiting case, where all consumers are switch-capable, non-dominant firms effectively exit in the mature phase.

While the model has many parameters, it is particularly interesting to consider the manner in which the proportion of switch-capable consumers, s, affects equilibrium behavior. We thus consider next the effect of an increase in the number of switch-capable consumers on the equilibrium level of entry:

Proposition 5: In any symmetric equilibrium of the three-stage game, the number of entrants is decreasing in the proportion of switch-capable consumers, s.

---

[12]This source of nonuniqueness would be eliminated if we imposed symmetry also at the entry stage, with firms randomizing with respect to the entry decision as well. We have elected to model deterministic entry strategies for simplicity; our qualitative conclusions also hold in the case of random entry strategies. In addition, our definition of equilibrium and our propositions are consistent with a variety of consumer beliefs as to investment when a non-equilibrium price is observed. Since sequential equilibrium is defined in terms of strategies and beliefs, this represents an additional source of multiple equilibria (although alternative beliefs do not alter the equilibrium outcome). Footnote 9 gives a particular set of beliefs that supports the described equilibrium.

A proof is found in the Appendix. Similar arguments establish that the equilibrium level of entry is increasing in δ and decreasing in k and E.

A simple measure of shakeout is the reduction in market share obtained by a non-dominant firm between the competitive and mature phases. If we measure market share by the proportion of consumers that a firm captures, then the term s/N^* captures the extent of shakeout in the symmetric equilibrium. We then have:

Corollary 2: In any symmetric equilibrium of the three-stage game, the extent of shakeout is increasing in the proportion of switch-capable consumers, s.

This result follows readily from Proposition 5. Similarly, the extent of shakeout is increasing in k and E and decreasing in δ.

Proposition 5 and Corollary 2 are easily understood. As the fraction of switch-capable consumers grows, the benefit of being the low-price leader also rises, since this firm then experiences an even greater expansion in sales volume in the mature phase. As a consequence, competitive-phase price rivalry between firms becomes more intense when consumer switching costs are less pronounced, and this is reflected in a lower level of expected profits. To maintain zero equilibrium profits, therefore, it must be that fewer firms enter as s increases. An immediate implication is then that the extent of shakeout, s/N^*, rises with s, as each non-dominant firm begins with a large market share (since N^* is small) and loses a large proportion of that market share to the low-price leader (since s is large).

VI. Implications for Predatory Pricing

A major concern of antitrust law is *predatory pricing*, in which a predatory firm engages in temporary price-cutting in order to drive out a rival, and then raises its price later. The first part of this basic predatory pattern emerges in the symmetric equilibria of our model, in that the low-price leader of the competitive phase obtains repeat-business benefits that place rivals at a market-share disadvantage in the mature phase. Interestingly, the second part of the predatory sequence occurs as well, since the low-price leader will necessarily choose a higher price in the mature phase. Despite this resemblence to the predatory pattern, however, we argue in this section that it may be inappropriate to view the low-price leader's pricing behavior as predatory.

We begin by showing that the low-price leader raises its price in the mature phase. In fact, equilibrium prices rise through time for *all* firms, whether or not they choose the lowest price in the competitive phase. To see this, observe from Figure 2 that $\tilde{V}_P > 0$ for $P < P^*(0)$, and so

$$(16).\quad \tilde{V}_P(P,\hat{\rho}(P)) = V_P(P,\tilde{I}(P,\hat{\rho}(P)),\hat{\rho}(P)) = \Pi_P(P,\hat{I}(P),1/N) > 0$$

for $P < P^*(0)$. But we also know that the firm's mature-phase price, $R^*(\hat{I}(P))$, satisfies

$$(17).\quad \Pi_P(R^*(\hat{I}(P)),\hat{I}(P),F) \equiv 0$$

for all $F > 0$. Since Π is a concave function of P, it thus must be that $P < R^*(\hat{I}(P))$ when $P < P^*(0)$, which proves the following proposition:

Proposition 6: In any symmetric equilibrium of the three-stage game, with probability one, a firm's price rises from the competitive to the mature phase.

Intuitively, prices are bid down during the competitive phase for the right to enjoy large market share and profit-maximizing pricing in the mature phase.

Although actual exit of rival firms in the mature phase does not occur in our model, the pattern of behavior in equilibrium is sufficiently close to the predatory sequence to suggest a violation of antitrust laws by the low-price leader. Thus, it becomes important to ask whether the leader's price cutting ought to be regarded as predation. The large literature on the law and economics of predation, exemplified by Areeda and Turner (1975), Williamson (1977), and many others, emphasizes that behavior is predatory when it circumvents market mechanisms that reward efficient firms. This standard is clearly enunciated by Posner (1976, p. 188), who defines predatory pricing as "pricing at a level calculated to exclude an equally or more efficient rival."

In our model, the low-price leader's price-cutting emerges precisely as a means of communicating to consumers that it has made the largest investment in cost reduction, and thus that it is the most efficient firm. Far from circumventing the market mechanism, introductory price cuts and the corresponding repeat-business mechanism constitute a natural market response to the problem of communicating efficiency in the presence of imperfect price information. Since price reductions by the low-price leader cannot work to exclude equally or more efficient rivals, it seems inappropriate to regard the leader's behavior as predatory.[13]

A number of practical legal tests have been proposed to assist in determining whether or not behavior is predatory. Despite the apparent absence of predation in our model, a naive application of some of these tests would condemn the low-price leader. Baumol (1979), for example, proposes that a firm engages in predation when it raises its price following the exit of a rival. In our setting, all firms raise their prices following the shakeout at the end of the competitive phase, so that the leader may superficially run afoul of the Baumol rule.

The most influential standard, introduced by Areeda and Turner, states that predatory pricing should be inferred if and only if an alleged predator chooses a price strictly less than its short-run marginal cost. This test exerts a perverse effect in our model, since it operates to condemn the leader only in circumstances that are of greatest benefit to consumers. The

[13]Both Areeda and Turner and Williamson recognize that "promotional pricing" by newly-established firms may warrant exception from antitrust laws, based on the idea that such price cuts communicate the attributes of new products and thereby overcome consumer loyalty to incumbent firms. Our model demonstrates that promotional pricing can serve an analogous purpose in a newly-emerging market, in which one of the new entrants obtains a long-run advantage through seemingly predating on its rivals.

following proposition clarifies this point by relating competitive-phase pricing to the discount parameter, δ:

Proposition 7: In any symmetric equilibrium of the three-stage game,

(i). If δ is sufficiently small, then in the competitive phase, each firm chooses $P > c(\hat{I}(P))$ with probability one.

(ii). For any $\varepsilon > 0$, if δ is sufficiently large, then the low-price leader chooses $P < c(\hat{I}(P))$ with probability exceeding $1-\varepsilon$.

Intuitively, small values of δ are associated with steep discounting of the mature phase, leading to weak rivalry in the competitive phase. Prices are then reduced only a small amount from monopoly levels, and are certain to lie above marginal costs. For large δ, in contrast, competitive-phase prices are extremely responsive to prospects for capturing repeat business, and the low-price leader is driven to violate the Areeda-Turner rule with very high probability. Thus, the Areeda-Turner rule condemns the leader precisely when rivalry is sharpest and most beneficial to consumers, despite the fact that the salient features of the leader's behavior are not sensitive to δ.

Other major predation tests do not exert such a perverse effect in the model.[14] Generally speaking, it is appropriate to view intense price rivalry in new-product markets as symptomatic of firms' efforts to establish the attractiveness of their products in the eyes of consumers, and thereby to obtain repeat-business benefits. The shakeout that follows the competitive phase is simply a manifestation of consumer learning, as opposed to predatory monopolization.[15]

---

[14]The standards of Posner, who modifies the Areeda-Turner rule by focusing on long-run marginal cost, and Ordover and Willig (1981), who consider a hypothetical-exit criterion, would not be violated in our model. Williamson's rule applies explicitly to an established firm that faces new entry, and so it is not relevant in our setting.

[15]As a caveat to this conclusion, it is important to recognize that we do not establish an explicit no-predation benchmark against which to compare the social surplus generated in the symmetric equilibrium outcomes. Using social surplus as a standard, it is possible that equilibrium outcomes with many entrants, high level of

VII. Conclusion

The retail industry has undergone important changes in recent years, as giant retailers, emphasizing low prices, advanced selling technologies and huge sales volumes, have battled for leadership, while less-efficient companies have fallen by the wayside. We offer in this paper a framework for understanding this process. The key features of our model are that consumers face search costs, requiring them to use historic prices as proxies for current prices, and that retail-firm technology embodies increasing returns to scale.

Our main arguments are consistent with rational behavior, both for firms and consumers. To illustrate the key ideas and to confirm their compatibility with rational behavior, we have developed a three-stage equilibrium model of the retail industry. This model is special in several respects, but it lays bare the forces that drive our conclusions, making it apparent that our central results would hold under a variety of alternative modeling approaches.

The model also offers some guidance with respect to public policy in the retail industry. In particular, even though the dominant firm's price rises through time, and initially may be below marginal cost, we argue that an interpretation of predatory pricing may be inappropriate, since the dominant firm is also the most-efficient (lowest-cost) firm in the market. Standard tests of predatory pricing can incorrectly condemn the leader, with possibly counterproductive implications for consumer welfare.

---

investment and very steep competitive-phase price reductions might have lower social surplus than less-competitive outcomes associated with strict predation rules, as a consequence of excessive rent dissipation through duplicative investment activities. Here, the antitrust laws might operate to stem excessive rivalry, which is a purpose somewhat different than that contemplated in the earlier literature.

Appendix

Proof of Lemma 2: Using (6) and Assumption 2, we have that:

(A1). $V_{PP}(P,I,\rho) = \Pi_{PP}(P,I,1/N) < 0.$

Next, observe that:

(A2). $$V_{II}(P,I,\rho) = \Pi_{II}(P,I,1/N) + \delta[\rho\Pi^*_{II}(I,(1-s)/N + s) + (1-\rho)\Pi^*_{II}(I,(1-s)/N)]$$
$$< \Pi_{II}(P,I,1/N)$$
$$< 0,$$

where the final inequality follows since $\Pi_{II} < 0$ under Assumption 2 and where the first inequality holds since:

$$\Pi^*_{II}(I,F) = \Pi_{II}(R^*,I,1/N) + \Pi_{IR}(R^*,I,1/N)R^*_I$$
$$= \Pi_{II}(R^*,I,1/N) + \Pi_{IR}(R^*,I,1/N)[-\Pi_{IR}(R^*,I,1/N)/\Pi_{RR}(R^*,I,1/N)]$$
$$< 0,$$

where this inequality follows from Assumption 2.

Note next that:

(A3). $$V_{PI}(P,I,\rho) = \Pi_{PI}(P,I,1/N)$$
$$= -c'(I)\, D'(P)\, (1/N)$$
$$< 0.$$

Putting (A1-3) together, we now have that:

(A4). $$V_{PP}V_{II} - (V_{PI})^2 = \Pi_{PP}V_{II} - (\Pi_{PI})^2$$
$$> \Pi_{PP}\Pi_{II} - (\Pi_{PI})^2$$
$$> 0,$$

where the final inequality uses Assumption 2. Q.E.D.

Proof of Lemma 3: Part (i) is established in (A3) above; part (ii) is immediate from (6); and part (iv) follows immediately from (6) and (5). As for part (iii), use (6) to get:

$$V_{I\rho}(P,I,\rho) = \Pi^*_I(I,(1-s)/N + s) - \Pi^*_I(I,(1-s)/N) > 0$$

if $\Pi^*_{IF}(I,F) > 0$. This is true, since:

$$\Pi^*_{IF}(I,F) = -c'D > 0,$$

where the inequality derives from Assumption 1. Q.E.D.

Proposition A: There exists no pure-strategy equilibrium for the two-stage game.

Proof: Suppose that $T > 1$ firms tie and select the lowest price, L, in the competitive-phase of a pure-strategy equilibrium. These firms then also choose some optimal investment level, I, and they earn some expected discounted profit, S. By deviating to $(L-\varepsilon,I)$ for $\varepsilon > 0$ and small, however, a particular low-priced firm earns an even greater expected discounted profit, $S' > S$, since a small price cut has little effect on competitive-phase profits but increases the firm's mature-phase market share from the putative equilibrium level $(1-s)/N + s/T$ to the deviant level $(1-s)/N + s$, where this increase results in discretely higher expected discounted profit under (5). Thus, there can be at most one low-priced firm in a pure-strategy equilibrium.

Given this, high-priced firms can not be deterred from selecting $(P^*(0),I^*(0))$. For if instead $P \neq P^*(0)$ were selected by a high-priced firm, then that firm could deviate to $(P^*(0),I^*(0))$ and earn greater expected discounted profit, since:

$$V(P,\tilde{I}(P,0),0) < V(P^*(0),I^*(0),0) \leq V(P^*(0),I^*(0),\hat{\rho}(P^*(0)).$$

Thus, high-priced firms select $(P^*(0),I^*(0))$ and earn $V^*(0)$. The low-priced firm thus can select any $P < P^*(0)$ and still "win." Since $P^*(1) < P^*(0)$, the low-priced firm thus selects $(P^*(1), I^*(1))$ and earns $V^*(1)$ in any pure-strategy equilibrium. But this is contradictory, since a high-priced firm would then deviate to $(P^*(1) - \varepsilon,I^*(1))$ and earn approximately $V^*(1)$, which exceeds $V^*(0)$. Q.E.D.

Proposition B: In any symmetric equilibrium of the two-stage game, no particular price P is played with positive probability.

Proof: Suppose to the contrary that some competitive-phase price P is played with positive probability by a firm in the competitive-phase of a symmetric equilibrium. Since firms' strategies are symmetric, there is then a positive probability that a firm selecting this price will tie with other firms and be one of several low-priced firms. In particular, if the firm ties with M-1 other firms, then its future market share is $(1-s)/N + s/M$. Let I be the investment level chosen by a firm in conjunction with the price P. Consider now a deviation, in which a given firm makes only a small change in its overall strategy, by selecting $(P - \varepsilon, I)$ whenever its original strategy required it to select (P,I). Note that the number of point masses must be countable, and so $\varepsilon > 0$ can be found such that ε is small and $P - \varepsilon$ is played with zero probability under the hypothesized equilibrium strategy. This deviation has neglible effect on competitive-phase profit, but it offers a discrete increase in mature-phase profit, since in all events in which the firm would have tied as a low-price leader, it is now *the* low-priced leader, with the associated discretely larger future market share, $(1-s)/N + s$. Q.E.D.

Proposition C: In any symmetric equilibrium of the two-stage game, the highest price that could ever be charged is the monopoly price for a firm with zero likelihood of being the low-priced firm, i.e., $\bar{P} = P^*(0)$.

Proof : Suppose that $\bar{P} \neq P^*(0)$. Then a firm selecting $\bar{P}$ earns $V(\bar{P},\tilde{I}(\bar{P},0),0) < V(P^*(0),I^*(0),0) \leq V(P^*(0),I^*(0),\hat{\rho}(P^*(0)))$. Thus, a contradiction arises, since prices in an interval below $\bar{P}$ earn approximately $V(\bar{P},\tilde{I}(\bar{P},0),0)$, which is strictly less than what would be earned with a deviation to $(P^*(0),I^*(0))$. Q.E.D.

Proposition D: In any symmetric equilibrium of the two-stage game, there can be no gaps in the firms' price distribution.

Proof: Suppose a gap, (P_1,P_2), exists such that $\hat{G}(P_1) = \hat{G}(P_2)$ and $P_L < P_1 < P_2 < P^*(0)$. Suppose further that (P_1,P_2) is the largest such gap, in that, for all $\varepsilon > 0$, $\hat{G}(P_1 - \varepsilon) < \hat{G}(P_1)$ and $\hat{G}(P_2 + \varepsilon) > \hat{G}(P_2)$. Mixing then requires indifference, or $\tilde{V}(P_1,\hat{\rho}(P_1)) = \tilde{V}(P_2,\hat{\rho}(P_2))$, where $\hat{\rho}(P_1) = [1 - \hat{G}(P_1)]^{N-1} = [1 - \hat{G}(P_2)]^{N-1}$. Given that $\tilde{V}$ is strictly concave, however, two distinct prices can yield the same expected profit at a given ρ only if $P_1 < P^*(\hat{\rho}(P_1)) < P_2$. But then a deviation to $(P^*(\hat{\rho}(P_1)),I^*(\hat{\rho}(P_1)))$ yields a higher discounted expected profit, since $V^*(\hat{\rho}(P_1)) = \tilde{V}(P^*(\hat{\rho}(P_1)),\hat{\rho}(P_1)) > \tilde{V}(P_1,\hat{\rho}(P_1))$. Q.E.D.

Proof of Proposition 4 Applying the envelope theorem and differentiating $V^*(0)$ with respect to N, we obtain:

(A5). $\frac{dV^*(0)}{dN} = \Pi^*_F(I^*(0),1/N)\ (-1/N^2) + \delta\Pi^*_F(I^*(0),(1-s)/N)\ [-(1-s)/N^2] < 0,$

where the inequality follows from (5). Thus, N^* is unique. Q.E.D.

Proof of Proposition 5: Using (15) and (A5), differentiating $V^*(0) - E = 0$ with respect to s, and letting $V^*_N \equiv \frac{dV^*(0)}{dN}$, we obtain:

(A6). $\frac{\partial N^*}{\partial s} = \delta\, \Pi^*_F(I^*(0),(1-s)/N^*) / [NV^*_N] < 0,$

which establishes the proposition. Q.E.D.

Proof of Proposition 7: Put $\underline{c} = \lim_{I\to\infty} c(I)$. From (6) it is easy to see that $\tilde{V}(P,1)$ and $\tilde{V}(P,0)$, as well as $\tilde{I}(P,1)$ and $\tilde{I}(P,0)$, become arbitrarily close to one another, uniformly over N and $P \geq \underline{c}$, as $\delta \to 0$. As can be seen from Figure 1, it follows that P_L converges to $P^*(0)$ and $\hat{I}(P_L)$ converges to $I^*(0)$ as $\delta \to 0$, uniformly over N. Since $P^*(0) > c(I^*(0))$, we have that $P > c(\hat{I}(P))$ for every $P \in [P_L,P^*(0))$, for every N, when δ is sufficiently small. This proves (i).

As for (ii), note that the free-entry condition $\tilde{V}(\underline{c},\hat{\rho}(\underline{c})) = E$ may be rewritten (using linearity of $\Pi(P,I,F)$ in F):

$$\Pi(\underline{c},\tilde{I}(\underline{c},\hat{\rho}(\underline{c})),1/N^*) - k\tilde{I}(\underline{c},\hat{\rho}(\underline{c})) + \delta\Pi^*(\tilde{I}(\underline{c},\hat{\rho}(\underline{c})),1)[(1-s)/N^* + \hat{\rho}(\underline{c})s] = E.$$

Since $\Pi^*(\tilde{I}(\underline{c},\hat{\rho}(\underline{c})),1) > 0$, it follows that $N^*\to\infty$ and $\hat{\rho}(\underline{c})\to 0$ as $\delta\to\infty$. The probability that the low-price leader chooses $P \leq \underline{c}$ is given by:

$$1 - (1 - \hat{G}(\underline{c}))^{N^*} = 1 - \hat{\rho}(\underline{c})(1 - \hat{G}(\underline{c}))$$

which must approach unity as $\delta\to\infty$. Q.E.D.

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