

**BID-ASK SPREADS WITH INDIRECT COMPETITION**  
**AMONG SPECIALISTS**

by

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*Abstract:* We examine the bid-ask quotes offered by specialists (or dealers) who face indirect competition from other specialists who trade in related assets. In the context of a simple model where investors have mean-variance preferences, we characterize the equilibrium bids and asks quoted by  $K$  specialists in  $N$  assets, where some specialists may control more than one asset. We compare the equilibrium spreads as the number (and factor structure) of the assets each specialist controls is varied. It is shown that for some constellations of initial portfolio holdings and asset covariance it is socially preferred to have competing specialists, while for others it is socially preferred to have their actions coordinated (or to have one specialist control several assets). In a simple factor model, we show how the optimal specialist control structure depends on whether the assets trade as substitutes or complements. In some situations it is beneficial to have specialist power concentrated within industries, in other situations, across industries, and in yet other situations, not to be concentrated at all.

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## Bid-ask spreads with indirect competition among specialists

### 1. Introduction

This paper is about indirect competition between specialists. We argue, however, that it relates to financial markets more generally, when competition between dealers or traders is imperfect. Competition between traders may be considered imperfect whenever entry to the trading place is limited by the number of seats or when the set of available prices is restricted.<sup>3</sup>

At the New York Stock Exchange (NYSE) bid and ask prices for individual stocks are set by specialists, who enjoy the exclusive right to determine those prices. At the opening of trading the imbalance of market orders is private information of the specialist, who determines a “fair” price to clear all market orders and can add a market order. This exclusive right to determine opening prices grants market power to the specialist. Furthermore, the specialist has full knowledge of the limit order book and gets to see incoming orders on the electronic routing system before anyone else. In turn, the specialist is committed to quote buying and selling prices and to transact standard quantities at those prices. Recent empirical work suggests that specialists can exploit their market power profitably in short term trading.<sup>4</sup>

Despite this “monopoly” position in determining prices for a particular security, the specialist faces potential competition from trades which circumvent the specialist,<sup>5</sup> from trade in related securities, and from other sources. In the case of the NYSE, Demsetz (1968) argues that competitive pressure from related and rival markets will avoid excessive spreads. “Competition of several types will keep the observed spread close to cost. The main types of competition emanate from (1) rivalry for the specialist’s job, (2) competing markets, (3) outsiders who submit limit orders rather than market orders, (4) floor traders who may bypass the specialist by crossing buy and sell orders themselves, (5) and other specialists” (Demsetz, 1968, p.43). In this paper we study indirect competition between specialists in a simple model to identify conditions under which such competition narrows bid-ask spreads and improves welfare.

Since assets are differentiated in their risk characteristics, typically, the correlation structure of asset returns determines the assets’ substitutability in investment portfolios. Positively correlated assets are substitutes, while negatively correlated assets are complements which investors would like to hold in tandem. So highly positively correlated assets tend to be highly substitutable and, accordingly, specialists trading in such shares may emerge as close competitors. Consequently, and more generally, the underlying correlation structure of assets determines the specialists’ market power. So, when all assets are “closely correlated”

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<sup>3</sup> For instance, recent work by Christie and Schultz (1994) and Christie, Harris, and Schultz (1994) suggests that market makers on NASDAQ were colluding to maintain large bid-ask spreads.

<sup>4</sup> See, for instance, Hasbrouck and Sofianos (1993) and Madhavan and Smidt (1993).

<sup>5</sup> For a discussion of the unorganized search market see Gehrig (1993).

(in the sense that each asset can be closely approximated by a portfolio of the remaining assets), indeed Demsetz's conjecture (5) may be right and regulators should be less concerned about the monopoly power of independent specialists.

This basic analysis, however, is complicated by two factors. First, the substitutability or complementarity of assets depends not only on the correlation of their payoffs, but also on the initial distribution of asset holdings. Depending on the initial asset holdings, positively correlated assets can trade as complements. Second, each specialist unit<sup>6</sup> typically controls many stocks rather than just one. Thus concentrating the debate about a monopolistic dealer in isolation is misleading. The *ownership structure* of the specialist rights in conjunction with the *correlation structure* of the underlying assets and the (initial) *distribution* of asset holdings are crucial in determining the competitiveness of stock trading. We shall demonstrate that the initial distribution of assets and the correlation structure of the underlying assets jointly determine whether trades in given securities behave like complements or substitutes.

We find that there are constellations in which joint ownership of specialist rights is in the public as well as in the private interest. These contradict the simple intuition that less concentrated control will always lead to greater competition and thus higher welfare. For example, in situations where the demands for two assets are complementary, a single specialist dealing in both assets will charge lower spreads than competing specialists. The single specialist sees the benefit from the fact that a lower spread for one asset stimulates demand for the other asset, while competing specialists do not internalize this externality. In such a situation, having a single specialist control both assets offers a strict Pareto improvement compared to having competing specialists. This type of situation (complementary demands for two assets) could arise with negatively correlated assets and investors who want to increase or decrease the overall size of their portfolios. It could also arise with positively correlated assets, in situations where investors are currently holding unbalanced (i.e., other than market) portfolios. The trading environment, characterized by the correlation structure of the underlying assets and the nature of portfolio imbalances, determines, whether trades in assets are complements or substitutes. When trades behave like complements it is privately and socially desirable to have a single specialist trading in both securities. When trades behave like substitutes independent ownership is socially preferred.

The right to become a specialist in a given stock is allocated to a member of the exchange by general consensus among the exchange's member firms<sup>7</sup>. Since the number of seats in a financial market is limited, typically there are far fewer firms owning the specialists' rights than there are securities traded.<sup>8</sup> In an

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<sup>6</sup> On the NYSE in 1990 specialists controlled on average more than 4 stocks each. In addition, specialists were clustered into groups (averaging about 10 specialists each) called specialist units.

<sup>7</sup> For example, rule 103B (NYSE, Constitution and Rules, April 1st, 1991) reads: "Securities listing on the Exchange will be allocated to specialist units according to such policies as are established and made known to the membership from time to time".

<sup>8</sup> In December 1990 a total of 1774 companies were listed at the NYSE. Their securities were allocated to 421 individual specialists, who each belonged to one of 46 specialist units. (See 1991 NYSE factbook.)

exchange in which financial firms concentrate on industries with similar risk characteristics, one would typically expect a less competitive outcome and higher bid-ask spreads than in a market in which member firms hold diversified specialist rights. As we show, however, such a statement hinges on the initial allocation of assets.<sup>9</sup>

Furthermore, the competitiveness of a protected market, like the NYSE, will depend on the correlation of assets traded in the protected market to assets traded in other markets, like NASDAQ for example. In this case, the (socially) optimal ownership structure in the protected market depends on the correlation structure of assets traded in other markets.

Our approach also suggests that competition for listings will entail important consequences for trading. The number and the type of firms listed in a given (protected) market may substantially affect its liquidity.<sup>10</sup> We therefore give another potential explanation for the NYSE's concern about not being able to attract NASDAQ listed firms like Intel, Microsoft, Apple and Novell: Besides the obvious interest in direct trading revenues in those securities (and the precedence of large firms choosing not to list on the NYSE) there may be an important indirect interest in relaxing competition for other NYSE quoted computer stocks, and/or stimulating demand for complementary stocks listed on the NYSE.

The literature about the determinants of bid and ask prices in specialist markets largely concentrates on the costs of inventory holdings<sup>11</sup> and informational differences between traders and specialists<sup>12</sup>. Market structure is considered an important determinant of bid and ask prices as far as direct competition between dealers in the same security is concerned<sup>13</sup>. The only previous study of the role of imperfect competition among specialists in determining bid-ask spreads is by Hagerty (1991).<sup>14</sup> Hagerty examined monopolistic competition among specialists selling securities correlated with non-tradable endowment risks faced by investors. She shows that, under certain conditions, the bid-ask spreads go to zero and that the market becomes asymptotically efficient as the number of securities becomes large. She also shows that spreads are larger for securities with lower idiosyncratic risks (better hedging characteristics), and that with a fixed cost of entry there may be too many securities offered in the market relative to the social optimum. In Hagerty's model, securities are independently distributed about the non-traded endowment, their main purpose is hedging of that risk, and they are held in zero net supply. The demand for securities in her model is driven by their correlation with the non-traded initial endowment: their ability to eliminate the idiosyncratic risk therein; hence, her securities always trade as substitutes. Our contribution is in identifying how the trading and

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<sup>9</sup> Obviously, when the initial allocation is efficient, specialists will not enjoy any market power under either market organization. We show that for different initial distributions of securities the desirability of single ownership versus independent ownership may be completely reversed.

<sup>10</sup> See Gehrig, Stahl, Vives (1993) for a related point.

<sup>11</sup> See e.g. Garman (1976), Amihud, Mendelson, (1980) and Ho, Stoll (1981).

<sup>12</sup> See e.g. Stoll (1985, section VI) and Glosten, Milgrom (1985).

<sup>13</sup> See e.g. Roell (1988) or Dennert (1993).

<sup>14</sup> There has been work on the role of heterogenous information in price formation in imperfectly competitive multi-security markets, e.g. Admati (1985) and Caballé and Krishnan (1994).

competitive pressures on specialists depend on the correlation of asset payoffs, and their initial distribution among the trading population. Our model differs from Hagerty's (1991) in its basic design. In her model securities are best thought of as futures contracts. Our assets are all in non-zero net supply, are traded, and are held in initial portfolios. Thus our model is best thought of as a stock market. This permits an analysis of trading complementarities and thus introduces new facets to the competitive picture.

The analysis is performed in a highly stylized static model. Investors, characterized by mean-variance utility functions, have initial portfolio imbalances and try to trade towards their desired holdings. They cannot trade directly in a Walrasian market but need to trade through intermediaries, who are specialists in a protected market and therefore the sole supplier of the transaction services. (In section 7 we consider the effect of free entry of intermediaries for some assets.)

The assumption of a protected asset market is extreme and only serves the purpose to insulate indirect competition among specialists from sources of direct intra-asset competition. By submitting limit orders sufficiently patient traders can actually compete with specialists. Impatient traders, typically, prefer to submit market orders that are transacted immediately. In this vein Gehrig (1993) analyzes direct competition between a specialist and an underlying frictional market of direct exchange. He shows that the specialists' market power is affected by the strength of the trading friction and that in equilibrium traders with a high urgency of trade will trade with the specialist while traders with a low urgency of trade will enter the frictional market. Trades with the specialist are executed immediately while trades in the frictional market are delayed in some states of nature. Our approach is orthogonal to this analysis and can be understood as a situation when direct trade is frictional and traders are sufficiently impatient in a more general setting.

The basic liquidity model is outlined in section 2. In section 3 we characterize the benchmark competitive equilibrium where bid-ask spreads are set to zero and prices are chosen to clear markets. In section 4 we provide a general solution for bids and asks charged by any configuration of specialists controlling  $N$  assets. That is, some specialists may control several assets. In Section 5, to draw some intuition from the general solution, we specialize to the case of two assets and two alternative distributions of initial endowments. We demonstrate how these distributions and the asset covariance structure affect equilibrium bid-ask spreads of two independent, rival specialists, and how these spreads compare to the spreads charged by a single specialist controlling both assets. Another specialization is adopted in section 6 where asset returns are assumed to be characterized by a factor structure. Specifically, we compare different control structures when industry factors are present to understand whether it is more beneficial to have specialist control coordinated within industries, or across industries.. Section 7 discusses competition between an exchange with limited entry (such as the NYSE or NASDAQ) and a competitive (free entry) market. In section 8, we allow for general risk aversion and trading on the side of the specialists, and show that the solution to the individual specialist's profit maximization problem under risk aversion can be made arbitrarily close to the solution of the match-making specialists, by increasing the number of buyers and sellers. Hence, in large markets,

the analysis of sections 4–7 is a reasonable approximation for the case of risk averse specialists. All proofs appear in an appendix.

## 2. The Basic Model

Trading opportunities are created by the incongruence of initial and desired portfolio holdings. The divergence between initial and desired portfolios can be viewed as being caused by liquidity events (see Grossman and Miller (1988)). Trading can take place at the official market only.<sup>15</sup>

### *Timing*

There are two periods. Assets are traded in the first period. Payoffs are realized in the second period and agents consume.

### *Assets*

There is one riskless asset which serves as a numeraire and has a terminal payoff normalized to 1. There are  $N$  risky assets with random terminal payoffs  $\tilde{x}_j$ ,  $j \in \{1, 2, \dots, N\}$ .  $\mu = (\mu_1, \dots, \mu_N)$  is the vector of expected terminal payoffs, and  $\Sigma$  is the covariance matrix, where  $\sigma_{ij}$  is the covariance between the terminal payoffs of assets  $i$  and  $j$ . We assume that  $\Sigma$  is nonsingular.

### *Specialists*

Each risky asset is traded by a specialist. There are  $K \leq N$  specialists and we let  $N_k \subset \{1, 2, \dots, N\}$  denote the set of assets controlled by specialist  $k$ . The specialist assigned to security  $j$  determines bid and ask prices  $(b_j, a_j)$ .

Specialists maximize their trading revenue and must choose bids and asks to clear the market.

The requirement that specialists must clear the market is made for convenience. We wish to isolate the effects of indirect competition on pricing, and do not wish to complicate the analysis with the personal inventory and investment considerations of the specialist. In section 8 we prove that the solution to this constrained problem is arbitrarily close to that of the unconstrained problem (where specialists may take positions in the assets in addition to setting bids and asks), provided that specialists have some risk aversion and there is a large enough number of investors.

### *Investors*

There are two types of investors and  $n$  investors of each type. Except for section 8 it will not matter what  $n$  is, so until then we set  $n = 1$ . Investors of type 1 are initially endowed with portfolios of the risky

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<sup>15</sup> Gehrig (1993) and Yanelle (1989) provide models with concurrent direct and indirect trade.

assets  $e^1 \in \mathbb{R}_{\geq 0}^N$  and investors of type 2 are initially endowed with  $e^2 \in \mathbb{R}_{\geq 0}^N$ . Let  $e$  be the  $N$  dimensional vector, where  $e_j := e_j^1 + e_j^2 > 0$  is the total stock of asset  $j$ .

Differences in the initial portfolios generate potential gains from trade. As in Grossman and Miller (1988), the initial portfolios are allocated by some exogenous event, which may be viewed as a liquidity event. The demands for trade are independent of the initial holdings of the risk-free asset (given the mean-variance preferences), so without loss of generality we set those holdings to be zero.

Given risk aversion, investors' desired asset holdings typically will differ from their endowments. Investors' have mean-variance utility functions with constant absolute risk aversion parameters  $\tau^i \geq 0$ , i.e. investor  $i \in \{1, 2\}$  maximize expected utility of wealth  $\bar{W}^i$ :

$$EU(\bar{W}^i) = E\bar{W}^i - \tau^i \text{Var}\bar{W}^i. \quad (1)$$

These preferences are consistent, for instance, with negative exponential utility and normally distributed asset payoffs.

Let  $q_j^i$  denote the final holdings of asset  $j$  by investor  $i$ . At given bid and ask prices final wealth is determined by:

$$\bar{W}^i = \sum_j [q_j^i \bar{x}_j + \max(e_j^i - q_j^i, 0) b_j + \min(e_j^i - q_j^i, 0) a_j] \quad (2)$$

### 3. The Competitive Benchmark Case

It is useful to consider the competitive outcome as a benchmark. The standard competitive equilibrium requires single market clearing prices  $p_j = b_j = a_j$  for each security.

In this case, maximization of expected utility yields the following demand system for assets:

$$q^i = \frac{1}{2\tau^i} \Sigma^{-1} (\mu - p) \quad , \quad i = 1, 2. \quad (3)$$

In a competitive equilibrium market demand equals market supply and thus, equilibrium prices are:

$$p^c = \mu - \frac{2\tau^1 \tau^2}{\tau^1 + \tau^2} \Sigma e, \quad (4)$$

Equilibrium requires  $p_j \in [0, \mu_j]$  for each asset  $j$ , which requires  $\frac{2\tau^1 \tau^2}{\tau^1 + \tau^2} \Sigma e \in X_j[0, \mu_j]$ . We shall maintain this assumption for the remainder of the analysis.

When specialists possess market power, they can demand premia and competitive pricing will fail to hold. This is the subject of the next section.

#### 4. Strategic Specialists: A General Solution

When entry into the organized market is limited specialists enjoy market power. In principle, they can exercise it in order to maximize transaction revenues and to structure their own asset portfolio. In the next four sections we concentrate on the case in which specialists maximize revenues from trading and cannot hold net positions.

Denote trades of investor  $i = 1, 2$  in asset  $j$  by  $\theta_j^i := q_j^i - e_j^i$ . So  $\theta_j^i > 0$  denotes a buy and  $\theta_j^i < 0$  denotes a sell order.

Specialist  $k$  who controls assets  $N_k \subset \{1, 2, \dots, N\}$  faces the optimization problem

$$\max_{(a_j, b_j), j \in N_k} \sum_{j \in N_k} R_j \quad \text{subject to} \quad \theta_j^1 + \theta_j^2 = 0 \quad \text{for each } j \in N_k, \quad (5)$$

where  $R_j = (\max(\theta_j^1, 0) + \max(\theta_j^2, 0)) a_j + (\min(\theta_j^1, 0) + \min(\theta_j^2, 0)) b_j$ .

The direction of trade depends on the position of the desired portfolio holdings relative to the initial allocation. There is zero trade when the initial portfolios happen to be identical with the desired portfolios. To jointly solve for the (pure strategy) Nash equilibria of the game where specialists choose bids and asks, one has to find the sets of bids and asks which solve the maximization problem of each specialist, taking the other prices (and their effects on demands) as given. One also has to check that the demands of investors at the quoted prices are consistent with the bids and the asks. That is, one has to verify that investors' equilibrium demands are nonnegative at the ask, if they are trading at the ask, and nonpositive at the bid, if they are trading at the bid.

The following proposition characterizes the general solution to the problem with  $K$  specialists and  $N$  assets, where  $K \leq N$ . For simplicity, in the sequel we shall concentrate on the case of identical investor risk aversion  $r_1 = r_2 = r$ .

Since the determination of who will sell and who will buy is dependent on the covariance matrix, the endowments, and the equilibrium prices, we cannot specify which prices will be bids and which will be asks beforehand. Thus, we specify  $p^1$  to be the prices that investors of type 1 trade at, and  $p^2$  to be the prices that investors of type 2 trade at. In section 5, when we examine particular endowments, specialist structures, and asset structures, we can identify which prices are bids and which are asks.

##### **Proposition 4.1**

*In a (non-cooperative) price setting game between specialists the Nash equilibrium<sup>16</sup> prices satisfy  $\theta_j^1(p_j^1 - p_j^c) \geq 0$  for each  $j$  and:*

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<sup>16</sup> As is clear in the proof, this solution is unique if demands are non-zero. If at the solution demands turn out to be zero for some assets, then equilibrium is still almost unique: there may be a continuum of prices in those assets (any lower bids and higher ask spreads) which result in the same demands.



$$-(p^2 - p^c) = p^1 - p^c = rA^{-1}(e^2 - e^1)$$

where  $p^c$  is the vector of competitive prices given in (4) and  $A$  is the matrix with entries  $A_{ij} = 2(\Sigma^{-1})_{ij}$  if  $i = j$  or  $i$  and  $j$  are controlled by the same specialist (including  $i = j$ ), and  $A_{ij} = (\Sigma^{-1})_{ij}$ , otherwise. If  $p_j^i > p_j^c$ , then  $p_j^i$  is an ask and  $p_j^i < p_j^c$  then  $p_j^i$  is a bid.

The corresponding vector of equilibrium bid-ask spreads is

$$|p^1 - p^2| = |2rA^{-1}(e^2 - e^1)|. \quad (6)$$

Proof: See the appendix.

There are several things to notice about the general solution. First, bid and ask prices are symmetric around the competitive price. This is a result of the market clearing restriction and the simple linear demand structure. As one might expect, the ask is always higher than the competitive price, while the bid is always lower. Second, the price spread is increasing in risk aversion. This reflects the specialists' response to the investors' more inelastic demands which result from larger potential gains from trade. Third, the spread is "roughly" increasing in the initial difference in endowments, but the exact effect depends on the structure of  $A$  and thus on the covariance structure of the assets' payoffs. Fourth, we can interpret the structure of  $A$  as reflecting the importance of cross price elasticities. The basic entries of  $A$  reflect the general interplay of cross price elasticities in the pricing solution of an imperfectly competitive market. The fact that some entries are doubled, reflects the fact that a specialist who controls more than one asset cares about the effect of each price on each of the assets that he or she controls.

Further direct interpretation of the above solutions is difficult in this abstract form, so in order to obtain additional intuition and make comparisons across correlation structures and endowments, we specialize to the case of two assets in the next section, and then analyze a four asset example with an underlying factor structure in Section 6.

empirically.

## 5. The Two Asset Case: Monopoly vs. Independent Specialists

Examining the case of two assets and specific initial portfolio holdings, we now simplify the general solution presented in Section 4. The two (representative) types of initial portfolio holdings on which we concentrate are:

$$\text{initial portfolio holding (A)} \quad e^1 = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}, \quad e^2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\text{initial portfolio holding (B)} \quad e^1 = \begin{pmatrix} e_1 \\ 0 \end{pmatrix}, \quad e^2 = \begin{pmatrix} 0 \\ e_2 \end{pmatrix}$$

In initial portfolio holding (A), investors differ in the level of their holdings only, and in equilibrium investor 1 will sell both assets while investor 2 will buy both assets. (A) represents a situation in which investors need to change the sizes of their portfolios, but do not need to rebalance them. In initial portfolio holding (B), investors have exactly one of the two assets, and in equilibrium each investors will wish to buy one asset and sell the other. Thus (B) represents a situation in which investors are primarily rebalancing their portfolios.

These different portfolio holdings will affect the strategic properties of trades in the two risky assets. For example, consider the case of almost perfectly correlated assets. Under initial portfolio holding (A) investor 1 desires to trade his risky position with investor 2 whose risk bearing capacity is underutilized. Under initial portfolio holding (B) the desire to trade is fairly small since both investors basically hold identical portfolios and there are little gains from diversification. In this case, it appears that specialist market power should be larger in (A) than in (B). As the correlation coefficient between the assets decreases, the diversification incentive becomes stronger and eventually investors are more eager to trade in initial portfolio holding (B) than (A), since both can reduce the riskiness of their portfolios. In such cases the specialist market power is larger in (B) than in (A).

Denote the spread in prices for asset  $j$ ,  $a_j - b_j$ , by  $s_j$ .

### Independent Specialists

First we consider the case of two independent specialists in the two risky assets. Specialist 1 controls asset 1 and specialist 2 controls asset 2.

#### Corollary 5.1

*In a (non-cooperative) price setting game between two specialists, the Nash equilibrium prices and spreads are, provided  $\rho \leq \min\left(\frac{2e_1\sigma_1}{e_2\sigma_2}, \frac{2e_2\sigma_2}{e_1\sigma_1}\right)$ :*

*In the case of initial portfolio holding (A) :*

$$\begin{pmatrix} a_1^{N,A} \\ a_2^{N,A} \end{pmatrix} = \begin{pmatrix} p_1^c \\ p_2^c \end{pmatrix} + \frac{1}{4 - \rho^2} \begin{pmatrix} 2 - \rho^2 & -\rho\frac{\sigma_1}{\sigma_2} \\ -\rho\frac{\sigma_2}{\sigma_1} & 2 - \rho^2 \end{pmatrix} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (7.a)$$

$$\begin{pmatrix} b_1^{N,A} \\ b_2^{N,A} \end{pmatrix} = \begin{pmatrix} p_1^c \\ p_2^c \end{pmatrix} - \frac{1}{4 - \rho^2} \begin{pmatrix} 2 - \rho^2 & -\rho\frac{\sigma_1}{\sigma_2} \\ -\rho\frac{\sigma_2}{\sigma_1} & 2 - \rho^2 \end{pmatrix} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (7.b)$$

$$\begin{pmatrix} s_1^{N,A} \\ s_2^{N,A} \end{pmatrix} = \frac{2}{4 - \rho^2} \begin{pmatrix} 2 - \rho^2 & -\rho\frac{\sigma_1}{\sigma_2} \\ -\rho\frac{\sigma_2}{\sigma_1} & 2 - \rho^2 \end{pmatrix} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (7.c)$$

In the case of initial portfolio holding (B):

$$\begin{pmatrix} a_1^{N,B} \\ a_2^{N,B} \end{pmatrix} = \begin{pmatrix} p_1^c \\ p_2^c \end{pmatrix} + \frac{1}{4 - \rho^2} \begin{pmatrix} 2 + \rho^2 & -3\rho\frac{\sigma_1}{\sigma_2} \\ -3\rho\frac{\sigma_2}{\sigma_1} & 2 + \rho^2 \end{pmatrix} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (8.a)$$

$$\begin{pmatrix} b_1^{N,B} \\ b_2^{N,B} \end{pmatrix} = \begin{pmatrix} p_1^c \\ p_2^c \end{pmatrix} - \frac{1}{4 - \rho^2} \begin{pmatrix} 2 + \rho^2 & -3\rho\frac{\sigma_1}{\sigma_2} \\ -3\rho\frac{\sigma_2}{\sigma_1} & 2 + \rho^2 \end{pmatrix} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (8.b)$$

$$\begin{pmatrix} s_1^{N,B} \\ s_2^{N,B} \end{pmatrix} = \frac{2}{4 - \rho^2} \begin{pmatrix} 2 + \rho^2 & -3\rho\frac{\sigma_1}{\sigma_2} \\ -3\rho\frac{\sigma_2}{\sigma_1} & 2 + \rho^2 \end{pmatrix} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (8.c)$$

where  $\rho$  is the correlation coefficient  $\frac{\sigma_{12}}{\sigma_1\sigma_2}$ .

Proof: The expressions follow directly from Proposition 4.1. In case A, the bids correspond to  $p^1$  and the asks to  $p^2$ . So we need to verify that  $\theta^2 \geq 0$ . Calculating  $\theta^2$  in equilibrium (given the expression for  $\theta^i$  in the proof of Proposition 4.1, and substituting for  $p^2$  from (7.a)):

$$\theta^2 = \frac{1}{2} \begin{pmatrix} 1 \\ 4 - \rho^2 \end{pmatrix} \begin{pmatrix} 2e_1 + \rho\frac{\sigma_2}{\sigma_1}e_2 \\ 2e_2 + \rho\frac{\sigma_1}{\sigma_2}e_1 \end{pmatrix}.$$

This expression is nonnegative (and our assumption concerning the signs of  $\theta^1$  and  $\theta^2$  is correct) precisely when

$$\rho \geq \max \left( \frac{-2e_1\sigma_1}{e_2\sigma_2}, \frac{-2e_2\sigma_2}{e_1\sigma_1} \right).$$

In case B,  $b_1^{N,B} = p_1^1$  and  $b_2^{N,B} = p_2^2$ . So we need to verify that  $\theta_1^2 \geq 0$  and  $\theta_2^2 \leq 0$ . Noting that

$$\theta^2 = \frac{1}{2r} \Sigma^{-1} \begin{pmatrix} \bar{\mu}_1 - \bar{a}_1 \\ \bar{\mu}_2 - \bar{b}_2 \end{pmatrix} - \begin{pmatrix} 0 \\ e_2 \end{pmatrix}$$

and using (8.a), (8.b) and (4) we calculate  $\theta^2$  in equilibrium:

$$\theta^2 = \frac{1}{2} \begin{pmatrix} 1 \\ 4 - \rho^2 \end{pmatrix} \begin{pmatrix} 2e_1 - \rho\frac{\sigma_2}{\sigma_1}e_2 \\ -2e_2 + \rho\frac{\sigma_1}{\sigma_2}e_1 \end{pmatrix}.$$

$\theta_1^2 \geq 0$  and  $\theta_2^2 \leq 0$  precisely when

$$\rho \leq \min \left( \frac{2e_1\sigma_1}{e_2\sigma_2}, \frac{2e_2\sigma_2}{e_1\sigma_1} \right).$$

Q.E.D.

The condition on  $\rho$ ,  $\rho \leq \min \left( \frac{2e_1\sigma_1}{e_2\sigma_2}, \frac{2e_2\sigma_2}{e_1\sigma_1} \right)$  ensures that the stated prices reflect the bids and asks. This conditions is not very restrictive and is always satisfied if, for instance,  $e_1 = e_2$  and  $\sigma_1 = \sigma_2$ . If this condition is violated, the only change in the Corollary will be a relabelling of which expressions correspond to bids and which ones correspond to asks.

Corollary 5.1 provides additional intuitive aspects of the equilibrium bids and asks. First, note that as  $\rho \rightarrow 1$  competition intensifies and bid-ask spreads decline, as assets become more substitutable. Second, spreads are larger for  $\rho < 0$  than for  $\rho > 0$  of the same magnitude, reflecting the complementary nature of negatively correlated assets and the substitutable nature of positively correlated assets. Third, we now get a clearer look at the entries of  $A$  from Proposition 4.1. Notice that the diagonal terms involve expressions of  $\frac{\rho\sigma_i}{\sigma_j}$ . These terms are thus based on the regression coefficients of one asset on the other. These expressions are essential in determining the demands for the assets, and thus enter into cross price elasticity calculations and the equilibrium solution. From these we can see, in addition to the above remarked comparative statics in  $\rho$ , that the spread for an asset is declining in the ratio of its variance relative to the variance of the other asset (assuming a positive correlation, with just the reverse for a negative correlation). The higher its relative variance, the less attractive the asset is in the portfolio.

Interestingly, for correlated assets ( $\rho \neq 0$ ) specialists' price quotes differ in the two trading environments. Substituting from (4) into (7.c) and (8.c) we find:

**Corollary 5.2**

*Let  $\sigma_1 = \sigma_2$ . Then  $s^{N,A} > s^{N,B}$  when  $\rho > 0$ ,  $s^{N,A} < s^{N,B}$  when  $\rho < 0$ , and  $s^{N,A} = s^{N,B}$  when  $\rho = 0$ .*

The competitive prices are independent of how the endowments are split among investors and thus are the same in (A) and (B). However, in the presence of market power, the distribution of initial asset holdings matters. It is clear from Corollaries 5.1 and 5.2, that although in both (A) and (B) the spreads narrow as  $\rho$  increases (the assets become more substitutable), that the spreads under (B) are more sensitive to  $\rho$  than under (A). The spread  $s^{N,B}$  is larger than  $s^{N,A}$  for negative values of  $\rho$ , but the spread  $s^{N,B}$  diminishes more rapidly than  $s^{N,A}$  as  $\rho$  is increased, and ultimately  $s^{N,B}$  is smaller than  $s^{N,A}$ .

To understand this result, we need to look at how the initial asset holdings affect the demands of the investors. It is those demands to which the specialists ultimately respond. When assets are positively correlated, the initial endowment of type 1 investors in case (A) is riskier than the initial endowment of type 1 investors in case (B). This can be seen since  $\frac{\text{Var}(e_1^1 + e_2^1)}{\text{E}(e_1^1 + e_2^1)} = \frac{(1+\rho)\sigma^2}{\mu}$  and  $\frac{\text{Var}(e_1^1)}{\text{E}(e_1^1)} = \frac{\sigma^2}{\mu}$  (where these are the variances in resulting wealth if no trading occurs, given the endowments). Accordingly, when the initial endowment position is riskier, investor 1 has a larger need to sell asset 1. Hence, the supply of asset 1 is less elastic. By an analogous argument investor 2's demand for asset 1 is less elastic when his or her initial position is less risky. Thus, when assets are positively correlated the specialist in security 1 enjoys more market power in case (A) and when assets are negatively correlated market power is larger in case (B).

**Joint Profit Maximization: A Monopolistic Specialist**

It would seem that competition between specialists would be impaired by the fact that the specialist

seats are owned by a few specialist units. In 1990, for example, at the NYSE 1741 different common stocks were traded by 421 specialists, who again belonged to 46 specialist units.<sup>17</sup> So at the NYSE, on average, a specialist exercised rights in just over 4 stocks and a specialist unit exercised specialist rights in about 38 stocks in 1990. Let us now examine how the competitiveness of an exchange depends on the distribution of stocks across specialists and specialist units.

A firm owning specialist rights in two stocks may prevent them from competing intensely, particularly for highly substitutable stocks. In this sense, joint ownership may destroy the benefits of competition. However, in the case of complementary securities, the monopolist may actually also charge lower prices and enhance welfare. By pricing assets, which behave like complements appropriately, aggregate trading volume can be increased. The monopolist specialist unit can take advantage of the aggregate trading volume by charging lower spreads than independent units. In this way joint ownership may reduce trading costs, enhance trading revenues and overall welfare accordingly.

We can examine the predictions of Proposition 4.1 in the case of a single specialist who jointly controls the prices of two assets or two specialists who choose prices to maximize the sum of their revenues:

**Corollary 5.3**

*When specialists maximize joint profits the optimal bid-ask spreads in the case of initial portfolio holding (A) are:*

$$\begin{pmatrix} a_1^{M,A} \\ a_2^{M,A} \end{pmatrix} = \begin{pmatrix} p_1^c \\ p_2^c \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (9.a)$$

$$\begin{pmatrix} b_1^{M,A} \\ b_2^{M,A} \end{pmatrix} = \begin{pmatrix} p_1^c \\ p_2^c \end{pmatrix} - \frac{1}{2} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (9.b)$$

$$\begin{pmatrix} s_1^{M,A} \\ s_2^{M,A} \end{pmatrix} = \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (9.c)$$

*and in the case of initial portfolio holding (B) are:*

$$\begin{pmatrix} a_1^{M,B} \\ a_2^{M,B} \end{pmatrix} = \begin{pmatrix} p_1^c \\ p_2^c \end{pmatrix} + \frac{1}{2(1-\rho^2)} \begin{pmatrix} 1+\rho^2 & -2\rho\frac{\sigma_1}{\sigma_2} \\ -2\rho\frac{\sigma_2}{\sigma_1} & 1+\rho^2 \end{pmatrix} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (10.a)$$

$$\begin{pmatrix} b_1^{M,B} \\ b_2^{M,B} \end{pmatrix} = \begin{pmatrix} p_1^c \\ p_2^c \end{pmatrix} - \frac{1}{2(1-\rho^2)} \begin{pmatrix} 1+\rho^2 & -2\rho\frac{\sigma_1}{\sigma_2} \\ -2\rho\frac{\sigma_2}{\sigma_1} & 1+\rho^2 \end{pmatrix} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (10.b)$$

$$\begin{pmatrix} s_1^{M,B} \\ s_2^{M,B} \end{pmatrix} = \frac{1}{1-\rho^2} \begin{pmatrix} 1+\rho^2 & -2\rho\frac{\sigma_1}{\sigma_2} \\ -2\rho\frac{\sigma_2}{\sigma_1} & 1+\rho^2 \end{pmatrix} \begin{pmatrix} \mu_1 - p_1^c \\ \mu_2 - p_2^c \end{pmatrix} \quad (10.c)$$

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<sup>17</sup> See NYSE factbook, 1991.

Proof: This follows from Proposition 4.1, setting  $b^{M,A} = p^1$ ,  $b^{M,B} = (p_1^1, p_2^2)$ , and verifying that  $\theta^i p^i \geq 0$ .

Now we are ready to compare the equilibrium prices and equilibrium spreads across different forms of market organization and the various initial portfolio holdings.

#### **Corollary 5.4**

Let  $\sigma_1 = \sigma_2$  and  $e_1 = e_2$ .

*In initial portfolio holding (A), when  $\rho > 0$  equilibrium spreads in the case of independent specialist units are smaller than in the case of a single specialist, i.e.  $s_z^{N,A} < s_z^{M,A}$  for  $z = 1, 2$ . When  $\rho < 0$ , equilibrium spreads in the case of independent specialist units are larger than in the case of a single specialist, i.e.  $s_z^{N,A} > s_z^{M,A}$  for  $z = 1, 2$ .*

*In initial portfolio holding (B), when  $\rho > 0$  equilibrium spreads in the case of independent specialist units are larger than in the case of a single specialist unit, i.e.  $s_z^{N,B} > s_z^{M,B}$  for  $z = 1, 2$ . When  $\rho < 0$ , equilibrium spreads in the case of independent specialist units are smaller than in the case of a single specialist unit, i.e.  $s_z^{N,B} < s_z^{M,B}$  for  $z = 1, 2$ .*

Proof: See the appendix.

We can couple the finding of Corollary 5.2 with that of Corollary 5.4 (assuming  $\sigma_1 = \sigma_2$  and  $e_1 = e_2$ ) to deduce that when  $\rho > 0$ :  $s_z^{M,A} > s_z^{N,A} > s_z^{N,B} > s_z^{M,B}$  for  $z = 1, 2$ ; and when  $\rho < 0$ :  $s_z^{M,B} > s_z^{N,B} > s_z^{N,A} > s_z^{M,A}$  for  $z = 1, 2$ .

Corollary 5.4 identifies situations where a single specialist unit trading in both stocks, will yield lower bid-ask spreads in both markets than would be chosen by two independent specialists. In fact, when social welfare is measured as the sum of specialists' trading profits and investors' expected utility, social welfare may be higher, when the pricing of the two securities is coordinated (controlled by the same specialist unit). The specialists' profits are simply transfers of the risk free asset from investors, and so total societal welfare is decreasing in the bid-ask spread, since investor welfare is decreasing in the bid-ask spread. The socially preferable ownership structures are summarized as follows:

#### **Corollary 5.5**

Let  $\sigma_1 = \sigma_2$  and  $e_1 = e_2$ .

*Under initial portfolio holding (A): for  $\rho > 0$  independent ownership of the two specialists is socially preferable, while for  $\rho < 0$  single ownership is socially preferable, and*

*Under initial portfolio holding (B): for  $\rho > 0$  single ownership of the two specialists is socially preferable, while for  $\rho < 0$  independent ownership is socially preferable.*

In some of the above cases we can make stronger statements about the socially preferred structure: we can strictly Pareto rank them. When specialists choose their bid ask spreads to jointly maximize their profits, they profit more than they would acting independently (provided  $\rho \neq 0$ ). In some such cases, the joint maximization solution also results in lower spreads which are better for the investors as well. Concentration of ownership in one specialist unit strictly Pareto dominates independent ownership either under initial portfolio holding (A) and negative correlation, or under initial portfolio holding (B) under positive correlation. Hence, both the correlation of the underlying assets, as well as the distribution of assets across investors will determine the social desirability of a monopolistic (or cartellized) market organization relative to a more competitive organization with independent specialists.

What really matters for the social desirability of monopoly are the properties of the demand functions for transactions. These depend both on the initial portfolio holdings and the correlation structure of the assets. Trades in assets 1 and 2 behave like substitutes in initial portfolio holding (A) when  $\rho > 0$  and in initial portfolio holding (B) when  $\rho < 0$ , and then it is socially preferable to have independently acting specialists. To see this we examine the desired trades. The proof of Proposition 4.1 reveals that in the case of initial portfolio holding (A) transactions demand  $\theta$  can be written as:<sup>18</sup>

$$\theta_1 = \frac{1}{2r(1-\rho^2)\sigma_1^2\sigma_2^2} (\sigma_2^2(\mu_1 - a_1) - \sigma_{12}(\mu_2 - a_2)) \quad (11.a)$$

$$\theta_2 = \frac{1}{2r(1-\rho^2)\sigma_1^2\sigma_2^2} (-\sigma_{12}(\mu_1 - a_1) + \sigma_1^2(\mu_2 - a_2)) \quad (11.b)$$

In the case of initial portfolio holding (B) transactions demand reads:

$$\theta_1 = \frac{1}{2r(1-\rho^2)\sigma_1^2\sigma_2^2} (\sigma_2^2(\mu_1 - a_1) - \sigma_{12}(\mu_2 + a_2)) \quad (12.a)$$

$$\theta_2 = \frac{1}{2r(1-\rho^2)\sigma_1^2\sigma_2^2} (-\sigma_{12}(\mu_1 + a_1) + \sigma_1^2(\mu_2 - a_2)) \quad (12.b)$$

These formulations demonstrate that in case (A), trades in assets 1 and 2 behave like complements when  $\rho < 0$  and like substitutes when  $\rho > 0$ . Just the reverse is true in case (B). Thus since under initial portfolio holding (A) when  $\rho < 0$  and under initial portfolio holding (B) when  $\rho > 0$  it is better to have joint maximization by the specialists.

Empirically negative correlation among assets may seem uncommon. In the case of positive correlation, the substitutable or complementary nature of the demand for the assets then depends on the portfolio imbalances. In this case, the model predicts that the endowment structure still decisively affects equilibrium spreads. When trading is dominated by trading assets in tandem, (as in an index – case (A)) independent

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<sup>18</sup> These formulations stipulate bid prices consistent with equilibrium.

ownership is socially preferred, while single ownership yields lower spreads when stocks are predominantly traded on the basis of individual assets (case (B)). An exception is the case where one asset is a derivative security on another. For instance, the demands for a security and an option on that security are often complementary. In such cases, our results would suggest that, in the presence of imperfect competition, it would be beneficial to have the same specialists control both markets.

We should mention that the analysis we have provided is an interim one, where the covariance structure and initial portfolio imbalances are known. Ideally, as these may change over time (and one may not wish to continuously reallocate specialist rights) one would want to perform an ex ante analysis which incorporates the likelihood of various trading complementarities.

The result of Corollary 5.5 parallels the finding of Spence (1976) that monopoly may be socially desirable in the case of complementary products. In a trading framework the *complementarity of trades* matters, not necessarily the *complementarity in payoffs* of the underlying assets. Still, the correlation structure of the underlying assets is an important factor in determining the degree of market power an individual specialist can exert; it is just not the only factor.

Similar results about the social desirability of monopoly in situations of imperfect competition are obtained by Stahl (1982) and Wolinsky (1983) in settings with consumer search costs. These authors find that a monopolist might charge lower prices because of the benefits from the associated increase in local demand (market expansion effect). Our results are driven by a different externality, but come to similar conclusions concerning the desirability of monopoly. In some of the situations described in corollary 5.5, lower spreads increase aggregate trading volume and the monopolist benefits most. In these situations joint ownership is socially optimal.



## 6. Factor Structures and Optimal Specialist Structures

In this section we further explore the dependence of the optimal specialist control structure on the underlying covariance of asset payoffs and the initial portfolio holdings. Here, we expand the analysis of the previous section to allow for four assets whose covariance structure is derived from a simple factor structure.<sup>19</sup> The factor structure of asset payoffs represents two industries, with two assets (or firms) in each industry. There is a factor common to all four assets, and a separate factor common to each of the two different industries. Each asset also has an idiosyncratic component. One can consider specialist control structures, ranging from independent specialists to a single specialist. In particular, in the case of some joint control, one can examine whether it is better to have specialists coordinated within industries or across industries.

### Industry Factors

Consider the factor structure

$$\begin{aligned}\tilde{x}_1 &= \tilde{f} + \tilde{f}_I + \tilde{\epsilon}_1 \\ \tilde{x}_2 &= \tilde{f} + \tilde{f}_I + \tilde{\epsilon}_2 \\ \tilde{x}_3 &= \tilde{f} + \tilde{f}_J + \tilde{\epsilon}_3 \\ \tilde{x}_4 &= \tilde{f} + \tilde{f}_J + \tilde{\epsilon}_4\end{aligned}$$

where  $\tilde{f}$  is a common factor and  $\tilde{f}_I$  and  $\tilde{f}_J$  are industry specific factors of industry  $I$  and  $J$ .  $\tilde{\epsilon}_i$  are idiosyncratic shocks (with zero mean). Factors are independent and orthogonal to each other and the idiosyncratic shocks. Furthermore, with  $\text{var}(\tilde{f}) = \phi^2 > 0$ ,  $\text{var}(\tilde{f}_I) = \gamma^2 > 0$ ,  $\text{var}(\tilde{f}_J) = \gamma^2 > 0$ , and  $\text{var}(\tilde{\epsilon}_i) = \tau^2 > 0$ , the variance-covariance matrix  $\Sigma$  exhibits positive correlation among all assets:

$$\Sigma = \begin{pmatrix} \phi^2 + \gamma^2 + \tau^2 & \phi^2 + \gamma^2 & \phi^2 & \phi^2 \\ \phi^2 + \gamma^2 & \phi^2 + \gamma^2 + \tau^2 & \phi^2 & \phi^2 \\ \phi^2 & \phi^2 & \phi^2 + \gamma^2 + \tau^2 & \phi^2 + \gamma^2 \\ \phi^2 & \phi^2 & \phi^2 + \gamma^2 & \phi^2 + \gamma^2 + \tau^2 \end{pmatrix}$$

We consider four alternative control structures: independent ownership (N) where there are four independent specialists, single ownership (M) where one specialist (or set of coordinated specialists) control all of the assets, and two structures where there are two specialists who each control two assets. This last situation

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<sup>19</sup> According to the arbitrage pricing theory (Ross, 1976) only the systematic risk of assets is priced. This builds on the implicit assumption that idiosyncratic risk is eliminated when the number of available assets is large. In competitive markets arbitrage ensures that asset prices are determined as weighted averages of the prices for factor risk. In imperfectly competitive markets, however, the ability to arbitrage can be severely impeded. In this section, we illustrate the determination of the bid-ask spread in imperfectly competitive markets (proposition 4.1), when the payoffs of the underlying assets are characterized by a simple factor structure.

can take two different forms: When each specialist controls both assets of an industry the control structure which we call industry ownership (I), and when each specialist controls one asset in each industry we call the structure cross-industry ownership (X). In the cross-industry ownership case, we assign one specialist to assets 1 and 3, and the other specialist to assets 2 and 4.

We determine the equilibrium spreads  $s^N$ ,  $s^M$ ,  $s^I$ , and  $s^X$  for each specialist ownership structure, under four alternative initial endowment vectors which are characterized by  $e = e^1 - e^2$ . The spreads in each asset are identical given the symmetry of demands for each asset. The statement that societal welfare is monotonic in size of the spreads again applies (as do the implications for Pareto ranking noted following Corollary 5.5).

**Corollary 6.1**

Let  $2r(4\phi^2 + 2\gamma^2 + \tau^2) < \min_{k=I,J}[E\bar{f} + E\bar{f}_k]$ .

a) When  $e' = (1, 1, 1, 1)$  the equilibrium spreads satisfy:  $s^N < s^X < s^I < s^M$ .

b) When  $e' = (1, 1, -1, -1)$  and

if  $\gamma^2(4\phi^2 + 2\gamma^2) \geq (\phi^2 - \gamma^2)\tau^2$  the equilibrium spreads satisfy:  $s^X < s^N \leq s^M < s^I$ .

if  $\gamma^2(4\phi^2 + 2\gamma^2) < (\phi^2 - \gamma^2)\tau^2$  the equilibrium spreads satisfy:  $s^X < s^M < s^N < s^I$ .

c) When  $e' = (1, -1, 1, -1)$  the equilibrium spreads satisfy:  $s^I = s^M < s^N < s^X$ .

d) When  $e' = (1, -1, -1, 1)$  the equilibrium spreads satisfy:  $s^I = s^M < s^X < s^N$ .

Proof: See the appendix.

Corollary 6.1 shows us that various control structures may turn out to be optimal, depending on the initial asset holdings.

Again, we can interpret the above results in terms of the complementarity of the associated demands for each asset. In case a), all of the assets trade as substitutes. In this case it is best to have independent specialists. Furthermore, conditional on having only two specialists, it is preferable to have them set across industries as these are weaker substitutes. In case b), assets within an industry trade as substitutes and across industries they trade as complements. In this case it is best to have two specialists set up across industries. The specialists will compete inside each industry, but will adjust spreads downward for some of the complementarities which exist across industries. Here, having independent specialists is better than having industry ownership, as assets are substitutes within an industry. The spreads are largest for industry ownership since the competitive pressure is lower and complementarities in trades are not accounted for. Hence, both joint ownership and independent ownership outperform industry ownership. Joint ownership partially internalizes the complementarities and independent ownership increases competition. It turns out that the relative advantage of joint ownership relative to independent ownership depends on the relative

strength of the force of competition relative to the importance of the complementarity in trade.<sup>20</sup> In case c), assets within an industry trade as complements and across industries some of them trade as substitutes and others as complements. In this case it is best to have industry ownership as the specialists will compete across industries, but adjust spreads downwards for some of the complementarities which exist within the industries. For the given endowment structure competition between the two industry specialists is just the same as the additional complementarity consideration which a monopolist accounts for across industries (for example between assets 1 and 4), and so the spreads for industry ownership are the same as under joint ownership. Competition reduces equilibrium spreads in the case of independent ownership relative to cross-industry ownership. Finally, case d) is similar to case c), except for the assignment of which assets the cross industry specialists control. Here, they control the complements and thus do better than the independent specialists. In case d) assets are stronger complements (due to their higher correlation) within an industry than across industries, which explains why industry ownership dominates cross-industry ownership.

Note that the monopolist structure is never optimal in a strict sense in Corollary 6.1. Although a monopolist internalizes some of the complementarities, the monopolist also controls substitutes. Given the covariance structure associated with the factor structure in this section, it is impossible for all assets to trade as complements. Thus, we always find another ownership structure, in particular one which concentrates only complements in the hands of the same specialist, (weakly) dominating a monopolist.

Corollary 6.1 reiterates the fact that the direction of trades crucially affects specialists' market power and, hence, ultimately the costs of trading. In other words, the desirability of one control structure over another depends very much on the nature of the trading activities and the interrelationships of the demands. The determination of an optimal ownership structure for a specialist market will have to take into account empirical complementarities between trades in different assets.

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<sup>20</sup> The complementarity in trade is particularly important when  $\gamma^2 < \phi^2$  and  $\tau^2 > \frac{2(2\phi^2 + \gamma^2)\gamma^2}{\phi^2 - \gamma^2}$ .

## 7. Ownership Structure in Competing Exchanges

Our previous analysis relies on barriers to entry. There may be reasons to restrict entry into securities trading on organized markets. Dennert (1993), for example, shows that in an asymmetric information framework, insiders' opportunities to hide information increases in the number of market makers, which causes bid-ask spreads to widen, as the number of market makers increases. The introduction of the Intermarket Trading System (ITS) did not cause an erosion of NYSE trading volumes or revenues, despite the fact that NYSE-quoted stocks could be traded elsewhere. This suggests that there are segments in securities markets, which are effectively protected from direct competition. On the other hand, entry seems rather easy in other segments of securities markets. In this section we analyze situations under symmetric information, where an open market with unrestricted entry and a protected market with limited entry co-exist. In particular, we are interested in the consequences of the indirect competition induced by the correlation of assets. If one asset is traded in a protected market and the other asset is traded in a competitive market, how do the resulting prices compare to the situation we have analyzed so far?

Security 1 is traded by a specialist and security 2 is traded on a competitive dealership market with free (and costless) entry. Price competition<sup>21</sup> for trades ensures that  $a_2 = b_2 = p$ . Notice, however, that the price  $p$  typically depends on the monopolist's price choices  $(a_1, b_1)$  and therefore may differ from the competitive price  $p_2^c$ . Competition in the competitive segment does not necessarily yield the Walrasian outcome in that market segment. As the following Proposition shows, however, it does in equilibrium.

### Proposition 7.1

*Under both initial portfolio holdings (A) and (B), the equilibrium prices when a specialist controlling 1 faces a competitive market in 2 are characterized by  $a_2 = b_2 = p_2^c$  and*

$$a_1^C = p_1^c + \frac{1}{2}(\mu_1 - p_1^c) - \frac{\sigma_{12}}{2\sigma_2^2}(\mu_2 - p_2^c) \quad (13.a)$$

$$b_1^C = p_1^c - \frac{1}{2}(\mu_1 - p_1^c) + \frac{\sigma_{12}}{2\sigma_2^2}(\mu_2 - p_2^c) \quad (13.b)$$

$$s_1^C = \mu_1 - p_1^c - \frac{\sigma_{12}}{\sigma_2^2}(\mu_2 - p_2^c). \quad (13.c)$$

Proof: See the appendix.

Summarizing from Corollaries 5.1 and 5.3 and Proposition 7.1:

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<sup>21</sup> Strictly speaking, intermediaries can sell any unwanted inventory of stocks in further rounds of trading. This assumption rules out non-Walrasian equilibria in the case of inelastic demand as discussed by Stahl (1988) and Yanelle (1989).

### Corollary 7.2

Let  $\sigma_1 = \sigma_2$  and  $e_1 = e_2$ .

When  $\rho > 0$ ,

$$s_1^{M,A} > s_1^{N,A} > s_1^C > s_1^{N,B} > s_1^{M,B},$$

and when  $\rho < 0$ ,

$$s_1^{M,B} > s_1^{N,B} > s_1^C > s_1^{N,A} > s_1^{M,A}.$$

The above analysis is only suggestive, since the nature of competition in the protected market will depend on correlations among numerous assets traded in the protected market and competitive markets. Thus, in a multi-market context the question about the desirability of different ownership structures is likely to be a difficult one. While the correlation of the underlying assets traded in a given exchange is an important piece of information, additionally information about the cross correlations with assets traded elsewhere may be important, as well as information about initial portfolio holdings. All these variables jointly determine, whether trades in a given set of assets are substitutes or complements. When these trades behave like complements, i.e. an increase in the spread of one asset reduces demand for the other asset(s), joint control of the specialist rights may be beneficial.

## 8. Risk Averse Specialists in Large Markets

Throughout this paper we have assumed that specialists were constrained to clear their markets. In this section, we briefly discuss the specialists' behavior in situations where they may hold their own portfolios of the assets, may short sell the asset, and maximize their expected utility of the trading revenue plus the value of their own holdings. We argue that with large numbers of traders, the specialists' behavior will be as we assumed in the previous sections. That is, given any level of risk aversion, the difference between supply and demand (which is the change in the specialist's portfolio) goes to zero as a percentage of the overall supply (or demand) as the number of investors they face becomes large. Trading revenue becomes the dominant factor in the specialist's calculation, and thus their maximization problem is as if they maximized trading revenue subject to balancing supply and demand.

Starting with a given economy, replicate it  $n$  times and consider a specialist setting prices for a given asset in that replicated economy. Assume that investors of the same type are acting identically, and let  $q_b$  be the amount bought by the specialist at the bid and  $q_a$  be the amount sold by the specialist at the ask, for each replication. We can write the specialist's expected utility as

$$EU^s = nq_a a - nq_b b + (e^s + nq_b - nq_a)\mu - r^s(e^s + n(q_b - q_a))^2\sigma^2,$$

where  $e^s$  and  $r^s$  are the specialist's initial endowment and risk aversion parameter, respectively. Maximizing  $EU^s$  is equivalent to maximizing

$$q_a a - q_b b + \left(\frac{e^s}{n} + q_b - q_a\right)\mu - r^s \left(\frac{e^s}{n} + 2e^s(q_b - q_a) + n(q_b - q_a)^2\right)\sigma^2. \quad (14)$$

Notice that  $q_a a - q_b b + \left(\frac{e^s}{n} + q_b - q_a\right)\mu$  is bounded when varying  $a$  and  $b$  (individual investors are risk averse and so  $q_a$  and  $q_b$  are bounded). Thus, when solving (19) as  $n$  becomes large, the solution necessarily involves setting  $q_b - q_a$  close to 0, as this enters the second term proportionally to  $n$ . The problem thus approaches maximizing  $q_a a - q_b b$  subject to  $q_a = q_b$ .<sup>22</sup>

In the above argument, the specialist only holds asset 1. Provided that the assets are not perfectly correlated, allowing the specialist to hold other assets will not affect the reasoning. What is critical to the argument is that the market be large in a sense determined by the level of risk aversion of the specialist. In thinner markets, or if the specialist is not very risk averse, the specialist's own portfolio becomes an important part of the bid-ask spread determination.

Of course, our model is static and does not capture the dynamic optimization problem which a specialist faces. If trades are spread out over time, then the inventory control problem becomes a non-trivial part of the bid-ask selection. Our focus on trading profits is meant to complement existing research on the role of inventory control (and that on the role of adverse selection) in the determination of bid-ask spreads.

## 9. Concluding Comments

In this paper we argue that the ownership structure of intermediaries decisively affects the competitiveness of markets whenever intermediaries exercise some degree of market power. We argue that the complementarity of trades may be an important criterion in allocating ownership rights to intermediaries. Securities that trade as complements should be traded by the same intermediary and securities that trade as substitutes should be traded by independent intermediaries. Whether securities trade as substitutes or complements cannot be determined on the basis of empirical correlations of asset returns alone. Rather, the empirical cross-price reactions of trades (for example, equations 11 and 12) are the relevant criterion. Hence, also the empirical nature of trades should determine, whether stocks and options should be traded on the same exchange (by the same intermediaries) or in independent markets (by independent intermediaries).

Furthermore, our analysis suggests that stocks with similar risk return characteristics (and widely dispersed share ownership) may exhibit quite different patterns of trade complementarity depending on whether they are included in an actively traded index or not.

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<sup>22</sup> To be careful: for any  $\epsilon > 0$  there exists  $n_\epsilon$  such that any solution to maximizing  $EU^s$  for  $n \geq n_\epsilon$  involves  $|q_b - q_a| < \epsilon$ . The theorem of the maximum ensures that the solution to maximizing  $EU^s$  subject to  $|q_b - q_a| = 0$  is continuous as we free up the constraint.

## Appendix

### Proof of Proposition 4.1

We solve the problem first by assuming that specialists may specify a price  $p^1$  for investors of type 1, and also a price  $p^2$  for investors of type 2. We then show that these prices will correspond to bids and asks, and that investors will not want to trade at the other investors' prices.

First, note that for any Nash equilibrium  $p^1, p^2$ , and any asset  $j$  it may turn out that  $\theta_j^1 = 0 = \theta_j^2$  and that at least one of these demands is constrained (i.e. the unconstrained demand is of the wrong sign at  $p_j^i$ ). We can then find an equivalent equilibrium where  $p_j^i$  is set so that the unconstrained demand is 0 for each  $i$ . This fact allows us to work with unconstrained demands in the revenue maximization problem, and then check that the demands are of the correct sign.

We begin by using the market clearing restriction to rewrite the specialists' revenue expressions. The demand of investor  $i$  facing prices  $p^i$  is

$$\theta^i = \frac{1}{2r} \Sigma^{-1} (\mu - p^i) - e^i.$$

From (4) it follows that  $\frac{1}{r} \Sigma^{-1} (\mu - p^c) = e$ , so we can rewrite the market clearing equation,  $\theta^1 = -\theta^2$  as

$$\frac{1}{2r} \Sigma^{-1} (2p^c - p^1 - p^2) = 0$$

or

$$p^1 - p^c = -(p^2 - p^c).$$

Under market clearing, the revenue from asset  $j$ ,  $R_j = \theta_j^1 p_j^1 + \theta_j^2 p_j^2$ , can be rewritten as

$$R_j = \theta_j^1 p_j^1 - \theta_j^1 (-p_j^1 + 2p_j^c)$$

or

$$R_j = 2\theta_j^1 (p_j^1 - p_j^c).$$

Now let us consider any specialist unit  $k$  controlling assets  $N_k \subset N$ . For their choice of prices to be part of a Nash equilibrium, they must maximize  $\sum_{j \in N_k} \theta_j^1 (p_j^1 - p_j^c)$ , taking the prices of the other specialists as given. The necessary first order conditions for this problem require that for each  $j \in N_k$

$$\frac{(\Sigma^{-1})_j}{2r} (\mu - p^1) - e_j^1 - \sum_{j' \in N_k} \frac{(\Sigma^{-1})_{j'j}}{2r} (p_{j'}^1 - p_{j'}^c) = 0,$$

where  $(\Sigma^{-1})_j$  is the  $j$ -th row of  $\Sigma^{-1}$  and  $(\Sigma^{-1})_{j'j}$  is the entry in the  $j'$ -th row and  $j$ -th column of  $\Sigma^{-1}$ .

Combining the first order conditions for all the specialists and each asset, we find that

$$\frac{(\Sigma^{-1})}{2r} (\mu - p^c) - e^1 - \frac{A}{2r} (p^1 - p^c) = 0,$$

where  $A$  is the matrix with entries  $A_{ij} = 2(\Sigma^{-1})_{ij}$  if  $i = j$  or  $i$  and  $j$  are controlled by the same specialist (including  $i = j$ ), and  $A_{ij} = (\Sigma^{-1})_{ij}$ , otherwise. Therefore,

$$p^1 - p^c = A^{-1}((\Sigma^{-1})(\mu - p^c) - 2re^1).$$

Since by (4),  $\frac{1}{r}\Sigma^{-1}(\mu - p^c) = e$ , we find that

$$p^1 - p^c = rA^{-1}(e^2 - e^1),$$

as stated in Proposition 4.1.

Next, we check the second order conditions to assure that these conditions are sufficient to have the prices maximize revenues. The second order conditions require that  $-A$  is negative definite, or that  $A$  is positive definite. We can write  $A = \Sigma^{-1} + X$ , where  $X$  is the matrix with entries  $X_{ij} = (\Sigma^{-1})_{ij}$  if  $i = j$  or  $i$  and  $j$  are controlled by the same specialist (including  $i = j$ ), and  $X_{ij} = 0$ , otherwise. We know that  $\Sigma$  is positive definite since it is a nonsingular covariance matrix, which implies that  $\Sigma^{-1}$  is also positive definite.<sup>23</sup> It follows that  $X$  is also positive definite.<sup>24</sup> Now notice that  $x^t Ax = x^t(\Sigma^{-1} + X)x = x^t \Sigma^{-1} x + x^t X x > 0$ , so  $A$  is positive definite.

Finally, if  $\theta_j^i(p_j^i - p_j^c) \geq 0$  for all  $i$  and  $j$ , then it follows that investors are buying at the ask and selling at the bid. To conclude the proof, since we have assumed that investor type  $i$  can only trade at  $p^i$ , we must check that no investor would rather reverse some subset of their trades and trade at the other investor type's prices. Suppose to the contrary, that for some investor reversing trades in some assets results in a higher expected utility. Suppose investor type 1's optimal trade would include trading some subset of assets at  $p_j^2$  (rather than  $p_j^1$ ). It follows that investor type 1 could have had an even higher expected utility performing the same set of trades at  $p_j^1$ <sup>25</sup> which contradicts the fact that  $\theta_j^1$  is the unconstrained expected utility maximizing profile of trades for investors of type 1.

## Proof of Corollaries 5.2 and 7.2

Under the hypothesis of the corollary, let  $\sigma_1 = \sigma_2 = \sigma$  and let  $e_1 = e_2 = e$ . Substitute from (4) into (7.c) (8.c) (9.c) (10.c) and (13.c) to find:  $s_z^{N,A} = r\sigma^2 e(1 - \rho^2)(\frac{4+2\rho}{4-\rho^2})$ ,  $s_z^{N,B} = r\sigma^2 e(1 - \rho^2)(\frac{4-2\rho}{4-\rho^2})$ ,  $s_z^{M,A} = r\sigma^2 e(1 + \rho)$ ,  $s_z^{M,B} = r\sigma^2 e(1 - \rho)$ , and  $s_1^C = r\sigma^2 e(1 - \rho^2)$ . The rest follows directly.

<sup>23</sup> A symmetric matrix is positive definite if and only if all of its eigenvalues are positive. The eigenvalues of a symmetric matrix's inverse are the reciprocals of the eigenvalues of the matrix. Thus a positive definite symmetric matrix's inverse is also positive definite.

<sup>24</sup> A symmetric matrix is positive definite if and only if the determinants of all of the leading principal minors are positive. Since this holds for  $\Sigma^{-1}$ , and  $X$  is obtained by symmetrically deleting some off diagonal entries from  $\Sigma^{-1}$ , it also holds for  $X$ .

<sup>25</sup> For example, if  $p_j^2 < p_j^c$  then the investor is selling asset  $j$ , and from market clearing we know that  $p_j^1$  is a higher price ( $p_j^c < p_j^1$ ). These trades would violate the buy at the ask, sell at the bid requirement.



### Proof of Corollary 6.1

The proof is by application of Proposition 4.1 and involves straightforward algebra. We provide only the essential steps.

Write

$$\Sigma^{-1} = \frac{1}{d} \begin{pmatrix} a & b & c & c \\ b & a & c & c \\ c & c & a & b \\ c & c & b & a \end{pmatrix}$$

where,

$$\begin{aligned} a &:= (\phi^2 + \gamma^2 + \tau^2)(2\phi^2 + 2\gamma^2 + \tau^2) - 2\phi^4 > 0 \\ b &:= -(\phi^2 + \gamma^2)(2\phi^2 + 2\gamma^2 + \tau^2) + 2\phi^4 < 0 \\ c &:= -\phi^2\tau^2 < 0 \\ d &:= \tau^2((2\phi^2 + 2\gamma^2 + \tau^2)^2 - 4\phi^4) > 0 \end{aligned}$$

For each control structure the inverse matrix  $(A^O)^{-1}$ ,  $O = N, M, I, X$  can be determined as follows.

$$(A^N)^{-1} = \frac{d}{D^N} \begin{pmatrix} 2(2a^2 + ab - c^2) & -2ab - b^2 + 2c^2 & -c(2a - b) & -c(2a - b) \\ -2ab - b^2 + 2c^2 & 2(2a^2 + ab - c^2) & -c(2a - b) & -c(2a - b) \\ -c(2a - b) & -c(2a - b) & 2(2a^2 + ab - c^2) & -2ab - b^2 + 2c^2 \\ -c(2a - b) & -c(2a - b) & -2ab - b^2 + 2c^2 & 2(2a^2 + ab - c^2) \end{pmatrix}$$

where  $D^N = (2a - b)(2a + b - 2c)(2a + b + 2c)$

$$(A^I)^{-1} = \frac{d}{D^I} \begin{pmatrix} 2a^2 + 2ab - c^2 & -2ab - 2b^2 + c^2 & -c(a - b) & -c(a - b) \\ -2ab - 2b^2 + c^2 & 2a^2 + 2ab - c^2 & -c(a - b) & -c(a - b) \\ -c(a - b) & -c(a - b) & 2a^2 + 2ab - c^2 & -2ab - 2b^2 + c^2 \\ -c(a - b) & -c(a - b) & -2ab - 2b^2 + c^2 & 2a^2 + 2ab - c^2 \end{pmatrix}$$

where  $D^I = 4(a - b)(a + b + c)(a + b - c)$

$$(A^X)^{-1} = \frac{d}{D^X} \begin{pmatrix} 2(4a^3 - ab - 5ac^2 + 2bc^2) & -4a^2b + b^3 + 8ac^2 - 5bc^2 & \dots & \dots \\ -4a^2b + b^3 + 8ac^2 - 5bc^2 & 2(4a^3 - ab - 5ac^2 + 2bc^2) & \dots & \dots \\ -2c(-2ab + b^3 + 4a^2 - 3c^2) & -c(-2ab + b^3 + 4a^2 - 3c^2) & \dots & \dots \\ -c(-2ab + b^3 + 4a^2 - 3c^2) & -2c(-2ab + b^3 + 4a^2 - 3c^2) & \dots & \dots \end{pmatrix}$$

where  $D^X = (2a + b - 3c)(2a + b + 3c)(2a - b + c)(2a - b - c)$

$$(A^M)^{-1} = \frac{d}{D^M} \begin{pmatrix} a^2 + ab - 2c^2 & -ab - b^2 + 2c^2 & -c(a-b) & -c(a-b) \\ -ab - b^2 + 2c^2 & a^2 + ab - 2c^2 & -c(a-b) & -c(a-b) \\ -c(a-b) & -c(a-b) & a^2 + ab - 2c^2 & -ab - b^2 + 2c^2 \\ -c(a-b) & -c(a-b) & -ab - b^2 + 2c^2 & a^2 + ab - 2c^2 \end{pmatrix}$$

where  $D^M = 2(a-b)(a+b+2c)(a+b-2c)$

On the basis of these matrices for each scenario of endowments equilibrium spreads are determined. The ranking of spreads follows from straightforward comparison, substituting for  $a$ ,  $b$ , and  $c$ .

a)  $e' = (1, 1, 1, 1)$ :

$$s^N = \frac{2rd}{2a+b+2c}, s^I = \frac{2rd}{2(a+b+c)}, s^X = \frac{2rd}{2a+b+3c}, s^M = \frac{2rd}{2(a+b+2c)}$$

b)  $e' = (1, 1, -1, -1)$ :

$$s^N = \frac{2rd}{2a+b-2c}, s^I = \frac{2rd}{2(a+b-c)}, s^X = \frac{2rd}{2a+b-3c}, s^M = \frac{2rd}{2(a+b-2c)}$$

c)  $e' = (1, -1, 1, -1)$ :

$$s^N = \frac{2rd}{2a-b}, s^I = \frac{2rd}{2(a-b)}, s^X = \frac{2rd}{2a-b+c}, s^M = \frac{2rd}{2(a-b)}$$

d)  $e' = (1, -1, -1, 1)$ :

$$s^N = \frac{2rd}{2a-b}, s^I = \frac{2rd}{2(a-b)}, s^X = \frac{2rd}{2a-b-c}, s^M = \frac{2rd}{2(a-b)}$$

Finally, we need a condition that guarantees that all bids are positive and no ask exceeds  $\mu$ . Given the symmetry of bids and asks about the competitive price, it follows that bids are positive and asks do not exceed  $\mu$  if  $p^c - \frac{1}{2}s^O \geq 0$  and  $p^c + \frac{1}{2}s^O \leq \mu$  for  $O = I, N, M, X$  and the initial endowment allocation. By inspection one readily verifies that  $\frac{1}{2}s^O < r(4\phi^2 + 2\gamma^2 + \tau^2)$ . Given that  $p^c = \mu - r\Sigma e$  and  $\Sigma e \leq 4\phi^2 + 2\gamma^2 + \tau^2$  (where the latter is a vector), it follows that equilibrium bid prices are positive when  $\mu - r\Sigma e \geq r(4\phi^2 + 2\gamma^2 + \tau^2)$  and ask prices do not exceed  $\mu$  when  $\mu - r\Sigma e \leq \mu - r(4\phi^2 + 2\gamma^2 + \tau^2)$ . These conditions are implied by the assumption of Corollary 6.1.

### Proof of Proposition 7.1.

Notice that if  $\theta_1^2 \leq 0$  maximizes  $EU^2$ , then  $\theta_1^1 \leq 0$  maximizes  $EU^1$ ,<sup>26</sup> for both initial portfolio holdings. Thus, since the specialist in 1 will not choose to only buy the asset at an optimum, it must be that  $\theta_1^2 \geq 0$  and  $\theta_2^2 \leq 0$ .

<sup>26</sup> This is true, given that there is only one price for 2.

Under initial portfolio holding (A)

$$\theta^2 := \frac{1}{2r} \Sigma^{-1} \begin{pmatrix} \bar{\mu}_1 - \bar{a}_1 \\ \bar{\mu}_2 - \bar{p} \end{pmatrix}$$

$$\theta^1 := -\frac{1}{2r} \Sigma^{-1} \begin{pmatrix} \bar{\mu}_1 - \bar{b}_1 \\ \bar{\mu}_2 - \bar{p} \end{pmatrix} - \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

and under initial portfolio holding (B)

$$\theta^2 := \frac{1}{2r} \Sigma^{-1} \begin{pmatrix} \bar{\mu}_1 - \bar{a}_1 \\ \bar{\mu}_2 - \bar{p} \end{pmatrix} - \begin{pmatrix} 0 \\ e_2 \end{pmatrix}$$

$$\theta^1 := -\frac{1}{2r} \Sigma^{-1} \begin{pmatrix} \bar{\mu}_1 - \bar{b}_1 \\ \bar{\mu}_2 - \bar{p} \end{pmatrix} - \begin{pmatrix} e_1 \\ 0 \end{pmatrix}$$

where  $\bar{p} = p - p_2^c$ . Market clearing (in both cases) in the market for asset  $y$  implies

$$\bar{p} = \frac{\sigma_{12}}{\sigma_1^2} \frac{\bar{a}_1 + \bar{b}_1}{2}$$

In equilibrium (in both cases), the specialist will quote prices which satisfy  $\bar{a}_1 + \bar{b}_1 = 0$ . (Notice that the claim from the proof of Proposition 4.1 holds in this case, so the specialist will clear the market for 1.) Thus,  $\bar{p} = 0$ . In either case, by market clearing and  $\bar{a}_1 = -\bar{b}_1$

$$R_1 = \theta_1^2 \bar{a}_1 + \theta_1^1 \bar{b}_1 = 2\theta_1^2 \bar{a}_1.$$

Thus

$$\bar{a}_1 = \operatorname{argmax} (\sigma_2^2 (\bar{\mu}_1 - \bar{a}_1) - \sigma_{12} (\bar{\mu}_2)) \bar{a}_1.$$

The rest follows directly.

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