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APPROXIMATE COMPETITIVE EQUILIBRIA IN LARGE ECONOMIES

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Abstract: We examine market-clearing prices and allocations in economies where agents' demand functions are undominated relative to their beliefs about other agents' actions. For sufficiently large economies and give certain restrictions on beliefs, the resulting allocations are nearly competitive.

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1 Introduction

We investigate the validity of the price-taking assumption inherent in the definition of perfect competition. We use a model in which consumers, who have private information about their individual preferences, choose (or report) a demand function, not necessarily their true demand. Prices are chosen to clear markets, and goods are allocated according to reported demands. We study whether market-clearing prices and allocations of the reported economy are close to the competitive equilibria of the true economy. We employ fairly weak behavioral assumptions about how agents choose their demands: each agent forms beliefs about other agents' actions. These beliefs are represented by a set of (possible) strategy profiles for the rest of the economy, which can be seen as the support of some distribution over other agents' actions. Agents select demand functions that are undominated with respect to their beliefs. For example, if each agent's beliefs include all possible actions by other agents, then the reported economy may be formed by any set of demands that are undominated in pure strategies. In contrast, if each agent's beliefs are the singleton of demands reported by other agents, then the reported economy is a Nash equilibrium.

We prove that the reported economy is close to the true economy, provided that (a) consumers believe that they have, individually, a small influence on equilibrium prices, and (b) consumers' beliefs are sufficiently rich. Proviso (a) guarantees that there are only small gains to be made from grossly misrepresenting one's true demand near potential equilibrium prices. Consumers, however, may report any demand at prices that should not occur according to their beliefs. Proviso (b) ensures that individuals will report seriously for a wide range of prices.

Our conditions are, in some sense, necessary as well. With regard to (a): if agents believe that they can influence prices considerably, then they will exercise that influence and only by chance will the market-clearing allocation of the reported economy resemble that of the true economy. With regard to (b): if agents' beliefs are very concentrated, for instance a singleton, then the reported economy may have many Walrasian allocations that do not resemble those of the true economy. This phenomenon can occur even when beliefs are concentrated around the true economy.

We also explore the consequences of replacing (b) with the assumption that consumers' beliefs about the reported economy are not completely incorrect: the actual reported economy belongs to the belief-set. Although under this alternative hypothesis the reported economies do not necessarily approximate the true economy, it is still possible to prove that the equilibrium allocations of the reported economies approach a Walrasian allocation of the true limit economy.

Relation to the Literature

In economies with few individuals, any single agent has a considerable influence on the composition of the reported economy and therefore on the equilibrium prices. Hurwicz (1972) showed that manipulations are attractive in small economies, and so it is clear that a large numbers argument is necessary to justify price-taking behavior.

The presumption that in large economies individuals may have almost no influence on equilibrium prices was explored by Roberts and Postlewaite (1976). They provided an example where a single agent has substantial price influence, even as the economy becomes arbitrarily large. They went on to argue, however, that such examples are exceptional, and that if the limit economy is regular (i. e., the equilibrium-price correspondence is finite valued and continuous in a neighborhood of the limit economy), then the the benefits from manipulation become arbitrarily small as the economy grows.

The fact that the gains from manipulation shrink as the economy grows is suggestive, but not convincing as a justification for the use of the competitive model. First, even though deviations from price-taking behavior may only result in small utility gains, a consumer's optimal reported demand may be far from price-taking. Second, even if individual reported demands are close to price-taking, it is possible that, when aggregated, these demands lead to prices and allocations that are substantially different from the competitive equilibria of the true economy.[1]

Jackson (1992) analyzes the first problem and shows that behavior on the individual level does indeed approximate competitive behavior as the economy becomes large, again provided that the limit economy is regular.

The second problem of whether aggregate behavior in large economies approximates price-taking behavior is addressed by Thompson (1979) and Otani and Sicilian (1982, 1990). Thompson (1979) shows by construction that the set of Nash equilibria (of a game where agents submit demand functions) can be quite large and different from the competitive allocations, even as the economy grows.[2] Otani and Sicilian (1990) show that the set of Nash equilibria is generally very large and does not collapse to competitive behavior in the limit. Otani and Sicilian (1990, Proposition 3) also show, however, that this negative result is overturned if agents can only report smooth demands and the sequence of Nash equilibrium reported economies converges to a regular economy. In such a situation the limit Nash allocation is a competitive allocation of the limit economy.

One might be hopeful that the Nash requirement (and the smoothness requirement, as we shall discuss) of the Otani and Sicilian result can be weakened without destroying the result itself. The information requirements for a Nash equilibrium in a large economy are quite strong. In fact, one might conjecture that ignorance in a large economy would help bring about competitive behavior, as without specific knowledge of other agents' actions it is difficult for agents to know how to manipulate prices and allocations.

If one moves to the other extreme from Nash behavior and explores complete ignorance as captured by strategy-proofness (dominant strategy incentive compatibility), the results are quite negative. Barbera and Jackson (1992) show that if strategy-proof behavior is

[1] See Gul and Postlewaite (1992 discussion point 3) for an exposition of this point.

[2] The implementation literature has considered the possibility of explicitly designing a mechanism to result in Walrasian allocations. See the concluding remarks for more discussion.

required on the part of individuals, then the only allocations that can be achieved are far from efficient, much less competitive, even in arbitrarily large economies. The requirement that allocations be strategy-proof is, however, very strong and difficult to satisfy.

Behavioral assumptions between the extremes of the complete information of Nash equilibrium and the complete ignorance of strategy-proofness are more likely to provide positive results. We confirm that approximately competitive behavior obtains in situations where agents have incomplete information represented by sets of beliefs (satisfying certain restrictions) and choose actions that are undominated relative to those beliefs. A restriction on the set of beliefs (that agents believe they cannot influence prices significantly) retains the part of the regularity assumption that generated the positive results of Roberts and Postlewaite (1976) and Otani and Sicilian (1990), and allows those results to hold under more general behavioral and informational assumptions.

Finally, let us discuss the relation of our work to a recent paper by Gul and Postlewaite (1992), who also explore behavior in large economies with incomplete information. They show that if all agents are independently drawn from a finite number of types (whose price-taking demands differ sufficiently), then as the economy is replicated, there exists an incentive compatible and individually rational allocation rule that is almost efficient.[3] The allocation that agents receive is actually the competitive one from an artificial economy, which is near the actual one with high probability. This suggests that approximate competitive behavior is possible in large economies. The Gul and Postlewaite work, however, leaves two questions open. First, the finite types assumption is critical to their approach (as they acknowledge in their remark 2). The basic intuition is that the influence of any single agent on the equilibrium price is reduced arbitrarily for a sufficiently large replication of the economy. Thus, although announcing an incorrect type, i. e., misrepresenting

[3]For simplicity, the reader may consider a player's type to be the player's preference-endowment pair or, equivalently, the player's price-taking excess demand. The Gul and Postlewaite model is more general in the externalities that it admits.

the demand function, will not change significantly the equilibrium price, it will result in a consumption bundle which differs significantly from the one stipulated by the true demand function. As the economy is replicated, any incorrect type that can be potentially announced will eventually result in a loss of utility to the agent. Since there are only a finite number of types, the optimal strategy for any agent eventually becomes reporting the true demand. If the type space is permited to be infinite, however, it is not obvious that the results will still hold: each agent's optimal strategy will approach price-taking behavior but will not equal it. Again, when aggregated across agents such slight deviations might result in a substantial aggregate deviation. Here, we show that the aggregate behavior will converge in an infinite model. Second, it is not clear that all the equilibrium allocations of a given mechanism are converging to competitive ones. That is, Gul and Postlewaite show that there exists an incentive compatible allocation rule, but do not examine whether other equilibria (non–truthful ones) might arise when such a rule is used (see their remark 4). We show that all of the undominated actions will converge under certain restrictions on beliefs.

After introducing the main definitions in the next section, we present an example that clarifies the role of regularity (and the smoothness of demands) in the existing literature and that motivates our results. Section 4 presents the main Theorems.

2 Definitions

Preferences, Demands and Competitive Equilibrium

There is a finite number of goods, l. Preferences are represented by a complete and transitive binary relation R, defined on $I\!R^l_+ \times I\!R^l_+$. (For simplicity, consumption sets are assumed to be $I\!R^l_+$.) The notation xRy is interpreted as meaning $x \in I\!R^l_+$ is weakly preferred to $y \in I\!R^l_+$. The strict relation implied by R is denoted P (xPy if not yRx). Preferences are continuous, monotonic, and strictly convex.[4] The set of all such preferences is denoted

[4]A preference relation P is monotonic if xPy whenever $x_k \geq y_k$ for all $k \in \{1, 2, \ldots, l\}$ and $x \neq y$.

$\mathcal{P}$.

A demand function d associated with an endowment $e \in I\!R^l_+$ is a continuous map from Δ_{++} (the unit simplex of strictly positive prices in $I\!R^l_+$) into $I\!R^l_+$, such that $p \cdot d(p) \leq p \cdot e$ for all $p \in \Delta_{++}$. To insure that competitive equilibria exist we also require that $\|d(p^n)\| \longrightarrow \infty$ as p^n approaches the boundary of Δ_{++}. The set of all such demand functions is denoted $D(e)$.[5] Let $D = \{(d, e) : e \in I\!R^l_+,\ d \in D(e)\}$. We endow the space of demand functions with the topology of uniform convergence on compact sets.[6]

Let μ be a simple probability measure defined on D. The interpretation of $\mu(\{(d, e)\})$ is that a proportion $\mu(\{(d, e)\})$ of the agents in a finite economy have endowment e and are acting according to the demand function d. $M(D)$ denotes the set of all Borel probability measures μ on D. $M(D)$ is endowed with the topology of weak convergence. We denote $B(\mu, 1/k)$ the open ball of radius $1/k$ around μ with respect to the Prohorov metric ρ.[7]

For any collection of demand functions, the prices at which the aggregate excess demand is zero, i. e., the competitive equilibrium prices, are solely determined by the distribution of demands; the number of demand functions plays no role (Hildenbrand (1974), Proposition 4, page 114). The competitive equilibrium *price correspondence* Π that selects the set of equilibrium prices for each distribution of demands $\mu \in M(D)$ is therefore defined by:

$$\Pi(\mu) = \{p \in \Delta_+ \mid 0 \in \int_D (d(p) - e)\, d\mu\}.$$

Representing Consumer Behavior

Let A be a finite set of consumers' names, and let λ be the normalized counting distribution on A.[8] An *economy* $\mathcal{E}(A)$ is a measurable map $\mathcal{E} : A \to I\!R^l_+ \times \mathcal{P}$ that assigns to each

[5] Given the properties of preferences any demand correspondence that maximizes preferences is single valued and continuous at $p \gg 0$ and empty otherwise.

[6] D is endowed with the product topology: $(d^k, e^k) \longrightarrow (d, e)$ if $d^k \longrightarrow d$ and $e^k \longrightarrow e$.

[7] A definition of the Prohorov metric can be found, for instance, in Billingsley (1968) p. 237.

[8] For any $C \subset A$, $\lambda(C) = \#C/\#A$ where $\#$ represents the number of elements in a given set.

agent $a \in A$ an endowment $e_a \in I\!R^l_+$ and a preference relation $R_a \in \mathcal{P}$. The preference-endowment distribution $\bar{\theta} \in M(\mathcal{P} \times I\!R^l_+)$ of the economy $\mathcal{E}(A)$ is defined as $\bar{\theta} = \lambda \circ \mathcal{E}^{-1}$, that is, $\bar{\theta}(C) = \lambda(\mathcal{E}^{-1}(C))$ for any measurable set $C \subset \mathcal{P} \times I\!R^l_+$.

The *competitive* or *price-taking* demand function of an agent $a \in A$ is the one generated by a's preferences, given a's endowment, when a takes prices as given. It is denoted $\bar{d}_a$ and defined by $p \cdot \bar{d}_a(p) \leq p \cdot e_a$ and $\bar{d}_a(p)\ R_a\ y$ for all y such that $p \cdot y \leq p \cdot e_a$.

Consumers report (or act) according to some demand function which is not necessarily their "competitive" demand. The collection of demands reported by all agents in the economy constitutes a "reported" economy. Let r be the map that assigns to each consumer $a \in A$ a reported demand–endowment pair $(d_a, e_a) \in D$. Endowments are assumed to be verifiable, and so e_a is the agent's true endowment. Preferences, however, are not observed and so d_a may differ from the price-taking demand $\bar{d}_a$. The reported economy $\mu \in M(D)$ is defined by $\mu = \lambda \circ r^{-1}$. If all agents report their price-taking demands ($r(a) = (\bar{d}_a, e_a)$ for all $a \in A$), the resulting reported economy μ is the same as the "true" demand-endowment distribution, denoted $\bar{\mu}$, corresponding to the consumers in A. The market-clearing prices for the economy $\mathcal{E}(A)$ when agents report their price-taking (competitive) demands is denoted by $\Pi(\mathcal{E}(A)) = \Pi(\bar{\mu})$.

Given their preferences, it may be in agents' best interest to act or report a demand function other than their price-taking (competitive) one. Agents choose their reported demands in response to their beliefs about how their reports will influence equilibrium prices. Suppose, for instance, consumer a believes that the distribution of reported demands is given by $\mu_d \in M(D)$ when a reports d. In deciding on a demand to report, a evaluates how consumption varies in response to different reports. Thus, when a reports d the equilibrium price may be $p \in \Pi(\mu_d)$ providing a with a consumption $d(p)$. A different demand d' may result in a different equilibrium price $p' \in \Pi(\mu_{d'})$ and consumption $d'(p')$.

The beliefs of a given consumer a are represented by a collection B^a of triples (μ, N_μ, p_μ) where $\mu \in M(D)$, N_μ is a neighborhood of μ, and p_μ is a continuous function from N_μ to

Δ_{++} such that $p_\mu(\nu) \in \Pi(\nu)$ for all $\nu \in N_\mu$.

The continuous price function p_μ represents the prices that consumer a expects to see in equilibrium when μ is the reported economy. It also determines what consumer a perceives as his or her influence on prices. When a acts according to demand d, the anticipated equilibrium prices become $p_\mu(\mu_d)$, provided $\mu_d \in N_\mu$. If by changing reported demands consumer a affected a considerable change in the reported distribution μ (i.e. $\mu_d \notin N_\mu$), then the expected equilibrium price would no longer be given by the function p_μ. When there are many consumers, the report of any single individual will not significantly affect the reported economy.

Since the equilibrium price correspondence need not be lower hemi-continuous, there may not exist a continuous selection p_μ on any neighborhood of a given reported economy μ. If any agent believed that such an economy could arise, then small changes in his or her report would produce considerable changes in prices. Our definition of beliefs rules this out.

Representing beliefs by a collection of measures and price functions captures the uncertainty a consumer faces. Beliefs B^a may be thought of as representing the support of some prior distribution. However, we are deliberately vague and do not define any distributions over the price functions, since we employ weak assumptions about behavior that do not require such definitions. We will only assume that agents' actions are undominated relative to their beliefs. For any set of beliefs B and any $\delta > 0$, we define

$$B_\delta = \{(\mu, N_\mu, p_\mu) \in B \mid p_\mu(\mu) \geq \delta\, 1\},$$

where 1 is the unit vector in $\mathbb{R}^l$.

Given an agent a with endowment e_a, preferences R_a, and beliefs B^a, we say that $d \in D(e_a)$ *δ-dominates* $d' \in D(e_a)$ for a *relative to* B^a if, for every $(\mu, N_\mu, p_\mu) \in B^a_\delta$ such that $\mu_d \in N_\mu$ and $\mu_{d'} \in N_\mu$,

$$d(p_\mu(\mu_d))\ R_a\ d'(p_\mu(\mu_{d'})),$$

and there is some $(\mu, N_\mu, p_\mu) \in B^a_\delta$ such that $\mu_d \in N_\mu$ and $\mu_{d'} \in N_\mu$ and

$$d(p_\mu(\mu_d))\ P_a\ d'(p_\mu(\mu_{d'})).$$

If no $d \in D(e_a)$ δ-dominates $d' \in D(e_a)$ for a relative to B^a, then we say that $d' \in D(e_a)$ is δ-*undominated* for a relative to B^a.

As δ becomes smaller, the set of undominated demands increases. Similarly, as the collection B^a becomes larger, the set of undominated demands increases.

Finally, we define the sequences of economies for which our results apply. Let A^k be a sequence of finite sets and let λ^k be the normalized counting distribution on A^k. The sequence of economies $\mathcal{E}^k(A^k)$ is *purely competitive* (Hildenbrand (1974), pp. 136-137) if

(i) $\# A^k \longrightarrow \infty$,

(ii) the corresponding sequence of preference-endowment distributions $\bar{\theta}^k$ converges to the limit distribution $\bar{\theta} \in M(\mathcal{P} \times \mathbb{R}^l_+)$,

(iii) the sequence of mean endowments converges to the mean endowment of the limit, $\int_{\mathbb{R}^l_+} e_a\, d\theta^k \longrightarrow \int_{\mathbb{R}^l_+} e_a\, d\theta$ and $0 \ll \int_{\mathbb{R}^l_+} e_a\, d\bar{\theta} \ll \infty$.

A purely competitive sequence of economies represents the "same" economy (ii and iii), with an increasing degree of competition (i). Given a competitive sequence of economies, we will refer to $\bar{\theta}$ as the true limit economy.

3 An Example

In this section we present an example that illustrates the problems arising when individuals successfully manipulate prices.

Hurwicz (1979b), Thomson (1979), and Otani and Sicilian (1982, 1990) showed that if agents act according to any demand function, then the set of Nash equilibria is large and does not collapse to competitive behavior as the economy is replicated. Otani and Sicilian (1990) also show that if agents can only report smooth demand functions and the reported economies converge to a regular economy, then the Nash equilibrium allocations approximate competitive outcomes. This leaves unclear the role of the smoothness assumption in obtaining convergence to the competitive behavior, independent of the role of that assumption in facilitating the regularity of the limit economy. The constructive proofs showing noncompetitive behavior rely on demands with kinks in them.

Below, we present an example of a sequence of economies where all agents submit linear demands resulting in Nash equilibrium allocations that are distinct from competitive allocations, even in the limit. This example shows that smoothness of demands is not sufficient to obtain competitive behavior. And as our Theorems will show, nor is it necessary. Thus we argue that it is the continuity of equilibrium price functions at the limit that is critical to obtaining convergence. In essence, this continuity ensures that agents believe they have no influence on prices for large economies.

We construct a replica sequence of economies with two goods. All agents have the same preferences represented by the utility function $u(x_1, x_2) = x_1 x_2$. Agents differ only in their endowments, which are either of type $e^a = (0, 1)$ or $e^b = (1, 0)$. Economy k has k agents of each type. The price-taking demands of the agents are

$$\bar{d}^a(p_2) = (\frac{p_2}{2}, \frac{1}{2}) \text{ and } d^b(p_2) = (\frac{1}{2}, \frac{1}{2p_2}),$$

where the normalization of prices $p_1 = 1$, allows us to state everything in terms of $p_2 \in I\!R_{++}$. (Dividing by $1 + p_2$, the new prices are in Δ_+.) The unique competitive equilibrium price is $p_2 = 1$, with all agents consuming $1/2$ unit of each good.

The following constitutes a Nash equilibrium for each k of the game where agents submit smooth demand functions: $d_2^a(p_2) = \frac{4k}{3(2k-1)} - \frac{5}{9(2k-1)}p_2$ and $d_2^b(p_2) = \frac{2k+1}{3(2k-1)} - \frac{5}{9(2k-1)}p_2$, where demands are stated for the second good and demand for the first good is determined by budget balance ($d_1^a(p_2) = p_2(1 - d_2^a(p_2))$ and $d_1^b(p_2) = 1 - p_2 d_2^b(p_2)$). The resulting market-clearing price is $p_2 = 6/5$ with resulting allocations $x^a = (\frac{2}{5}, \frac{2}{3})$ and $x^b = (\frac{3}{5}, \frac{1}{3})$. Agents of type a are better off and agents of type b are worse off than in the competitive equilibrium.

To verify that this is an equilibrium, notice that from budget balance we can write $u^a = (p_2 - p_2 x_2^a)x_2^a$ and $u^b = (1 - p_2 x_2^b)x_2^b$. Given the demands of the other agents, the market-clearing equation as a function of agent a's equilibrium allocation x_2^a and the price p_2 is

$$k = x_2^a + (k-1)[\frac{4k}{3(2k-1)} - \frac{5}{9(2k-1)}p_2] + k[\frac{2k+1}{3(2k-1)} - \frac{5}{9(2k-1)}p_2],$$

which simplifies to

$$p_2 = \frac{9}{5}x_2^a.$$

We can rewrite u^a as $\frac{9}{5}((x_2^a)^2 - (x_2^a)^3)$. The optimal choice is then $x_2^a = \frac{2}{3}$. *Any* demand schedule for a which results in $x_2^a = \frac{2}{3}$, given the demands of the other agents, is then optimal. The suggested schedule $d_2^a(p_2) = \frac{4k}{3(2k-1)} - \frac{5}{9(2k-1)}p_2$ is thus a best response.

We can perform similar calculations for agents of type b. The market-clearing equation as a function of the equilibrium allocation x_2^b and the price p_2 is

$$k = x_2^b + k[\frac{4k}{3(2k-1)} - \frac{5}{9(2k-1)}p_2] + (k-1)[\frac{2k+1}{3(2k-1)} - \frac{5}{9(2k-1)}p_2]$$

which simplifies to

$$p_2 = \frac{9}{5}x_2^b + \frac{3}{5}.$$

We can rewrite u^b as $u^b = x_2^b - \frac{9}{5}(x_2^b)^3 - \frac{3}{5}(x_2^b)^2$. The optimal choice is then $x_2^b = \frac{1}{3}$. Any demand schedule for b which results in $x_2^b = \frac{2}{3}$, given the demands of the other agents, is then optimal. Again, the suggested schedule is a best response.

Notice that in this Nash equilibrium, the slope of the aggregate excess demand function faced by any agent in any economy is constant; agents perceive that they have the same influence on price all along the sequence. Individual (reported) demand functions, however, become increasingly steep and converge to constant demands. (The slope of the individual demands tends to zero, which, with the standard representation of prices in the vertical axis, corresponds to a steeper line.) Thus, the limit of the aggregate demand function is not equal to the aggregate of the limit demand functions.

This example points out that requiring smoothness of the reported demand curves is not sufficient to generate competitive behavior (and again, Theorems 1 and 2 will show that it is not necessary either). The Nash equilibrium reported demands in this example are in fact linear and the preferences are as well-behaved as possible. This example, however, relies critically upon the anticipation that other agents will act according to the reported demands. The reported demands of other agents provide any given agent with a choice of which allocation he or she desires. Without any uncertainty, the shape of the demand schedule that an agent submits is irrelevant to that agent, except for the requirement that it pass through the desired allocation. This allows us to choose carefully the slope of the demand of the agent through this optimal allocation, to support the equilibrium behavior of other agents.

The implication is that if we introduce some uncertainty, examples such as the one above might not survive, as the shape of the demand schedule is no longer irrelevant to an agent. Uncertainty alone, however, will not suffice, as each agent may still believe that he or she has a substantial influence on price, even in a very large economy. This points to the regularity assumptions as the key factor to obtaining a convergence result.

Gul and Postlewaite (1992) conjecture that if agents are "informationally small," the inefficiencies that arise due to asymmetric information will be small. Our assumption that agents believe they have little influence on aggregate prices (and the regularity assumptions in the articles referenced) captures a notion of informational smallness.

One of Gul and Postlewaite's objectives is to find and formalize the correct notion of informational smallness. They conclude from their results that an agent is informationally small if the incremental effect of the agent's private information on the demand for any good is a small proportion of the aggregate endowment of that good. The example above and the results of Otani and Sicilian (1990) show that the characterization of informational smallness advanced by Gul and Postlewaite will not suffice to produce approximately competitive behavior when agents can choose from an infinite set of demands. Gul and Postlewaite's proposed definition works for them because in their model individuals only have finitely many possible demands to report. In the example presented, as the economy is replicated, the true demand of any single individual, the individual's private information, will be a small fraction of the total endowment. Agents, however, can still have considerable influence on prices independent of the size of the economy: it is the actions of other agents that determine the influence that any individual has on prices.

4 Approximation Theorems

Theorem 1 shows that if consumers believe that they have, individually, a small influence on prices, then as the economy grows agents' undominated demands converge to their competitive demands and the distribution of reported demands converges to the true demand distribution.

Theorem 1 requires that consumers' beliefs satisfy the following characteristics. Let a be any consumer in economy $\mathcal{E}(A)$ reporting or acting according to a demand $d_a \in D(e_a)$. Then,

- consumer a knows the size of the economy when choosing an action, i. e.,

$$\forall(\mu, N_\mu, p_\mu) \in B^a,\ \mu = \lambda \circ r^{-1} \text{ for some report } r : A \to D,$$

  and

- consumer a has *complete* beliefs:

$$\forall p \gg 0,\ \exists(\mu, N_\mu, p_\mu) \in B^a \text{ such that } \mu_{d_a} \in N_\mu \text{ and } p = p_\mu(\mu_{d_a}).$$

Our results are stated in terms of a competitive sequence of economies. It is not necessary that consumers know the exact number of agents in the economy: it suffices that consumers realize that the economy is increasing in size.

The assumption of complete beliefs holds that consumer a, acting according to a demand $d_a(\cdot)$, has beliefs diverse enough to explain any realization of prices.

Given any positive number $\delta > 0$, Δ_δ is the set $\{p \in \Delta \mid p \geq \delta 1\}$. Let $D_\delta(e)$ denote the restriction of $D(e)$ to demand functions defined on Δ_δ and let $D_\delta = \{(d, e) : e \in I\!R^l_+,\ d \in D_\delta(e)\}$. For any $\mu \in M(D)$, μ_δ represents the distribution induced by μ on D_δ.[9]

Theorem 1 *Let $\mathcal{E}^k(A^k)$ be a purely competitive sequence of economies and $\bar{\mu}^k$ the corresponding "true" demand-endowment distributions. Suppose that each agent $a^k \in A^k$ knows the size of the economy, has complete beliefs B^{a^k}, and announces or acts according to a demand function d^k_a which is δ-undominated relative to B^{a^k}. If (a) and (b) are satisfied where:*

(a) $\forall \{a^k\}_k$, $a^k \in A^k$, $\exists \gamma > 0$, $K > 0$ such that $k > K$, $(\mu, N_\mu, p_\mu) \in B^{a^k}$, and $\mu', \mu'' \in B(\mu, 1/k)$ implies that $\mu', \mu'' \in N_\mu$ and $\|p_\mu(\mu') - p_\mu(\mu'')\| < \gamma\, \rho(\mu', \mu'')$;

(b) $\forall a^k \in A^k$, $\forall d, d' \in D(e_{a^k})$, and $\forall (\mu, N_\mu, p_\mu) \in B^{a^k}$, if $d(q) = d'(q)$, $q = p_\mu(\mu_d)$, and $\mu_d, \mu_{d'} \in N_\mu$, then $p_\mu(\mu_d) = p_\mu(\mu_{d'}) = q$;

then

1. $\mathcal{E}^k(a^k) \longrightarrow (R, e) \;\Rightarrow\; \sup_{p \in \Delta_\delta} \|d^k_{a^k}(p) - d^k_{a^k}(p)\| \longrightarrow 0$;

2. $\mu^k_\delta \Rightarrow \bar{\mu}_\delta$, where μ^k is the sequence of reported economies, and $\bar{\mu}$ is the limit of the "true" demand-endowment distributions ($\bar{\mu} = \lim_{k \to \infty} \bar{\mu}^k$). [10]

[9] Let $g : D \longmapsto D_\delta$, $g(d, e) = (d', e)$, where d' is the restriction of d to Δ_δ. Then, for any set $E \subset D_\delta$, $\mu_\delta(E) = \mu \circ g^{-1}(E)$.

[10] If the preference-endowment distributions θ^k converge to θ (as guaranteed by the purely competitive hypothesis), then the "true" demand-endowment distributions $\bar{\mu}^k$ converge to $\bar{\mu}$, the demand-endowment distribution generated by θ. This can be seen as an application of Lemma 2 below.

Condition (a) is a key assumption about consumers' beliefs: consumers in large economies believe that individually, they have a small influence on prices. Announcing a demand d transforms any reported economy μ into μ_d. As the size of the economy increases, the influence of a single consumer on the reported economy vanishes: $\rho(\mu, \mu_d) < \frac{1}{\# A^{\kappa}}$ (which follows from the definition of the Prohorov metric). According to (a) consumers believe that their small influence on the reported economy will not result in a significant change in prices: as the size of the economy increases, $\mu_d \in X_\mu$, and therefore reporting d may be evaluated using the equilibrium selection $p_\mu(\mu_d)$. The Lipschitz condition in (a) implies that small changes in μ will not effect arbitrarily large changes on the equilibrium price according to consumers' beliefs.

Hypothesis (b) states that irrelevant changes in a reported demand do not change the expected equilibrium prices: if a consumer believes that reporting the demand d results in a price q, changing the demand for prices different from q should not affect the consumer's expected prices (provided the global report μ_d is not significantly altered). Notice, however, that consumers may still believe that the same reported economy μ may generate many different prices: there may exist many triples $(\mu, X_\mu, p_\mu) \in B^a$ and $(\mu, X'_\mu, p'_\mu) \in B^a$ with $p_\mu \neq p'_\mu$.

Theorem 1 has two conclusions. First, consider any sequence of consumers with characteristics converging to a given preference-endowment pair. Any corresponding sequence of undominated demands will converge uniformly to the true price-taking demand. Second, the reported economy approximates the true economy. In large economies, consumers' reports are close to truthful, and aggregating those reports does not result in a significant error. We may conclude that any sequence of equilibria of the reported economies will approximate, in a subsequence, a competitive equilibrium of the true limit economy.

In selecting their announced demands, consumers take into account only the effect of their announcement on prices, given their beliefs about the reported economy. Theorem 1 requires that consumers form complete beliefs—i. e., consumers' beliefs contain a varied collection of reported economies, a collection that may generate, roughly, any price. Suppose, for istance, that instead consumer a is certain about the behavior of the rest of the economy: B^a is a singleton. In selecting a demand, a only cares about the "expected" prices implicit in B^a. Thus, a is indifferent when choosing among demands that only differ in "unexpected" prices. Therefore, a's report need not converge to his or her price-taking demand (at all prices) as the economy becomes larger.

Without the completeness assumption, undominated demands will converge to the true demand for sequences of expected prices, but they may not do so for other prices. In Theorem 2, we replace the completeness assumption with the condition that beliefs be "correct", i. e., consumers' expected prices *include* the market-clearing prices of the reported economy. We prove in this case that the equilibrium allocations of the reported economies converge in distribution to some equilibrium allocation of the true economy.

Theorem 2 *Let $\mathcal{E}^k(A^k)$ be a purely competitive sequence of economies and $\bar{\mu}^k$ the corresponding "true" demand-endowment distributions. Suppose that each agent $a^k \in A^k$ knows the size of the economy, and announces or acts according to a demand function d_a^k, which is δ-undominated relative to a's beliefs B^{a^k}. Let μ^k be the sequence of reported economies. Suppose $\Pi(\mu^k) \subset \Delta_\delta$, and hypotheses (a) and (b) in Theorem 1 hold. In addition, for all reported economies μ^k and $p^k \in \Pi(\mu^k)$, every consumer $a \in A^k$ has beliefs $(\mu^k, X_{\mu^k}, p_{\mu^k}) \in B_a^k$ such that $p^k = p_{\mu^k}(\mu^k)$. Then, for any sequence of market-clearing prices of the reported economies $p^k \in \Pi(\mu^k)$,*

1. *$\mathcal{E}^k(a^k) \longrightarrow (R, e) \;\Rightarrow\; \|d_{a^k}^k(p^k) - \bar{d}_{a^k}^k(p^k)\| \longrightarrow 0$;*
2. *for any $a^k \in A^k$ let $c^k(a^k) = d_a^k(p^k)$ and for any $a \in A$ let $c(\bar{d}_a, e_a) = \bar{d}_a(p)$. In a subsequence, $p^k \longrightarrow p$ with $p \in \Pi(\bar{\mu})$, $\bar{\mu} = \lim_{k \longrightarrow \infty} \bar{\mu}^k$, and $\lambda^k \circ (c^k)^{-1} \Rightarrow \bar{\mu} \circ c^{-1}$.*

Theorem 2 has two conclusions. Conclusion *1* states that the reported demands approximate the competitive demands at the market-clearing prices of the reported economy as the size of the economy increases. Since beliefs need not be complete, the behavior of the reported demands at other prices cannot be determined. Conclusion *2* states that (in a subsequence) the equilibrium allocations of the reported economies converge in distribution to an equilibrium allocation of the true limit economy. As the size of the economy increases, the equilibrium allocations $c^k(\cdot)$ of the reported economies are defined on different spaces of agents. Conclusion *2* compares the distribution in the commodity space of the equilibrium allocations.[11] Although the equilibrium allocations of the reported economies approximate equilibrium allocations of the true limit economy, it is still possible that the reported economies do not converge at all, or that they do not approximate the true economy. This is so because the reported demands need not converge to the competitive demands at out-of-equilibrium prices.

Before providing the proof of Theorem 1, we give a brief outline. We use two main lemmas to prove Theorem 1. Lemma 1 states that $d^k \rightarrow \bar{d}$ uniformly. The intuition behind Lemma 1 rests on the assumption that individually, consumers believe they have a small influence on prices in sufficiently large economies. Given the strict convexity of preferences, in order for a reported demand to be optimal in a small neighborhood around a price, the demand must be near the price-taking demand. The proof of the Lemma identifies a demand function which would dominate the announced demand otherwise. The rest of the proof of Lemma 1 must deal with handling all expected prices simultaneously, and must account for the fact that consumers' characteristics may change as the economies change. Lemma 2 then links the individual behavior to the aggregate. It shows that if reported demands converge uniformly over agents, i. e., conclusion *1* holds, then the reported economy converges weakly to the true limit economy. To apply Lemma 2 directly requires that reported demands be linked to true preferences and endowments. It is possible, however, that two agents with the same preferences and endowments hold different beliefs, and therefore, report different

[11] See Hildenbrand (1974) pp. 153-154 for further discussion of the use of an allocation's distribution as the relevant information in economies with a large number of agents.

undominated demands. (This could happen even if beliefs did not differ between agents.) By Lemma 1, agents with the same characteristics report nearly the same demands in large economies. With this observation, Lemma 2 is applied and conclusion *2* is established.

It is worth pointing out that our assumptions on beliefs are only used to obtain conclusion *1*. Conclusion *2* follows from *1* and the fact that the sequence of economies is purely competitive. Thus, other environments where *1* holds will also have *2* as a conclusion.

Proof of Theorem 1 We begin with a Lemma proving that $d^k \to \bar{d}$ uniformly. Since $\bar{d}^k \to \bar{d}$ uniformly as well, conclusion *1* follows.

Lemma 1 *Let $\mathcal{E}^k(a^k) = (R^k, e^k)$ for all k and suppose $(R^k, e^k) \longrightarrow (R, e)$. Let d^k be any δ-undominated demand relative to B^{a^k} and $\bar{d}$ be the competitive demand for an agent with characteristics (R, e). Then $d^k \longrightarrow \bar{d}$ uniformly on Δ_δ.*

Proof of Lemma 1 First, for each fixed k we define a family of demand functions $\tilde{d}_n^k$ indexed by the integer $n > 0$. We show that if no $\tilde{d}_n^k$ (for any n) δ-dominates d^k with respect to B^{a^k}, then $d^k(\cdot)$ converges to $\bar{d}(\cdot)$ uniformly on Δ_δ.

Given n, k and $q \in \Delta_{++}$, let

$$z(q) = \begin{cases} 0 & \text{if } \|d^k(q) - \bar{d}^k(q)\| \leq 1/n \\ \frac{(\|d^k(q) - \bar{d}^k(q)\| - 1/n)}{1/n} & \text{if } 1/n < \|d^k(q) - \bar{d}^k(q)\| < 2/n \\ 1 & \text{if } \|d^k(q) - \bar{d}^k(q)\| > 2/n. \end{cases}$$

Define,

$$\tilde{d}_n^k(q) = z(q)\, \bar{d}^k(q) + (1 - z(q))\, d^k(q).$$

For any k and n, the demand $\tilde{d}_n^k$ is in $D(e^k)$. Notice also that for all q, we have that

$$\|\tilde{d}_n^k(q) - \bar{d}^k(q)\| \leq 1/n, \text{ and} \tag{1}$$

$$\tilde{d}_n^k(q) \neq d^k(q) \Rightarrow \|d^k(q) - \bar{d}^k(q)\| > 1/n. \tag{2}$$

Second, we prove that there is a positive integer N, such that for any $n^* > N$ and for any sequence $\{(\nu^k, N_{\nu^k}, p_{\nu^k})\}_k$, with $(\nu^k, N_{\nu^k}, p_{\nu^k}) \in B_\delta^{q^k}$ $\forall k$, there exists K so that $k > K$ implies

$$\tilde{d}^k_{n^*}(p_{\nu^k}(\nu^k_{\tilde{d}^k_{n^*}}))\ R^k\ d^k(p_{\nu^k}(\nu^k_{d^k})). \tag{3}$$

Note that for all k sufficiently large, (a) implies that $\nu^k_{\tilde{d}^k_{n^*}}, \nu^k_{d^k} \in N_{\nu^k}$.

Suppose (3) does not hold. Then, for any fixed n there exists a sequence $\{(\nu^k, N_{\nu^k}, p_{\nu^k})\}_k$ with $(\nu^k, N_{\nu^k}, p_{\nu^k}) \in B_\delta^{q^k}$ $\forall k$, such that

$$d^k(p_{\nu^k}(\nu^k_{d^k}))\ P^k\ \tilde{d}^k_n(p_{\nu^k}(\nu^k_{\tilde{d}^k_n})) \tag{4}$$

for all k sufficiently enough. Given n, select k_n so that $k_n > k_{n-1}$, and (4) holds. In the resulting subsequence $\{(\nu^{k_n}, N_{\nu^{k_n}}, p_{\nu^{k_n}})\}_{k_n}$, we observe that

$$d^{k_n}(p_{\nu^{k_n}}(\nu^{k_n}_{d^{k_n}}))\ P^{k_n}\ \tilde{d}^{k_n}_n(p_{\nu^{k_n}}(\nu^{k_n}_{\tilde{d}^{k_n}_n}))\ \forall k_n. \tag{5}$$

Taking a further subsequence, we know that $p_{\nu^{k_n}}(\nu^{k_n}_{d^{k_n}}) \longrightarrow p$, $p \in \Delta_\delta$. Hypothesis (a) implies that $p_{\nu^{k_n}}(\nu^{k_n}_{\tilde{d}^{k_n}_n}) \longrightarrow p$. For all k_n large, $p_{\nu^{k_n}}(\nu^{k_n}_{d^{k_n}})$ and $p_{\nu^{k_n}}(\nu^{k_n}_{\tilde{d}^{k_n}_n})$ must be strictly positive because $p \gg 0$. Thus, there is a compact set containing the consumption bundles $d^{k_n}(p_{\nu^{k_n}}(\nu^{k_n}_{d^{k_n}}))$ and $\tilde{d}^{k_n}_n(p_{\nu^{k_n}}(\nu^{k_n}_{\tilde{d}^{k_n}_n}))$ for all sufficiently large k_n. In a subsequence, $d^{k_n}(p_{\nu^{k_n}}(\nu^{k_n}_{d^{k_n}}))$ converges to a consumption bundle x. From (1), $\tilde{d}^{k_n}_n(p_{\nu^{k_n}}(\nu^{k_n}_{\tilde{d}^{k_n}_n})) \longrightarrow \bar{d}(p)$. Then, (5) yields $x\ \ R\ \ \bar{d}(p)$. Since $d^{k_n}(p_{\nu^{k_n}}(\nu^{k_n}_{d^{k_n}}))$ is a demand bundle, $p_{\nu^{k_n}}(\nu^{k_n}_{d^{k_n}}) \cdot d^{k_n}(q^{k_n}) \leq p_{\nu^{k_n}}(\nu^{k_n}_{d^{k_n}}) \cdot e^{k_n}$ for all k_n. Therefore, $p \cdot x \leq p \cdot e$. Since preferences are strictly convex and $x\ R\ \bar{d}(p)$, it must be that $x = \bar{d}(p)$. We have established that

$$d^{k_n}(p_{\nu^{k_n}}(\nu^{k_n}_{d^{k_n}})) \longrightarrow \bar{d}(p). \tag{6}$$

Since $(R^k, e^k) \longrightarrow (R, e)$, $\bar{d}^k \longrightarrow d$ uniformly on compact sets. Hence, $\|d^k(p_{\nu^k}(\nu^k_{d^k})) - \bar{d}^k(p_{\nu^k}(\nu^k_{d^k}))\| < 1/n$ for all large k. Then, the definition of $\tilde{d}^k_n$ implies that $\tilde{d}^k_n(p_{\nu^k}(\nu^k_{d^k})) = d^k(p_{\nu^k}(\nu^k_{d^k}))$ for large k. This contradicts hypothesis (b) and (4). We conclude that there is an N so that $n^* > N$ satisfies (3).

Third, we prove that for each $n^* > N$ there exists K such that $k > K$ implies that $\forall(\nu^k, N_{\nu^k}, p_{\nu^k}) \in B_\delta^{a^k}$

$$d^k(p_{\nu^k}(\nu^k_{d^k}))\ I^k\ \bar{d}^k_{n^*}(p_{\nu^k}(\nu^k_{\bar{d}^k_{n^*}})), \tag{7}$$

where I^k denotes indifference. Since d^k is δ-undominated with respect to $B_\delta^{a^k}$, it follows that for each k either

$$d^k(p_{\nu^k}(\nu^k_{d^k}))\ I^k\ \bar{d}^k_{n^*}(p_{\nu^k}(\nu^k_{\bar{d}^k_{n^*}})),$$

for all $(\nu^k, N_{\nu^k}, p_{\nu^k}) \in B_\delta^{a^k}$, or there exists $(\nu^k, N_{\nu^k}, p_{\nu^k}) \in B_\delta^{a^k}$ such that

$$d^k(p_{\nu^k}(\nu^k_{d^k}))\ P^k\ \bar{d}^k_{n^*}(p_{\nu^k}(\nu^k_{\bar{d}^k_{n^*}})).$$

If the last statement were true for infinitely many k, we could construct a subsequence contradicting (3). This establishes (7).

Finally, let $p^k \longrightarrow p$, $p^k \in \Delta_\delta$ for all k. Since beliefs are complete, there exists $\nu^k \in B_\delta^{a^k}$ with $p^k = p_{\nu^k}(\nu^k_{d^k})$. The same argument leading to (6) shows, using (7) that $d^k(p^k) \longrightarrow \bar{d}(p)$. Given the continuity of $\bar{d}$ and the compactness of Δ_δ, this implies that $d^k(\cdot)$ converges to $\bar{d}(\cdot)$ uniformly on Δ_δ.

Q.E.D.

Lemma 2 *Let $\bar\theta^k \in M(\mathcal{P} \times \mathbb{R}^l_+)$ for $k = 1, 2, \ldots,$ and $\bar\theta^k \Rightarrow \bar\theta$, $\bar\theta \in M(\mathcal{P} \times \mathbb{R}^l_+)$. Denote by E^k and E the supports of $\bar\theta^k$ and $\bar\theta$ respectively. Let $h^k : E^k \longmapsto D_\delta$, $k = 1, 2, \ldots,$ and $h : \mathcal{P} \times \mathbb{R}^l_+ \longmapsto D_\delta$ be measurable. Suppose*

$$(R^k, e^k) \in E^k,\ (R^k, e^k) \to (R, e),\ (R, e) \in E,\ \Rightarrow\ h^k(R^k, e^k) \to h(R, e).$$

Then, $\bar\theta^k \circ (h^k)^{-1} \Rightarrow \bar\theta \circ h^{-1}$.

Proof We must show that for any continuous bounded function g,

$$\int_Y g(y)\, d(\bar\theta^k \circ (h^k)^{-1}) \longrightarrow \int_Y g(y)\, d(\bar\theta \circ h^{-1}).$$

or equivalently, that

$$\int_{\mathcal{P}\times \mathbb{R}^l_+} g(h^k(R,e))\, d\bar{\theta}^k \longrightarrow \int_{\mathcal{P}\times \mathbb{R}^l_+} g(h(R,e))\, d\bar{\theta}. \tag{8}$$

Extend h^k outside E^k by setting $h^k(R,e) = h(R,e)$, $\forall(R,e)$ not in E^k. The value of the integrals above does not change. Then, for any sequence $(R^k,e^k) \in \mathcal{P} \times \mathbb{R}^l_+$, with $(R^k,e^k) \to (R,e)$, $h^k(R^k,e^k) \to h(R,e)$. The conclusion that $\bar{\theta}^k \circ (h^k)^{-1} \Rightarrow \bar{\theta} \circ h^{-1}$ then follows from Billingsley (1968) Theorem 5.5.

Q.E.D.

We now complete the proof of conclusion 2 in Theorem 1. Define $f^k : A^k \times E^k \mapsto D_S$ as follows: if $(R^k,e^k) = \mathcal{E}(a^k)$, then $f^k(a^k,R^k,e^k) = r^k(a^k)$, otherwise, $f^k(a^k,R^k,e^k) = (\bar{d}^k,e^k)$. Let $f(R,e) = (\bar{d}(\cdot),e)$, where $\bar{d}^k$ and $\bar{d}$ are the competitive demands restricted to Δ_S of any agent with characteristics (R^k,e^k) and (R,e), respectively.

From Lemma 1, we have that $f^k(a^k,R^k,e^k) \longrightarrow f(R,e)$ provided $(R^k,e^k) \longrightarrow (R,e)$.

Let $\theta^k = \lambda^k \circ g^{k^{-1}}$, where $g^k : A^k \mapsto A^k \times \mathcal{P} \times \mathbb{R}^l_+$ is defined by $g^k(a^k) = (a^k, \mathcal{E}^k(a))$. Thus, the marginal distribution of θ^k on $\mathcal{P} \times \mathbb{R}^l_+$ is $\bar{\theta}^k$, the preference-endowment distribution corresponding to $\mathcal{E}^k$.

We now prove that $\mu^k_S \Rightarrow \bar{\mu}_S$. Notice that $\theta^k \circ (f^k)^{-1} = (\lambda^k \circ (g^k)^{-1}) \circ (f^k)^{-1} = \lambda^k \circ (f^k \circ g^k)^{-1}$. Since $(f^k \circ g^k)$ is the map r^k that assigns to each agent $a \in A^k$ the reported demand-endowment pair $(d^k_a, e^k_a) \in D_S$, $\mu^k_S = \theta^k \circ (f^k)^{-1}$. It is straightforward from the definitions of $\bar{\theta}$ and f that $\bar{\mu}_S = \bar{\theta} \circ f^{-1}$.

Thus, we must verify that for any continuous bounded function h on D_S

$$\int h(d)\, d(\theta^k \circ (f^k)^{-1}) \longrightarrow \int h(d)\, d(\bar{\theta} \circ f^{-1}),$$

or equivalently,

$$\int h(f^k(a,R,e))\, d\theta^k \longrightarrow \int h(f(R,e))\, d\bar{\theta}.$$

Notice that

$$\int \bar{h}^k(R,e)\,d\theta^k \geq \int h(f^k(a,R,e))\,d\theta^k \geq \int \underline{h}^k(R,e)\,d\theta^k. \tag{9}$$

where $\bar{h}^k(R,e) = \sup_{a\in A^k} h(f^k(a,R,e))$ and $\underline{h}^k(R,e) = \inf_{a\in A^k} h(f^k(a,R,e))$.

Let $(R^k,e^k) \longrightarrow (R,e)$. Since $\bar{h}^k(R^k,e^k) = h(f^k(a^k,R^k,e^k))$ for some consumer a^k (recall that A^k is finite), Lemma 1 implies that $\bar{h}^k((R^k,e^k)) \longrightarrow h(f(R,e))$. By a similar argument, $\underline{h}^k((R^k,e^k)) \longrightarrow h(f(R,e))$. Since the marginal distribution of θ^k on $\mathcal{P}\times I\!R^l_+$ is $\bar{\theta}^k$, we observe that

$$\int \bar{h}^k(R,e)\,d\theta^k = \int \bar{h}^k(R,e)\,d\bar{\theta}^k \text{ and } \int \underline{h}^k(R,e)\,d\theta^k = \int \underline{h}^k(R,e)\,d\bar{\theta}^k.$$

Lemma 2 then implies that

$$\int \bar{h}^k(R,e)\,d\bar{\theta}^k \longrightarrow \int h(f(R,e))\,d\bar{\theta} \text{ and } \int \underline{h}^k(R,e)\,d\bar{\theta}^k \longrightarrow \int h(f(R,e))\,d\bar{\theta}.$$

Taking limits on (9) we obtain the desired result, thus establishing that $\mu^k_S \Rightarrow \bar{\mu}_S$.

Q.E.D.

The proof of Theorem 2 follows the proof of Theorem 1 closely.

Proof of Theorem 2 Conclusion *1* follows from a slight change in Lemma 1: completeness of beliefs is used in the last paragraph of the proof of that Lemma to ensure that there are beliefs that support any sequence of prices p^k. Conclusion *1* in Theorem 2 only applies to sequences p^k for which such beliefs exist.

To prove conclusion *2* we use Lemma 2 (modified so that h^k and h map into allocations rather than functions). Since $p^k \in \Delta_S$, there exists a convergent subsequence $p^k \longrightarrow p$. (We abuse notation by not distinguishing the subsequence.)

We first prove that the consumption bundles converge in distribution to the competitive ones and then that $p \in \Pi(\bar{\mu})$.

Define $f^k : A^k \times E^k \longmapsto I\!R^l$ as follows: if $(R^k,e^k) = \mathcal{E}(a^k)$, then $f^k(a^k,R^k,e^k) = c^k(a^k)$, otherwise, $f^k(a^k,R^k,e^k) = \bar{d}^k(p^k)$. Let $f(R,e) = \bar{d}(p)$ where $\bar{d}^k$ and $\bar{d}$ are the competitive demands of any agent with characteristics (R^k, e^k) and (R, e) respectively. It follows from l that $f^k(a^k, R^k, e^k) \longrightarrow f(R, e)$ provided $(R^k, e^k) \longrightarrow (R, e)$.

Let $\theta^k = \lambda^k \circ g^{k^{-1}}$, where $g^k : A^k \longmapsto A^k \times \mathcal{P} \times I\!R^l_+$ is defined by $g^k(a^k) = (a^k, \mathcal{E}^k(a))$. Thus, the marginal distribution of θ^k on $\mathcal{P} \times I\!R^l_+$ is $\bar{\theta}^k$, the preference-endowment distribution corresponding to $\mathcal{E}^k$.

We now prove that $\lambda^k \circ (c^k)^{-1} \Rightarrow \bar{\mu} \circ c^{-1}$. Notice that $\theta^k \circ (f^k)^{-1} = (\lambda^k \circ (g^k)^{-1}) \circ (f^k)^{-1} = \lambda^k \circ (f^k \circ g^k)^{-1}$. Since $(f^k \circ g^k)$ is the map c^k that assigns to each agent $a \in A^k$ the equilibrium consumption $d^k_a(p^k)$, $\lambda^k \circ (c^k)^{-1} = \theta^k \circ (f^k)^{-1}$. It is straightforward from the definitions of $\bar{\theta}$ and f that $\bar{\mu} \circ c^{-1} = \bar{\theta} \circ f^{-1}$. The same steps employed to prove conclusion 2 in Theorem 1 establish the desired result.

Finally, we show that $p \in \Pi(\bar{\mu})$. Since $p^k \in \Pi(\mu^k)$,

$$E[c^k] = \int_{A^k} c^k(a)\, d\lambda^k = \int_{A^k} e^k_a\, d\lambda^k.$$

Since $c^k(\cdot)$ converges in distribution to $c(\cdot)$ and the sequence of economies is purely competitive,

$$\lim_{k \to \infty} E[c^k] = E[c] = \int_{\boldsymbol{R}^l_+} e_a\, d\bar{\mu}.$$

This proves that p is a market-clearing price for $\bar{\mu}$.

Q.E.D.

5 Concluding Remarks

1. The assumption that the per capita endowments converge, condition (iii) in the definition of a purely competitive sequence of economies, is not used in Theorem 1. This assumption is used in Theorem 2 to prove that the limit of equilibrium prices of the reported economies is a competitive equilibrium price of the true limit economy, since in Theorem 2 the reported economies need not converge.

2. Our results suggest that as long as consumers believe they have a small influence on prices, even if they do not behave as price takers, the resulting equilibrium price and allocations will be nearly competitive. Although we set up the Theorems in terms of competitive sequences of economies, the basic intuition would still hold in a single economy provided agents believed that their influence on prices was small. In order to assess the validity of such an assumption on beliefs, however, one must check that it is consistent with the environment in question. For instance, if the per capita endowments of the economies in Theorem 1 did not converge, a single agent could receive a constant fraction of the total endowment in all economies. Although Theorem 1 would still hold, the agent receiving the large endowment, if aware of the situation, may be able to influence prices. Thus, without the convergence of per capita endowments, our assumptions on beliefs may be unrealistic. Notice that the per capita endowments do converge if, for instance, an economy is replicated.

3. Our results show that the equilibrium allocations of the reported economies converge to a Walrasian allocation of the true limit economy. Since the Walrasian correspondence need not be lower hemi-continuous at the limit economy, the equilibrium allocations of the reported and true economies might not be close to each other along the sequence if the true limit economy happens to be one for which the Walrasian correspondence is not lower hemi continuous. Of course, the two equilibrium allocations will be close to each other when the Walrasian correspondence is hemi-continuous at the limit economy.

4. We believe that assumptions (a) and (b) in both Theorems—i. e., individuals believe that they have little influence on prices—could be replaced by assuming that for all consumers in the limit economy the belief-set B^a is a compact set of regular economies, and that the preference-endowment-belief distribution converges. In this set-up, it is natural to require that consumers report only smooth demands to keep the reported economies within the class of smooth economies. Since regular economies constitute an open dense set in the space of smooth economies, it is not unreasonable to assume that belief-sets are formed of regular economies.[12] To proceed along these lines, the proof of Lemma 1 would have to be modified. As it stands now, the proof constructs a dominating demand, $\tilde{d}^k$, that is not smooth.

5. In the model we considered, markets always clear. Our set-up might be useful to analyze some non-balanced mechanisms: consumers in a given economy are asked to report their demands. A potentially different price is assigned to each consumer by computing a market-clearing price for the economy without that consumer. Agents are allocated the consumption bundle that results from their reported demands and their assigned price.[13] It is clearly a dominant strategy to report one's price-taking demand, since the price that each consumer faces is unaffected by his or her announced demand. In situations where the individual prices are close to a Walrasian price, the resulting allocations would be close to competitive. Proving that there are individual prices close to a Walrasian one, and that the aggregate imbalance is not too large, might be achieved by methods similar to the ones we have used here.

6. Although we assume that individual consumers form beliefs about the actions of all other consumers, it need only be the case that agents form beliefs about their own influence on prices. We have formally set up the assumption otherwise for simplicity.

7. When consumers' preferences are common knowledge (or at least satisfy the non-exclusive information requirement[14] that any given agent's information is redundant given the pooled information of the remaining agents), then there are mechanisms for which the set of Nash equilibria coincide exactly with the set of Walrasian equilibria, even for small economies.[15] However, it seems reasonable to expect that individual preferences will be, at

[12] See for instance, Appendix 2.3 in Hildenbrand (1974).

[13] We have heard this mechanism suggested by several people, but have not been able to find any reference for it. Thus we think of it as a "folk" mechanism, but are quite happy to stand corrected.

[14] The non-exclusive information requirement was used, for instance, in Blume and Easley (1983, 1990), Postlewaite and Schmeidler (1986), and Palfrey and Srivastava (1987).

[15] For exact Nash implementation of the Walrasian correspondence, see for example Hurwicz (1979a) and

least partly, private information. Thus in order to justify the price-taking assumption, one must allow for the possibility of incomplete information, as we have done here.

Palfrey and Srivastava (1986) and Mas-Colell and Vives (1993) have examined implementation with incomplete information in large economies. Both use mechanisms drawing on the law of large numbers. Palfrey and Srivastava (1986) study environments where the replication is such that any agent's information is almost surely redundant in the limit, and can thus implement rules as in the case with non-exclusive information. Mas-Colell and Vives (1993) study an implementation problem in economies with a continuum of agents and relate the incentive compatibility results of continuum economies with those of approximating sequences of economies. Given a fixed and known distribution over agents' types, they prove an upper hemi-continuity property of Bayesian equilibria for continuous mechanisms. Using this result they show that if a continuous mechanism implements uniquely a Walrasian allocation in the continuum economy, then the Bayesian equilibria of the approximating finite economies yield an allocation that is almost competitive with probability close to one. They demonstrate such a continuous mechanism.

Our approach differs from both of these in that we are not using distributional assumptions over the types of agents, and so our "mechanism" (the Walrasian one) is not tied to any particular distribution or any replication procedure. Instead we identify restrictions on individual beliefs for which the Walrasian mechanism works.

Schmeidler (1980). The survey on complete information implementation by Moore (1992) provides references for some of the more recent work in this area. When information is incomplete, the incentive compatibility requirements preclude such results generally (see the examples in Palfrey and Srivastava (1987)).

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