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A CHARACTERIZATION OF EFFICIENT, BAYESIAN INCENTIVE COMPATIBLE MECHANISMS

by

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Abstract

A mechanism that is both efficient and incentive compatible in the Bayesian-Nash sense is shown to be payoff-equivalent to a Groves mechanism at the point in time when each agent has just acquired his private information. This equivalence result simplifies the question of whether or not an efficient, Bayesian incentive compatible mechanism can satisfy other desired objectives, for the search for an appropriate mechanism can be restricted to the family of Groves mechanisms. The method is used to extend a result of Myerson and Satterthwaite on the inefficiency of bilateral bargaining to a multilateral setting.

1. Introduction

Achieving efficiency when economic agents strategically pursue their individual self-interest is a fundamental economic problem. This paper concerns the case in which agents have private information upon which efficient choice depends. If efficiency is to be achieved, then rules (or a mechanism) must be devised so that it is in each agent's self-interest to reveal what he knows. Two notions of strategic behavior are considered in this paper: Bayesian-Nash and dominant strategy equilibrium. Two large literatures have developed concerning the same economic problems but distinguished by which of these two solution concepts is assumed. The main result of this paper connects these two literatures. Through this connection, a number of questions that concern efficiency and incentive compatible revelation in the Bayesian-Nash sense are simplified by being reduced to questions about Groves mechanisms, which are a familiar family of mechanisms in which efficient choice is sustained as a dominant strategy equilibrium. Because the Groves mechanisms are so well understood, this reduction both simplifies proofs and provides intuition. The value of this method is demonstrated in this paper in the context of mechanisms for trading when traders privately know their own preferences.

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1.1. The social choice problem. The model considered here is basic in the mechanism design literature. An element of a set A (the set of social alternatives) can be selected for n economic agents. Each agent i receives utility directly from the choice of a social alternative according to his valuation $v_i(a,t_i)$, where a is in A and t_i (agent i's type) is a parameter that agent i knows privately. Agent i's utility function $u_i(\bullet)$ is quasilinear in the social alternative and money, i.e.,

$$u_i(x_i,a,t_i) = v_i(a,t_i) - x_i,$$

where x_i is monetary transfer from agent i. Utility is thus transferable and each agent is risk neutral. The utility of each agent is normalized so that his utility is zero if no social alternative is chosen.

Following Harsanyi (1967-68), the strategic use of private information is modeled as a noncooperative game of incomplete information. The parameter t_i is an element of a probability space T_i with measure μ_i, where μ_i models the beliefs of every other agent about the type of agent i. All of the above is common knowledge among the agents. An independent, private value model is thus studied in this paper.

Let $t \equiv (t_1, \ldots, t_n)$ denote a vector of types. A revelation mechanism specifies a social alternative a and a vector of monetary transfers (x_i) as a function of the reported vector of types of the agents. The mechanism is incentive compatible (IC) if honest reporting of types defines a Bayesian-Nash equilibrium. This paper characterizes IC revelation mechanisms such that a social alternative that maximizes the sum of the valuations is chosen when the agents honestly report their types. Specifically, it is assumed that a function a(t) exists such that $a(t) \in \text{argmax} \sum_i v_i(a,t_i)$ for each t, and a mechanism is efficient (EF) if a(t) is selected when t is the vector of types.[3] A revelation mechanism is acceptable if it is both IC and EF.

Time plays an important role in this paper. The play of a noncooperative game of incomplete information can be divided into three temporal stages according to the state of knowledge among the agents: at the ex ante stage, each agent knows only the distribution of the types of all agents; at the

[3] Choosing an alternative to maximize the sum of the valuations given t is a necessary and sufficient condition for Pareto optimality if the budget is required to balance ex post (i.e., the transfers must sum to zero for each t). Because a weaker notion of budget-balancing is discussed in this paper, and because much of the discussion concerns IC and EF independently of other constraints on the transfers, "efficient" is used a bit loosely here, though consistent with the literature.

interim stage, each agent has learned his own type but still knows only the distribution of the types of his opponents; at the ex post stage, the types of all agents are common knowledge. In the spirit of Holmström and Myerson's (1983) classification of efficiency according to these stages, constraints can also be classified in this way: an ex ante constraint is stated in terms of the distributions of the agents' types; an interim constraint is stated in terms of the type of at most one agent and the distributions of types of his opponents; an ex post constraint is stated in terms of a vector of types of all of the agents. In this terminology, IC is an interim constraint and EF is an ex post constraint. Also, given an acceptable mechanism, define agent i's interim expected utility and his interim expected transfer as his expected utility and his expected transfer at the interim stage as functions of his type t_i.

1.2. The results. The Groves mechanisms are a family of acceptable mechanisms in which honest revelation of one's type is a dominant strategy,[4] which is a more demanding notion of incentive compatibility than IC. The equivalence result states that from an interim perspective any acceptable mechanism is payoff-equivalent to some Groves mechanism. Formally, the interim expected utility and transfer of each agent in any acceptable mechanism is totally determined by the well-known formula for the transfers in the family of Groves mechanisms. Through the Revelation Principle[5], the interim expected utility and transfer of an agent as functions of his type in any efficient Bayesian-Nash equilibrium of any mechanism is thus the same as in a Groves mechanism.

This equivalence result is an analogue for Bayesian incentive compatibility of a well-known result of Green and Laffont (1977) for dominant strategy incentive compatibility. Green and Laffont proved that an EF revelation mechanism in which honest revelation is a dominant strategy for each agent is necessarily a Groves mechanism. It is surprising that this strong characterization result extends to incentive compatibility in the Bayesian sense because Bayesian incentive compatibility seems so much weaker than dominant strategy incentive compatibility. Even though a dominant strategy equilibrium is attractive as a solution concept because it is behaviorally far more plausible than a Bayesian-Nash equilibrium, theorists moved from the dominant solution concept to the Bayesian-

[4] See, for instance, Groves (1973), or Green and Laffont (1977). While this is the most common name for this family of mechanisms, they originated also in the work of Clarke (1971) and Vickrey (1961). The reference to Vickrey is especially pertinent, given the discussion of trading mechanisms later in this paper.

[5] See Fudenberg and Tirole (1991) for discussion and references on this topic.

Nash solution concept largely because so little could be explained using dominant strategies. With the weaker Bayesian concept of incentive compatibility, a greater variety of behavior could be modeled. For the classical notion of ex post efficiency, however, and from the interim perspective, the equivalence result shows that no freedom is gained by relaxing the incentive compatibility constraint in this way.

The equivalence result suggests the following technique, which is a refinement of the Revelation Principle for a class of problems. Suppose one wishes to determine whether or not in some economic problem a mechanism exists with an efficient Bayesian-Nash equilibrium that also satisfies some other desirable constraints (e.g., budget-balancing, or individual rationality). The Revelation Principle simplifies this existence question by reducing the search to the family of acceptable mechanisms that satisfy the other constraints. If these other constraints are interim or ex ante in nature, then the equivalence result implies that a mechanism with these properties can exists if and only if a Groves mechanism can have these properties. This allows one to go a step beyond the Revelation Principle in simplifying the existence question by assuming an explicit formula for the transfers with unknown constants; the existence question then becomes whether or not the constants can be solved for to satisfy all of the desired constraints. This reduction can greatly simplify this existence question.[6]

The equivalence result is proven in the next section. In section 3, this technique is applied to investigate a bargaining model that Myerson and Satterthwaite (1983) studied in the bilateral case. Here, the number of traders on each side of the market is arbitrary. Features of this model are identified that determine whether or not efficient trade can be incentive compatible given that interim individual rationality and an ex ante budget constraint must also be satisfied. A Groves mechanism is used to establish existence when a mechanism with these properties can exist. Instances of this multilateral model have been analyzed before using lengthy calculational arguments; a purpose of this section is to illustrate how the equivalence result permits the general case to be analyzed in a simple, intuitive fashion that avoids calculation. In section 4, insight from the analysis of the

[6] This technique is apparent in McAfee (1991) and in Makowski and Mazetti (1993), though it is not developed in either of these papers as a general and simple method.

Myerson-Satterthwaite model is used to develop three examples. One example clarifies the positive role that correlation among the agents' types can play in mechanism design. The other two examples reveal some odd properties of a class of acceptable trading mechanisms as the role of incomplete information in the marketplace diminishes. These examples provide new insight into the plausibility of efficient trade when traders do not know each others' preferences.

2. The Equivalence Result

The equivalence result follows from the conclusion of the envelope theorem. I begin by sketching its proof in the special case in which the space T_i of possible types of agent i is an interval $[\underline{t}_i, \bar{t}_i]$ of the real line. A formal discussion follows below in section 2.3.

2.1. The model and a special case. Throughout the paper, t_i will denote agent i's type and t_i^* will denote his reported type. Let $U_i(t_i^* \mid t_i)$, $V_i(t_i^* \mid t_i)$, and $X_i(t_i^*)$ denote (respectively) the interim expected utility, valuation, and transfer of agent i in a revelation mechanism given his type t_i, his report t_i^*, and honest reporting by all other agents[7]:

$$U_i(t_i^* \mid t_i) \equiv E_{t_{-i}}[\, u_i(x_i(t_i^*,t_{-i}),a(t_i^*,t_{-i}),t_i)],$$

$$V_i(t_i^* \mid t_i) \equiv E_{t_{-i}}[v_i(a(t_i^*,t_{-i}),t_i)],$$

$$X_i(t_i^*) \equiv E_{t_{-i}}[\, x_i(t_i^*,t_{-i})].$$

Notice that $U_i(t_i^* | t_i) = V_i(t_i^* | t_i) - X_i(t_i^*)$. Also, let $U_i(t_i) \equiv U_i(t_i | t_i)$ and $V_i(t_i) \equiv V_i(t_i | t_i)$. Incentive compatibility thus means:

(IC) $\quad U_i(t_i) \equiv U_i(t_i \mid t_i) \geq U_i(t_i^* \mid t_i)$ for all $t_i, t_i^* \in \Omega_i$.

Recall that the family of Groves mechanisms are defined by the formula for the transfers

(1) $\quad x_i(t) = -\sum_{j \neq i} v_j(a(t),t_j) + k_i,$

where k_i is a real number.[8] The basic Groves mechanism is the member of this family in which

[7] As usual, t_{-i} denotes the vector of types of all but the ith agent.

[8] The family of Groves mechanisms also includes the case in which the constant k_i in (1) is replaced by an arbitrary function of the reported types of all but the ith agent. The freedom to add an

each k_i equals zero.

Consider now the case of $T_i = [\underline{t}_i, \bar{t}_i]$. With appropriate differentiability assumptions, the envelope theorem holds and implies that

$$\frac{dU_i(t_i)}{dt_i} = \frac{\partial U_i(t_i^* = t_i \mid t_i)}{\partial t_i} = \frac{\partial V_i(t_i^* = t_i \mid t_i)}{\partial t_i},$$

where the partial derivatives are taken with respect to t_i as agent i's type, not as his report. It follows that

(2) $$U_i(t_i) = U_i(\underline{t}_i) + \int_{[\underline{t}_i,\, t_i]} \partial V_i(t_i^* = \tau_i \mid \tau_i)/\partial t_i \, d\tau_i,$$

where τ_i is a dummy variable.

The usual approach upon reaching (2) is to reduce it by calculation. Here, two observations are made that give rise to the equivalence result. First, the integrand in (2) depends only upon the fundamentals of the social choice problem (i.e, the distributions of the agents' types, the efficient choice rule a(t), and the agents' valuation functions), and not upon any feature of the revelation mechanism (i.e., the transfer functions (x_i)). Incentive compatibility is thus so strong as a constraint that it determines each agent's interim expected utility up to a constant (the value of $U_i(\bullet)$ at $t_i = \underline{t}_i$ or at any other value of t_i). Second, each Groves mechanism satisfies (2) because it is acceptable, and the constants $U_i(\underline{t}_i)$ can be freely adjusted by ranging over the family of Groves mechanisms. The family of Groves mechanisms thus spans the entire set of interim utility functions of acceptable mechanisms, which is the equivalence result.

The value of this result is illustrated in section 2.2 below by a simple proof of a result of Myerson and Satterthwaite (1983). Two additional constraints on revelation mechanisms are needed for this example and later in the paper. A revelation mechanism is <u>interim individually rational</u> (IIR) if an agent's interim expected utility is nonnegative whatever the value of his type:

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arbitrary function of this kind instead of a constant is significant only from the ex post perspective, for from the interim perspective only the expected value of the function matters. For this reason I work mostly with the subfamily of the family of Groves mechanisms that is defined by (1).

(IIR) $U_i(t_i) \geq 0$ for all t_i.

A revelation mechanism is ex ante budget-balancing (EABB) if the expected value of the sum of the transfers is nonnegative:

(EABB) $E[\, \Sigma_i\, v_i(a(t),t_i) - U_i(t_i)] = E[\Sigma_i\, x_i(t)] \geq 0.$

The first term in (EABB) expresses a resource constraint: the total amount of utility received ex ante by the agents cannot exceed the total valuation created by efficient social choice. Given (2), it is typically easier to work with (EABB) in this form because it is stated in terms of the fundamentals of the social choice problem and not in terms of the transfers. An acceptable mechanism is desirable if it satisfies both IIR and EABB.

The subsidy required by an acceptable mechanism is defined as:

$$|\min \{0, E[\, \Sigma_i\, v_i(a(t),t_i) - U_i(t_i)]\}\,|.$$

In words, if the mechanism satisfies EABB, then the required subsidy is zero; if the agents are to receive more utility than the social choice problem allows, then the required subsidy is the absolute value of the shortfall.

2.2. Example: the inefficiency of bilateral trade. A seller has an indivisible item that he may sell to a buyer. The seller's type is his cost c for the item and the buyer's type is his value v for it. The seller receives $x_s - c$ if he sells the item and receives a payment of x_s, while the buyer receives $v - x_b$ when he buys the item and pays x_b. Each trader's utility is zero if he fails to trade. The seller's cost c is distributed according to the distribution F(•) and the buyer's value v is distributed according to the distribution G(•). The distributions F(•) and G(•) have continuous densities f(•) and g(•), respectively, and the support of each of these densities is the unit interval [0,1].[9] The set A consists of two elements, the "trade" and the "no trade" alternatives. Efficiency requires that the item is traded if and only if $v \geq c$.[10]

[9] Myerson and Satterthwaite consider the case in which the supports of these densities may be distinct but overlapping intervals. This case will be covered by the discussion of the multilateral version of this problem in section 3.

[10] For simplicity, I ignore the formalities required to describe the case of ties among types (e.g., whether or not trade should occur when $v = c$), which occur with probability zero.

I now prove that a desirable mechanism can not exist. Formulas equivalent to (2) for the seller's and the buyer's interim expected utilities are derived by Myerson and Satterthwaite (p. 269-270) and also follow from Theorem 1 below. The equivalence result thus holds. Because IIR and EABB are constraints on the interim expected utility functions, an acceptable mechanism can satisfy both of these constraints if and only if some Groves mechanism can satisfy them. The search for a desirable mechanism can thus be restricted to this special family.

The basic Groves mechanism in this setting is a "two-price" mechanism in which the buyer pays c and the seller receives v when $v \geq c$ and trade occurs, with no transfers made when the item is not traded. Ex post, each trader in the basic Groves mechanism receives the entire gains from trade from every transaction. The family of Groves mechanisms described by (1) includes additional taxes k_s and k_b on the seller and the buyer that are made regardless of whether or not the item is traded. It follows that in a Groves mechanism, $E_c[U_s(c)] = \Gamma - k_s$ and $E_v[U_b(v)] = \Gamma - k_b$, where Γ is the ex ante expected gains from trade. Notice that Γ equals $E_t[\sum_i v_i(a(t),t_i)]$ in the general model. The ex ante budget constraint (EABB) is thus $-\Gamma + k_b + k_s \geq 0$, which implies $k_b + k_s > 0$. The implications of IIR conclude the argument. The seller with cost equal to one and the buyer with value equal to zero never trade, and hence $U_s(1) = -k_s$ and $U_b(0) = -k_b$. IIR implies that $k_s, k_b \leq 0$, which contradicts $k_b + k_s > 0$. Q.E.D.

This proof provides intuition behind the result and also identifies the key components of arguments of this kind in other settings. Ignoring for the moment the constants k_s and k_b, IC requires that each trader receive ex ante the entire expected gains from trade Γ, which leads by EABB to a required subsidy of size Γ. The question then becomes whether the taxes k_s and k_b can be sufficiently large to fund this subsidy ($k_b + k_s \geq \Gamma$). IIR is then used to bound the taxes k_s and k_b by examining the welfare of the worst-off type of each agent. Because the worst-off traders in this example cannot be taxed at all, the subsidy cannot be funded and a desirable mechanism can not exist. In other settings, the interim expected utility of the worst-off type of each individual is sufficiently large that the subsidy can be funded without violating IIR. In such cases, a Groves mechanism demonstrates the existence of a desirable mechanism.[11]

[11] Notice that the proof identifies Γ as the minimal subsidy required by an acceptable, IIR

The remainder of section 2 is organized as follows. The key step to the equivalence result is formula (2), which expresses an agent's interim expected utility as a function of the social choice problem and not of the transfers. The generality of (2) is examined in the next subsection. The main objective is to avoid the assumption that an agent's interim expected transfer is a differentiable function of his type, which is needed for the envelope theorem. While this may seem like a modest assumption, a purpose of the equivalence result is to facilitate the proof that mechanisms with desired properties either do or do not exist, and proofs of nonexistence are most meaningful if the transfers are not restricted in any way. A standard argument is used to show that no assumptions on the transfers are needed to establish (2) for a model that includes most examples in the literature.

2.3. The general case. Two assumptions are now made on the social choice problem:

(3) the set Ω_i of possible types of agent i is a connected, open subset of R^{n_i};

(4) the interim expected valuation $V_i(t_i^* \mid t_i)$ of each agent is continuously differentiable in both his type t_i and his report t_i^*.

It will be clear from the argument below that (3) can be relaxed so that Ω_i includes the boundary of the open set (as is common in the literature). With this in mind, assumptions (3) and (4) are satisfied by most models in the literature, including all that are discussed below.

Theorem 1. *Consider an acceptable mechanism. For any choices τ_i, τ_i^* of agent i's type,*

$$(5) \qquad U_i(\tau_i) = U_i(\tau_i^*) + \int_C D_{t_i} V_i(t_i^* = \tau \mid t_i = \tau)\, d\tau,$$

where C is a smooth curve from τ_i^* to τ_i within Ω_i and $\tau \in R^{n_i}$ is a dummy variable.

Notice that (5) and the definition of $U_i(\tau_i)$ can be combined to solve for the agent i's interim expected transfer in terms of the social choice problem:

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mechanism in this bargaining problem. Myerson and Satterthwaite (1983, p. 272) computed the minimal required subsidy but they did not identify it as Γ. This was recognized, however, by McAfee (1991, p. 57). This interpretation will be important in the examples of section 4.

$$X_i(\tau_i) = V_i(\tau_i) - U_i(\tau_i) = V_i(\tau_i) - U_i(\tau_i^*) + \int_C D_{t_i} V_i(t_i^* = \tau \mid t_i = \tau)\, d\tau.$$

This formula and (4) together imply that $X_i(\bullet)$ is differentiable. Differentiability of the interim expected transfer function is thus now a consequence, rather than a hypothesis, of the model.

Proof of Theorem 1. The proof follows an argument from Myerson (1981). Let $\rho \in R^{n_i}$ denote a unit vector and let $s \in R$. The constraint IC implies that for $\tau_i \in \Omega_i$,

$$U_i(\tau_i) \geq U_i(\tau_i + s\rho|\tau_i) \text{ and } U_i(\tau_i + s\rho) \geq U_i(\tau_i|\tau_i + s\rho).$$

Combining these inequalities produces

$$U_i(\tau_i|\tau_i + s\rho) - U_i(\tau_i) \leq U_i(\tau_i + s\rho) - U_i(\tau_i) \leq U_i(\tau_i + s\rho) - U_i(\tau_i + s\rho\ |\tau_i),$$

which, after canceling the interim expected transfers, becomes

(6) $$V_i(\tau_i\,|\tau_i + s\rho) - V_i(\tau_i) \leq U_i(\tau_i + s\rho) - U_i(\tau_i) \leq V_i(\tau_i + s\rho) - V_i(\tau_i + s\rho\ |\tau_i).$$

Divide each expression by s and take the limit as $s \to 0$. By (4), the left- and the right-hand terms both converge to the derivative of $V_i(t_i^*\,|t_i)$ with respect to t_i in the direction of ρ at $t_i^* = t_i = \tau_i$.[12] It follows that $D_{t_i}U_i(\tau_i) = D_{t_i}V_i(t_i^* = \tau_i \mid t_i = \tau_i)$, which implies (5) . Q.E.D.

Equation (5) states that acceptability determines each agent i's interim expected utility function up to a constant, i.e., its value at some particular type τ_i^*. The Groves mechanisms are acceptable, and one can freely vary the value of each agent i's interim expected utility at a particular type τ_i^* by changing the constant k_i in formula (1). The equivalence result thus follows.

Theorem 2 (Equivalence Result). *The Groves mechanisms are acceptable, and the interim expected utility functions of the agents in any acceptable mechanism are the same as in some Groves mechanism.*

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12 After dividing by s, the right-hand side of (6) equals $[V_i(\tau_i+s\rho) - V_i(\tau_i)]/s + [V_i(\tau_i) - V_i(\tau_i+s\rho\ |\tau_i)]/s$. The limit of the first term is the total derivative of $V_i(t_i \mid t_i)$ with respect to t_i in the direction of ρ at $t_i = \tau_i$, and the limit of the second term is the partial derivative of $V_i(t_i^* \mid t_i)$ with respect to t_i^* in the direction of ρ at $t_i^* = t_i = \tau_i$. Subtracting produces the desired result.

2.4. Desirable mechanisms. The equivalence result unifies the study of acceptable mechanisms. To illustrate this, an inequality that is necessary and sufficient for the existence of a desirable mechanism is now derived. This inequality is the cornerstone of the analysis in several papers that investigate the existence of such mechanisms in a variety of special cases of the model. The point of Theorem 3 is that this inequality can be derived in a general setting by a simpler and more meaningful argument than the various computational approaches taken in these papers.

In the basic Groves mechanism, let $\underline{U}_i \equiv \inf \{U_i(t_i) \mid t_i \in \Omega_i\}$. In words, $\underline{U}_i$ is the greatest lower bound on agent i's interim expected utility (where $\underline{U}_i \equiv -\infty$ if $U_i(\bullet)$ is not bounded below).

Theorem 3. *The minimal subsidy required by an acceptable, IIR mechanism is*

(7) $\quad |\min\{0, -(n-1)E_t[\Sigma_i v_i(a(t),t_i)] + \Sigma_i \underline{U}_i\}|.$

A desirable mechanism therefore exists if and only if:

(8) $\quad (n-1)\, E_t[\Sigma_i v_i(a(t),t_i)] \leq \Sigma_i \underline{U}_i.$

Examples of the use of (8) to determine the existence of desirable mechanisms include Crampton et. al. (1987, p. 618, ineq. (I)), McAfee (1991, p. 56, ineq. (10)), Makowski and Mazzetti (1993, p. 454, ineq. (E)) and Myerson and Satterthwaite (1983, p. 272).

Proof of Theorem 3. The equivalence result implies that the subsidy required by any acceptable mechanism is the same as the subsidy required by some Groves mechanism. The expected value of the sum of the transfers in a Groves mechanism is

$$E_t[\Sigma_i x_i(t)] = -E_t[\Sigma_i \Sigma_{j\neq i} v_j(a(t),t_j)] + \Sigma_i k_i$$

$$= -(n-1)E_t[\Sigma_i v_i(a(t),t_i)] + \Sigma_i k_i.$$

IIR implies that $\underline{U}_i \geq k_i$ for each agent i. Consequently, an acceptable, IIR mechanism requires a subsidy of at least the amount in (7) to operate. Conversely, if (8) holds, then the Groves mechanism with $k_i = \underline{U}_i$ for $1 \leq i \leq n$ is desirable. Q.E.D.

Theorem 3 explains a mystery concerning desirable mechanisms in those cases in which they exist. When (8) holds, McAfee (1991) and Makowski and Mazzetti (1993) each produce a simple, straightforward revelation mechanism that is desirable. The mystery is that the mechanism in each of these papers implements efficiency in dominant strategies rather than the weaker solution concept of Bayesian-Nash equilibrium that underlies the necessary and sufficient condition (8). The mystery is resolved by the equivalence result, and more specifically, by the proof of the sufficiency of (8); and though it is not noted in these papers, the dominant strategy mechanisms that they produce are Groves mechanisms, tailored to the features of the specific models.[13]

2.4. Qualifications. I conclude this section by noting some limits on the generality of the equivalence result:

- The equivalence between acceptable mechanisms and the Groves mechanisms holds from the interim perspective and not the ex post perspective. The set of acceptable mechanisms contains the Groves mechanisms as a proper subset from the ex post perspective, for acceptable mechanisms can satisfy ex post constraints that Groves mechanisms fail to satisfy.[14]

- The equivalence result is not true in a general Bayesian model with correlated types. Examples have been found in this case of acceptable mechanisms that are not Groves mechanisms.[15] The proof of equivalence breaks down at formula (5) for interim expected utility, which depends upon the transfers in the case of correlated types.

- The equivalence result generalizes to the case in which each agent i's utility function is of the form $u_i(x_i,a,t_i) = v_i(a,t_i) - y(a)x_i$, where the same function $y(\bullet)$ enters each agent's utility function in this

[13] This can be shown by properly selecting k_i in (1) as a function of the reports of all except the ith trader. A similar argument shows that the two-price mechanism discussed in section 3 is a Groves mechanism.

[14] For instance, d'Aspremont and Gérard-Varet (1979) show that efficient, ex post budget-balancing and ex ante individually rational mechanisms may be Bayesian incentive compatible, but such mechanisms do not generally exist if dominant strategy incentive compatibility is required.

[15] For instance, McAfee and Reny (1992) showed that desirable mechanisms can exist in the Myerson-Satterthwaite model of bilateral bargaining if the types are correlated and drawn from the same interval. A Groves mechanism is acceptable in this case, but it cannot satisfy both IIR and EABB.

manner.[16] It thus does not require the absence of income effects. The equivalence result does depend upon the private value assumption, for if agent j's valuation function depends upon agent i's privately known type, then a Groves mechanism will not in general be dominant strategy incentive compatible, and dominant strategy incentive compatible and efficient mechanisms may not even exist (e.g., see Williams and Radner (1988)).

- The equivalence result does not hold if the set Ω_i of agent i's types is finite. It is worth noting that Green and Laffont (1977) assumed that the set of possible utility functions of an agent is "dense" in a particular sense in order to characterize all efficient and dominant strategy incentive compatible mechanisms as Groves mechanisms. As mentioned earlier, the equivalence result is an analogous characterization for efficient, Bayesian incentive compatible mechanisms. Quasilinear utility is assumed in each result and the assumption of a continuum of types in the equivalence result replaces the denseness assumption of Green and Laffont's result.

3. A Multilateral Bargaining Problem

The usefulness of the equivalence result is illustrated in this section by a simple analysis of the multilateral Myerson-Satterthwaite bargaining problem[17]. Necessary and sufficient conditions will be determined for the existence of a desirable mechanism. There are now m buyers, each of whom has one unit of the good to sell, and n sellers, each of whom wants to buy one unit. Let v_i denote the value of the ith buyer and let c_j denote the cost of the jth seller. The utility of a buyer or a seller is the same as in the bilateral case analyzed in section 2. The buyers' values are independently drawn from $[\underline{v}, \overline{v}]$ according to the distribution $G(\bullet)$ and the sellers' costs are independently drawn from $[\underline{c}, \overline{c}]$ according to the distribution $F(\bullet)$. The distributions $G(\bullet)$ and $F(\bullet)$ have continuous density functions $g(\bullet)$ and $f(\bullet)$ with supports $[\underline{v}, \overline{v}]$ and $[\underline{c}, \overline{c}]$ (respectively). It is assumed that $(\underline{v}, \overline{v}) \cap (\underline{c}, \overline{c}) \neq \emptyset$, so it is not common knowledge at the interim stage that the item should or should not be traded.

Desirable mechanisms exist if and only if inequality (8) is satisfied. Inequality (8) is so complex in

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16 See Bergstrom and Cornes (1983) for a discussion of this topic.

17 I refer to the bargaining problem in this way because Myerson and Satterthwaite were the first to address it without reference to any one procedure for bargaining. The problem itself, however, appeared earlier in Chatterjee and Samuelson (1983) who studied the outcome of a particular set of rules for bargaining in this setting.

this multilateral bargaining problem, however, that it can be difficult to determine directly from it what features of the bargaining problem insure that it holds. I thus begin by deriving an inequality that, while equivalent to (8), is much simpler in that many of the terms in (8) cancel. The equivalent inequality is derived by considering a particular subfamily of the Groves mechanisms, which are now described.

The two-price mechanism is a revelation mechanism that operates as follows:

(i) the reported values/costs are ordered in a list, $s_{(1)} \leq s_{(2)} \leq \ldots \leq s_{(m+n)}$;

(ii) buyers whose reported values are at least as large as $s_{(m+1)}$ obtain items from sellers whose reported costs are at or below $s_{(m)}$, and all other traders do not trade[18];

(iii) a buyer pays $s_{(m)}$ and a seller receives $s_{(m+1)}$ when they trade.

While it can be shown that the two-price mechanism is a Groves mechanism, it is straightforward to verify directly that it is acceptable. Individualized taxes are now added to the two-price mechanism:

(iv) the ith buyer pays b_i and the jth seller pays s_j whether they trade or not, where each b_i and s_j is a constant.

The individualized taxes $(b_i)_{1 \leq i \leq m}$ and $(s_j)_{1 \leq j \leq n}$ do not alter the incentive compatibility or the efficiency of the mechanism. These taxes generalize the two-price mechanism sufficiently so that the interim expected utility functions of traders in any acceptable mechanism is the same as in the two-price mechanism with a proper choice of the taxes. The existence of a desirable mechanism can thus be explored simply by examining the two-price mechanism together with these individualized taxes.

The advantage of studying this family of mechanisms instead of all other Groves mechanisms comes from the simple characterization of the budget and the individual rationality constraints that it permits. For any sample of values/costs, the two-price mechanism requires a subsidy equal to

[18] This fully defines the allocation of the items except in the case of $s_{(m)} = s_{(m+1)}$. Such ties occurs with probability zero once it is shown that the mechanism is IC. See Rustichini et. al. (1994) for a way to complete the definition of this mechanism using random allocation in the case of ties so that honest reporting is a dominant strategy for each trader.

$s_{(m+1)} - s_{(m)}$ times the number of items traded in that sample. Letting $H(s_{(m)}, s_{(m+1)})$ denote the expected number of trades given the values of $s_{(m)}$ and $s_{(m+1)}$, the subsidy required by the two-price mechanism is

(9) $$E[H(s_{(m)}, s_{(m+1)})(s_{(m+1)} - s_{(m)})],$$

which is positive because $(\underline{v}, \bar{v}) \cap (\underline{c}, \bar{c}) \neq \emptyset$. With the addition of the individualized taxes, the budget constraint (EABB) becomes

(10) $$E[H(s_{(m)}, s_{(m+1)})(s_{(m+1)} - s_{(m)})] \leq \Sigma_{1 \leq i \leq m} t_i + \Sigma_{1 \leq j \leq n} u_j,$$

i.e., the individualized taxes must be sufficiently large to fund the subsidy required by the two-price mechanism. IIR is also simple to characterize. Buyer i's interim expected utility is minimized in the two-price mechanism at $v_i = \underline{v}$ and seller j's interim expected utility is minimized at $c_j = \bar{c}$. These minimal interim expected utilities bound the individualized taxes. Letting $\underline{U}_b$ denote a buyer's interim expected utility at $\underline{v}$ and $\underline{U}_s$ a seller's interim expected utility at $\bar{c}$ in the two-price mechanism, IIR is equivalent to $\underline{U}_b \geq t_i$ for each buyer i and $\underline{U}_s \geq u_j$ for each seller j. Substituting into (10) produces

(11) $$E[H(s_{(m)}, s_{(m+1)})(s_{(m+1)} - s_{(m)})] \leq m\underline{U}_b + n\underline{U}_s.$$

Inequality (11) is a necessary and sufficient condition for the existence of a desirable mechanism. The proof of Theorem 4 uses this inequality to determine how the existence of such mechanisms depends upon m, n, and the intervals $[\underline{v}, \bar{v}]$ and $[\underline{c}, \bar{c}]$.

Theorem 4. *A desirable mechanism does not exist if either:*

(A) $[\underline{v}, \bar{v}] = [\underline{c}, \bar{c}]$; *or*

(B) $m = n$.

A desirable mechanism exists if either:

(C) $\underline{v} > \underline{c}$ *and n is sufficiently larger than m; or*

(D) $\bar{v} > \bar{c}$ *and m is sufficiently larger than n.*

In (C) and (D), how much larger one side of the market must be than the other for the existence of a desirable mechanism depends upon the distributions F and G. The results of Makowski and Mazetti (1993) on the possibility of efficiency in monopoly are thus special cases of (C) and (D). The Crampton et. al. result (1988, Prop. 2, p. 621) on the inefficiency of monopoly is a special case of (A), and the Myerson-Satterthwaite result (1983, Cor. 1) on the inefficiency of bilateral trade is a special case of (B).[19] These results and the work of McAfee (1991) all show that the existence of desirable mechanisms depends upon features of the trading problem such as the number of traders, the distributions, whether or not the commodity is divisible, etc.. The next obvious task is to carefully classify the features of a trading problem that permit the existence of a desirable mechanism. Theorem 4 contributes to this task.

Proof of Theorem 4. The values $\underline{U}_b$ and $\underline{U}_s$ of the "worst-off" buyer and seller in the two-price mechanism are nonnegative. With (11) in mind, I begin by noting the cases in which these values are positive. A buyer with value equal to $\underline{v}$ trades only when $\underline{v}$ is at least as large as m other value/costs. Because $\underline{v}$ is the lowest possible value of any buyer, a buyer with value $\underline{v}$ trades with positive probability only if there are at least m sellers (i.e., $n \geq m$) and sellers' costs can be below $\underline{v}$ (i.e., $\underline{v} > \underline{c}$). It follows that

(12) $\underline{U}_b > 0 \Leftrightarrow \underline{v} > \underline{c}$ and $n \geq m$.

A similar argument shows that

(13) $\underline{U}_s > 0 \Leftrightarrow \bar{v} > \bar{c}$ and $m \geq n$.

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[19] It is worth noting that (A) and (B) of Theorem 4 are stronger than the results of Myerson-Satterthwaite and Crampton et. al. in that inefficiency is proven here using a weaker notion of the budget constraint: these earlier papers assume that the transfers must balance ex post, while here they must only balance ex ante. This is significant because it helps to clarify a result concerning the effect of correlation between the buyer's value v and the seller's cost c in the bilateral case upon the possibility of achieving efficiency. McAfee and Reny (1992) proved that a desirable mechanism exists if v and c are correlated in a particular way. While they interpreted their result as showing that the Myerson-Satterthwaite result may not hold if v and c are correlated, it could just as well be attributed to their relaxation of the budget constraint. The proof given above thus completes their argument by showing that it is correlation alone that drives their result.

(A) The case of $[\underline{v}, \bar{v}] = [\underline{c}, \bar{c}]$. It follows from (12) and (13) that $\underline{U}_b = \underline{U}_s = 0$, and so (11) does not hold.[20]

(B) The Case of m = n. The required (9) in the two-price mechanism in this case equals:

$$E[\, H(s_{(m)}, s_{(m+1)})\,(s_{(m+1)} - s_{(m)}) \mid \underline{v} > s_{(m)}] \bullet F(\underline{v})^m$$

$$+ E[\, H(s_{(m)}, s_{(m+1)})\,(s_{(m+1)} - s_{(m)}) \mid \underline{v} \le s_{(m)},\ s_{(m+1)} \le \bar{c}\,] \bullet \Pr\{\underline{v} \le s_{(m)}, s_{(m+1)} \le \bar{c}\}$$

$$+ E[\, H(s_{(m)}, s_{(m+1)})\,(s_{(m+1)} - s_{(m)}) \mid \bar{c} < s_{(m+1)}] \bullet [1 - G(\bar{c})]^m.$$

The middle term in this sum is positive. If $\underline{v} > s_{(m)}$, then the costs of all m sellers are below $\underline{v}$ and the values of all m buyers are above $s_{(m)}$. It follows that $H(s_{(m)}, s_{(m+1)}) = m$ and $\underline{v} = s_{(m+1)}$. Similarly, if $\bar{c} < s_{(m+1)}$, then $H(s_{(m)}, s_{(m+1)}) = m$ and $\bar{c} = s_{(m)}$. The expected deficit in the two-price mechanism is thus more than

$$mE[\,\underline{v} - s_{(m)} \mid \underline{v} > s_{(m)}] \bullet F(\underline{v})^m + mE[\,s_{(m+1)} - \bar{c} \mid \bar{c} < s_{(m+1)}] \bullet [1-G(\bar{c})]^m,$$

which equals $m(\underline{U}_b + \underline{U}_s)$. It follows that (11) does not hold.

(C) The case of $\underline{v} > \underline{c}$ and n sufficiently larger than m. If $\underline{v} > \underline{c}$ and n > m, then it follows from (12) and (13) that positive taxes can be collected from buyers but not from sellers. Inequality (11) thus reduces to

(14) $\quad E[\, H(s_{(m)}, s_{(m+1)})\,(s_{(m+1)} - s_{(m)})\,] \le m\,\underline{U}_b.$

Holding the number m of buyers fixed, the result is now proven by showing that (14) holds as n becomes large. Note first that $H(s_{(m)}, s_{(m+1)}) \le m$ because no more than m items can be traded. It is thus sufficient to prove that

(15) $\quad E[s_{(m+1)} - s_{(m)}] \le \underline{U}_b.$

As the number n of sellers becomes large, it becomes increasingly likely that both $s_{(m+1)}$ and $s_{(m)}$ will

[20] Note also that $E[\, H(s_{(m)}, s_{(m+1)})\,(s_{(m+1)} - s_{(m)})]$ is the minimal subsidy required for an acceptable, IIR mechanism in this case. This will be used in an example in section 4.

be near $\underline{c}$; formally, $s_{(m+1)}$ and $s_{(m)}$ converge in probability to $\underline{c}$ as n goes to infinity. It is thus clear that the left-hand side of (15) converges to zero and the right-hand side converges to $\underline{v}$ - $\underline{c}$. For sufficiently large n, (15) (and therefore (11)) must hold.

The proof of (D) is similar and is therefore omitted. Q.E.D.

Both IIR and EABB are crucial in the proofs of nonexistence, for existence follows if either constraint is appropriately relaxed.[21] It is thus worth noting that most real trading mechanisms satisfy stronger constraints than IIR and EABB. Ex post budget-balancing (i.e., the transfers sum to zero ex post) is common, and, if a trader fully knows his preferences as trade occurs,[22] then ex post individual rationality is also the rule (i.e., a trader is not required ex post to accept a trade he does not want). Parts (A) and (B) of Theorem 4 are thus especially strong results because IIR and EABB are weaker constraints than what real mechanisms typically satisfy. Conversely, the Groves mechanisms used to prove existence of desirable mechanisms in the environments described in (C) and (D) are of questionable significance because IIR and EABB seem weak in this setting. Further work is thus needed to convincingly argue that efficiency is in a practical sense attainable in such environments.[23]

4. Three Examples of Bargaining

Three examples are examined in this section that concern acceptable, IIR mechanisms in the generalized Myerson-Satterthwaite bargaining problem. Each example rests upon an interpretation of the minimal subsidy required for such mechanisms that follows from the equivalence result. The first two examples show how odd these mechanisms can seem from the perspective of two benchmarks of microeconomics: the cases of (i) trade with complete information, and (ii) trade among a large number of traders. The examples suggest that superior models of trading mechanisms can

21 This is obvious in the case of EABB. d'Aspremont and Gérard-Varet (1979) prove existence for the case in which ex ante individual rationality replaces IIR.

22 This is modeled here by the private value assumption.

23 It is also worth noting that IIR and EABB as opposed to ex post individual rationality and budget-balancing appear to be crucial to results that show that IC and EF are attainable if the types of agents are correlated. See McAfee and Reny (1992), Crémer and McLean (1988) and Wilson (1993) for discussions of this issue.

be found by dropping EF as a constraint and imposing EABB. The third example provides insight into the positive role that correlation among the agents' types can play in the existence of desirable mechanisms.

4.1. Example: approaching complete information. This example concerns the limit of the independent private value model as it approaches the case of complete information. As in section 2.2, it is assumed that there is one trader on each side of the market and that the buyer's value and the seller's cost are drawn from the same interval $[\underline{v}, \bar{v}] = [\underline{c}, \bar{c}]$. Consider a sequence of pairs of distributions $((F_i, G_i))$ such that the density of the seller's cost f_i and the density of the buyer's value g_i converge to mass points at $\bar{v}$ and $\underline{c}$, respectively (see Figure 1). In the limit, it is common knowledge that the buyer's value is $\bar{v}$, the seller's cost is $\underline{c}$, and the item should be transferred. The analysis in section 2.2 holds for each pair (F_i, G_i) in the sequence. Consider next the sequence whose ith term is the minimum subsidy required for an acceptable, IIR mechanism in the bargaining problem defined by the ith pair of distributions (F_i, G_i). Because the ith term in this sequence is the ex ante expected gains from trade in the bargaining problem given by (F_i, G_i), it is clear that this sequence of minimal subsidies is increasing and converges to $\bar{v} - \underline{c}$, the expected gains from trade in the limiting case of complete information. Thus, no matter how close the bargaining problem is to complete information, a desirable mechanism does not exist.

This example is curious because there exist simple, plausible mechanisms that are IC, IIR, EABB but not EF and that converge in the expected way as the limiting case of complete information is approached. Consider, for instance, a fixed-price mechanism[24] in which the price is set at $p^* = (\bar{v} + \underline{c})/2$. It is easy to see that this mechanism is IC, IIR, EABB, and additionally is EF in the limiting case of complete information. If this fixed-price mechanism is used, then the gains from trade that are not achieved clearly converge to zero as complete information is approached.[25]

It is not surprising that a desirable mechanism exists in the limiting case of complete information,

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24 For the fixed price of p^* and reported v^* and c^*, trade occurs in this mechanism when $v^* \geq c^*$ and the buyer pays p^* to the seller when the item is traded (and otherwise no payment is made). See Hagerty and Rogerson (1985) for further discussion.

25 The fixed-price mechanism is an especially simple example of an mechanism with these properties. More sophisticated mechanisms may attain higher levels of efficiency for the bargaining problems along the sequence of pairs of distributions $((F_i, G_i))$.

for bargaining is straightforward in this case. What is surprising is that the subsidy required for attaining the first best can jump discontinuously upwards if even the smallest amount of uncertainty about whether or not the item should be traded is introduced. The example points to the counterintuitive nature of results that can be derived by insisting upon first-best efficiency in situations of incomplete information. All acceptable, IIR mechanisms are discontinuous at the familiar case of complete information, while IC, IIR, EABB, but not EF mechanisms can converge as the case of complete information is approached in a manner consistent with intuition about the effect of a small amount of uncertainty upon bargaining.

4.2. Example: approaching perfect competition. The minimal subsidy required for an acceptable, IIR mechanism will be examined as the number of traders becomes large in the symmetric case of $[\underline{v}, \overline{v}] = [\underline{c}, \overline{c}]$, $m = n$, and $F = G$. As shown in the proof of Theorem 4 (B), the minimal subsidy in this case is $E[H(s_{(m)}, s_{(m+1)})(s_{(m+1)} - s_{(m)})]$. $H(s_{(m)}, s_{(m+1)})$ equals the expected number of the m values/costs at or below $s_{(m)}$ that are sellers' costs; in the case of $m = n$ and $F = G$, $H(s_{(m)}, s_{(m+1)})$ equals $m/2$. The minimal subsidy thus equals $(m/2)\, E[s_{(m+1)} - s_{(m)}]$. In the case of $F = G$ it is known that $E[s_{(m+1)} - s_{(m)}] \geq k/m$ for some value of $k > 0$ that depends upon F.[26] The minimal subsidy of an acceptable, IIR mechanism is thus bounded below by a positive constant for all markets in this sequence, no matter how large.

If a continuum of buyers whose values are distributed according to F(•) and a continuum of sellers whose costs are distributed according to F(•) is postulated, then trade at a price equal to the median of F(•) (i.e., the competitive price) is efficient; desirable mechanisms thus exist in this continuum market. Markets with a continuum of traders are fundamental in microeconomic thought because they model perfect competition in which trade is efficient. The meaningfulness of this idealized form of trade depends upon the proper convergence of markets with a finite number of traders as the size of the market increases. The example illustrates a discontinuity of the subsidy required for an acceptable, IIR mechanism as the perfectly competitive model is approached: the subsidy does not converge as the number of traders on each side of the market increases to infinity to the "zero subsidy" that is needed for efficiency in the perfectly competitive case.

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26 This follows from the methods discussed in David (1981, p. 34-35) together with the assumption that F is C^1 with compact support.

This discontinuity is curious because there exist IC, IIR and EABB mechanisms in which the portion of the potential gains from trade that are not achieved in the finite market converges to zero as the number of traders increases to infinity. This can be true of very simple mechanisms, and the convergence to efficiency may be quite fast.[27] Mechanisms that are IC, IIR and EABB but not EF thus can be continuous at the limiting perfectly competitive market, while mechanisms that are acceptable, IIR but not EABB are necessarily discontinuous at the limiting case; the loss from using an inefficient mechanism can become arbitrarily small as the market increases in size, while the cost of achieving first best efficiency does not. This is analogous to Example 4.1 in which an IC, IIR and EABB mechanism exists that properly converges as the case of complete information is approached, while no sequence of acceptable, IIR but not EABB mechanisms can have this property. While first-best efficiency can be attained if there is incomplete information by relaxing the budget constraint, the mechanisms that achieve first-best efficiency have counterintuitive features as either of these two benchmark cases of classical microeconomics is approached.

4.3. Example: approaching perfect correlation. This example provides insight into the effect of correlation between the buyer's value v and and the seller's cost c upon the existence of desirable mechanisms. Consider a pair of intervals $[\underline{c}_1, \bar{v}_1]$ and $[\underline{c}_2, \bar{v}_2]$ such that $\bar{v}_1 < \underline{c}_2$. At the ex ante stage, it is common knowledge that the buyer's value v and the seller's cost c will both lie within the same interval, but it is not known in which of these two intervals they will lie. The probability that they both lie in $[\underline{c}_j, \bar{v}_j]$ is $p_j > 0$ (where $p_1 + p_2 = 1$). At the interim stage, the conditional beliefs are as in Example 4.1. This is depicted in Figure 2.

A sequence (K_i) of joint distributions of v and c is now defined such that each distribution K_i has these properties. For $j = 1, 2$, consider first a sequence $((F_{ij}, G_{ij}))$ of pairs of distributions on $[\underline{c}_j, \bar{v}_j]$ such that (F_{ij}) converges to a point mass at $\underline{c}_j$ and (G_{ij}) converges to a point mass at $\bar{v}_j$. The joint distribution K_i of v and c is defined by the probabilities p_1 and p_2 together with the following property of the conditional probabilities:

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27 In Rustichini et. al. (1994), for instance, it is shown that in trade at a market-clearing price (i.e., the k-double auction) the portion of the potential gains from trade that are not achieved in the market with m buyers and m sellers is $O(1/m)$ (or $O(1/m^2)$ on a per capita basis). The sense in which this is fast is discussed in the paper. It is also worth noting that both the k-double auction and the fixed-price mechanism mentioned in example 4.1 are individually rational and budget-balancing in the ex post sense, which are stricter and more realistic than IIR and EABB.

(16) for all v, c ∈ $[\underline{c}_j, \bar{v}_j]$, $K_i(c|v) = F_{ij}(c)$ and $K_i(v|c) = G_{ij}(v)$.

The limit of the sequence (K_i) is a case in which v and c are perfectly correlated, and in which at the the interim stage it is common knowledge that the item should be traded.

The analysis of this example is simple because of its similarity at the interim stage to Example 4.1. The methods of this paper can be used to show that the minimum subsidy required by acceptable, IIR mechanisms converges upwards as i increases to $p_1(\bar{v}_1 - \underline{c}_1) + p_2(\bar{v}_2 - \underline{c}_2)$, which is the ex ante expected gains from trade in the limiting case of perfect correlation. A desirable mechanism cannot exist for finite i, while such mechanisms do exist in the limiting case of perfect correlation.[28]

The meaning of this example is revealed by noting that it can be generalized to the case in which there are an arbitrary number q of nonintersecting intervals in which both v and c may lie. Notice that by choosing a larger q and smaller intervals, the buyer's value v and the seller's cost c become more highly correlated and a trader's observation of his own value/cost becomes more informative about the value/cost of the other trader. The argument from the case of two intervals generalizes, however, to show that a desirable mechanism does not exist for a given q and sufficiently large i, regardless of the size of q. Despite the large amount of "learning" about each other's value/cost that may occur as a trader learns his own value/cost, bargaining in this framework is necessarily inefficient if it is acceptable, IIR, and EABB, regardless of how close the case of perfect correlation is approached.

Crémer and McLean (1988) and McAfee and Reny (1992) have shown that a desirable mechanism may exist if the types are correlated and the beliefs satisfy a nondegeneracy condition. The point of this example is not to question their results, for it does not satisfy their nondegeneracy condition[29]; the point is to question an intuition for the results. Intuitively, it seems plausible that correlation is beneficial because it means that each trader acquires information about the value/cost

[28] The fixed-price mechanism generalizes as follows to show the existence of the desired mechanism: if the reported value and cost are in the same interval $[\underline{c}_j, \bar{v}_j]$, then trade occurs at a price of $(\underline{c}_j + \bar{v}_j)/2$; if the reported value and cost are in different intervals, then trade does not occur and no payments are made.

[29] The nondegeneracy condtion requires that the conditional beliefs F(•|v) and G(•|c) vary as v and c (respectively) change, which is not satisfied here because of (16).

of the other trader as he learns his own value/cost. In this way, the inefficiency caused by the uncertainty about each other's value/cost might be dissipated. This intuition is incorrect. The example instead reinforces the lesson of the proofs in these papers, which show that correlation can be beneficial because it expands the set of "bonuses and penalties" that can be imposed upon an agent at the interim stage for reporting different types.[30] The means of influencing the incentives of the agents may thus expand as correlation is introduced, which thereby may enlarge the set of constraints that an IC mechanism can satisfy.

5. Conclusion

The problem of achieving efficiency when agents have incomplete information is central in mechanism design. By showing that an efficient, Bayesian incentive compatible mechanism is payoff-equivalent at the interim stage to a Groves mechanism, the equivalence result connects work on this problem that uses the Bayesian-Nash solution concept to earlier work that uses the dominant strategy solution concept. The Groves mechanisms are now so well understood that the equivalence result can be said to provide intuition for results in Bayesian mechanism design. As demonstrated here by a generalization of a result by Myerson and Satterthwaite (1983), the equivalence result is also a useful tool for determining the existence of incentive compatible and efficient mechanisms subject to interim constraints.

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30 In the case of correlation, an agent's type affects his beliefs and hence the probabilistic weights he attaches to different outcomes. The expected outcome associated with different reports may thus depend in the case of correlation upon the agent's true type (unlike the case of independence, in which it does not). The set of conditional expected payoffs that can be imposed upon an agent thus expands as correlation is introduced to the extent that each agent's conditional beliefs depends upon his type (i.e, the nondegeneracy condition). This expansion does not occur in the example because of (16).

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Captions for Figures

Figure 1. A sequence of pairs of probability distributions $((F_i,G_i))$ is considered such that the support of each of the distributions F_i, G_i is the interval $[\underline{c}, \overline{v}]$. The respective density functions f_i, g_i are graphed above. The limit of the sequence (F_i) is the probability distribution in which $\underline{c}$ occurs with probability one and the limit of the sequence (G_i) is the probability distribution in which $\overline{v}$ occurs with probability one. The arrows indicate how the densities change as i increases.

Figure 2. The buyer and the seller know at the ex ante stage that with probability p_j the buyer's value v and the seller's cost c will both lie in $[\underline{c}_j, \overline{v}_j]$, and that v and c will be independently drawn from $[\underline{c}_j, \overline{v}_j]$ according to the probability distributions G_{ij} and F_{ij} (respectively). The densities g_{ij} and f_{ij} of these distributions are graphed above. At the interim stage, each trader knows his own value/cost and that the other trader's value/cost lies within the same interval. As i changes, the distributions G_{ij} and F_{ij} change as in Figure 1.

Figure 1

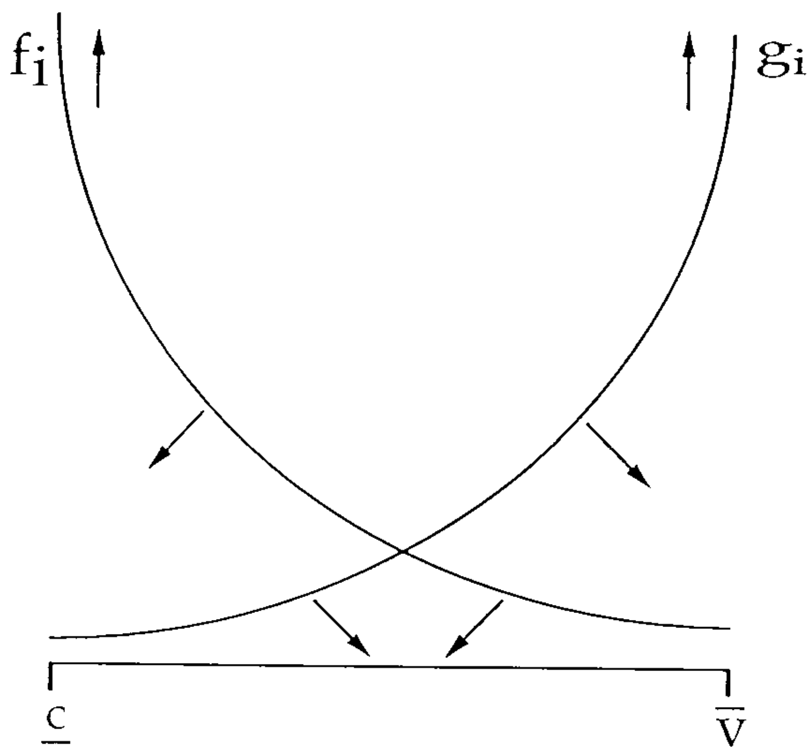
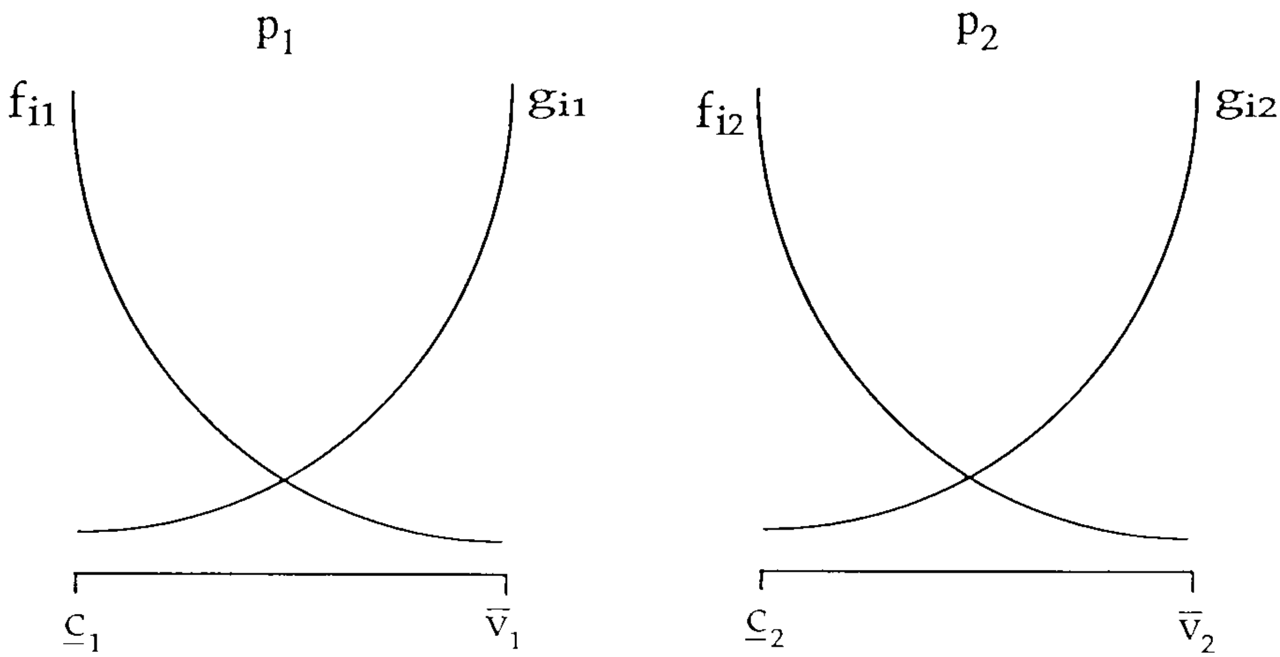


Figure 2

ex ante:



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