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LEARNING BY DOING AND PROTECTION OF AN INFANT-INDUSTRY

by

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Abstract: This paper addresses an optimal tariff design to protect an infant-industry in the presence of learning effects. Firms decide how much to produce, taking into account learning effects induced by their current production, while the government decides on the level of tariff protection. In order to include different levels of bargaining power for each group of agents, each component of the welfare function is weighted with these weights taken as given. We solve the symmetric case without spillovers and fixed cost reduction due to accumlated output. Assuming that domestic and foreign production are imperfect substitutes for each other but perfect substitutes within each group, we use a complete linear demand system to represent domestic consumers' preferences. The analytic Markov Perfect Equilibria of this game is derived by solving a linear-quadratic differential game. The optimal tariff policy is characterized and compared to Spain's tariff policy on Iron and Steel for 1913.
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1 Introduction

The aim of this paper is to study the protection of an infant industry when learning effects are important. The common argument made in economic policy discussions is that protecting an industry in its early stages of development allows that industry's firms to compete later. This is because of the cost savings induced by protecting the domestic market. Even though some recent literature partially supports this view, traditional theory rejects any type of trade restricting policy because it fails to achieve maximum welfare from a world perspective. The present model studies some features of an optimal protection policy from the developing country's point of view. We will show that even in a simplified framework, a time consistent tariff protection may serve to ensure development of the infant-industry and enhances domestic welfare. However, this policy has its limits. A decreasing tariff profile is required in order to compensate for learning exhaustion with increasing competition.

In addition to this theoretical goal, we address an issue that has historical interest *per se*. Historically, tariff policies have been widely used to protect infant or non-infant industries in order to promote their development. This has been particularly prevalent in the iron and steel industry, a key sector in any industrializing country. However, the performance of these tariff policies has been very different in each case. For instance, while the Imperial tariff may have developed the German iron and steel industry by imposing intermediate levels of protection, the prohibitive Spanish tariffs isolated the Spanish iron and steel industry from the European market limiting development possibilities.

The application of Industrial Organization models to International Trade issues has enabled rich theoretical advances in this area, leading to results that sometimes contradict traditional views of trade policy. Among the new views, there are several arguments in favor of protection. Brander and Spencer (1981) point out the possible benefits of a tariff for a country of any size because the terms of trade effect will be absorbed, at least partially, by the foreign monopolist instead of being passed on to domestic consumers. Venables (1985) argues that a tariff can lower domestic prices by increasing competition among domestic firms: the tariff raises home market's profitability which promotes entry. In contrast, Grossman and Horn (1988) show a negative effect on welfare from either permanent or temporary protection when there exists some informational barrier, that is, when consumers do not know output quality until they have purchased the good and form expectations about firms' quality choice according to firm's past behavior.

The most common argument in trade policy in favor of protection deals with the existence of dynamic economies of scale at the industry or firm level. In this case, tariff protection induces higher domestic production and therefore cost savings which provide the domestic industry with a better cost position relative to its foreign competitors. This is also the mainstream focus of the research in this field. The common reasoning is outlined in Brander and Spencer's (1983) model of strategic trade policy: in the presence of government intervention, domestic Stackelberg leadership becomes a credible strategy. Along the same line of reasoning, Brander and Krugman (1983) develop a reciprocal dumping duopoly model with static economies of scale (downward–sloping marginal cost). In this model, if the government protects its domestic market, the domestic firms will reduce costs by increasing production, and therefore, as the foreign firm has reduced its respective production, the domestic firm will be able to compete in the foreign market.

As Krugman (1984) has shown all these models can be interpreted as different ways of promoting exports through protecting imports, whether they include static economies of scale, international R&D competition or dynamic economies of scale (learning by doing). As many policy makers use to claim, Krugman notes that benefits from a future cost advantage may motivate government protection. However, Gruenspecht (1988) points out the possibility that firm behavior is more collusive in this environment, and as a consequence, firms respond to protection in a manner that frustrates government efforts to promote higher domestic production in order to reduce costs through learning.

The optimal tariff policy derived in the present model does not suffer from this caveat. The government does not commit to some level of protection that firms may take as given while colluding. Instead, the tariff policy is contingent on industry performance. Excessive collusion that leads to a low rate of cost reduction will be compensated by lower levels of tariff protection in order to increase competition.

Another major caveat to the traditional arguments for protection is the assumption of unilateral government intervention. In order to broaden the strategic aspects of trade policy design there exists a growing literature that addresses either the strategic interaction among governments[1] or between governments and the industry[2] as this paper does. Within this literature, the work of Matsuyama (1990) applies relatively closely, although his modelling is not appropriate for addressing the dynamic features of learning by doing. Matsuyama presents an infinitely repeated game where at each stage the government decides whether the industry is going to be protected one more period, and the only firm in the industry decides whether to invest in the current period or to delay that decision. In his model it is assumed that the government always prefers liberalization to protection, but it is always willing to delay liberalization for another period if this would induce the domestic firm to invest. Similarly, the firm prefers to invest in order to prepare for competition if liberalization seems very likely, but choosing not to invest can be a profitable strategy if this induces the government to postpone trade liberalization. Under these reasonable assumptions on players' preferences Matsuyama shows that the game has a unique stationary subgame perfect equilibrium in mixed strategies.

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[1] See Bagwell and Staiger (1990) on enforcement of trade agreements where tariff levels are determined in a repeated game framework among governments.

[2] For instance Dixit and Kyle (1985) analyze different government policies in the standard entry deterrence and promotion model extended to include the international issues; Staiger and Tabellini (1987) consider the time consistency of protection policies in a game between the firms in an industry and the government; and finally Anderson (1992) addresses the strategic behavior of exporting firms and the government in presence of voluntary export restraints.

An especially nice feature of Matsuyama's model is the recursivity of the game. After one period, if the government chose to protect and the firm did not invest, the game is identical to the one played one period before. However, if learning by doing effects are considered, there exists at least one state variable (the level of unit cost and/or the accumulated output) that differs from the previous period due to production. Therefore, it is not appropriate to work with time-independent strategies when we consider the existence of dynamic economies of scale.

At this point, there are two options: we could reformulate Matsuyama's model to include a dynamic effect while considering only a finite horizon. However, this would not be a realistic approximation to the empirical issue addressed herein[3]. Moreover, time consistency issues cannot be appropiately addressed within a finite horizon specification. Alternatively, we will define a more complex repeated game with state–dependent strategies and look for a Markov Perfect Equilibrium. This case will use a continuous time formulation of the game specifically to benefit from a result of dynamic programming in order to establish the time consistency of the equilibrium strategies.

The intuition of Matsuyama's model is retained even in this dynamic framework. The government wants to liberalize in order to maximize domestic welfare. But, due to learning effects, welfare maximization over time requires the establishment of a tariff to protect the domestic industry. The optimal tariff will depend on industry performance. Given a low learning effect, the optimal policy will reduce the tariff to increase competition and avoid excessive domestic monopoly power. On the other hand, domestic firms prefer a monopolistic position, but the possibility of foreign competition induces them to increase production above the static profit maximization level in order to reduce cost and be able to compete later. Hence, if producers

[3] The iron and steel industry is characterized by the existence of economies of scale and a learning process so that domestic firms are not competitive against foreign firms inmediately after investing in fixed capital. In addition see the criticism of Tornell (1987) in Matsuyama (1990).

have the ability to influence the government, the government's willingness to liberalize may be lower than that required by the optimum solution. The dynamics have their origin in the decreasing speed of learning induced by a downward slopping convex fixed cost function over accumulated output[4].

In addition to the works previously referred to, there are a few very interesting studies that calibrate monopolistic competition models applied to international issues. The works of Dixit (1988) and Irwin (1991) share common features with this paper's model, such as the fixed number of firms and the conjectural variation approach. Baldwin and Krugman (1988) address the same problem of learning economies but their set up is static which allows them to avoid the use of Markov strategies. The isomorphism between learning economies and static economies of scale is no longer true under general conditions such as discounting and/or *closed loop* strategies. Our model generates the same qualitative results as those of Baldwin and Krugman (1988), Krugman (1984), or Stokey (1986), and in addition it shows the dynamic evolution of the production decision and tariff protection.

This paper attempts to build a dynamic model of optimal tariff design when there exists learning economies. Two principles will guide the construction of this model. First, simplicity: the nature of the class of models that we suggest is complex enough to get lost in technical issues very easily. This is the reason why we choose a very particular specification for demand and the learning curve. Second, our aim to carry out an empirical application based on this model. It is therefore essential to develop a model whose parameters can be easily identified in order to carry out a simple calibration exercise. The paper is organized as follows. In section 2 the model and its assumptions are described. In section 3 the model is presented in detail and the optimality conditions and features of the Markov Perfect Equilibria are derived. In section 4 the development of the Spanish Iron and Steel Industry

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[4] Demand does not induce any dynamic effect because of its stationary linear specification. Welfare gains from protection may be higher than those highlighted by this model if, in addition, learning induces marginal cost reductions, and/or if demands grows along time.

is reviewed and the theoretical assumptions are linked to historical facts, which justifes the application of this model to this case of study. In section 5 we proceed with the calibration of the model: the applied tariff is compared to the optimal policy derived from the theoretical model. Some comparative statics exercises are also carried out. Finally some welfare loss measures to Spanish government departure from the optimal policy are provided. In section 6 we summarize conclusions.

2 A Concise Description of the Model

The game consists of $n + 1$ players: n firms and the government of a small country. There is no entry or exit into or out of this industry. The problem to be addressed is protection of an infant -industry. Firms ask for protection to have time to reduce total costs and later to be better positioned to compete with foreign firms. Total cost declines due to learning by doing. As pointed out earlier, this is the major difference between ours and Matsuyama's model, and it renders it impossible to apply the recursivity feature of his model.

The model is structured using a continuous time specification. The only state variable of the model is the vector of accumulated outputs for each firm in the industry. Denote the realization of this vector at time t as y^t. The level of total cost is assumed to depend on accumulated output. For simplicity, assume constant marginal cost. For each player, time t strategies are contingent on the state of the game. Production is the firm's control variable. The only choice variable for the government is the tariff level. A firm's objective in each period is to maximize its expected discounted profits. The government maximizes the weighted sum of consumer surplus, total profits and tariff revenues. For simplicity, we will assume that foreign firms, who produce a slightly differentiated good, behave competitively. In addition assume that they have exhausted their respective learning processes. This assumption allows us to ignore strategic effects between domestic and foreign firms as well as foreign firms' investment consideration of output decisions. Foreign firms compete while domestic firms supply a differentiated good in a monopolistically competitive regime and are subject to learning effects.

Up to this point we have introduced some deviations from Matsuyama's model that may be interesting to discuss more explicitly. Consider first, the existence of dynamic economies of scale through the process of learning by doing. The existence of a learning curve has been observed in several industries since World War II[5]. The effects of a learning curve on market performance under different competitive regimes has been studied by both Spence (1981) and Fudenberg and Tirole (1983). While Spence analyzes an infinite horizon, continuous time, *open loop* solution under assumptions of monopoly and perfect competition, Fudenberg and Tirole focus on differences between the *open loop* and *closed loop* solution in a two period model. Fudenberg and Tirole (1983) use a general formulation for the learning curve where Spence (1981) uses an exponential learning specification and Lieberman (1984) uses a logarithmic one. As will become evident, technical requirements of our differential game force us to assume that learning does not affect marginal cost. Marginal cost is assumed to be constant: learning only reduces fixed cost. Stokey (1986) studies the dynamics of an industry under the assumption of complete spillovers in learning which reduces marginal cost instead of fixed cost. It is shown in this infinite horizon environment that there exists a unique symmetric Nash Equilibrium within the space of continuous strategies. A compensating policy is suggested to favour production in early stages of the industry's lifecycle because of the existence of learning externalities. Shifting learning effects from marginal to fixed cost, as the present model does, enables us to find a closed form solution for the equilibrium strategies and to derive some propositions on features of one possible compensating policy.

A second deviation of our model is that we allow for the existence of more than one firm in the industry. This requires that we specify the industry's conduct and the nature of the solution[6]. The solution can be symmetric or asymmetric. In order to keep the model tractable we will

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[5] See for instance Lieberman (1984) and Fudenberg and Tirole (1986, §2b).

[6] On this issue see Helpman and Krugman (1989, §8).

solve only the symmetric case[7]. We also assume that there is no learning spillover effects. This together with the symmetry assumption characterizes an n vector of state variables with identical accumulated output along the equilibrium path.

In relation to the industry's conduct we adopt a conjectural variation approach. The perceived marginal revenue to each firm depends on the value of a specific parameter which represents the different cases of Cournot, Bertrand, collusion or perfect competition. This parameter represents each firm's belief over its competitors' behavior in response to the firm's action. This approach is a useful way to discriminate among industry conduct at the calibration stage. However, it introduces limitations on comparative statics exercises because of the lack of optimizing behavior over the conjectural variation parameter. The conjectural variation approach is adopted to solve different cases of the stage game, and the corresponding parameter is then held constant over the time horizon of the game.

The pattern of production by firms and the government's design of the tariff policy is constructed to be optimal. The optimal control paths derived here are dynamic best response functions for each set of agents, that is, the government and the firms as a whole[8]. Given the government's optimal tariff policy (defined on a vector of state variables), firms choose their optimal output paths symmetrically (which also depends on the same vector of state variables), including in such computation their symmetric, common knowledge belief on their competitors' responses. For these strategies to be a Nash Equilibrium, the government's strategy must also be a best response to firms' strategies as described above. Markov perfection requires that these strategies be a perfect equilibria for any time and state[9].

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[7] For a n firms asymmetric version of this model we have to solve numerically $2(n+1)$ Riccati equations.

[8] See Hanig (1986, §3.1), pp.86-88.

[9] See Fudenberg and Tirole (1986), p.19.

Finally, the model includes explicit distributive considerations, which are taken as given for the social planner's problem. The intent is not to model government preferences explicitly as resulting from interacting pressure groups. Rather we will carry out a sensitivity analysis after we parameterize the model in order to fit the data and to test the traditionally assumed existence of strong interest groups[10]. Let us summarize now the considerations on the government's objective function. Let CS^t denote the consumer surplus derived from the demand specification. Π^t is industry profits and R^t represents the government's tariff revenues. The stage social welfare function is then defined as:

$$W^t = CS^t + \alpha^t \Pi^t + \beta^t R^t$$

where $\beta^t \geq \underline{\beta}$ is an index of government's relative preference for tariff revenues at time t[11]. Therefore, if $\beta^t > 1$ the government outweights tariff revenues. The parameter $\alpha^t \geq 0$ is an index of government's relative preference for producers surplus at time t; $\alpha^t = \beta = 1$ represents the standard case of a social welfare maximizing government. However, if $\alpha^t > 1$ the government is partially captured by producers, and if $\alpha^t < 1$ by consumers. By searching for the values of α^t and β^t that best fit actual values for each market structure outcome of the model, we can approximate the relative political power of pressure groups and government tariff preferences.

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[10] The political economy of pressure groups dates back at least to Olson (1968). An extensive overview of this literature and its applications to government intervention analysis is provided by Noll (1989). On the political economy of protection see also Baldwin (1984) and (1985), Mayer (1984), and Pincus (1977), and especially Hillman (1989).

[11] $\underline{\beta}$ is defined in appendix A; it is the minimum value of β^t such that government's optimal control problem is well defined.

3 A Linear–Quadratic Differential Game of Industry Protection with Dynamic Economies of Scale

3.1 Demand System

Assume that domestic and foreign production are considered imperfect substitutes for each other by domestic consumers but perfect substitutes within each category. Domestic firms produce a homogeneous good and they compete in a differentiated product market with foreign firms. Let $X^t = \sum_{i=1}^{n} x_i^t$ denote the domestic industry production and let M^t denote imports at time t. Assume a quadratic utility function with symmetric cross–effects for domestic consumers such that own effects dominate cross effects, that is, a strictly concave function of the form:

$$U(X^t, M^t) = Q_0^t + a_x X^t + a_m M^t - \frac{1}{4}\left[b_x (X^t)^2 + b_m (M^t)^2 + 2k X^t M^t\right]$$

where all parameters a_x, a_m, b_x, b_m, k are strictly positive. The condition $DetH[U(X^t, M^t)] = \frac{1}{4}(b_x b_m - k^2) > 0$ ensures the utility function to be strictly concave. At each time, t, consumers maximize $U(X^t, M^t)$ subject to the monetary constraint $I^t = Q_0^t + \tilde{P}_x X^t + \tilde{P}_m M^t$, where $U(X^t, M^t)$ is a money valued utility function and Q_0^t represents the aggregate consumption of a competitive numeraire good. This implies that the Lagrange multiplier of this constrained maximization problem is equal to one in any period. Let $\tilde{P}_x^t$ and $\tilde{P}_m^t$ represent the domestic market price for domestic production and imports in each period. Since we consider only the case of an import tariff, τ^t, we have:

$$\tilde{P}_x^t = P_x^t \tag{1}$$

$$\tilde{P}_m^t = P_m^t + \tau^t \tag{2}$$

where P_x^t and P_m^t are the world prices of the domestic and the imported good respectively. Therefore the first order necessary conditions for the unique solution to this problem are:

$$P_x^t = a_x - \frac{1}{2} b_x X^t - \frac{1}{2} k M^t \tag{3}$$

$$P_m^t + \tau^t = a_m - \frac{1}{2} k X^t - \frac{1}{2} b_m M^t \tag{4}$$

Let $P^t = (P_x^t, P_m^t)$. Using Cramer's rule, demands for domestically produced goods and imports as functions of the tariff level can be written as follows:

$$X^t(P^t, \tau^t) = X^t(P^t, 0) + \frac{2k\tau^t}{b_x b_m - k^2} = X^t(P^t, 0) + \mu_x \tau^t \geq 0 \qquad (5)$$

$$M^t(P^t, \tau^t) = M^t(P^t, 0) - \frac{2b_x\tau^t}{b_x b_m - k^2} = M^t(P^t, 0) - \mu_m \tau^t \geq 0 \qquad (6)$$

with $\mu_x > 0$, $\mu_m > 0$. Finally, consumer surplus is given by:

$$CS(X^t, M^t) = U(X^t, M^t) - Q_0^t - \tilde{P}_x^t X^t - \tilde{P}_m^t M^t \qquad (7)$$

3.2 Cost Function

Assume that the learning effect only reduces firms' fixed costs and that firm costs are additively separable in accumulated output and current production, which implies that marginal cost is constant over time. The fixed cost is a positive, strictly decreasing and strictly convex function defined on $[0, y^*]$ and constant on $[y^*, \infty)$, for some large level of accumulated output y^*. In particular, we adopt the following additively separable specification:

$$C_i^t(y_i^t, x_i^t) = \begin{cases} c_0 + c_1 y_i^t + \frac{1}{2}c_2(y_i^t)^2 + c_3 x_i^t & \text{if} \quad y_i^t \leq y^* \\ c_0 + c_1 y^* + \frac{1}{2}c_2(y^*)^2 + c_3 x_i^t & \text{if} \quad y_i^t \geq y^* \end{cases} \qquad (8)$$

where x_i^t represents output and y_i^t represents accumulated output of firm i at time t. Assumptions on the shape of the fixed cost function allow us to impose two restrictions on the admissible values of the parameters of this function: $c_1 < 0$ and $c_2 > 0$.

3.3 The Firm's Problem

In an infinite horizon game, each firm's problem is to maximize the present value of its own profits given its competitors' behavior, the government's tariff, while considering the learning effects induced by current production. This problem can be stated as:

$$\max_{x_i^t} u_i = \int_0^\infty \pi_i^t(y^t, x^t, \tau^t)e^{-rt}dt$$

$$s.t. \qquad \dot{y}_i^t = \frac{dy_i^t}{dt} = x_i^t$$

$$y_i(0) = y_i^0$$

Given the production decisions of the rest of the competitors this is a standard linear–quadratic differential game. Hence, we face a dynamic programming problem that can be solved using Pontryagin's maximum principle. The necessary conditions for this problem depend on $\hat{x}_i^t = \hat{x}_i^t(y_1^t, \ldots, y_n^t)$ and $\hat{\tau}^t = \hat{\tau}^t(y_1^t, \ldots, y_n^t)$, the optimal control for firm i and government's optimal tariff respectively, at time t[12]. This captures the interaction of firms' strategies and the government's policy over the game horizon. This effect makes firm i's co–state variable depend on the government's tariff policy and competitors' actions. In order to simplify and aggregate these first order conditions, we impose symmetry and constant conjectural variations over the horizon of the game:

$$\frac{\partial x_j^t}{\partial x_i^t} = \begin{cases} 1 & \text{if } j = i \\ \gamma & \text{if } j \neq i \end{cases}, \qquad \frac{\partial \tau^t}{\partial x_i^t} = \frac{n}{\mu_x} > 0 \quad \forall i$$

The last expression is obtained by imposing symmetry on consumers' domestic demands (5), so that $\frac{\partial \tau^t}{\partial x_i^t} = n\frac{\partial \tau^t}{\partial X^t}$, i.e., the conjectural variation of each firm's output decision on the government's optimal tariff choice are the same and, in fact, are equal to equilibrium beliefs. After rearranging the firms' necessary conditions this can be written as:

$$a_x - \frac{1}{2}\left[b_x(1+\theta) + k\frac{\mu_m}{\mu_x}\right]X^t - \frac{1}{2}kM^t(\tau^t) - c_3 + \frac{\lambda^t}{n} = 0 \qquad (9a)$$

$$\dot{\lambda}^t = r\lambda^t - (nc_1 + c_2Y^t) + \frac{1}{2}b_x n[2n\theta - \theta - 1]X^t\frac{\partial \hat{X}^t}{\partial Y^t}$$
$$+ \frac{1}{2}k\mu_m(X^t + \tau^t\mu_x)\frac{\partial \hat{\tau}^t}{\partial Y^t} \qquad (9b)$$

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[12] The complete derivation of players optimal strategies is presented in appendix A.

which uses $\sum_{j=1}^{n} \frac{\partial x_j^t}{\partial x_i^t} = 1 + \gamma(n-1)$ to define $\theta = [1+\gamma(n-1)]/n$, the aggregated version of the conjectural variation parameter. Table 1 presents the values of each conjectural variation parameter under different competition regimes.

Table 1

Regime	γ	θ
Cournot	0	$\frac{1}{n}$
Bertrand	$\left[\frac{-1}{n-1}, 0\right]$	$\left[0, \frac{1}{n}\right]$
Competition	$\frac{-1}{n-1}$	0
Collusion	1	1

3.4 The Government's Problem

The government's problem is to maximize the present value of a weighted sum of consumer surplus, industry profits and tariff revenues, given the optimal industry production strategy and considering the aggregate learning effects induced by its tariff policy:

$$\max_{\tau^t} u_{n+1} = \int_0^{\infty} \left[CS^t(y^t, x^t, \tau^t) + \alpha^t \Pi^t(y^t, x^t, \tau^t) + \beta^t R^t(y^t, x^t, \tau^t)\right] e^{-rt} dt$$

$$s.t. \qquad \dot{Y}^t = \frac{dY^t}{dt} = X^t$$

$$Y(0) = Y^0$$

Because of the demand and cost assumptions, this is also consistent with a standard linear-quadratic differential game structure given the production decisions of the industry. Now, denote by $\hat{X}^t(Y^t) = \sum_{i=1}^{n} \hat{x}_i^t(y^t)$ the optimal choice of X^t for the industry as a whole at time t. Then, the solution must satisfy the following generalized Hamilton–Jacobi conditions:

$$\frac{\partial H_{n+1}^t}{\partial \tau^t} = \Gamma_0(X^t(0), M^t(0), \alpha^t, \beta^t) + \Gamma_1(\beta^t)\tau^t + \mu_x \tilde{\lambda}^t = 0 \tag{10a}$$

$$\dot{\tilde{\lambda}}^t = r\tilde{\lambda}^t + \alpha^t(c_1 + \frac{c_2}{n}Y^t) \tag{10b}$$

As it is shown in appendix A, dynamic optimality conditions for the government are equivalent to the standard one player case. This implies that the government's necessary conditions to establish the optimal tariff are the same for either the *open loop* or the *closed loop* equilibrium of this problem.

3.5 Theoretical Results

In solving this model, we assume perfect information, which implies that each player knows the history of the game, i.e., the previous realizations of the state vectors, $y^s \in R^n$, and control variables, $(x^s, \tau^s) \in R^{n+1}$, $\forall s \leq t$. Focusing on smooth equilibria[13], a differential game equilibrium of the model is a set of functions $\{a_i^t(y^t)\} = \{(\ldots, x_i^t(y^t), \ldots), \tau^t(y^t)\}$ such that for any time and state, a player's strategy maximizes its payoff from that time on. Applying dynamic programming, the differential game equilibrium solves the generalized Hamilton Jacobi conditions. This system of partial differential equations are the first order conditions of the corresponding Hamiltonian for each player. Such a system is not easily solved except in the case of some particular functional specifications such as the linear quadratic case of this model in which the players are the government and any representative symmetric firm of the industry. Furterhmore, the motion equation must be linear and the objective function must also be quadratic in the state and control variables.

It may be useful at this point to discuss the nature of the solutions that will result from this model. The Nash equilibrium of any differential

[13] The equilibrium payoffs need to be continuous and almost everywhere differentiable functions of the state variables. This is the case for this model for any linear solution, given the cost specification (8). See Starr and Ho (1969).

game will generally depend on the structure of the players' information sets. In the *open loop* equilibria, players' information sets are limited to the initial state of the game. In this case, players simultaneously commit themselves to the entire path of actions over the game's horizon. By contrast, in the *closed loop* equilibria players have perfect recall, so that their actions depend on the complete history of the game[14], and more importantly, strategies are time consistent. In a stochastic game like this one, the state follows a Markov process in the sense that the probability distribution over next period's state is a function of the current state and actions, and hence, the history at t can be summarized by y^t. Markov strategies depend only on the state of the system and player's information sets includes only the payoff relevant history[15]. A Markov Perfect Equilibrium (MPE) is a profile of Markov Strategies that yields a Nash equilibrium in every proper subgame[16].

The above digression enables us to obtain the *open loop* and the MPE from our generalized Hamilton Jacobi conditions. For the *open-loop* case, each of these Hamilton Jacobi conditions do not differ from those of the one agent dynamic programming problem because each co state variable does not depend on the remaining players' strategies since these are simultaneously chosen at the beginning of the game. By contrast, the MPE is a Subgame Perfect Equilibria in Markov strategies, that is, strategies that only depend on the state, and which capture the interaction among players over the game horizon. In this case, co state variables depend on opponents' actions[17].

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[14] See Fudenberg and Tirole (1991, §13.4.1) and Hanig (1986, §2).

[15] See Maskin and Tirole (1993) for further details on history equivalence and Markov equilibrium properties.

[16] A perfect equilibrium of a stochastic game allows players' strategies to be a function of the entire history while Markov perfection requires that for each player and in each period the strategies be uniquely defined by the value of the history, that is, the resulting state of the game at each stage. In addition, this game is stationary because motion equations do not depend on t; the stage payoff functions are defined as the present value of functions that do not depend directly on t. See Fudenberg and Tirole (1991, §13.3.1).

[17] See Başar and Olsder (1982, §6.5) and Fudenberg and Tirole (1991, §13.3.2).

The solution to equations (9) and (10) provide the MPE directly. If we further impose $\frac{\partial x_j^t}{\partial y_i^t} = \frac{\partial \tau^t}{\partial y_i^t} = 0, \forall\, i, j, t$, these equations also solve the *open loop* equilibrium. Therefore, the optimal government and induced industry strategies can be found from conditions (9) and (10). Using (5)-(6) they may be written:

$$\hat{X}^t = \frac{2a_x - kM^t(\tau^t) - 2c_3 + \frac{2}{n}\lambda^t}{b_x(2+\theta)} \tag{11a}$$

$$\hat{\tau}^t = \frac{\Gamma_0(X^t(0), M^t(0), \alpha^t, \beta^t) + \mu_x\tilde{\lambda}^t}{-\Gamma_1(\beta^t)} \tag{11b}$$

The solution to this linear quadratic game is found by assuming that the co-state variables are linear in the state, so that there exists a closed form strategy equilibria of the game:

$$\lambda^t(Y^t) = \phi_0 + \phi_1 Y^t \tag{12a}$$

$$\tilde{\lambda}^t(Y^t) = \tilde{\phi}_0 + \tilde{\phi}_1 Y^t \tag{12b}$$

As a consequence, the optimal strategies are also linear in the state:

$$\hat{X}^t(Y^t) = \frac{2a_x - kM^t(0) - k\mu_m \dfrac{\Gamma_0(\cdot) + \mu_x(\tilde{\phi}_0 + \tilde{\phi}_1 Y^t)}{\Gamma_1(\beta^t)} - 2c_3 + \dfrac{2}{n}(\phi_0 + \phi_1 Y^t)}{b_x(2+\theta)} \tag{13a}$$

$$\hat{\tau}^t(Y^t) = \frac{\Gamma_0(\cdot) + \mu_x(\tilde{\phi}_0 + \tilde{\phi}_1 Y^t)}{-\Gamma_1(\beta^t)} \tag{13b}$$

To solve the game, differentiate the proposed solution (12) making use of the fact that $\dot{Y}^t = X^t$. Later, substitute (12) into the right hand side of (9b) and (10b) using $\hat{X}^t(Y^t)$, $\hat{\tau}^t$, $\frac{\partial X^t}{\partial Y^t}$, and $\frac{\partial \tau^t}{\partial Y^t}$ according to (13). This produces two sets of two linear equations in Y^t. Equating the coefficients of the corresponding equations generates four, nonlinear Riccati equations that determine ϕ_0, ϕ_1, $\tilde{\phi}_0$, and $\tilde{\phi}_1$. These equations must be satisfied by any linear MPE production tariff path. To obtain the *open loop* solution the procedure is similar except that $\frac{\partial X^t}{\partial Y^t} = \frac{\partial \tau^t}{\partial Y^t} = 0$ is imposed in place of (13). The Riccati equations are derived in detail in appendix A.

A general proof of the existence of this solution is provided by Lukes (1971). Given existence, we still may wonder whether there may exist another nonlinear MPE. Given the linear quadratic structure of the game, some transversality condition, and the linear specification of $\hat{X}^t(Y^t)$ and $\hat{\tau}^t(Y^t)$, this linear solution is unique within the space of analytic functions of the state variable if $T < \infty$[18], but uniqueness has not been proved for infinite horizon differential games. However, it is possible to determine whether the unique linear solution for the finite horizon case is also an MPE for the infinite horizon case.

PROPOSITION 1: *Strategies* (13) *constitute an infinite horizon MPE if* $\phi_1 \leq 0$ *and* $\hat{\phi}_1 \leq 0$.

PROOF: See appendix A.

In the infinite horizon case an MPE must also satisfy the following transversality conditions:

$$\lim_{t\to\infty} \lambda^t(Y^t)e^{-rt} = \lim_{t\to\infty} \hat{\lambda}^t(Y^t)e^{-rt} = 0 \tag{14}$$

By (12), these conditions hold whenever Y^t is bounded but this is not the case in this model. It is sufficient to assume that the optimal accumulation output path Y^t is a function of exponential order less than r. Then the transversality conditions (14) are fulfilled by implicitly imposing an upper bound for each period's production relative to the actual accumulated output. This is a reasonable restriction that will always hold when the learning effect is exhausted. Hence, provided the model's demand system is stationary, each period's production will be constant and the ratio X^t/Y^t decreases as production continues.

We are already in disposition to discuss the features of the optimal tariff policy. The results are presented in form of propositions.

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[18] See Papavassilopoulos and Cruz (1979) for a complete proof of this statement.

PROPOSITION 2: *An increasing tariff policy cannot be time consistent in an infinite horizon game.*

PROOF: Suppose that the tariff is increasing. Differentiating (13*b*) yields:

$$\frac{\partial \tilde{\tau}^t(Y^t)}{\partial Y^t} = \frac{\mu_x \tilde{\phi}_1}{-\Gamma_1(\beta^t)}$$

For this derivative to be positive, $sign(\tilde{\phi}_1) \neq sign(\Gamma_1(\beta^t))$. Substituting (5)–(6) into the implicit definition of $\Gamma_1(\beta^t)$ in (10*a*), and using $\beta^t \geq \beta > \frac{1}{2}$, it is straightforward to show that $\Gamma_1(\beta^t) < 0$. Then, in order for the optimal tariff to increase with accumulated output, it must be the case that $\tilde{\phi}_1 > 0$. But then, the conditions of Proposition 1 are violated. Hence, this tariff policy cannot be an MPE in the infinite horizon case. ■

From the proof of Proposition 1, it is obvious that Proposition 2 is also true regardless of the symmetric oligopoly assumption of the actual solution. Since the game does not have a unique endpoint in an infinite horizon set up, there may be other non linear equilibrium strategies. Proposition 2 has proved that within the space of linear strategies, an increasing tariff policy will never be a time consistent policy in the infinite horizon case, although it may constitute an MPE for some finite horizon cases[19]. The intuition is simple. If a government announces an increasing tariff, this is clearly a non-optimal strategy in the long run. When learning effects fall below some threshold, switching to a less restrictive trade policy is always a dominant strategy because it promotes competition and increases welfare by increasing consumer surplus despite a second order producer surplus loss. It is very likely that the transition rate to a less restrictive trade regime will be determined by α^t and β^t, the weights of each component of the social welfare function.

Proposition 2 applies even to the non-exhausted learning case, $y_i^\infty < y^*$. However in most industries, learning is eventually exhausted or not

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[19] The fact that this result holds for $\beta^t \geq \beta > \frac{1}{2}$ may not include the standard welfare maximizer government case, $\alpha^t = \beta^t = 1$, if $\beta > 1$ given the model parameters.

significative after some level of accumulated production. Let $T = \min_t\{t \mid y_i^t \geq y^*\}$. Beyond this point, investment considerations of output decisions dissapear and the relevant state is y^*, $\forall t \geq T$. Solving the firms' and government's problems for $y_i^t > y^*$, the MPE strategies are:

$$\hat{X}^t(Y^t) = X^t(Y^*), \qquad \tau^t(Y^t) = \tau^t(Y^*), \quad \forall t \geq T \tag{15}$$

PROPOSITION 3: *If the learning process is exhausted, $y_i^\infty > y^*$, and if the unique MPE for the finite horizon $[0, T]$ involves a non increasing tarif policy, then there exists a unique MPE for the infinite horizon game.*

PROOF: First, consider the case where $t \geq T$. Since the relevant state of the game becomes constant once learning has been exhausted, the MPE reduces to the infinitely repeated NE whose equilibrium strategies are represented by (15). Any other subgame perfect equilibrium (trigger strategy) for this infinitely repeated game does not qualify for Markov perfection. This is due to the fact that the payoff relevant history is the same over $[T, \infty)$. Hence, it is not possible to have different payoffs in each period when the state is common and strategies are state contingent. Therefore, industry output and the optimal tariff/subsidy are constant over time and are completely determined by the model's parameters so the MPE is unique over $[T, \infty)$. Second, if there exists a unique non increasing tariff policy equilibrium for $[0, T]$, it is a linear function of the state because of the linear-quadratic structure of the model. This unique non increasing tariff policy equilibrium solves the system of partial differential equations (9)–(10). It is straightforward to show that for any initial state of the game, (15) is the limit of (13) as $t \to T$. ■

Thus, if there exists a unique, non increasing tariff, linear MPE for the finite horizon in which learning occurs, there is also a unique, time consistent tariff policy for the infinite horizon game within the space of continuous strategies. Observe that the MPE strategies are continuous and generally non linear over $[0, \infty)$. Unless production and the tariff remain constant over the whole period, these continuous MPE strategies are kinked at $t = T$. This uniqueness result will be extensively used in the calibration of the model.

Time consistency of the optimal policy design has been extensively studied over the past few years. The work of Staiger and Tabellini (1987) is the most closely related to optimal tariff protection policy. In a different framework, they conclude that the optimal trade policy must be time inconsistent, which provides unexpected protection in order to maximize redistributive effects in favor of individuals with high marginal utility of income. Protection results from the government's inability to precommit to free trade. Moreover, they show that any time consistent policy involves an excessive amount of protection.

It is worth noting several differences between this work and our model. Our model does not deal with the distributive effects of tariff protection, even when they are modelled as exogenous determinants of the government's objective function. Instead, this paper presents a very particular situation where there exists an infant industry that shows important learning effects in the early stages of development. In this case, and in contrast with most of the works dealing with time consistency of optimal policies, there may be an optimal time consistent tariff policy that does not require the government's precommitment to future liberalization. This derives from the fact that the policy is a Markov strategy that depends on only one state variable which captures firms' learning effects. We have also determined the limits of a tariff policy to be time consistent in an infinite horizon framework. Therefore, we obtain the opposite result in which the policy provides excessive protection compared to Staiger and Tabellini's work. In our case a time inconsistent tariff policy provides excessive protection as compared to a time consistent one. Where the former shows an increasing tariff path, the later will (generally) decrease.

Finally, our model provides an analytic characterization of firms' strategies along an infinite horizon MPE. In general, industry output is not restricted to evolve in a particular way together with the state, but it is clearly determined for each parameterization of the model. It is plausible to expect that when learning effects are higher, production will increase faster than when learning is exhausted, but this result will depend on the model's parameters. Observe that differentiating (13a) we get:

$$\hat{x}_1 = \frac{\partial \hat{X}^t(Y^t)}{\partial Y^t} = \frac{k\mu_m\mu_x \dfrac{\tilde{\phi}_1}{\Gamma_1(\beta^t)} + \dfrac{2}{n}\phi_1}{b_x(2+\theta)}$$

Given its linear quadratic structure, the model generates a constant rate of change of industry production with respect to accumulated output. It follows that given $\beta^t > \frac{1}{2}$, industry output will always decrease with the state when $\phi_1 < 0$ and $\tilde{\phi}_1 > 0$ or when $\tilde{\phi}_1 < 0$ and ϕ_1 is negative enough; specifically $\phi_1 < -nk\mu_m\mu_x\tilde{\phi}_1/(2\Gamma_1(\beta^t)) < 0$. Therefore, even for an infinite horizon, time consistent, and linear MPE, industry production shows ambiguous dynamics[20].

4 Protection of the Spanish Iron and Steel Industry

4.1 Historical Overview

The slow development of the Spanish Iron and Steel industry started in the south of Spain, at the beginning of the second quarter of the nineteenth century. Recovery of industrial production, once Spain's economy adjusted to the definitive loss of the colonial markets, allowed development of such business. However, the increase in the production level was insufficient to provide a basis for fast development of the sector. After fifty years of upstart attempts, the industry located finally in the Basque Country, near the French border on the northern coast of Spain. This location decision resulted from the reliance of the Bessemer steel making process which turned the coal endowment of the area the best of Europe. It was not based on the existence of an entrepreneurial class or any kind of previous capital accumulation in that area.

[20] Observe that a negative sign for this derivative will ensure the fulfillment of transversality conditions (14).

English production of iron and steel mainly employed the acid Bessemer converter and the acid Martin Siemens open-heart process which could not use phosphoric iron ore[21]. This forced English firms to import iron ore from Sweden (Orebo and Norrbotten), Italy (Liguria and Elba), Algeria, and Spain. The Basque Country supplied up to one third of the English imports of iron ore (this represented up to 91% of Spanish production). Only after 1920 did the importance of the Spanish provinces decline[22]. English capital was invested in the Basque iron ore industry in order to provide the English iron and steel industry with this necessary input. Companies such as *Orconera Iron Ore Co.*, *Somorrostro Iron Ore Co.*, and sixty two others invested more than five million pounds in the Basque Country before the Great War. The importance of this investment was due not only to the high quality of the iron, but also to the "endowment complementarity" which allowed Spanish based firms to avoid backhaul problems. Trade took place in both directions: the Basque Country exported iron ore and ships carried coal at very low cost when they returned from Wales and Northeastern England. The economic integration between the Basque Country and England was stronger than it was with the rest of Spain. This process generated very important capital accumulation that resulted in rapid growth of the Spanish iron and steel industry, and in the development of important financial institutions in the last third of the century which further enhanced the industrialization process[23]. The Basque Country arose as one of two regions that sustained the economic industrialization of Spain. The other, Catalonia, specialized primarilly in textile production. The Spanish iron and

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[21] By 1900, 90% of the open-heart and 71% of the Bessemer English steel was produced using acid methods. See Carr and Taplin (1962), p.237.

[22] Basic Bessemer steelmaking declined rapidly after 1894 which made British industry more dependent on the Spanish ore. After 1890, Lancashire and Cumberland (Britain's own hematite mining districts) also began importing the Spanish ore. See Allen (1979) and Pearl (1978, §12).

[23] The owners or tenants of the iron ore mines were also the owners or major shareholders of the iron and steel firms. For more information on the relationship between iron ore exports and the industrialization of the Basque Country, see Gonález (1981) and Shaw (1977).

steel industry grew significantly since the beginning of the twentieth century but not as much as in other European regions which were also integrated with the English trade in coal and iron. The First World War temporarily increased the demand for these products. Indeed, the old furnaces in the south of Spain restarted production. And, in 1917, construction of another big firm began, but *Altos Hornos del Mediterráneo* did not operate until 1923, and then with high excess capacity.

The oligopolistic market structure evident in Spain is a common feature of the iron and steel industry elsewhere as well. However, the Spanish industry was the most concentrated of all European countries. Only in Spain did one firm produce over 60% of the nation's output. As a consequence, price agreements among producers were prevalent, beginning as early as 1886. And, partial agreements persisted until the foundation of the *Central Siderúrgica* in 1907, a producers union which controlled 100% of the Spanish production.

The development of this sector took place after the construction of the rail network (at least of its peak phase) which generated a huge demand for steel products. At that time, they were mainly supplied by French firms. Before the Liberal Revolution of 1868, steel producers had begun asking for protection. They succeeded only after over twenty years, when their interest coincided with those of other producers. Producers from several industries as well as landowners' interests influenced the Tariff Act of 1891. Given the low stage of industrialization in Spain at that time, and the loss of landowner income due to massive grain imports after 1870 (which was due to large reductions in transportation costs after the Crimean War), the idea of protecting the domestic market to facilitate industrialization became very popular. The initial protective tariff was so high that most imports dried up. But unfortunately, industrialization did not happen as fast as it was expected. Industrial profits and landowners' rents increased significantly, but this did not imply substantial growth in demand for iron and steel products. New tariffs were imposed in 1906 and 1922 in order to avoid competition from highly productive foreign firms.

Since the end of the 19th century, the demand for iron and steel goods exceeded supply. After the establishment of the protection tariff, iron and steel firms followed a less agressive production strategy and benefited from their monopolistic power. Increasing protectionism was a dominant market feature beginning in the last decade of the eighteenth century. The effective tariff level in 1913 was as high as in the rest of Europe in 1930. The iron and steel tariff was 30% higher than the mean of European tariff in 1913 and 250% higher in 1927[24].

As a result, steel producers became one of the most economically important and politically influential groups in Spain. This interest group constituted a solid organization with financial power, that turned into the leading industry asking for protection as a method of rent seeking instead of competing for foreign markets to increase the scale of production and profits as they had done before. Given the industry concentration (economic and geographic) it is reasonable to assume a high homogeneity of preferences among members although firm characteristics were quite different. The group was lead by *Altos Hornos de Vizcaya* (AHV). It is commonly accepted that *Altos Hornos de Vizcaya* behaved as price leader in the producers union *Central Siderúrgica*, and the remaining firms comprised a competitive fringe, taking prices as given. The development of the sector generated so much liquidity that it was the origin of today's most important financial institution in Spain. Political influence reached the point that some of the firm managers had simultaneous responsibilities at the government, as General Directors of Tariffs and Customs, or even as Ministers of the Government[25].

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[24] Most of the data provided here on the Spanish iron and steel industry can be found in Fraile (1991) and Nadal (1975).

[25] Pablo de Alzola, president of AHV, was also appointed president of the commission of enquiry set up in 1904 to report on the revision of the 1891 tariff. See Harrison (1978), pp.84-85.

4.2 Key Assumptions of the Model

In this section the theoretical model is justified as being applicable to the analysis of the protectionist policy of the Spanish iron and steel industry in the first third of the 20th century by tying the model's assumptions to historical facts. Though this is not a completely homogeneous period, some of the model's parameters can be estimated using information from these years that show stable patterns. Parameterization of the model will be focused on 1913, a year that represents the end of a long homogeneous period of industrial development and tariff protection. Following the order of appearance of the model's assumptions, we have:

a) No entry-exit. This hypothesis is completely satisfied before WWI. Most firms started producing in the last quarter of the 19th century. During the war, a few firms entered the industry to benefit from the strong demand in that period. And firms also took advantage of the restrictive tariff policy and the subsequent expansive economic policy of Primo de Rivera's Dictatorship. Some of these firms had their origin in previously small related business or, as in the case of Altos Hornos del Mediterráneo, in Altos Hornos de Vizcaya itself. However (as Table B.6 shows in appendix B), with this exception, none of the entrants achieved a market share above 2.5%. Hence, an analysis of the tariff policy in 1913 is consistent with the model but the implications for later periods should be interpreted more carefully.

b) Infant Industry. The Spanish firms had not exhausted the learning process, so that production decisions must also be considered investment decisions. The low mean size of the Spanish firms as compared with the world standard suggests that large cost reductions would be possible. While the international mean firms' capacity increased from 50000 tons/year to 500000 tons/years between 1900 and 1930, Spanish iron and steel average firm's capacity increased from 24000 tons/years at the beginning of the century to 66000 tons/year in 1930[26]. By the turn of the century, Spanish accumulated

[26] Moreover, as stated before, around 60% of this capacity was concentrated in *Altos Hornos de Vizcaya*. See Fraile (1991), pp.146–149.

production since 1860 reached 4786 th.tons of iron and 2787 th.tons of steel compared to the British accumulated production of 281201 th.tons of iron and 72449 thousands of tons (th.tons) of steel for the same period[27]. While this indicates the existence of unexploited static economies of scale, it also allows for the possibility of potential important learning by doing efficiency gains unless the learning curve is very steep in the first stages of industry development.

c) Learning only reduces fixed cost. Dynamic economies of scale reduces total cost in general. Economic analysis is traditionally focused on the effect of learning on marginal costs. This is the appropiate approach in a long run framework, but fixed cost reduction up to the optimal long run level is not neglected by this approach. The iron and steel industry is highly capital intensive. Furthermore, most productivity increases for variable factors over the first half of the century are explained by capital embodied technical improvements in furnaces and equipment[28]. And, one common and important feature of the iron and steel industry is the high share of fixed to total costs. Most improvements involve the employment of more fixed factors such as increasing the size and height of blast furnaces, mechanization of handling and stocking, or addition of mixers, gas cleaners, electric steelmaking, etc. Pratten (1971) has shown that scale expansion of the British industry in 1960's reduced fixed cost more than variable cost. It seems reasonable to assume that this effect is even larger in the initial stages of industrial development. However increasing productivity, which took place during the first third of the century should also be accounted for. For instance, the wage/ton ratio of cast iron ration at AHV reduced 33% between 1902 and 1921. And labor productivity in the Spanish steel industry increased 60% between 1916 and 1930 although it decreased 14% in

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[27] See Carreras (1989), pp.200-201 for the Spanish case and Mitchell (1992), pp.448-456 for the British case.

[28] See Pearl (1978) for a detailed survey of major technical advances in steelmaking between 1900 and 1950.

the iron industry[29]. Therefore, the optimal tariff policy derived herein should at least to be considered an upper bound since variable costs do decrease over this period. Using the same reasoning, calibration of the model will provide a lower bound for welfare losses due to departure from the optimal policy.

d) Constant instantaneous returns to scale. The assumption of constant marginal cost is justified by the important excess capacity that characterized the Spanish iron and steel industry during this period. Even during the peak demand war period and despite the dictatorship induced expansion of the 20s, the average production/capacity usage ratio was only 72% for iron and 67% for steel. See Table B.1.

e) Cournot competition. Economic historians commonly agree that the *Central Siderúrgica* allowed for the cartelization of the industry through price agreements. However, signed agreements involved both price agreements and market quotas according to chapter 7 of González (1985). The adopted conjectural variation approach should capture, at least in some limited sense, the strategic elements of firms' decisions indicating whether price or quantity competition was more likely to be the main control variable[30].

f) Tariff policy vs. other trade restrictive policies. Spain's iron and steel exports constituted only 5% of domestic production between 1908 1930. In the peak war demand period, it rose to 15%. However, imports amounted to 18% of domestic production for the same period reaching a maximum of 49% in 1921. These facts definitively characterized Spain as an importing country. Until the end of the Spanish Civil War, the government did not participate directly in the production of iron and steel. Nor was there a generalized subsidy policy. Instead, there existed important government contracts during this period: the replacement of the Spanish fleet after the

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[29] See Fernández de Pinedo (1992), p.136 and Fraile (1991), p.155.

[30] Fraile (1991, §5) applies the standard static Stackelberg price leader oligopoly model to explain the welfare implications of the leadership of AHV into this producers union. The major problem with his approch is that it does not account for dynamic issues that necessarily arise in a period over fifty years.

Spanish American war awarded by the Maura Government and Primo de Rivera's Public Works Policy are good examples of this[31]. However, tariff policy was by far the most extensive trade restricting policy employed.

g) Competitive foreign firms. International cartelization in the iron and steel industry was widespread by the mid twenties, but prior to the first World War, there were only minor differences between national and international prices for the European major producers (United Kingdom, Germany, France, Belgium Luxembourg)[32]. Moreover, the model assumes that strategic effects are ignored by foreign firms. This may be justified by the small size of the Spanish iron and steel industry as compared to the European market and in particular relative to the British market which was the major foreign supplier[33]. Table B.4 shows the relative size of all the Spanish imports over the British production; the mean is slightly above 1%[34].

h) Foreign firms do not reduce cost. The world steel industry expanded during the last quarter of the 19th century as a result of radical technical innovations: the Bessemer process (1869) that required high grade hematite ores, the Martin Siemens process (1869) that allowed for the use of scrap iron, and the Thomas process (1879) that made possible the

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[31] see Harrison (1978, § 5).

[32] See Svennilson (1954, §7).

[33] This is a common opinion although difficult to illustrate with appropiate data. The *Memorandum on the Iron & Steel Industry* of the League of Nations (1927) provides a joint measure for Spain and Portugal imports. Between 1913 and 1925, British iron and steel products increased its share of these two countries imports from 45.3% to 61.5%. See also Fraile (1991), p.176.

[34] This measure has to be considered an upper bound since we are accumulating all the Spanish imports on the British market. With the data provided by the Department of Overseas Trade (1928), Spanish imports from the United Kingdom represented only 0.23% of British production in 1913 and 0.36% in 1928. According to the League of Nations (1927), the share of Spanish purchases of British exports increased from .86% in 1913 to 1.37% in 1925.

use of phosphorous ores. Altogether, these represented 90% of European production at the beginning of the war. The increase in iron and steel production between 1900 1950 was due to no such striking inventions but rather to increases in the scale of operation: improvements of larger furnaces and associated equipment, a much higher driving rate, developments in the manufacture of coke, the utilization of waste gas and greater fuel economy, and the discovery, exploitation, and transportation of more plentiful, richer, and cheaper ores. The American industry was by far the innovative leader. However, most of these technological advances can be ignored for the present case study. The technological gap was particularly important between the American and the British iron and steel industry[35]. The British industry (Spain's major foreign supplier) was characterized by a low degree of concentration and a slow technological diffusion speed which made it clearly obsolete as compared to its Continental competitors by the outbreak of the war[36]. In addition to the increase in production of steel products, there was a major process of substitution of steel for wrought iron. This important factor of modernization in the steel industry had been already exhausted by the British industry in 1913[37]. Finally, most innovations took place after the war, so that this assumption may be reasonably applied for the analysis of 1913[38].

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[35] As a matter of fact, by 1900 the American/British ratio of labor productivity was 2.5:1 for open heart and 6:1 for Bessemer steel.

[36] See Carr and Taplin (1962, §22), Pearl (1978), and Svennilson (1954, §7).

[37] This substitution process was particularly early and sharp for the British industry. Between 1883 and 1895 the percentage of pig iron converted into wrought iron shrinked from 70% to 5%. See Schubert (1978), p.61.

[38] For instance, large scale exploitation of Lake Superior ore started in 1892 and by 1900 it supplied 75% of iron ore requirements of the American industry. Coke consumption per ton of pig iron reduced from 21.5 cwt (hundredweight = 112 pounds in U.K.) to 13 cwt between 1900 and 1950 but the mean British consumption exceeded 30 cwt between 1900 and 1930. By–products methods of coke production (tar, benzole, ammonia, and naphta) were not generally in use until the end of World War I. Technical difficulties in increasing the size and driving rate of blast-furnaces were not overcome until that date either. Agglomeration of iron

i) Product differentiation. This is a difficult point to justify because of the lack of accurate and detailed information. Spain can be considered a small importing country with a low development level during the relevant period. Tables B.3 and B.4 show the relative importance of Spanish production and the level of imports and exports relative to domestic production. As can be seen, the importance of exports declines. It is realistic to assume that import products were those technically more difficult to produce. As tariffs increased, imports of elaborated products (e.g., from Germany and Belgium) fell sharply according to the League of Nations (1927).

j) Symmetry. It is evident that firms in the Spanish iron and steel industry were not homogeneous. This was discussed earlier and data is detailed in Table B.6. The largest firm by far was *Altos Hornos de Vizcaya* (AHV) established in 1902 as a merger between *Altos Hornos de Bilbao*, *La Vizcaya*, and *La Iberia*. Its market share fell from 71% to 52% between 1900 and 1930[39]. This is the main feature distinguishing the Spanish industry from the rest of the European industries: It was highly concentrated around one only firm. However, we analyze the symmetric case by using a hypothetical symmetric firm industry that equals the actual asymmetric solution. For each N firm asymmetric equilibrium there is an equivalent hypothetical n-firm symmetric equilibrium that is defined through the Herfindahl Index. Let s_i^t be the firm i's actual share of market sales at time t. Then the hypothetical n^t is defined by:

$$H^t = \sum_{i=1}^{N^t} (s_i^t)^2 = \sum_{i=1}^{n^t} \frac{1}{(n^t)^2} = \frac{1}{n^t}$$

This method is also used by Dixit (1985). Table B.6 reports Herfindahl indexes for different years and markets. They will be used later in the parameterization of the model.

ore by heat treatment began before 1900 but pelletizing of fine ores was developed by 1936, and only 14% of the burden charged to British blast-furnaces by 1950 was sintered. Lastly, automatic charging did not start until mid 1920s.

[39] The decrease is not that important if we consider that *Altos Hornos del Mediterráneo* was linked to AHV.

k) No spillovers. According to Fraile (1991), p.145, Spanish producers knew about innovations in the industry and international transfer of technology was allowed most of the time. However, location, product specialization and differences in scale among firms justify the assumption that learning economies did not spread through the industry[40].

5 Model Calibration

All data used to estimate the parameters of the model are detailed in appendix B. Since there is not data available on tariff revenues by products, X, the production of iron and steel is derived by adding up the Spanish production of iron (IRON) and steel (STEEL). Imports of iron and steel products, M, are obtained in the same way (SMI+SMS). Domestic and foreign prices for these aggregated goods are computed as weighted averages of the reported prices for each good (SPIP, SPSP, UKIP, UKSP). A representative measure of the applied tariff is added to the weighted average of foreign iron and steel products in order to obtain their value in Spain's domestic market. Over the first third of the century, the 1891, 1906, and 1922 Tariff Acts apply. We choose the 2nd tariff on the item: "Wrought Iron and Steel Products which exceeds 100 Kg." as the repesentative tariff for each subperiod. It has values of 50, 100, and 200 Pts./ton[41]. Table 2 shows the estimates of the model parameters.

Demand equations (3) – (4) have been estimated for 1907–1928 by Iterative Three Stage Least Squares constrained to satisfy symmetric cross effects. The price of iron and steel products shows two clearly different

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[40] González (1985, §4) provides a complete descriptive analysis on differences in production, labor productivity, prices, and product specialization of Spanish iron and steel industry between 1870–1913.

[41] We chose the 2nd tariff instead of the 1st because it applied to most European countries for which trade agreements were established. Also, the ratio between tariff revenues from imports of iron and steel products and quantity imported is 92.83 Pts./ton for 1913 according to the Estadísticas de Comercio Exterior de España. The chosen tariff shows the closest approximation to this ratio.

Table 2

Parameter	Estimate	t statistic
a_x^0	615.714501	4.231574
a_x^1	274.016070	3.040705
b_x	0.990809	2.656650
k	0.450731	1.690828
a_m^0	500.114550	2.874538
a_m^1	345.732890	1.978608
b_m	1.775377	0.572433
c_0	658.429820	0.747573
c_1	-0.185449	-0.908474
c_2	2.092E-5	1.189104
c_3	1.409824	0.638194

patterns over this sample: they steadily grow up until the end of the 1910s but sharply decline since the beginning in 1920. A dummy variable has been introduced in order to capture this effect so that each demand intercept is a_i^0 up to 1919 and $a_i^0 + a_i^1$ from 1920 on. As can be seen, all parameters are positive and the concavity condition is fulfilled, $DetH[U(X^t, M^t)] = 0.389 > 0$. Cost parameters are obtained by estimating (8) using OLS for 1902 1916. The estimation is referred to AHV. Data on cost is not available; it has been inferred from the data on profits and estimated sales[42]. Cost parameters are not significant but they will be taken here as given in order to illustrate the model. Sign restrictions are fulfilled resulting in a convex decreasing fixed cost function. In addition to these estimates, it is necessary to specify the following model parameteres: $r = 0.04$ is the average return of the Spanish Public Debt between 1900 and 1923, $n = 124$ is the inverse Herfindahl Index

[42] AHV's production of iron and steel, x_i^t, is computed as a share of the Spanish total production. This share evolves linearly according to the domestic market shares provided in Table B.6 for few years. Accumulated output, y_i^t adds each year production on an initial value of 5534.4515 th.tons, i.e., 79.61% of the iron and 61.87% of the steel produced in Spain between 1860 and 1900.

for year 1913 as shown in Table B.6. finally, $X^t(0)$ and $M^t(0)$ are computed in each case by solving $(3) - (4)$ for $\tau^t = 0$. Furthermore, α and β, the weights of the planner's social welfare function, are constrained to be equal to one for most of the cases.

Table 3 shows the solutions of the model under different assumptions. Parameters ρ_0, ρ_1, $\tilde{\rho}_0$, and $\tilde{\rho}_1$ are the unknowns of the Riccati equations as defined in appendix A[43]. $\hat{x}_0$, $\hat{x}_1$, $\hat{\tau}_0$, and $\hat{\tau}_1$ are respectively, the intercepts and slopes of the optimal aggregate production and tariff strategy as implicitly defined by equation (13). Welfare components are shown in 1913 millions of Pts.; $\tau(Y)$ is Pts./ton of iron and steel imports. Optimal domestic production and imports are measured in thousand of tons. Finally, I_W is an index of the relative (static) welfare for each situation. The reference value corresponds to $\theta = 0.463/124$. The bottom of Table 3 presents the best response strategies and induced welfare components for the actual accumulated output in 1913 which is 16712.1 th.tons.

The conjectural variations approach followed here allows us to determine for which value of θ firms' perceived marginal revenue equals their marginal cost. In this case is $\theta = 0.463/124$, the value for which optimal domestic production equals actual domestic production for 1913. Tables C1 to C5 in appendix C presents different stages of computation that approximate this result. They also illustrate the nature of the solution as θ varies between 0 and 1. It is worth noting that the chosen value of θ corresponds to a fairly competitive regime according to Table 1. This is the first important empirical result of this paper: it contradicts the general belief about competitiveness of the Spanish iron and steel industry in the first third of the century.

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[43] Riccati equations have been solved using version 2.0 of Mathematica. The estimates of Table 2 were computed using version 7.03 of MicroTSP.

Table 3. Solutions

	θ	α	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\tilde{\tau}_0$	$\tilde{\tau}_1$
Open Loop	0	1	-567.45	0.000523	4.58	-0.000004	490.578	0.000003	60.368	-0.000002
	0	1	5.14E+08	4.346312	-4.14E+06	-0.035051	2.95E+06	0.024975	-1.89E+06	-0.015945
	0.5	1	-568.94	0.000523	4.59	-0.000004	490.569	0.000003	60.373	-0.000002
	0.5	1	6.42E+08	5.433021	-5.18E+06	-0.043815	3.69E+06	0.031220	-2.36E+06	-0.019932
	1	1	-569.93	0.000523	4.60	-0.000004	490.564	0.000003	60.377	-0.000002
	1	1	7.71E+08	6.519729	-6.22E+06	-0.052578	4.43E+06	0.037464	-2.83E+06	-0.023919
	$0.463/n$	1	-567.46	0.000523	4.58	-0.000004	490.578	0.000003	60.368	-0.000002
	$0.463/n$	1	5.15E+08	4.354427	-4.15E+06	-0.035116	2.96E+06	0.025022	-1.89E+06	-0.015975
Closed Loop	$0.463/n$	1	35130.80	-4.043018	14473.05	-0.553800	2274.800	-0.085984	6642.245	-0.251930
	$0.463/n$	1	9.54E+08	4.737284	-21808.27	-0.000113	6.30E+06	0.031298	-9862.550	-0.000051
	$0.463/n$	1	67883.31	0.000523	4.52	-0.000004	942.982	0.000003	60.342	-0.000002
	$0.463/n$	1	71266.36	11.434891	4990.25	0.402153	1498.868	0.118611	2328.409	0.182944
	$0.463/n$	0.843037	139408.35	-6.731636	3.44	-0.000146	3317.869	-0.324110	5516.557	-0.324110
	$0.463/n$	0.843475	148434.78	-6.965996	3.18	-0.000138	8 3380.493	-0.152153	5479.524	-0.330485

	θ	β	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
Open Loop	0	1	490.63	60.34	49.45	66.18	98.11	2.98	167.27	63.79
	0	1	2.95E+06	-1.89E+06	2.40E+06	6.32E+09	-5.92E+09	4.53E+09	-4.13E+09	-1.57E+09
	0.5	1	490.62	60.34	49.44	66.18	98.11	2.98	167.26	63.78
	0.5	1	3.69E+06	-2.36E+06	3.00E+06	9.88E+09	-9.25E+09	-7.08E+09	-6.45E+09	-2.46E+09
	1	1	490.61	60.34	49.44	66.17	98.11	2.98	167.26	63.78
	1	1	4.43E+06	-2.83E+06	3.60E+06	1.42E+10	-1.33E+10	-1.02E+10	-9.29E+09	-3.54E+09
	$0.463/n$	1	490.63	60.34	49.45	66.18	98.11	2.98	167.27	63.79
	$0.463/n$	1	2.96E+06	-1.89E+06	2.41E+06	6.34E+09	-5.94E+09	-4.54E+09	-4.14E+09	-1.58E+09
Closed Loop	$0.463/n$	1	837.83	2431.96	0.00	173.88	88.36	0.00	262.24	100.00
	$0.463/n$	1	6.30E+06	-9863.41	12688.47	9.85E+09	-1.97E+10	-1.25E+05	-9.83E+09	-3.75E+09
	$0.463/n$	1	943.03	60.31	49.48	231.89	49.66	2.98	284.53	108.50
	$0.463/n$	1	3481.11	5385.79	0.00	3001.68	-3943.48	0.00	-941.79	-359.14
	$0.463/n$	0.500102	837.70	100.00	0.00	173.82	88.39	0.00	262.21	99.99
	$0.463/n$	0.500095	837.70	-43.57	181.78	222.81	54.07	-7.92	268.96	102.56

There are two *open loop* equilibria for each value of θ. None of these solutions fulfill the requirements of Proposition 1 since ϕ_1 is always positive. *Open loop* solutions would need an additional institutional or technological source of commitment to be considered a credible equilibria. This source of commitment is difficult to find in the present framework. And, in addition, it would always lead to time inconsistent policies under the actual parameterization of the model. Therefore, we will focus on *closed loop* equilibria for the remainder of the paper. In order to ease the exposition, we will rely on the following technical result.

PROPOSITION 4: *For $\theta = 0.436/124$ and the estimated model parameters, there are four possible values of ϕ_0, ϕ_1, $\tilde{\phi}_0$, and $\tilde{\phi}_1$ that solve the MPE, but only one induces a time consistent tariff policy in an infinite horizon game. However, there is not any such time consistent equilibria for the symmetric Cournot oligopoly.*

PROOF: See appendix A.

Table 3 shows these four solutions for $\theta = 0.436/124$. Only one of them is such that $\phi_1 < 0$ and $\tilde{\phi}_1 < 0$. Therefore, in accordance with Proposition 3, there is a unique time consistent tariff policy for the infinite horizon game within the space of continuous strategies.

The behavior of the time consistent solution as a function of θ is depicted in Figure 1. Several iterative computations show that the solution of the Riccati equations is not time consistent in a narrow neighborhood of the symmetric Cournot oligopoly solution defined by $\theta = 1/124$. In particular, the solution is not time consistent between $\{\theta \mid \phi_1 = 0\} = 0.49395/124$ and $\max_\theta\{\theta \mid \tilde{\phi}_1 = 0\} = 1.51215/124$. Since the value of θ that equals the perceived marginal revenue and marginal cost falls outside this interval, we can rule out this interval of the conjectural variations parameter from consideration.

As Table 3 shows, the optimal level of tariff protection for the time consistent policy is much higher than that actuallly representative of 1913. This second empirical result also contradicts the traditional view of this

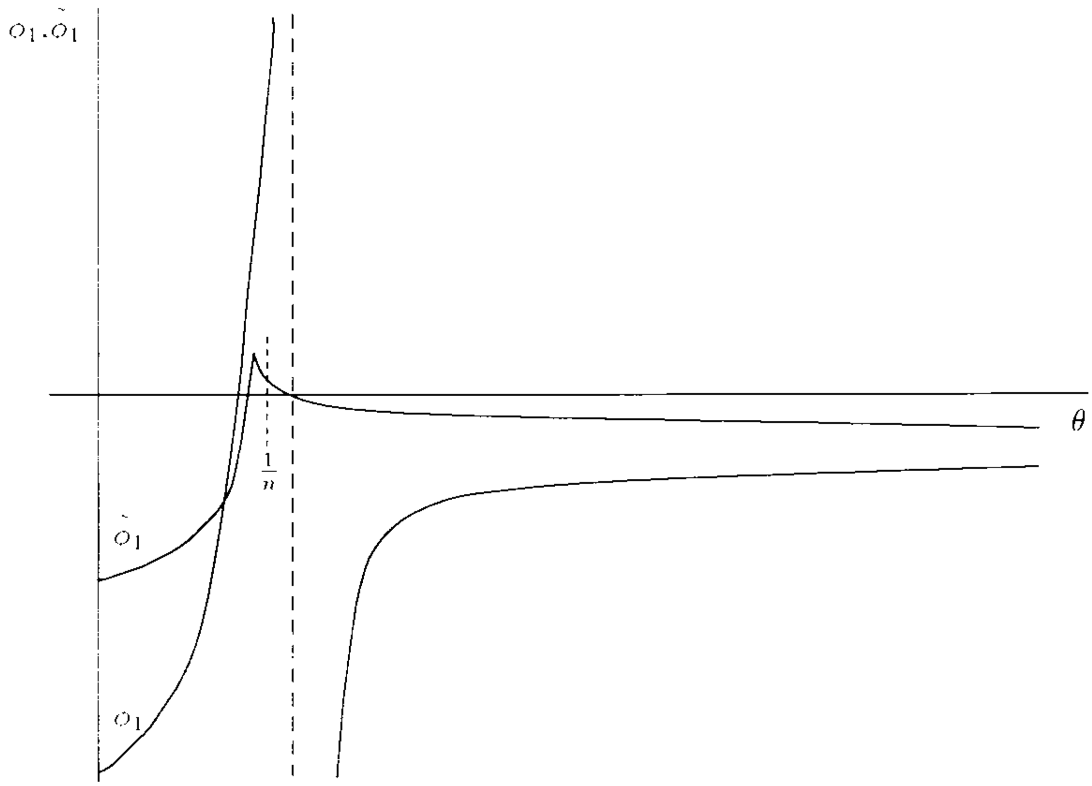

Figure 1

problem. However, this result was expected from the argument about the small average size of the Spanish firms compared to the European standards at that time. Using (6) and the model parameterization, it follows that any tariff level above 99.161 Pts./ton is prohibitive. Similarly, from (8) and the symmetry assumption, the aggregate level that for which learning is exhausted is $Y^* = -nc_1/c_2 = 1099220$ th.tons. Therefore, since learning is far from being exhausted (the aggregate accumulated production for year 1913 is 16712.1 th.tons), there still exists huge economies to exploit.

This model also allows us to measure distributional effects through the computation of α and β. Fraile (1991) justifies the traditional explanation of a "captured government" by iron and steel producers. The argument relies on arbitrary measures of pressure group's power. He also argues that tariff policy was used primarily as an instrument to raise revenues for the government as opposed to being ued to maximize welfare. None of these hypothesis are supported by the present calibration of the model. The advantage of the present method is that it obtains α and β as the result of intertemporal optimization by consumers, producers, and the government.

To compute α and β, in addition to the four Riccati equations (*A*.1) of appendix A, (13*a*) and (13*b*) must be included and equated to actual production and tariff levels. This is illustrated in the last two lines of Table 3. In the first of these two lines, production is fixed at 837.7 th.tons and tariffs are fixed at 100 Pts./ton, the actual levels for 1913. In addition to the effects on the rest of variables, it should be noted that both α and β are lower than 1. This result leads to the astonishing conclusion that the government is captured by consumers instead of by producers, and that it underweights tariff revenues into its objective welfare function.

However, in this case, the optimal tariff level is slightly above the prohibitive level. While this result is quite accurate for iron products, it does not match well for imports of steel products, which reached up to 26.48% of domestic production in that year. The model is then calibrated on the last line for a different scenario where (13*a*) is equated to the level of production for 1913, and (13*b*) is equated to a tariff level that generates the actual imports of iron and steel products for that year. Since a subsidy is required to generate 1913 levels of imports under the present parameterization of the model, consumer surplus rises while profits decline. This is due to the reduction in prices of iron and steel products forced by foreign competition.

Observe that the distributional parameters α and β remain quite stable even under such different scenarios. Nevertheless, these results must be interpreted carefully. Between these two scenarios τ goes from above the prohibitive level to negative values. The structure of the demand system may be driving this result because the actual estimation of demand parameters allows a relatively low prohibitive tariff level. In the first scenario β could be very low because τ is so high that imports are restricted, and tariff revenues are zero. By contrast, in the second scenario, β could be very low because τ has to be negative in order to allow for imports to reach their actual 1913 levels. However, this caveat does not apply to α. Under both scenarios, the applied tariffs are very low as compared to the socially efficient tariff level, so that it is difficult to argue that the government was captured by producers. On the contrary, since imports were in fact allowed while there still existed huge learning economies to exploit, we must conclude that the government was captured by consumers. This is correctly reflected in the estimates that imply an 18% higher valuation of consumer surplus over producer's profits in the social welfare function.

To complete the analysis, we must address how changes in the model's parameters affect the optimal production decision and tariff policy. In appendix C, tables C.5 through C.15 detail the effects of changes in the model's parameters (except c_0 which does not enter into the Riccati equations), for a neighborhood of $\pm 25\%$ of their estimated value. Even in the face of all these changes in the parameter values, the optimal tariff remains above the prohibitive levels in each case, so that optimal imports and tariff revenues are zero. All the comparative statics exercises are monotone along the range of variation in each parameter. Table 4 summarizes these results. This table shows the arc–elasticity of each item with respect to several parameters evaluated in a neighborhood of 5% about their estimated values.

Table 4. Comparative Statics

	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$CS(Y)$	$\Pi(Y)$	$W(Y)$
a_r	9.995		85.025		228.077	230.220	450.298	119.575	341.062
b_r	5.882	99.715	110.412	212.330	-280.868	-169.758	-457.167	195.667	-245.744
k	-12.596	0.032	-230.751	-225.317	-7.266	-135.465	-14.532	14.898	-4.623
a_m	-2.054		-17.530		-47.016	-47.457	-93.979	96.416	-29.940
b_m	6.978	-0.016	121.418	113.050	18.766	132.994	37.530	-38.276	11.958
c_1	-0.481		-0.199		-0.501	0.539	-1.003	4.533	0.863
c_2	-0.229	0.269	-0.223	0.121	-0.762	-0.813	-1.524	1.533	-0.494
c_3	0.042		-0.130		-0.384	-0.378	-0.768	-0.551	-0.695
r	-147.567	99.461	-48.889	99.758	-301.257	-303.955	-589.147	698.502	-193.274
α	-5.743	0.014	-3.919	0.006	-7.215	8.224	-14.429	14.765	-4.593
β	1.844	-0.029	201.286	199.988	2.449	2.815	4.897	-5.007	1.559

A shift in demand for domestic goods, a_x, increases the domestic market profitability so that optimal production is also higher. It also raises the optimal tariff by the same proportion as the increase in demand in order to mantain this increase in demand served by domestic firms and to extend the learning effect. All of the components of welfare increase but most of the gain accrues to consumers[44].

Since $b_x > 0$, the steeper is the demand for domestic goods, the lower is consumer willingness to pay. The argument is the opposite of the previous discussion of a_x, as is the effect on the optimal prouction and tariff levels. Welfare decreases, but while a decrease in the slope of domestic demand sharply reduces the consumer surplus, profits increase. This is easily explained by the elasticity of demand for domestic goods with respect to its own price which is less than 1. The reduction in domestic production without allowing imports raises producers revenues by a larger percentage, and therefore, profits increase.

Exactly the same argument applies to k, the degree of substitution between the domestic and foreign produced goods. The difference relies only on the magnitude of the effect of changes of parameters. An increase in the degree of substitution reduces consumer willingness to pay for domestic goods less than a decrease in the slope of domestic demand given the estimates of Table 2. This increase in the degree of substitution slightly reduces optimal domestic production, which allows for an important reduction in tariffs, even though consumers are now more willing to buy foreign goods. Profits increase by the same proportion as consumer surplus falls due to the inelastic domestic demand. Overall, welfare decreases.

Changes in demand parameters for foreign goods have just the opposite effects of the correspondig parameters on demand for domestic goods. An increase in a_m and a decrease in b_m raise the willingness to pay for foreign

---

[44] ϕ_1 and $\hat{\phi}_1$ are invariant to a_x because it does not enter the second and fourth Riccati equations of (A.1). Consequently, the slopes of the optimal linear strategies for both, production and tariff, are also unaffected. The same applies to a_m, c_1, and c_3.

goods. Since the demand system has only two goods, this implies that they lower the willingness to pay for domestic goods. The absolute values of the effect of changes in a_m and b_m on welfare components are significatively lower than the corresponding changes in a_x and b_x.

The following set of parameters refer to the cost function. All of them are inversely related to the optimal production decision, and therefore also to the optimal tariff and consumer surplus. Compared to the magnitude of demand parameters, cost parameters' effects are of second order. When the speed of learning increases ($c_1 < 0$ becomes smaller) production increases. Consumers benefit from a lower cost supply of domestic goods even when the tariff has raised. The reduction in the price of the domestic good also reduces producers profits because of the inelasticity of demand. This reduction offsets the increase in consumer surplus. An increase in the convexity of the fixed cost function c_2 causes cost to decline sharply, hence producers benefit more while learning lasts. However, the increase in profits does not compensate for the reduction in consumer surplus. Finally, any reduction in the marginal cost, c_3, increases optimal production levels, consumer surplus and even producers' profits.

Any change in the interest rate generates the most dramatic changes in all of the items reported in Table 4. Production is strongly and inversely related to r. An increase in r reduces production levels and consumer surplus, while profits raise sharply. As has been shown to be typical, the effect on consumer surplus dominates. The same happens to α but on a smaller scale, and the opposite holds for changes in β.

6 Concluding Remarks

The main contribution of this paper is to show that there exists an optimal time consistent tariff policy which ensures maximization of a discounted welfare function when there exists learning effects. Assuming that learning is limited to fixed cost reduction and that demand follows a simple linear structure, the optimal equilibrium strategies have been derived in closed form. This result allows us to prove the intuitive result that any time consistent tariff policy must involve a decreasing tariff in order to compensate for the exhaustion of the learning process with foreign competition. The optimal policy balances the actual loss in consumer surplus with future gains from lower costs, and when learning is exhausted, the excessive monopoly power of the domestic firms is offset by higher foreign competition.

The model has been applied to the case of the Spanish iron and steel industry before the outbreak of WWI. The theoretical assumptions of the model have been linked to this particular case. The calibration of the model shows surprising results as compared to the established interpretation of this problem. First, firms seems to behave more competitively than expected. Second, the learning effect is so far from being exhausted that even a higher tariff than the one actually applied is required. Third, the hypothesis that the government is captured by producers and that tariff revenues are used by the government as a way of rent seeking are rejected.

The usual caveats to any calibration exercise apply to the present work. This exercise should be understood as an illustration of the model, and its implications must be interpreted carefully. More general specifications of demand and cost functions may result in a better description of the facts that we addressed, but this can not be executed without incurring in some cost. Hence, for example, the more commonly considered case of declining marginal cost or a nonlinear demand system could be solved numerically as in Pakes and McGuire (1992). The disadvantage of this is that it is not possible to obtain analytic MPE strategies, so that the characterization of the time consistent tariff policy will be more cumbersome to obtain, if possible at all. Finally, a more detailed data set is required to improve the value of the empirical findings of the calibration exercise. This is especially the case for cost data.

Appendix A: Technical Results

<u>*Derivation of Optimal Controls*</u>

Given equations (3) (6) and (8), each firm's stage profit function is:

$$\pi_i^t = (a_x - \frac{1}{2}b_x \sum_{j=1}^{n} x_i^t - \frac{1}{2}kM^t)x_i^t - (c_0 + c_1 y_i^t + \frac{1}{2}c_2(y_i^t)^2 + c_3 x_i^t)$$

From this, the current Hamiltonian for firm i is:

$$H_i^t = (a_x - \frac{1}{2}b_x \sum_{j=1}^{n} x_i^t - \frac{1}{2}kM^t)x_i^t - (c_0 + c_1 y_i^t + \frac{1}{2}c_2(y_i^t)^2 + c_3 x_i^t) + \lambda_i^t x_i^t$$

Observe that the coefficient of $(x_i^t)^2$ is $-\frac{1}{2}b_x < 0$ which ensures that player i's optimal control is well defined. The solution must satisfy the following generalized Hamilton Jacobi conditions:

$$\frac{\partial H_i^t}{\partial x_i^t} = \left(a_x - \frac{1}{2}b_x X^t - \frac{1}{2}kM^t\right) - \frac{1}{2}b_x\left(\sum_{j=1}^{n}\frac{\partial x_j^t}{\partial x_i^t}x_i^t\right)$$

$$+\frac{1}{2}k\mu_m\frac{\partial \tau^t}{\partial x_i^t}x_i^t - c_3 + \lambda_i^t = 0$$

$$\dot{\lambda}_i^t = r\lambda_i^t - \frac{\partial H_i^t}{\partial y_i^t} - \sum_{j\neq i}^{n}\frac{\partial H_i^t}{\partial x_j^t}\frac{\partial \hat{x}_j^t}{\partial y_i^t} = r\lambda_i^t - (c_1 + c_2 y_i^t)$$

$$-\sum_{j\neq i}^{n}\left[-\frac{1}{2}b_x\left(x_i^t\sum_{h=1}^{n}\frac{\partial x_h^t}{\partial x_j^t} + X^t\frac{\partial x_i^t}{\partial x_j^t}\right)\frac{\partial \hat{x}_j^t}{\partial y_i^t}\right] + \frac{1}{2}k\mu_m\left(x_i^t + \tau^t\frac{\partial x_i^t}{\partial \tau^t}\right)\frac{\partial \hat{\tau}^t}{\partial y_i^t}$$

Imposing symmetry, $x_i^t = x_j^t, \forall i, j, t$ enables us to use the following identities:

$$X^t = nx_i^t \qquad Y^t = ny_i^t \qquad \lambda^t = n\lambda_i^t$$

Moreover, given a fixed number of firms n, symmetry also implies:

$$\dot{Y}^t = n\dot{y}_i^t = nx_i^t = X^t$$

$$\dot{\lambda}^t = n\dot{\lambda}_i^t$$

$$\frac{\partial \hat{X}^t}{\partial Y^t} = \frac{\partial(n\hat{x}_i^t)}{\partial y_i^t}\frac{\partial y_i^t}{\partial Y^t} = \frac{\partial \hat{x}_i^t}{\partial y_i^t}$$

$$\frac{\partial \hat{\tau}^t}{\partial Y^t} = \frac{\partial \hat{\tau}^t}{\partial y_i^t}\frac{\partial y_i^t}{\partial Y^t} = \frac{1}{n}\frac{\partial \hat{\tau}^t}{\partial y_i^t}$$

Using these expressions and firms' conjectural variation parameters, equation (10) is obtained by aggregation of the above necessary conditions.

We also must address the welfare function. The weights of the government's objective function, α^t and β^t, are assumed to be piecewise continuous functions of time. The components of the government's objective function are written as follows:

$$
\begin{aligned}
CS^t(y^t, x^t, \tau^t) =& \frac{1}{2}(b_x X^t + kM^t)X^t + \frac{1}{2}(kX^t + b_m M^t)M^t \\
& - \frac{1}{4}\left[b_x(X^t)^2 + b_m(M^t)^2 + 2kX^tM^t\right] \\
\Pi^t(y^t, x^t, \tau^t) =& n\pi_i^t(y^t, x^t, \tau^t) = (a_x - \frac{1}{2}b_x X^t - \frac{1}{2}kM^t)X^t \\
& - (nc_0 + c_1 Y^t + \frac{1}{2n}c_2(Y^t)^2 + c_3 X^t) \\
R^t(y^t, x^t, \tau^t) =& \tau^t M^t
\end{aligned}
$$

Therefore, the current Hamiltonian for the government becomes:

$$
\begin{aligned}
H_{n+1}^t =& \frac{1}{2}(b_x X^t + kM^t)X^t + \alpha^t[a_x - \frac{1}{2}b_x X^t - \frac{1}{2}kM^t]X^t \\
& - \frac{1}{4}\left[b_x(X^t)^2 + b_m(M^t)^2 + 2kX^tM^t\right] + \frac{1}{2}(kX^t + b_m M^t)M^t \\
& - \alpha^t[nc_0 + c_1 Y^t + \frac{1}{2n}c_2(Y^t)^2 + c_3 X^t] + \beta^t\tau^t M^t + \tilde{\lambda}^t X^t
\end{aligned}
$$

Differentiate this Hamiltonian to obtain:

$$
\begin{aligned}
\frac{\partial H_{n+1}^t}{\partial \tau^t} =& \alpha^t\mu_x(a_x - c_3) + [(\frac{1}{2} - \alpha^t)b_x\mu_x - \frac{1}{2}(1 - \alpha^t)k\mu_m]X^t(0) \\
& + [\frac{1}{2}(1 - \alpha^t)k\mu_x - \frac{1}{2}b_m\mu_m + \beta^t]M^t(0) + \mu_x\tilde{\lambda}^t \\
& + [-\frac{1}{2}k\mu_x\mu_m + \mu_m(\frac{1}{2}\mu_m b_m - 2\beta^t)]\tau^t = 0 \\
\dot{\tilde{\lambda}}^t =& r\tilde{\lambda}^t - \frac{\partial H_{n+1}^t}{\partial Y^t} - \frac{\partial H_{n+1}^t}{\partial X^t}\frac{\partial \hat{X}^t}{\partial Y^t} = r\tilde{\lambda}^t - \frac{\partial H_{n+1}^t}{\partial Y^t} - \frac{\partial H_{n+1}^t}{\partial \tau^t}\frac{\partial \tau^t}{\partial X^t}\frac{\partial \hat{X}^t}{\partial Y^t}
\end{aligned}
$$

where $\Gamma_0(X^t(0), M^t(0), \alpha^t, \beta^t)$ and $\Gamma_1(\beta^t)\tau^t$ are implicitly defined by the first optimality condition. Then, equation (10) follows from the above specification of government's Hamiltonian. Observe that the Envelope Theorem can be used to simplify the second generalized Hamilton Jacobi condition by substituting with the the first one, so that the government's co state variable does not depend directly on firms actions.

For this problem to be well defined, the coefficient of $(\tau^t)^2$ must be negative. Substituting (5) (6) into the current Hamiltonian expression and grouping terms, it can be shown that this condition holds whenever:

$$\beta^t \geq \beta = \frac{1}{2}\frac{b_x b_m}{b_x b_m - k^2} > \frac{1}{2}$$

Observe that β is determined entirely by demand parameters, so that, given the concavity of the utility function, it is possible that $\beta > 1$. Therefore the government's welfare maximization problem may not be well defined. In the present study case $\bar{\beta} = 0.56529$.

Proof of Proposition 1

For convenience, denote by e_i a $(n+1)\times 1$ vector with all its elements equal to zero except the i th, which is one. Similarly V_i denotes a null $(n+1)\times(n+1)$ matrix with a unit element in the i th position of the diagonal:

$$e_i' = (0, \cdots, 0, 1, 0, \cdots, 0), \qquad V_i = e_i \cdot e_i'$$

Drop the time superscripts for notational simplicity. Any finite horizon linear quadratic differential game can be written in matrix form as:

$$u_i = \int_0^T \left(\frac{1}{2} y' Q_i y + \frac{1}{2}\sum_{j=1}^{n+1} x_j' R_{ij} x_j + \frac{1}{2}\sum_{j=1}^{n+1} r_{ij}' x_j + q_i' y + f_i \right) e^{-rt} dt$$

$$+ \frac{1}{2} y'(T) S_i y(T)$$

$$\dot{y} = Ay + \sum_{j=1}^{n+1} B_j x_j$$

In general, the proposed linear solution (12) is:

$$\begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ \lambda_{n+1} \end{pmatrix} = \begin{pmatrix} o_{01} \\ \vdots \\ o_{0n} \\ \tilde{o}_0 \end{pmatrix} + \begin{pmatrix} o_{11} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & o_{1n} & 0 \\ 0 & \cdots & 0 & \tilde{o}_1 \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \\ Y \end{pmatrix} = \Omega_0 + \Omega_1 \begin{pmatrix} y \\ Y \end{pmatrix}$$

Next, define the following matrix of net effects of the state variables over the control variables:

$$Z = A - \sum_{i=1}^{n+1} B_i R_i^{-1} B_i' \Omega_i$$

Each of these matrices may be identified for the present model. A is an $(n+1) \times (n+1)$ null matrix. R_i is a diagonal matrix because each player only has one control variable; its elements are equal to $-b_x < 0$ for $i = 1, \ldots, n$, and $\Gamma_1(\beta^t) < 0$ for $i = n+1$ (see the derivation of optimal control paths in this appendix). $B_i = e_i$ in the symmetric and no spillover case for $i = 1, \ldots, n$, and $B_{n+1} = \mu_x / n > 0$ by symmetry and equation (5). Finally, $\Omega_i = \Omega_1 V_i = o_{1i} V_i$ for $i = 1, \ldots, n$. Therefore:

$$Z = \sum_{i=1}^{n} \frac{o_{1i}}{b_x} V_i - \frac{\mu_x}{n} \Gamma_1(\beta^t) \tilde{o}_1 V_{n+1} = \begin{pmatrix} \frac{o_{11}}{b_x} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \frac{o_{1n}}{n} & 0 \\ 0 & \cdots & 0 & -\frac{\mu_x}{n} \Gamma_1(\beta^t) \tilde{o}_1 \end{pmatrix}$$

Papavassilopoulos, Medanic, and Cruz (1979) have shown that if $\hat{x}^t$ and $\hat{\tau}^t$ satisfy the Riccati equations and the real parts of all eigenvalues of matrix Z are negative, a MPE for the finite horizon is also an MPE for the infinite horizon. In this model, Z is a diagonal matrix with o_{1i} on its diagonal multiplied by some positive constant (provided that the optimal control problems are well defined for each player). Then this asymptotic stability condition is equivalent to all co state variables being negatively related with accumulated output, i.e., $o_{1i} < 0$, for $i = 1, \ldots, n+1$, and $\tilde{o}_1 < 0$. Since the symmetric model collapses into a two players differential game (the government and any representative firm) with $n = 1$, this condition requires that $o_1 < 0$ and $\tilde{o}_1 < 0$. ■

Riccati Equations

For convenience, define the following parameters:

$$\sigma_1 = \frac{2a_r - kM^t(0) + k\mu_m \dfrac{\Gamma_0(X^t(0), M^t(0), \alpha^t, \beta^t)}{\Gamma_1(\beta^t)} - 2c_3}{b_r(2+\theta)}$$

$$\sigma_2 = \frac{k\mu_m\mu_r}{b_r(2+\theta)\Gamma_1(\beta^t)}$$

$$\sigma_3 = \frac{2}{b_r(2+\theta)n}$$

$$c_0 = \frac{b_r n[2n\theta - \theta - 1]}{2}$$

$$c_1 = \frac{k\mu_m\mu_r}{2\Gamma_1(\beta^t)}$$

Observe that using these coefficients $(13a)$ yields:

$$\hat{X}^t(Y^{-t}) = \sigma_1 + \sigma_2(\tilde{\phi}_0 + \tilde{\phi}_1 Y^{-t}) + \sigma_3(\phi_0 + \phi_1 Y^{-t})$$

$$\frac{\partial \hat{X}^t(Y^{-t})}{\partial Y^{-t}} = \sigma_2\tilde{\phi}_1 + \sigma_3\phi_1$$

Begin with the *closed loop* solution. First, differentiate the proposed solution for the co state variables (12):

$$\dot{\lambda}^t(Y^{-t}) = \phi_1 \dot{Y}^{-t} = \phi_1 X^t$$

$$\dot{\tilde{\lambda}}^t(Y^{-t}) = \tilde{\phi}_1 \dot{Y}^{-t} = \tilde{\phi}_1 X^t$$

Substituting $(13a)$ and making use of the above notation we get:

$$\dot{\lambda}^t(Y^{-t}) = \sigma_1\phi_1 + \sigma_2\phi_1\tilde{\phi}_0 + \sigma_2\phi_1\tilde{\phi}_1 Y^{-t} + \sigma_3\phi_1\phi_0 + \sigma_3\phi_1^2 Y^{-t}$$

$$\dot{\tilde{\lambda}}^t(Y^{-t}) = \sigma_1\tilde{\phi}_1 + \sigma_2\tilde{\phi}_1\tilde{\phi}_0 + \sigma_2\tilde{\phi}_1^2 Y^{-t} + \sigma_3\tilde{\phi}_1^2\phi_0 + \sigma_3\tilde{\phi}_1\phi_0 + \sigma_3\tilde{\phi}_1\phi_1 Y^{-t}$$

Now substitute (12) into (9b) and (10b) using (13) and its derivatives. This leads to:

$$
\begin{aligned}
\dot{\lambda}^t =& r\omega_0 + r\omega_1 Y^t - nc_1 - c_2 Y^t + \frac{1}{2} b_r n[2n\theta - \theta - 1]\big[\sigma_1 + \sigma_2(\tilde{\omega}_0 + \tilde{\omega}_1 Y^t) \\
& \cdots + \sigma_3(\omega_0 + \omega_1 Y^t)\big][\sigma_2 \tilde{\omega}_1 + \sigma_3 \omega_1] - \frac{1}{2} k \mu_m \big[\sigma_1 + \sigma_2(\tilde{\omega}_0 + \tilde{\omega}_1 Y^t) \\
& \cdots + \sigma_3(\omega_0 + \omega_1 Y^t) - \mu_r \frac{\Gamma_0 + \mu_r(\tilde{\omega}_0 + \tilde{\omega}_1 Y^t)}{\Gamma_1}\big] \frac{\mu_r \tilde{\omega}_1}{\Gamma_1} \\
\dot{\tilde{\lambda}}^t =& r\tilde{\omega}_0 + r\tilde{\omega}_1 Y^t + \alpha^t c_1 + \alpha^t \frac{c_2}{n} Y^t
\end{aligned}
$$

Equating coefficients gives the following set of Riccati nonlinear equations that must be satisfied by any MPE:

$$
\left.
\begin{aligned}
& \sigma_1 \omega_1 + \sigma_2[1 - \upsilon_0 \sigma_3]\omega_1 \tilde{\omega}_0 + \sigma_3[1 - \upsilon_0 \sigma_3]\omega_1 \omega_0 \\
& \quad \cdots + \upsilon_1[\sigma_1 - \mu_r \tfrac{\Gamma_0}{\Gamma_1}][\upsilon_0 \sigma_2 - \upsilon_1]\sigma_3 \omega_0 \tilde{\omega}_1^2 \\
& \quad \cdots + [\upsilon_1(\sigma_2 - \tfrac{\mu_r^2}{\Gamma_1}) - \upsilon_0 \sigma_2^2]\tilde{\omega}_0 \tilde{\omega}_1 - r\omega_0 && = \upsilon_0 \sigma_1 - nc_1 \\
& [\sigma_2 - 2\upsilon_0 \sigma_2 \sigma_3 + \upsilon_1 \sigma_3]\omega_1 \tilde{\omega}_1 + \sigma_3[1 - \upsilon_0 \sigma_3]\omega_1^2 \\
& \quad \cdots + [\upsilon_1(\sigma_2 - \tfrac{\mu_r^2}{\Gamma_1}) - \upsilon_0 \sigma_2^2]\tilde{\omega}_1^2 - r\omega_1 && = -c_2 \\
& \sigma_1 \tilde{\omega}_1 + \sigma_2 \tilde{\omega}_1 \tilde{\omega}_0 + \sigma_3 \tilde{\omega}_1 \omega_0 - r\tilde{\omega}_0 && = \alpha^t c_1 \\
& \sigma_2 \tilde{\omega}_1^2 + \sigma_3 \tilde{\omega}_1 \omega_1 - r\tilde{\omega}_1 && = \alpha^t c_2 / n
\end{aligned}
\right\} \quad (A.1)
$$

Working in the same way but assuming that $\frac{\partial X^t}{\partial Y^t} = \frac{\partial \tau^t}{\partial Y^t} = 0$, the *open loop* solution must satisfy the following equations:

$$
\left.
\begin{aligned}
\sigma_1 \omega_1 + \sigma_2 \omega_1 \tilde{\omega}_0 + \sigma_3 \omega_1 \omega_0 - r\omega_0 &= -nc_1 \\
\sigma_2 \omega_1 \tilde{\omega}_1 + \sigma_3 \omega_1^2 - r\omega_1 &= -c_2 \\
\sigma_1 \tilde{\omega}_1 + \sigma_2 \tilde{\omega}_1 \tilde{\omega}_0 + \sigma_3 \tilde{\omega}_1 \omega_0 - r\tilde{\omega}_0 &= \alpha^t c_1 \\
\sigma_2 \tilde{\omega}_1^2 + \sigma_3 \tilde{\omega}_1 \omega_1 - r\tilde{\omega}_1 &= \alpha^t c_2 / n
\end{aligned}
\right\} \quad (A.2)
$$

which is exactly (A.1) with $\upsilon_0 = \upsilon_1 = 0$.

Proof of Proposition 4

First, observe that regardless of the values of o_0 and $\tilde{o}_0$, any solution for o_1 and $\tilde{o}_1$ must fulfill the second and fourth Ricatti equations of (A.1) which only involves these two variables. Let rewrite them as follows:

$$\sigma_3 m_0 o_1^2 + m_2 o_1 \tilde{o}_1 + m_1 \tilde{o}_1^2 - r o_1 = M2 \tag{A.3}$$

$$\sigma_2 \tilde{o}_1^2 + \sigma_3 o_1 \tilde{o}_1 - r \tilde{o}_1 = M4 \tag{A.4}$$

where $r = 0.04$; $\sigma_2 = -0.131538$; $\sigma_3 = 0.00812416$; $m_0 = 1.03879$; $m_1 = 0.0653826$; $m_2 = -0.142805$; $M_2 = -2.092 \times 10^{-5}$, and $M_4 = 1.6871 \times 10^{-7}$, according to the model's parameters. This system cannot be solved in closed form. However, the proof of the existence of four solutions is almost inmediate. We can implicitly solve this system by equating the following expressions:

$$o_1 = \Phi_1(\tilde{o}_1) = \frac{-(m_3\tilde{o}_1 - r) \pm \sqrt{(m_2\tilde{o}_1 - r)^2 - 4\sigma_3 m_0 (m_1 \tilde{o}_1^2 - M_2)}}{2\sigma_3 m_0} \tag{A.3'}$$

$$o_1 = \Phi_2(\tilde{o}_1) = \frac{M_4 + r\tilde{o}_1 - \sigma_2 \tilde{o}_1^2}{\sigma_3 \tilde{o}_1} \tag{A.4'}$$

The intersections of these two hyperbolas solve $\Phi_1(\tilde{o}_1) = \Phi_2(\tilde{o}_1)$ which becomes a fourth degree polynomial in $\tilde{o}_1$. Therefore, this problem has four possible roots.

The proof of a unique infinite horizon, time consistent tariff policy will rely on the shape and relative positions of the roots of (A.3′) and (A.4′). It will be referred to the actual parameterization of the model.

According to the classification of conics, (A.3′) is a hyperbola since $4\sigma_3 m_0 m_1 < m_2^2$. It can easily be checked that this hyperbola has no asymptotes, since:

$$\lim_{\tilde{o}_1 \to \infty} \Phi_1(\tilde{o}_1) = \infty \quad \text{and} \quad \lim_{\tilde{o}_1 \to -\infty} \Phi_1(\tilde{o}_1) = -\infty$$

(A.3′) has no real roots. In addition it is not defined over the interval $I = [\varphi_l{}', \varphi_h{}'] = [-0.41762, -0.21058]$ where $(m_2\tilde{o}_1 - r)^2 - 4\sigma_3 m_0 (m_1 \tilde{o}_1^2 - M_2) \leq$

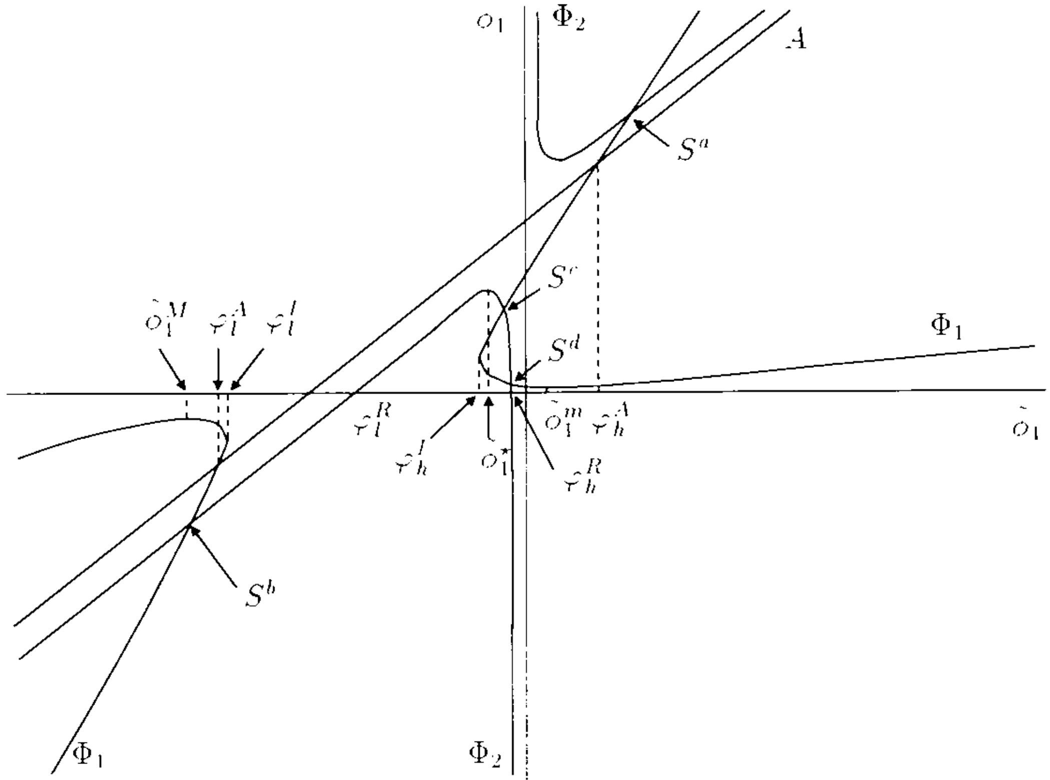

Figure A.1

0. The upper branch of this hyperbola achieves a local maximum ($\Phi_1 = -0.57573$) at $\tilde{\phi}_1^M = -0.62876 < \varphi_l{}^I$, while the lower branch achieves a local minimum ($\Phi_1 = 5.22525 \times 10^{-4}$) at $\tilde{\phi}_1^m = 5.70776 \times 10^{-4} > \varphi_h{}^I$. Finally, its intercepts are $\{\underline{\phi}_1^0, \overline{\phi}_1^0\} = \{5.23058 \times 10^{-4}, 4.73921\}$. Similarly, (A.4′) is also a hyperbola ($0 < \sigma_3^2$) with two asymptotes: the vertical axis and the line: $A(\tilde{\phi}_1) = (r - \sigma_2 \tilde{\phi}_1)/\sigma_3$, since:

$$\lim_{\tilde{\phi}_1 \to 0^+} \Phi_2(\tilde{\phi}_1) = \infty \qquad \lim_{\tilde{\phi}_1 \to 0^-} \Phi_2(\tilde{\phi}_1) = -\infty$$

$$\lim_{\tilde{\phi}_1 \to \infty} \frac{\Phi_2(\tilde{\phi}_1)}{\tilde{\phi}_1} = -\frac{\sigma_2}{\sigma_3} \qquad \lim_{\tilde{\phi}_1 \to \infty} \left[\Phi_2(\tilde{\phi}_1) + \frac{\sigma_2}{\sigma_3}\tilde{\phi}_1\right] = \frac{r}{\sigma_3}$$

(A.4′) has two real roots $\{\varphi_l{}^R, \varphi_h{}^R\} = \{-0.30409, -4.21781 \times 10^{-6}\}$, and achieves a local maximum ($\Phi_2 = 4.88691$) at $\tilde{\phi}_1^\star = -0.00113$. For convenience, Figure A.1 shows the functions $\Phi_1(\tilde{\phi}_1)$, $\Phi_2(\tilde{\phi}_1)$, and $A(\tilde{\phi}_1)$ that represent the solutions reported in Table 3. From Figure A.1 the proof is straightforward, but it will be presented formally here.

LEMMA A.1: *There exists only one solution in the first quadrant* $S^a = (\tilde{o}_1^a, o_1^a)$. *There is also a unique solution in the third quadrant* $S^b = (\tilde{o}_1^b, o_1^b)$.

PROOF: Observe that $\Phi_2(\tilde{o}_1) - A(\tilde{o}_1) = M_4/\sigma_3\tilde{o}_1$ is positive for $\tilde{o}_1 > 0$ and negative for $\tilde{o}_1 < 0$. That is, $A(\tilde{o}_1)$ is a lower bound of $\Phi_2(\tilde{o}_1)$ in the first quadrant but an upper bound in the third one. The solution to $A(\tilde{o}_1) = \Phi_1(\tilde{o}_1)$ is a quadratic function that has only two possible roots, $\{\varphi_l{}^A, \varphi_h{}^A\} = \{-0.47435, 0.00038\}$ for wich $A(\tilde{o}_1) = \{-2.75657, 4.92969\}$. Since $\varphi_h{}^A > 0 > \varphi_h{}^I$ and $A(\tilde{o}_1) < \Phi_2(\tilde{o}_1)\,\forall\tilde{o}_1 > 0$, it follows that $\tilde{o}_1^a > \varphi_h{}^A$ which uses $\Phi_1'(\varphi_h{}^A) > A'(\varphi_h{}^A)$ and the continuity of $\Phi_2(\tilde{o}_1)\,\forall\tilde{o}_1 > 0$. By the same reasoning $\varphi_l{}^A < \varphi_l{}^I < 0$ and $A(\tilde{o}_1) > \Phi_2(\tilde{o}_1)\,\forall\tilde{o}_1 < 0$ implies $\tilde{o}_1^b < \varphi_l{}^A$ which uses $\Phi_1'(\varphi_l{}^A) < A'(\varphi_l{}^A)$ and the continuity of $\Phi_2(\tilde{o}_1)\,\forall\tilde{o}_1 < 0$. Finally, since $\Phi_2(\tilde{o}_1)$ is continuous $\forall\tilde{o}_1 \neq 0$, $\Phi_2'(\varphi_h{}^A) > 0$ and $\Phi_2'(\varphi_l{}^A) > 0$ suffices for $o_1^a = \Phi_2(\tilde{o}_1^a) > \Phi_2(\varphi_h{}^A) > 0$, and $o_1^b = \Phi_2(\tilde{o}_1^b) < \Phi_2(\varphi_l{}^A) < 0$. ■

LEMMA A.2: *There is not any solution in the fourth quadrant.*

PROOF: Straightforward since $\min_{o_1}\{o_1 \mid o_1 = \Phi_1(\tilde{o}_1) \wedge \tilde{o}_1 \geq 0\} = \Phi_1(\tilde{o}_1^m) > 0$. ■

To complete the proof we must to account for the number of solutions in the second quadrant. It could be the case that the hyperbolas do not intersect or that they are tangent, which would imply either non real solutions or a double root. It is obvious from Figure A.1 that the number of solutions will be 0, 1, or 2 depending on the relative position and distance between the vertices of the two hyperbolas.

LEMMA A.3: *The equation* $\Phi_1(\tilde{o}_1) = \Phi_2(\tilde{o}_1)$ *has two solutions* $S^c = (\tilde{o}_1^c, o_1^c)$, $S^d = (\tilde{o}_1^d, o_1^d)$ *(or one double) s.t.* $\tilde{o}_1 < 0$ *whenever the following condition holds:* $\Phi_1(\varphi_h{}^I) \leq \Phi_2(\varphi_h{}^I)$.

PROOF: The lower branch of $\Phi_1(\tilde{o}_1)$ is monotone decreasing up to $\Phi_1(\tilde{o}_1^m) > 0$ for $\tilde{o}_1^m > 0$. The upper branch is monotone increasing in $\tilde{o}_1 \in [\varphi_h{}^I, \infty)$. Therefore both values of $\Phi_1(\varphi_h{}^R) > 0$. Then, since $\Phi_2(\tilde{o}_1)$ and each branch of $\Phi_1(\tilde{o}_1)$ is continuous and single valued for $\tilde{o}_1 \in [\varphi_h{}^I, \varphi_h{}^R]$

when the inequality of the condition is strict, there exists two solutions $\tilde{o}_1^c < \tilde{o}_1^d < \varphi_h{}^R < 0$ s.t. $\Phi_1(\tilde{o}_1^c) = \Phi_2(\tilde{o}_1^c) > \Phi_1(\tilde{o}_1^d) = \Phi_2(\tilde{o}_1^d) > 0$, and one double when it holds with equality. ■

It is evident that the condition stated in Lemma A.3 is sufficient but not necessary. Depending on the shape of the hyperbolas it is always possible to find a range for $\varphi_h{}^I$ s.t. $\Phi_1(\varphi_h{}^I) > \Phi_2(\varphi_h{}^I)$ and $\Phi_1(\varphi_h{}^I) < A(\varphi_h{}^I)$ that generates at least one double solution. For the present model the condition of Lemma 3 holds: $\Phi_1(-0.21058) = 0.58823 < 1.51404 = \Phi_2(-0.21058)$, so that, given the model parameters, the existence of at least four solutions has been already proved. From Lemma A.1 we have that $S^b = (\tilde{o}_1^b, o_1^b)$ is the only solution such that $\tilde{o}_1^b < 0$, $o_1^b < 0$. Therefore, only one solution fulfills the conditions of Proposition 1 for Markov Strategies to be MPE in an infinite horizon game.

It remains to show that there is not any time consistent equilibria for the symmetric Cournot oligopoly. Following the same procedure as before for $\theta = 1/124$, the actual model parameterization leads to $\sigma_2 = -0.131255$, $\sigma_3 = 0.00810664$, $m_0 = 0.506024$, $m_1 = -1.0674$, and $m_2 = -0.00263988$, while M_2 and M_4 remain unchanged. These values are such that the lower root of $\Phi_1(\tilde{o}_1) = 0$ is $\tilde{o}_1 = -0.00443$, which falls into the new interval defined by the roots of $\Phi_2(\tilde{o}_1) = 0$: $[\varphi_l{}^R, \varphi_h{}^R] = [-0.30475, -4.2178 \times 10^{-6}]$. Straightforward computation shows that for $\theta = 1/124$, $\Phi_1'(\tilde{o}_1) > \Phi_2'(\tilde{o}_1) > 0$, $\forall \tilde{o}_1 < 0$. Hence, by continuity, there is no solution to $\Phi_1(\tilde{o}_1) = \Phi_2(\tilde{o}_1)$ such that $\tilde{o}_1 < 0$. ■

Appendix B: Data and Sources

Table B.1

YEAR	CAP	IRON	STEEL	IRON/CAP	STEEL/CAP
1900	385.4	291.0	167.1	75.506	43.358
1901	440.2	323.9	166.2	73.580	37.756
1902	473.7	332.7	193.0	70.234	40.743
1903	507.3	378.0	222.9	74.512	43.938
1904	489.1	376.9	234.1	77.060	47.863
1905	470.8	382.7	257.2	81.287	54.630
1906	452.6	381.2	279.0	84.224	61.644
1907	454.4	349.2	289.2	76.849	63.644
1908	456.2	421.3	304.7	92.350	66.791
1909	458.4	417.0	299.2	90.969	65.271
1910	468.6	407.5	321.2	86.961	68.545
1911	478.8	408.9	327.3	85.401	68.358
1912	485.8	401.1	368.9	82.565	75.937
1913	485.8	444.9	392.8	91.581	80.856
1914	499.3	446.0	374.8	89.325	75.065
1915	506.2	440.3	388.6	86.981	76.768
1916	513.2	498.3	425.8	97.097	82.970
1917	544.0	369.6	394.4	67.941	72.500
1918	499.3	357.1	389.9	71.520	78.089
1919	509.7	276.1	392.6	54.169	77.026
1920	519.4	258.1	320.9	49.692	61.783
1921	582.1	252.9	287.1	43.446	49.321
1922	644.9	210.7	326.1	32.672	50.566
1923	708.1	389.1	459.4	54.950	64.878
1924	770.8	469.1	551.9	60.859	71.601
1925	854.8	506.4	648.7	59.242	75.889
1926	867.6	492.7	636.8	56.789	73.398
1927	880.3	587.7	728.6	66.761	82.767
1928	893.5	576.0	791.5	64.466	88.584
1929	1014.3	771.9	1021.7	76.102	100.730
1930	996.4	624.2	953.7	62.646	95.715

CAP: Spanish production capacity of iron and steel (thousand of tons); IRON: Spanish production of iron products (th.tons); STEEL: Spanish production of steel products (th.tons). IRON/CAP and STEEL/CAP are the percentage of production of iron and steel over available capacity. *Source:* Fraile (1991), p.121.

Table B.2

YEAR	SMI	SMS	IMP	VIMP	EXP	VEXP	TR
1900		62					
1901		37					
1902							
1903		44					
1904		33					
1905		31					
1906		29					
1907	4.81	42					
1908	4.27	36	74.71	31.83	49.89	8.13	8.49
1909	4.04	35	74.71	31.83	49.89	8.13	8.49
1910	5.32	36	74.71	31.83	49.89	8.13	8.49
1911	6.05	37	74.71	31.83	49.89	8.13	8.49
1912	5.96	54	109.97	41.79	37.27	4.47	11.39
1913	7.72	104	181.77	60.31	10.80	1.77	16.87
1914	10.34	48	95.26	34.10	40.39	7.80	9.40
1915	8.15	27	50.84	16.69	125.54	31.87	4.46
1916	30.60	35	92.88	23.18	118.32	35.95	4.68
1917	14.70	16	69.31	14.79	84.50	29.47	2.77
1918	6.45	11	43.26	11.13	23.92	18.40	3.22
1919	5.92	66	65.00	22.34	13.02		4.68
1920	14.52	137	173.41	53.96			15.27
1921	24.53	193	267.61	189.89			25.99
1922	14.68	177	254.89	159.43	9.17	5.21	
1923	6.63	134	237.47	149.63	6.63	4.85	
1924	3.14	126	251.88	156.34	26.73	12.20	
1925	3.89	146	275.74	126.08	7.24	7.34	
1926	3.51	94	178.00	89.61	5.71	3.17	
1927	12.33	102	238.37	80.01	1.54	4.16	
1928	15.55	130	239.29	121.57	2.32	2.95	
1929			334.24	120.05	2.80	3.70	
1930	4.79	68					

SMI: Spanish imports of iron (thousand of tons); SMS: Spanish imports of Steel (th.tons). *Source:* Fraile (1985) p.99. IMP: Spanish imports of iron and steel products (th.tons); VIMP: Nominal value of the Spanish imports (millions of Pts); EXP: Spanish Exports of iron and steel products (th.tons); VEXP: Nominal value of the Spanish exports (millions of Pts). TR: Tariff revenues from imports of iron and steel products (millions of Pts). *Source:* Estadísticas de Comercio Exterior de España.

Table B.3

YEAR	SMI/IRON	SMS/STEEL	IMP/PROD	EXP/PROD	AHVGP
1900		37.10			
1901		22.26			
1902					9.604
1903		19.74			9.078
1904		14.10			6.461
1905		12.05			6.577
1906		10.39			6.623
1907	1.38	14.52			8.687
1908	1.01	11.81	10.29	6.87	9.479
1909	0.97	11.70	10.43	6.97	10.023
1910	1.30	11.21	10.25	6.85	9.876
1911	1.48	11.30	10.15	6.78	9.624
1912	1.49	14.64	14.28	4.84	12.297
1913	1.73	26.48	21.70	1.29	12.371
1914	2.32	12.81	11.61	4.92	11.677
1915	1.85	6.95	6.13	15.15	14.655
1916	6.14	8.22	10.05	12.80	15.049
1917	3.98	4.06	9.07	11.06	
1918	1.81	2.82	5.79	3.20	
1919	2.14	16.81	9.72	1.95	
1920	5.63	42.69	29.95		
1921	9.70	67.22	49.56		
1922	6.97	54.28	47.48	1.71	
1923	1.70	29.17	27.99	0.78	
1924	0.67	22.83	24.67	2.62	
1925	0.77	22.51	23.87	0.63	
1926	0.71	14.76	15.76	0.51	
1927	2.10	14.00	18.11	0.12	
1928	2.70	16.42	17.50	0.17	
1929			18.64	0.16	
1930	0.77	7.13			

PROD=IRON+STEEL. See tables B.1 and B.2 for sources and definitions. All ratio series are percentages. AHVGP is AHV's gross profit (millions of Pts). *Source:* Fenández de Pinedo (1992), p.151.

Table B.4

YEAR	UKIRON	UKSTEEL	SMI/UK	SMS/UK	IMP/UK
1900	9104	4980		1.24	
1901	8056	4983		0.74	
1902	8818	4988			
1903	9078	5115		0.86	
1904	8834	5108		0.65	
1905	9762	5902		0.53	
1906	10347	6566		0.44	
1907	10276	6628	0.05	0.63	
1908	9202	5381	0.05	0.67	0.51
1909	9685	5976	0.04	0.59	0.48
1910	10173	6476	0.05	0.56	0.45
1911	9679	6566	0.06	0.56	0.46
1912	8891	6905	0.07	0.78	0.70
1913	10425	7787	0.07	1.34	1.00
1914	9067	7971	0.11	0.60	0.56
1915	8864	8687	0.09	0.31	0.29
1916	9062	9136	0.34	0.38	0.51
1917	9488	9873	0.15	0.16	0.36
1918	9253	9692	0.07	0.11	0.23
1919	7536	8021	0.08	0.82	0.42
1920	8164	9212	0.18	1.49	1.00
1921	2658	3762	0.92	5.13	4.17
1922	4981	5975	0.29	2.96	2.33
1923	7560	8618	0.09	1.55	1.47
1924	7424	8333	0.04	1.51	1.60
1925	6362	7504	0.06	1.95	1.99
1926	2497	3654	0.14	2.57	2.89
1927	7410	9243	0.17	1.10	1.43
1928	6716	8657	0.23	1.50	1.56
1929	7711	9791			1.91
1930	6291	7444	0.08	0.91	

UKIRON: British production of iron (thousand of tons); UKSTEEL: British production of steel (th.tons). *Source:* Mitchell (1992) pp.450 51 and 456 59. UKPROD=UKIRON+UKSTEEL. SMI/UK = SMI/UKIRON, SMS/UK = SMS/UKSTEEL, and IMP/UK = IMP/UKPROD are three different percentages that measure the importance of total Spanish imports relative to British production.

Table B.5

YEAR	UKIP	SPIP	UKSP	SPSP	PI	MPI	DPR
1900	127.1	105		188	96.7	101.0	4.51
1901	81.3	98		241	96.9	105.6	4.48
1902	79.4	85		209	94.7	102.2	4.42
1903	78.3	81		199	97.7	108.4	4.18
1904	73.5	86		131	99.5	108.9	4.21
1905	75.7	94	175.7	204	100.0	100.0	4.10
1906	73.2	95	181.1	226	97.3	95.9	3.97
1907	78.5	100	179.8	199	101.4	94.7	3.91
1908	70.8	92	175.3	180	98.6	89.3	3.86
1909	67.4	85	162.3	160	97.3	86.4	3.74
1910	67.2	95	162.6	230	98.2	94.1	3.76
1911	66.1	120	178.6	228	94.7	102.3	3.81
1912	71.6	78	195.8	144	99.4	101.9	3.79
1913	81.2	115	187.1	146	100.0	100.0	3.98
1914	92.9	100	177.0	196	98.4	87.5	4.22
1915	75.5	139	211.3	221	118.3	96.7	4.47
1916	97.9	235	265.1	372	141.0	130.5	4.31
1917	102.9	415	271.6	599	165.6	193.3	4.31
1918	116.5	625	250.5	800	204.9	260.9	4.12
1919	170.7	325	414.4	621	204.2	224.9	4.16
1920	235.9	357	572.7	747	221.8	346.3	4.44
1921	193.6	309	368.5	670	189.4	176.8	4.72
1922	123.0	240	315.5	474	177.3	214.9	4.62
1923	158.0	238	300.3	464	174.8	241.9	4.53
1924	145.5	212	308.8	525	183.4	225.3	
1925	122.6	209	271.6	290	189.2	192.4	
1926	125.0	191	275.7	272	180.8	153.3	
1927	101.8	178	272.1	280	173.3	141.7	
1928	92.3	160	252.1	274	168.5	137.3	
1929	107.8	175	291.2	252	172.4	180.1	
1930	135.6	179	366.2	255	173.0	194.6	

UKIP and UKSP are, respectively, British iron and steel prices (Pts/ton). Similarly SPIP and SPSP are Spanish prices (Pts/ton). *Sources:* British prices are available in Mitchell and Deane (1965), and Mitchell and Jones (1971). They have been converted into pesetas using the mean of the weakly exchange rate from "The Economist" provided by Fraile (1985), p.88. Prices of AHV are available in Carreras (1989), pp.226 227. These and the Spanish prices can be found in Fraile (1991), pp.172 173. PI and MPI are the Spanish price index and Spanish imports price index respectively. *Source:* Carreras (1989), pp.521 and 352 353. DPR is the rate of return of the Spanish Public Debt. *Source:* Martín (1985).

Table B.6

FIRM		1900	1913	1925	1929
A.H. Vizcaya	Total	71.33	69.63	53.29	52.22
(1902)	Iron	79.61	71.94	58.32	57.47
	Steel	61.87	67.33	49.24	48.18
A.H. Mediterráneo	Total			15.40	19.33
(1917)	Iron			15.62	19.33
	Steel			15.23	19.32
Duro-Felguera	Total	7.77	5.09	5.99	6.36
(1900)	Iron	4.78	4.77	4.31	6.89
	Steel	11.19	5.41	7.34	5.96
Fab. de Mieres	Total	14.07	4.40	3.18	3.42
(1879)	Iron	5.30	5.83	3.70	3.55
	Steel	24.09	2.96	2.77	3.31
Ind. Asturiana	Total	4.93	4.88	3.84	2.91
(1895)	Iron	9.24	4.97	4.52	3.09
	Steel		4.79	3.30	2.77
Comp. Basconia	Total		4.04	5.32	4.32
(1892)	Iron				
	Steel		8.12	9.59	7.65
S.A. Echevarría	Total			2.33	2.63
(1920)	Iron			2.39	2.14
	Steel			2.29	3.01
Material F.C. & C.	Total		1.27	1.61	1.38
(1881)	Iron				
	Steel		2.55	2.90	2.45
J.M. Quijano	Total	1.33	1.06	1.37	1.19
(1914)	Iron				
	Steel	2.85	2.14	2.47	2.11
Nueva Montaña	Total		5.73	3.95	3.27
(1899)	Iron		11.41	8.86	7.51
	Steel				
Others	Total	0.57	3.89	3.72	2.96
	Iron	1.06	1.10	2.28	
	Steel		6.70	4.87	5.24
Inverse	Total	284.59	124.06	354.57	475.54
Herfindahl	Iron	136.61	211.97	382.01	504.54
Indexes	Steel	713.67	146.97	420.77	510.94

Year of legal establishment is shown in parenthesis. Percentages of market share for each product and the Herfindahl indexes computed from the information provided by Merello (1939), for 1900, 1913, and 1929; and España (1927), for 1925. "Others" includes five (assumed) symmetric, small firms to compute the Herfindahl index.

Appendix C: Comparative Statics

Table C.1. θ

θ	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
0.0	2183.634	-23.70722	5423.98	-1.767749	1340.728	-0.425921	2525.717	-0.804169
0.1	1.52E+10	-0.50233	-11292.71	-0.000003	117567147	-0.003894	-5078.899	-0.000001
0.2	1.52E+10	-0.25808	-11300.48	-0.000004	112329320	-0.001910	-5082.431	-0.000001
0.3	1.57E+10	-0.18425	-11309.09	-0.000004	110790789	-0.001304	-5086.351	-0.000001
0.4	1.62E+10	-0.14902	11316.53	-0.000004	110076440	-0.001011	-5089.736	-0.000001
0.5	1.68E+10	-0.12863	-11322.79	-0.000004	109672460	-0.000838	-5092.584	-0.000001
0.6	1.75E+10	-0.11553	-11328.02	-0.000004	109416194	-0.000723	-5094.963	-0.000001
0.7	1.81E+10	-0.10652	-11332.38	-0.000004	109240466	-0.000642	-5096.946	-0.000001
0.8	1.88E+10	-0.10006	-11336.01	-0.000004	109112774	-0.000582	-5098.595	-0.000001
0.9	1.94E+10	-0.09528	11339.01	-0.000004	109015640	-0.000535	-5099.963	-0.000001
1.0	2.01E+10	-0.09166	-11341.50	-0.000004	108938924	-0.000497	-5101.093	-0.000001

θ	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
0.0	-5777.313	-10913.65	14026.07	77323.36	-1900.868	-153075.6	-77653.07	-29611.91
0.1	117567082	-5078.93	6594.89	3.42E+12	-6.85E+12	-33495.0	-3.42E+12	-1.31E+12
0.2	112329288	-5082.46	6599.39	3.13E+12	-6.25E+12	-33541.1	-3.13E+12	-1.19E+12
0.3	110790767	-5086.38	6604.38	3.04E+12	-6.08E+12	-33592.4	-3.04E+12	-1.16E+12
0.4	110076423	-5089.77	6608.69	3.00E+12	-6.00E+12	-33636.7	-3.00E+12	-1.14E+12
0.5	109672446	-5092.62	6612.32	2.98E+12	-5.96E+12	-33674.0	-2.98E+12	-1.14E+12
0.6	109416182	-5094.99	6615.35	2.97E+12	-5.93E+12	-33705.2	-2.97E+12	-1.13E+12
0.7	109240455	-5096.98	6617.87	2.96E+12	-5.91E+12	-33731.2	-2.96E+12	-1.13E+12
0.8	109112764	-5098.63	6619.97	2.95E+12	-5.90E+12	-33752.8	-2.95E+12	-1.12E+12
0.9	109015631	-5099.99	6621.72	2.94E+12	-5.89E+12	-33770.7	-2.94E+12	-1.12E+12
1.0	108938916	-5101.13	6623.16	2.94E+12	-5.88E+12	-33785.5	-2.94E+12	-1.12E+12

Table C.2a. θ

θ	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
0.0/124	2183.63	-23.70722	5423.98	-1.767749	1341	-0.425921	2525.72	-0.804170
0.1/124	2737.43	-20.83246	5595.09	-1.590319	1303	-0.361086	2603.56	-0.723455
0.2/124	3671.72	-17.57304	5867.44	-1.389131	1283	-0.296452	2727.45	-0.631932
0.3/124	5586.28	-13.71433	6389.50	-1.150928	1301	-0.228956	2964.95	-0.523571
0.4/124	11688.09	-8.70929	7949.27	-0.841925	1459	-0.151533	3674.50	-0.383002
0.5/124	594972.48	1.53274	-90933.85	-0.209467	-5226	-0.012103	-41308.58	-0.095289
0.6/124	-7.87E+08	5.45762	-6245.00	0.000039	-4924515	0.034174	-2782.64	0.000018
0.7/124	-2.43E+09	6.13415	-8481.52	0.000017	-14626185	0.036985	-3800.05	0.000008
0.8/124	-4.50E+09	7.00100	-9153.00	0.000010	-26138607	0.040703	-4105.52	0.000005
0.9/124	-7.19E+09	8.15115	-9420.49	0.000006	-40377500	0.045756	-4227.20	0.000003
θ	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
0.0/124	-5777	-10913.65	14026.067	77323	-1900.869	-153076	-77653	-29611.9
0.1/124	-4732	-9486.89	12208.923	58685	-1058.153	-115825	-58198	-22192.9
0.2/124	-3671	-7833.45	10103.085	40284	-652.137	-79142	-39510	-15066.8
0.3/124	-2526	-5785.02	7494.172	22242	-524.739	-43354	-21637	-8250.9
0.4/124	-1073	-2726.26	3598.495	5162	-438.237	-9810	-5086	-4939.6
0.5/124	-5428	-42901.06	54765.681	1271513	-48987.766	-2349506	-1029005	-392396.9
0.6/124	-4923944	-2782.34	3669.917	6001529046	-1.20E+10	-10211	-6.01E+09	-2.29E+09
0.7/124	-14625567	-3799.92	4965.928	5.297E+10	-1.06E+11	-18870	-5.30E+10	-2.02E+10
0.8/124	-26137926	-4105.44	5355.044	1.692E+11	-3.38E+11	-21985	-1.69E+11	-6.45E+10
0.9/124	-40376735	-4227.15	5510.055	4.038E+11	-8.08E+11	-23292	-4.04E+11	-1.54E+11

Table C.2b. θ

θ	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
1.0/124	-1.09E+10	9.75045	-9497.07	0.000004	-58875095	0.052908	-4262.04	0.000002
1.1/124	-1.61E+10	12.12579	-9428.21	0.000003	-84436113	0.063675	-4230.72	0.000001
1.2/124	-2.41E+10	16.02284	-9177.88	0.000002	-1.23E+08	0.081510	-4116.84	0.000001
1.3/124	-3.79E+10	23.59301	-8565.55	0.000001	-1.87E+08	0.116383	-3838.28	0.000001
1.4/124	-6.32E+10	44.65787	-6702.92	0.000001	-3.02E+08	0.213815	-2990.95	0.000000
1.5/124	2.57E+12	111.18675	26728.42	0.000000	1.20E+10	1.912447	12217.36	0.000000
1.6/124	2.37E+11	-57.15574	16092.57	-0.000000	1.07E+09	-0.258450	-7262.41	-0.000000
1.7/124	1.00E+11	-26.74390	-13348.16	-0.000001	441424648	-0.117663	-6013.95	-0.000000
1.8/124	6.66E+10	17.46572	-12529.05	-0.000001	285354648	-0.074821	-5641.32	-0.000000
1.9/124	5.16E+10	-12.97252	12142.97	-0.000001	215481431	-0.054148	-5465.69	-0.000001
2.0/124	4.32E+10	-10.32154	-11921.93	-0.000001	175787160	-0.042005	-5365.14	-0.000001

θ	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
1.0/124	-58874211	-4262.01	5554.448	8.585E+11	-1.72E+12	-23673	-8.59E+11	-3.27E+11
1.1/124	-84435048	-4230.70	5514.565	1.766E+12	-3.53E+12	-23330	-1.77E+12	-6.73E+11
1.2/124	-1.23E+08	-4116.83	5369.539	3.733E+12	-7.47E+12	-22105	-3.73E+12	-1.42E+12
1.3/124	-1.87E+08	-3838.27	5014.773	8.658E+12	-1.73E+13	-19248	-8.66E+12	-3.30E+12
1.4/124	-3.02E+08	-2990.95	3935.605	2.265E+13	-4.53E+13	-11771	-2.27E+13	-8.64E+12
1.5/124	1.20E+10	12217.36	0.000	3.550E+16	-7.10E+16	0	-3.55E+16	-1.35E+16
1.6/124	1.07E+09	-7262.41	9375.803	2.845E+14	-5.69E+14	-68091	-2.85E+14	-1.08E+14
1.7/124	441422682	-6013.95	7785.746	4.827E+13	-9.65E+13	-46823	-4.83E+13	-1.84E+13
1.8/124	285353398	-5641.33	7311.169	2.017E+13	-4.03E+13	-41245	-2.02E+13	-7.69E+12
1.9/124	215480527	-5465.70	7087.485	1.150E+13	-2.30E+13	-38738	-1.15E+13	-4.39E+12
2.0/124	175786458	-5365.15	6959.420	7.654E+12	-1.53E+13	-37338	-7.65E+12	-2.92E+12

Table C.3. θ

θ	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
0.40/124	11688.09	-8.709294	7949.27	-0.841925	1459.067	-0.151533	3674.502	-0.383002
0.41/124	13109.45	-8.092557	8309.22	-0.803846	1501.978	-0.142574	3838.245	-0.365679
0.42/124	14915.10	-7.441734	8769.34	-0.763662	1558.032	-0.133231	4047.563	-0.347399
0.43/124	17279.84	-6.750458	9379.41	-0.720979	1633.632	-0.123422	4325.090	-0.327982
0.44/124	20199.43	-6.010088	10228.36	-0.675264	1740.120	-0.113038	4711.288	-0.307185
0.45/124	25113.50	-5.208386	11491.57	-0.625760	1899.568	-0.101925	5285.933	-0.284666
0.46/124	32210.17	-4.326986	13565.92	-0.571335	2161.057	-0.089847	6229.583	-0.259907
0.47/124	44314.46	-3.335974	17561.57	-0.510139	2658.447	-0.076422	8047.250	-0.232068
0.48/124	68687.94	-2.180131	28016.54	-0.438763	3918.840	-0.060941	12803.338	-0.199598
0.49/124	135949.47	-0.732109	98521.79	-0.349341	11805.840	-0.041764	44876.997	-0.158919
0.50/124	594972.48	1.532736	-90933.85	-0.209467	-5226.214	-0.012103	-41308.58	-0.095289
θ	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
0.40/124	-1073.37	-2726.26	3598.49	5162.32	-438.24	-9810.43	-5086.35	-1939.61
0.41/124	-880.73	-2273.02	3021.24	3643.83	-404.21	-6867.34	-3627.72	-1383.38
0.42/124	-668.53	-1758.20	2365.56	2237.99	-354.26	-4159.12	-2275.38	-867.69
0.43/124	-429.01	-1156.17	1598.81	1025.56	-278.71	-1848.50	-1101.65	-420.10
0.44/124	-148.99	-422.42	664.30	179.06	-158.79	-280.61	-260.34	-99.28
0.45/124	196.19	528.57	0.00	9.53	22.88	0.00	32.42	12.36
0.46/124	659.53	1885.99	0.00	107.75	111.09	0.00	218.84	83.45
0.47/124	1381.28	4168.90	0.00	472.60	-175.24	0.00	297.36	113.39
0.48/124	2900.39	9467.63	0.00	2083.74	-2464.32	0.00	-380.58	-145.13
0.49/124	11107.87	42221.12	0.00	30562.71	-54380.37	0.00	-23817.66	-9082.53
0.50/124	-5428.48	-42901.06	54765.68	1271512.80	48987.77	-2349506	-1029005	-392396.8

Table C.4. θ

θ	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
0.460/124	32210.17	-4.326986	13565.92	-0.571335	2161.057	-0.089847	6229.583	-0.259907
0.461/124	33131.57	-4.233460	13848.51	-0.565560	2196.537	-0.088573	6358.133	-0.257280
0.462/124	34103.74	-4.138818	14150.23	-0.559715	2234.373	-0.087286	6495.389	-0.254621
0.463/124	35130.80	-4.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	-0.251930
0.464/124	36217.33	-3.946018	14819.22	-0.547810	2318.084	-0.084668	6799.721	-0.249205
0.465/124	37368.39	-3.847771	15191.30	-0.541743	2364.529	-0.083336	6968.986	-0.246445
0.466/124	38589.63	-3.748227	15592.24	-0.535596	2414.485	-0.081988	7151.379	-0.243649
0.467/124	39887.35	-3.647334	16025.45	0.529366	2468.352	-0.080623	7348.451	-0.240815
0.468/124	41268.61	-3.545035	16494.87	-0.523049	2526.594	-0.079241	7561.996	-0.237941
0.469/124	42741.34	-3.441270	17005.11	-0.516641	2589.749	-0.077841	7794.110	-0.235026
0.470/124	44314.46	-3.335974	17561.57	-0.510139	2658.447	-0.076422	8047.250	-0.232068
θ	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
0.460/124	659.53	1885.99	0.00	107.75	111.09	0.00	218.84	83.45
0.461/124	716.30	2058.45	0.00	127.09	107.27	0.00	234.36	89.37
0.462/124	775.65	2240.14	0.00	149.02	99.86	0.00	248.89	94.91
0.463/124	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
0.464/124	903.11	2634.98	0.00	202.03	72.16	0.00	274.19	104.56
0.465/124	971.82	2850.37	0.00	233.94	50.55	0.00	284.48	108.48
0.466/124	1044.30	3079.49	0.00	270.13	22.68	0.00	292.81	111.66
0.467/124	1120.97	3323.93	0.00	311.26	-12.46	0.00	298.79	113.94
0.468/124	1202.31	3585.50	0.00	358.06	-56.12	0.00	301.95	115.14
0.469/124	1288.86	3866.33	0.00	411.48	-109.77	0.00	301.71	115.05
0.470/124	1381.28	4168.90	0.00	472.60	-175.24	0.00	297.36	113.39

Table C.5. Comparative Statics: a_r

a_r	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
$0.75a_r$	33911.77	-4.043018	11350.16	-0.553800	1789.936	0.085984	5221.607	-0.251930
$0.80a_r$	34226.72	-4.043018	11984.48	-0.553800	1888.422	-0.085984	5510.168	-0.251930
$0.85a_r$	34498.84	-4.043018	12612.94	-0.553800	1985.997	-0.085984	5796.059	-0.251930
$0.90a_r$	34736.31	-4.043018	13236.65	-0.553800	2082.835	-0.085984	6079.792	-0.251930
$0.95a_r$	34945.36	-4.043018	13856.47	0.553800	2179.068	-0.085984	6361.754	-0.251930
$1.00a_r$	35130.80	-4.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	0.251930
$1.05a_r$	35296.41	-4.043018	15086.92	-0.553800	2370.110	-0.085984	6921.500	-0.251930
$1.10a_r$	35445.22	-4.043018	15698.48	-0.553800	2465.062	-0.085984	7199.707	-0.251930
$1.15a_r$	35579.66	-4.043018	16308.08	-0.553800	2559.709	-0.085984	7477.020	0.251930
$1.20a_r$	35701.71	-4.043018	16915.98	-0.553800	2654.092	-0.085984	7753.560	-0.251930
$1.25a_r$	35813.01	-4.043018	17522.40	-0.553800	2748.247	-0.085984	8029.431	-0.251930

a_r	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
$0.75a_r$	352.96	1011.33	0.00	30.86	22.21	0.00	53.07	20.24
$0.80a_r$	451.45	1299.89	0.00	50.48	42.20	0.00	92.68	35.34
$0.85a_r$	549.02	1585.78	0.00	74.66	58.66	0.00	133.33	50.84
$0.90a_r$	645.86	1869.51	0.00	103.33	71.77	0.00	175.09	66.77
$0.95a_r$	742.10	2151.48	0.00	136.41	81.64	0.00	218.05	83.15
$1.00a_r$	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
$1.05a_r$	933.14	2711.22	0.00	215.69	92.02	0.00	307.70	117.34
$1.10a_r$	1028.09	2989.43	0.00	261.81	92.66	0.00	354.48	135.18
$1.15a_r$	1122.74	3266.74	0.00	312.24	90.35	0.00	402.59	153.52
$1.20a_r$	1217.12	3543.28	0.00	366.94	85.11	0.00	452.05	172.38
$1.25a_r$	1311.27	3819.15	0.00	425.91	76.98	0.00	502.89	191.77

Table C.6. Comparative Statics: b_r

b_r	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
$0.75b_r$	34443.20	-3.035208	10481.11	0.298083	3076.614	-0.086036	6435.019	-0.180802
$0.80b_r$	34615.23	-3.236757	11278.88	-0.342973	2873.621	-0.086023	6486.474	0.195028
$0.85b_r$	34766.94	-3.438314	12077.01	-0.390990	2695.961	-0.086011	6532.070	-0.209254
$0.90b_r$	34901.75	-3.639877	12875.45	-0.442133	2539.137	-0.086001	6572.753	-0.223479
$0.95b_r$	35022.32	-3.841445	13674.14	-0.496403	2399.665	-0.085992	6609.275	-0.237705
$1.00b_r$	35130.80	-4.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	-0.251930
$1.05b_r$	35228.93	-4.244595	15272.14	-0.614323	2162.350	-0.085977	6672.154	-0.266155
$1.10b_r$	35318.11	-4.446174	16071.40	-0.677974	2060.543	-0.085970	6699.412	-0.280380
$1.15b_r$	35399.52	-4.647756	16870.79	-0.744751	1967.931	-0.085964	6724.354	-0.294605
$1.20b_r$	35474.12	-4.849341	17670.31	-0.814655	1883.316	-0.085959	6747.264	-0.308830
$1.25b_r$	35542.75	-5.050927	18469.94	-0.887685	1805.702	-0.085954	6768.381	-0.323055

b_r	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
$0.75b_r$	1638.78	3413.44	0.00	498.92	-69.70	0.00	429.22	163.68
$0.80b_r$	1436.00	3227.14	0.00	408.63	-13.69	0.00	394.94	150.61
$0.85b_r$	1258.54	3035.00	0.00	333.49	27.58	0.00	361.07	137.69
$0.90b_r$	1101.88	2837.94	0.00	270.67	56.98	0.00	327.65	124.94
$0.95b_r$	962.56	2636.73	0.00	218.03	76.68	0.00	294.71	112.38
$1.00b_r$	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
$1.05b_r$	725.50	2224.14	0.00	136.90	93.31	0.00	230.21	87.79
$1.10b_r$	623.80	2013.67	0.00	106.03	92.58	0.00	198.61	75.74
$1.15b_r$	531.29	1800.88	0.00	80.41	86.99	0.00	167.40	63.83
$1.20b_r$	446.76	1586.07	0.00	59.33	77.22	0.00	136.55	52.07
$1.25b_r$	369.23	1369.46	0.00	42.21	63.83	0.00	106.04	40.44

Table C.7. Comparative Statics: k

k	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{r}_0$	$\hat{r}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
$0.75k$	36076.44	-1.042749	27603.45	-1.040746	2295.173	-0.085981	9461.561	-0.355085
$0.80k$	35912.15	-1.042794	23954.23	-0.905966	2290.312	-0.085981	8764.276	-0.329708
$0.85k$	35735.74	-1.042844	20937.22	-0.794264	2285.829	-0.085982	8145.440	-0.307122
$0.90k$	35546.90	-1.042897	18415.18	-0.700656	2281.739	-0.085982	7592.008	-0.286863
$0.95k$	35345.36	-1.042955	16286.14	0.621436	2278.056	-0.085983	7093.687	-0.268564
$1.00k$	35130.80	-1.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	-0.251930
$1.05k$	34902.92	-1.043086	12916.83	0.495593	2271.994	-0.085985	6231.019	-0.236724
$1.10k$	34661.38	-1.043159	11571.58	-0.445142	2269.666	-0.085986	5854.565	-0.222751
$1.15k$	34405.86	-1.043237	10401.17	-0.401128	2267.848	-0.085987	5508.386	-0.209849
$1.20k$	34136.00	-1.043322	9376.90	0.362500	2266.578	-0.085988	5188.739	-0.197886
$1.25k$	33851.45	-1.043413	8475.71	-0.328413	2265.901	-0.085989	4892.482	-0.186749
k	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
$0.75k$	858.26	3527.34	0.00	182.46	83.74	0.00	266.20	101.51
$0.80k$	853.39	3254.17	0.00	180.39	84.88	0.00	265.28	101.16
$0.85k$	848.89	3012.79	0.00	178.50	85.91	0.00	264.41	100.83
$0.90k$	844.79	2797.92	0.00	176.78	86.83	0.00	263.61	100.52
$0.95k$	841.10	2605.42	0.00	175.24	87.65	0.00	262.88	100.25
$1.00k$	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
$1.05k$	835.01	2274.87	0.00	172.71	88.96	0.00	261.67	99.78
$1.10k$	832.66	2131.94	0.00	171.74	89.46	0.00	261.20	99.61
$1.15k$	830.83	2001.36	0.00	170.98	89.85	0.00	260.83	99.46
$1.20k$	829.54	1881.64	0.00	170.45	90.11	0.00	260.57	99.36
$1.25k$	828.84	1771.52	0.00	170.17	90.26	0.00	260.43	99.31

Table C.8. Comparative Statics: a_m

a_m	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
$0.75a_m$	35301.28	-4.043018	15105.95	-0.553800	2373.064	-0.085984	6930.157	-0.251930
$0.80a_m$	35268.70	-4.043018	14979.58	-0.553800	2353.444	-0.085984	6872.669	-0.251930
$0.85a_m$	35235.39	-4.043018	14853.10	-0.553800	2333.808	-0.085984	6815.136	-0.251930
$0.90a_m$	35201.32	-4.043018	14726.53	-0.553800	2314.155	-0.085984	6757.555	-0.251930
$0.95a_m$	35166.47	-4.043018	14599.84	-0.553800	2294.486	-0.085984	6699.925	-0.251930
$1.00a_m$	35130.80	-4.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	-0.251930
$1.05a_m$	35094.30	-4.043018	14346.14	-0.553800	2255.096	-0.085984	6584.512	-0.251930
$1.10a_m$	35056.92	-4.043018	14219.11	-0.553800	2235.373	-0.085984	6526.725	-0.251930
$1.15a_m$	35018.64	-4.043018	14091.96	-0.553800	2215.631	-0.085984	6468.882	-0.251930
$1.20a_m$	34979.43	-4.043018	13964.68	-0.553800	2195.869	-0.085984	6410.980	-0.251930
$1.25a_m$	34939.25	-4.043018	13837.26	-0.553800	2176.087	-0.085984	6353.018	-0.251930
a_m	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
$0.75a_m$	936.09	2719.88	0.00	217.05	62.37	0.00	279.42	106.55
$0.80a_m$	916.47	2662.39	0.00	208.05	68.32	0.00	276.37	105.39
$0.85a_m$	896.83	2604.86	0.00	199.23	73.90	0.00	273.13	104.15
$0.90a_m$	877.18	2547.28	0.00	190.59	79.10	0.00	269.69	102.84
$0.95a_m$	857.51	2489.65	0.00	182.14	83.92	0.00	266.06	101.46
$1.00a_m$	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
$1.05a_m$	818.12	2374.23	0.00	165.79	92.42	0.00	258.21	98.47
$1.10a_m$	798.40	2316.45	0.00	157.90	96.10	0.00	254.00	96.86
$1.15a_m$	778.66	2258.60	0.00	150.18	99.40	0.00	249.58	95.17
$1.20a_m$	758.90	2200.70	0.00	142.66	102.31	0.00	244.97	93.41
$1.25a_m$	739.11	2142.74	0.00	135.32	104.84	0.00	240.15	91.58

Table C.9. Comparative Statics: b_m

b_m	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
$0.75b_m$	34273.14	-1.043245	10079.22	-0.397281	2220.126	-0.085987	4643.443	-0.180728
$0.80b_m$	34496.02	-1.043186	10958.12	-0.428585	2234.281	-0.085986	5043.261	-0.194968
$0.85b_m$	34687.86	-1.043136	11836.93	-0.459889	2246.495	-0.085985	5443.045	-0.209209
$0.90b_m$	34854.72	-1.043092	12715.69	-0.491192	2257.141	-0.085985	5842.800	-0.223449
$0.95b_m$	35001.20	-1.043053	13594.39	-0.522496	2266.503	-0.085984	6242.532	-0.237690
$1.00b_m$	35130.80	-1.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	-0.251930
$1.05b_m$	35246.29	-1.042987	15351.67	-0.585103	2282.204	-0.085984	7041.941	-0.266170
$1.10b_m$	35349.85	-1.042960	16230.27	-0.616407	2288.851	-0.085983	7441.625	-0.280411
$1.15b_m$	35443.24	-1.042935	17108.84	-0.647711	2294.853	-0.085983	7841.296	-0.294651
$1.20b_m$	35527.88	-1.042912	17987.39	-0.679014	2300.298	-0.085983	8240.958	-0.308892
$1.25b_m$	35604.96	-1.042891	18865.92	-0.710318	2305.262	-0.085982	8640.612	-0.323132

b_m	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
$0.75b_m$	783.10	1623.10	0.00	151.90	98.69	0.00	250.59	95.56
$0.80b_m$	797.27	1784.93	0.00	157.45	96.30	0.00	253.75	96.76
$0.85b_m$	809.50	1946.73	0.00	162.32	94.08	0.00	256.39	97.77
$0.90b_m$	820.15	2108.49	0.00	166.62	92.02	0.00	258.64	98.63
$0.95b_m$	829.52	2270.24	0.00	170.45	90.12	0.00	260.56	99.36
$1.00b_m$	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
$1.05b_m$	845.24	2593.68	0.00	176.97	86.73	0.00	263.70	100.56
$1.10b_m$	851.89	2755.37	0.00	179.76	85.23	0.00	264.99	101.05
$1.15b_m$	857.90	2917.06	0.00	182.31	83.83	0.00	266.14	101.49
$1.20b_m$	863.35	3078.73	0.00	184.63	82.53	0.00	267.16	101.88
$1.25b_m$	868.32	3240.40	0.00	186.76	81.32	0.00	268.08	102.23

Table C.10. Comparative Statics: c_1

c_1	α_0	α_1	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
$0.75c_1$	35173.07	-4.043018	14480.25	-0.553800	2275.850	-0.085984	6645.520	-0.251930
$0.80c_1$	35164.61	-4.043018	14478.81	-0.553800	2275.640	-0.085984	6644.865	-0.251930
$0.85c_1$	35156.16	-4.043018	14477.37	-0.553800	2275.430	-0.085984	6644.210	-0.251930
$0.90c_1$	35147.71	-4.043018	14475.93	-0.553800	2275.220	-0.085984	6643.555	-0.251930
$0.95c_1$	35139.25	-4.043018	14474.49	-0.553800	2275.010	-0.085984	6642.900	-0.251930
$1.00c_1$	35130.80	-4.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	-0.251930
$1.05c_1$	35122.35	-4.043018	14471.61	-0.553800	2274.590	-0.085984	6641.589	-0.251930
$1.10c_1$	35113.89	-4.043018	14470.17	-0.553800	2274.380	-0.085984	6640.934	-0.251930
$1.15c_1$	35105.44	-4.043018	14468.73	-0.553800	2274.170	-0.085984	6640.279	-0.251930
$1.20c_1$	35096.99	-4.043018	14467.29	-0.553800	2273.960	-0.085984	6639.624	-0.251930
$1.25c_1$	35088.53	-4.043018	14465.85	-0.553800	2273.750	-0.085984	6638.969	-0.251930

c_1	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
$0.75c_1$	838.88	2435.24	0.00	174.31	87.36	0.00	261.67	99.78
$0.80c_1$	838.67	2434.59	0.00	174.22	87.56	0.00	261.78	99.83
$0.85c_1$	838.46	2433.93	0.00	174.14	87.76	0.00	261.90	99.87
$0.90c_1$	838.25	2433.28	0.00	174.05	87.96	0.00	262.01	99.91
$0.95c_1$	838.04	2432.62	0.00	173.96	88.16	0.00	262.12	99.96
$1.00c_1$	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
$1.05c_1$	837.62	2431.31	0.00	173.79	88.56	0.00	262.35	100.04
$1.10c_1$	837.41	2430.66	0.00	173.70	88.76	0.00	262.46	100.09
$1.15c_1$	837.20	2430.00	0.00	173.61	88.96	0.00	262.58	100.13
$1.20c_1$	836.99	2429.35	0.00	173.53	89.16	0.00	262.69	100.17
$1.25c_1$	836.78	2428.69	0.00	173.44	89.36	0.00	262.80	100.22

Table C.11. Comparative Statics: c_2

c_2	o_0	o_1	$\tilde{o}_0$	$\tilde{o}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
$0.75c_2$	35150.88	-4.040299	14481.13	-0.553632	2275.797	-0.085948	6645.920	-0.251854
$0.80c_2$	35146.87	-4.040843	14479.51	-0.553666	2275.598	-0.085955	6645.184	-0.251869
$0.85c_2$	35142.85	-4.041387	14477.89	-0.553699	2275.398	-0.085962	6644.449	-0.251884
$0.90c_2$	35138.83	-4.041931	14476.28	-0.553733	2275.199	-0.085970	6643.714	-0.251900
$0.95c_2$	35134.81	-4.042474	14474.66	-0.553766	2274.999	-0.085977	6642.979	-0.251915
$1.00c_2$	35130.80	-4.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	-0.251930
$1.05c_2$	35126.79	-4.043562	14471.43	-0.553833	2274.601	-0.085991	6641.510	-0.251945
$1.10c_2$	35122.77	-4.044105	14469.82	-0.553867	2274.401	-0.085998	6640.776	-0.251960
$1.15c_2$	35118.76	-4.044649	14468.21	-0.553900	2274.202	-0.086005	6640.042	-0.251976
$1.20c_2$	35114.75	-4.045193	14466.60	-0.553934	2274.003	-0.086013	6639.309	-0.251991
$1.25c_2$	35110.74	-4.045736	14464.98	-0.553967	2273.804	-0.086020	6638.575	-0.252006
c_2	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
$0.75c_2$	839.42	2436.91	0.00	174.54	88.02	0.00	262.56	100.12
$0.80c_2$	839.10	2435.92	0.00	174.41	88.09	0.00	262.49	100.10
$0.85c_2$	838.79	2434.93	0.00	174.27	88.16	0.00	262.43	100.07
$0.90c_2$	838.47	2433.94	0.00	174.14	88.22	0.00	262.37	100.05
$0.95c_2$	838.15	2432.95	0.00	174.01	88.29	0.00	262.30	100.02
$1.00c_2$	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
$1.05c_2$	837.51	2430.98	0.00	173.74	88.43	0.00	262.17	99.98
$1.10c_2$	837.19	2429.99	0.00	173.61	88.50	0.00	262.11	99.95
$1.15c_2$	836.87	2429.00	0.00	173.48	88.56	0.00	262.04	99.93
$1.20c_2$	836.55	2428.01	0.00	173.35	88.63	0.00	261.98	99.90
$1.25c_2$	836.23	2427.02	0.00	173.21	88.70	0.00	261.91	99.88

Table C.12. Comparative Statics: c_3

c_3	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
$0.75c_3$	35127.09	-4.043018	14477.75	-0.553800	2275.604	-0.085984	6644.541	-0.251930
$0.80c_3$	35127.83	-4.043018	14476.81	-0.553800	2275.443	-0.085984	6644.082	-0.251930
$0.85c_3$	35128.57	-4.043018	14475.87	-0.553800	2275.283	-0.085984	6643.622	-0.251930
$0.90c_3$	35129.31	-4.043018	14474.93	-0.553800	2275.122	-0.085984	6643.163	-0.251930
$0.95c_3$	35130.06	-4.043018	14473.99	-0.553800	2274.961	-0.085984	6642.704	-0.251930
$1.00c_3$	35130.80	-4.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	-0.251930
$1.05c_3$	35131.54	-4.043018	14472.11	-0.553800	2274.639	-0.085984	6641.785	-0.251930
$1.10c_3$	35132.29	-4.043018	14471.17	-0.553800	2274.478	-0.085984	6641.326	-0.251930
$1.15c_3$	35133.03	-4.043018	14470.23	-0.553800	2274.317	-0.085984	6640.867	-0.251930
$1.20c_3$	35133.78	-4.043018	14469.29	-0.553800	2274.156	-0.085984	6640.407	-0.251930
$1.25c_3$	35134.52	-4.043018	14468.35	-0.553800	2273.995	-0.085984	6639.948	-0.251930
c_3	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
$0.75c_3$	838.63	2434.26	0.00	174.21	88.48	0.00	262.69	100.17
$0.80c_3$	838.47	2433.80	0.00	174.14	88.46	0.00	262.60	100.14
$0.85c_3$	838.31	2433.34	0.00	174.08	88.43	0.00	262.51	100.10
$0.90c_3$	838.15	2432.88	0.00	174.01	88.41	0.00	262.42	100.07
$0.95c_3$	837.99	2432.43	0.00	173.94	88.38	0.00	262.33	100.03
$1.00c_3$	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
$1.05c_3$	837.67	2431.51	0.00	173.81	88.34	0.00	262.14	99.97
$1.10c_3$	837.51	2431.05	0.00	173.74	88.31	0.00	262.05	99.93
$1.15c_3$	837.34	2430.59	0.00	173.68	88.29	0.00	261.96	99.90
$1.20c_3$	837.18	2430.13	0.00	173.61	88.26	0.00	261.87	99.86
$1.25c_3$	837.02	2429.67	0.00	173.54	88.24	0.00	261.78	99.83

Table C.13. Comparative Statics: r

r	o_0	o_1	$\tilde{o}_0$	$\tilde{o}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
$0.75r$	54206.95	-3.038603	17064.68	0.415740	2678.212	-0.064572	7821.209	-0.189125
$0.80r$	49106.82	-3.239306	16372.51	-0.443341	2570.434	-0.068852	7506.332	-0.201681
$0.85r$	44788.84	-3.440114	15786.08	-0.470948	2479.141	-0.073133	7239.557	-0.214240
$0.90r$	41095.15	-3.641011	15284.13	-0.498561	2401.015	-0.077416	7011.217	-0.226801
$0.95r$	37906.37	-3.841983	14850.58	0.526178	2333.545	-0.081699	6813.990	-0.239365
$1.00r$	35130.80	-4.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	-0.251930
$1.05r$	32697.00	-4.244108	14141.87	0.581424	2223.275	-0.090269	6491.589	-0.264497
$1.10r$	30548.59	-4.445244	13849.44	-0.609052	2177.782	-0.094555	6358.556	-0.277065
$1.15r$	28640.59	4.646421	13589.65	-0.636682	2137.371	-0.098841	6240.374	-0.289634
$1.20r$	26936.70	-4.847634	13357.59	-0.664314	2101.276	-0.103128	6134.807	-0.302204
$1.25r$	25407.36	-5.048879	13149.25	-0.691948	2068.874	-0.107415	6040.033	-0.314775
r	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
$0.75r$	1599.08	4660.53	0.00	633.39	-363.03	0.00	270.36	103.10
$0.80r$	1419.78	4135.82	0.00	499.31	-205.01	0.00	294.30	112.23
$0.85r$	1256.93	3659.16	0.00	391.34	-89.11	0.00	302.23	115.25
$0.90r$	1107.23	3220.89	0.00	303.67	-5.74	0.00	297.93	113.61
$0.95r$	968.17	2813.70	0.00	232.19	51.81	0.00	284.00	108.30
$1.00r$	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
$1.05r$	714.69	2071.29	0.00	126.52	107.42	0.00	233.95	89.21
$1.10r$	597.57	1728.22	0.00	88.45	111.62	0.00	200.07	76.29
$1.15r$	485.52	1399.98	0.00	58.39	102.91	0.00	161.30	61.51
$1.20r$	377.79	1084.34	0.00	35.35	82.80	0.00	118.15	45.06
$1.25r$	273.74	779.47	0.00	18.56	52.47	0.00	71.03	27.09

Table C.14. Comparative Statics: α

α	ϕ_0	ϕ_1	$\tilde{\phi}_0$	$\tilde{\phi}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
0.75α	35638.66	-1.042874	14615.33	-0.553791	2289.955	-0.085982	6692.400	-0.251926
0.80α	35536.53	-1.042903	14586.80	-0.553793	2286.912	-0.085982	6682.334	-0.251927
0.85α	35434.68	-1.042932	14558.30	-0.553795	2283.875	-0.085983	6672.285	-0.251928
0.90α	35333.11	-1.042961	14529.85	-0.553796	2280.844	0.085983	6662.254	-0.251928
0.95α	35231.82	-1.042989	14501.43	-0.553798	2277.819	-0.085984	6652.241	-0.251929
1.00α	35130.80	-1.043018	14473.05	-0.553800	2274.800	-0.085984	6642.244	-0.251930
1.05α	35030.06	-1.043047	14444.71	-0.553801	2271.787	-0.085984	6632.266	-0.251931
1.10α	34929.59	-1.043076	14416.40	-0.553803	2268.779	-0.085985	6622.304	-0.251931
1.15α	34829.40	-1.043104	14388.13	-0.553805	2265.778	-0.085985	6612.359	-0.251932
1.20α	34729.48	-1.043133	14359.91	-0.553806	2262.782	-0.085985	6602.431	-0.251933
1.25α	34629.83	-1.043162	14331.71	-0.553808	2259.792	-0.085986	6592.521	-0.251934
α	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
0.75α	853.01	2482.18	0.00	180.24	84.97	0.00	265.20	101.13
0.80α	849.96	2472.11	0.00	178.95	85.67	0.00	264.62	100.91
0.85α	846.92	2462.04	0.00	177.67	86.36	0.00	264.03	100.68
0.90α	843.88	2452.00	0.00	176.40	87.03	0.00	263.43	100.46
0.95α	840.85	2441.97	0.00	175.13	87.70	0.00	262.84	100.23
1.00α	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
1.05α	834.81	2421.97	0.00	172.62	89.01	0.00	261.63	99.77
1.10α	831.79	2412.00	0.00	171.38	89.64	0.00	261.02	99.54
1.15α	828.79	2402.04	0.00	170.14	90.27	0.00	260.41	99.31
1.20α	825.78	2392.10	0.00	168.91	90.89	0.00	259.80	99.07
1.25α	822.79	2382.18	0.00	167.69	91.49	0.00	259.18	98.84

Table C.15. Comparative Statics: β

β	α_0	α_1	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\hat{x}_0$	$\hat{x}_1$	$\hat{\tau}_0$	$\hat{\tau}_1$
0.75β	34811.42	-1.043594	7190.08	-0.276917	2264.797	-0.085991	6608.695	-0.251945
0.80β	34917.59	-1.043402	8646.65	-0.332293	2268.125	-0.085989	6619.859	-0.251940
0.85β	34993.60	-1.043265	10103.24	-0.387670	2270.506	-0.085987	6627.845	-0.251936
0.90β	35050.71	-1.043162	11559.83	-0.443046	2272.294	-0.085986	6633.841	-0.251934
0.95β	35095.18	-1.043082	13016.44	-0.498423	2273.686	-0.085985	6638.508	-0.251932
1.00β	35130.80	-1.043018	14473.05	-0.553800	2274.800	0.085984	6642.244	-0.251930
1.05β	35159.96	-1.042966	15929.66	0.609176	2275.712	-0.085983	6645.303	-0.251929
1.10β	35184.28	-1.042922	17386.28	-0.664553	2276.473	-0.085983	6647.853	-0.251927
1.15β	35204.88	-1.042885	18842.90	-0.719930	2277.116	-0.085982	6650.011	-0.251926
1.20β	35222.54	-1.042854	20299.52	-0.775306	2277.668	-0.085982	6651.862	-0.251926
1.25β	35237.85	-1.042826	21756.14	-0.830683	2278.147	-0.085982	6653.466	-0.251925
β	$\hat{X}(Y)$	$\hat{\tau}(Y)$	$\hat{M}(Y)$	$CS(Y)$	$\Pi(Y)$	$R(Y)$	$W(Y)$	I_W
0.75β	827.70	2398.16	0.00	169.70	90.49	0.00	260.19	99.22
0.80β	831.07	2409.41	0.00	171.08	89.80	0.00	260.88	99.48
0.85β	833.48	2417.46	0.00	172.08	89.29	0.00	261.37	99.67
0.90β	835.29	2423.50	0.00	172.82	88.90	0.00	261.73	99.81
0.95β	836.70	2428.20	0.00	173.41	88.60	0.00	262.01	99.91
1.00β	837.83	2431.97	0.00	173.88	88.36	0.00	262.24	100.00
1.05β	838.75	2435.05	0.00	174.26	88.16	0.00	262.42	100.07
1.10β	839.52	2437.62	0.00	174.58	87.99	0.00	262.57	100.13
1.15β	840.17	2439.79	0.00	174.85	87.85	0.00	262.70	100.18
1.20β	840.73	2441.66	0.00	175.08	87.73	0.00	262.81	100.22
1.25β	841.22	2443.27	0.00	175.28	87.62	0.00	262.91	100.26

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