Electoral Competition with Outside Options: A Game Theoretic Analysis of Transition to Democracy.

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Introduction

During the last decade, at least 40 countries around the world have experienced major democratic reforms. According to Huntington (1991), this new wave of democratization is the third and most important in the history of the world. However, while in "old" democracies there is no doubt that parties will comply with electoral verdicts, this can not be taken for granted in the newly democratized countries. This situation makes electoral competition more uncertain and the political process more complex. How does this additional dose of uncertainty and complexity affect voting behavior? What kind of governments and policy outcomes will emerge? This paper provides a game theoretic framework to address these issues. We present a model of electoral competition in which there is a positive probability that the loser does not comply with or abide by the results of the election and chooses an outside option. Whether or not the outside option is exercised depends on the ability of the winner to compromise on policy grounds or portfolio distribution. The following are two examples of this pattern of political behavior, presented by Przeworski (1991) as characteristic of transition to democracy.

In February 1990, President de Klerk of South Africa decided to lift the ban on the African National Congress, the Pan Africanist Congress, and the South African Communist Party and began negotiations. The forum for the negotiations was the Conference for a Democratic South Africa and it took place in an atmosphere full of uncertainty and suspicion. The agreement called for a legally mandated five year government of National Unity regardless of the outcome of the 1994 elections, for a cabinet representation for all parties that won at least five percent of the vote, and a share of executive for the National Party. In October 1993, an Act was passed to create a multiparty executive body called Transitional Executive Council which became a kind of supercabinet.

In Nicaragua, despite winning the elections by a 15% margin in February 1990, Violetta Chamorro named a number of influential Contras to key government positions, while continuing a social pact with the Sandinistas. While accepting the electoral verdict, Daniel Ortega, leader of the Sandinistas, told reporters in March 1990 that the Revolution was "irreversible" and that "the majority in quality" would rule from below. Following this

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1 The first wave began in 1782 bringing into being 29 countries including the United States. The second wave followed the victory of the allies in World War II and reached its peak in 1962 with 36 countries.
verdict, the new government was confronted with a weak economy reeling from a series of clashes between peasants, who had benefited from Sandinistas' land policies, and demobilized Contras pressing for promised land and monetary compensation. This further illustrates and justifies that elections in these polities are always followed by some kind of political bargaining since there is a threat that the loser might "take to the streets."  

Voting behavior during democratic transition might appear to be very puzzling and enigmatic. In many countries, voters seem to prefer candidates from parties representing the least military threat: this was the case in South Africa, Nicaragua and Benin. The contrary was true from Guatemala and El Salvador where candidates with stronger military support have been elected. Furthermore, the diversity in democratization outcomes is striking and does not necessarily reflect a diversity in cultures or social institutions. Benin and Togo are countries with very similar social structure but democratization has been successful in Benin and failed in Togo. The same can be said about Ukraine and Russia. History, social structure and values are relevant in explaining political performance. However, these values arise not as "spontaneous generations" but as outcomes of strategies and actions of political agents. We need not only describe the historical factors of political development but also explain how the institutions they generate systematically affect collective behavior. History and culture constrain the choice set of voters and politicians and the degree of national unity only determines the potential for a successful democratic transition. A thorough analysis must focus on the types of actions and institutions which help to achieve a successful transition to democracy.

A key ingredient for this success seems to be the ability of parties to make compromises. According Dankwart Rustow, transition to democracy usually arises as outcome of an intense and obviously unwinnable struggle, when all parties become weary of interminable hostilities and key elites have

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2The 1800 election in the United States also provides an interesting illustration of the effect of threats of violence on political bargaining. (Illustration suggested by Keith Poole). This was the first peaceful transfer of power between opposing parties. In the 1789 constitution, there was no division between the ballots for president and the vice president. As a result, Thomas Jefferson and Aaron Burr both received the same number of electoral votes even though both were running mates. This threw the elections in the house of representatives where the federalists contrived to elect Burr rather Jefferson. During this period of controversy, some federalists leaders from the New England threatened to raise a militia and secede from the Union. This threat had an immediate effect on the bargaining to break the deadlock in the House of Representatives. Clearly, in this case outside options affected the political bargaining.
the will and the talent to negotiate a settlement.

A formal theoretic analysis of the political issues arising from transition to democracy are important for economic decision making. This is due to the fact that the political violence generated by a democratization crisis may be costly for economic activity and policy making. These costs may take a dimension of a natural disaster: shut down of education system, health services or transportation facilities, assassinations or emigration flows. This paper provides a model which sheds light on some of the enigmas of transition to democracy. Why in some countries do voters reward political extremism while in others they do not? How does the outside option of violence affect electoral outcome. Why in countries with similar social institutions do the outcomes of the democratization process differ so significantly? The model also explores the validity of the median voter theorem and Hotelling’s minimum differentiation principle as applied to newly democratic politics.

The model is based in the following assumptions:

1) Parties and voters are rational and have single-peaked policy preferences.

2) Voters are uniformly distributed around the median and there is one party on the left and one on the right of the political spectrum.

3) Each party has a political wing which conceives its political agenda. Each party also has a “military” wing, that is, a group of “activists” with military equipments, capable of initiating and organizing riots.

4) Winning the election gives the power to propose a policy compromise and the winner makes a take-it-or-leave-it policy offer.

While the first two assumptions are standard in the literature in formal political theory and rational choice comparative politics (Austen-Smith and Banks (1988), Baron (1993), Bates (1990)), the third and the fourth can be justified by empirical evidence. It is hardly imaginable that a party wins the elections and lets its opponent set the political agenda. Also the pressure of forming the new government and the cost of a suspension of the governmental activities makes parties very impatient in the post-election bargaining. This justifies modelling the process as a finite horizon game. Furthermore introducing renegotiation in the bargaining structure makes the equilibrium policy more costly to implement.

The paper is organized as follows: Section 1 describes the game. Section 2.1 analyses equilibrium behavior, assuming that outside options are exogenous and common knowledge. We find that voters might prefer the candidate with the least valuable outside option even if he is more “extremist”. The resulting policy outcome is “moderated” and there is no “riot”.

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We then extend the basic model in three directions. In section 2.2, we assume that the extent of the outside options values are private information and show that unless the costs associated with the outside options are "very" high, "riot" will be observed with positive probability and voters will lean towards the party with the most valuable outside option. In Section 3.1, outside options depend on vote share. Although this does not affect the voting outcome, voters can "fine tune" the policy outcome and make it coincide with the median voter ideal point. In Section 3.2, parties choose positions before the elections. In equilibrium they remain polarized and this platform divergence increases as the value of their opponent's outside option increases. Finally, Section 4 discusses the implications of the model for the design of democratic institutions.

1 Description of the Game

I analyze a political game in which a "strong" party $s$ and a "weak" party $w$ compete in a one dimensional policy space $P \subseteq \mathbb{R}$, for votes from a finite set of $N$ individuals. Each party has a policy position $b_k$ and an "outside option" of value $c_k$ where $k \in \{s, w\}.$ Voters are in odd number and are uniformly distributed around the median voter, $M$. Voter $i$'s most preferred policy is $\theta_i$ where $i \in [1, N]$. Party $w$ is positioned at the right and party $s$ is positioned at the left of median voter, that is $b_s < M < b_w$.

Time Sequence

The game starts with an election. At $t = 1$, each voter observes $b_k$ and $c_k$ and selects either $s$ or $w$. The party with the highest share of votes wins. At $t = 2$, the winner, for example $w$, makes an offer of policy compromise to the loser, for example $s$. It is a "take it or leave it" offer $y_c \in [b_w, \theta_M]$. At $t = 3$, the loser accepts or rejects the offer. If the loser accepts, the game ends with the proposed policy being implemented. A rejection triggers a conflict at $t = 4$. The winner for example $w$ can then opt to "fight" or "not fight". The "no fight" option leads to the end of the game and the loser's most preferred policy being implemented. Otherwise the conflict is eventually settled with a resulting policy compromise $\bar{y}_c \in [\theta_M, b_s]$. The resulting value of the outside option of $s$ is $c_s = (\bar{y}_c - b_s)^2$ and can be used to measure its "rioting".

\footnote{We can think of $b_k$ as being the product of the party's political wing while $c_k$ is the product of its military wing. In addition $s$ and $w$ refer to the strength of outside options.}
ability”. The hypothetical post-election conflict generates a cost \( c_a \) to the winning party and \( d_s \) to each voter. Party \( w \) is assumed to have a less valuable outside option and to generate less costly conflicts than the strong party; that is \( v_w \geq v_s, c_s \geq c_w \) and \( d_s \geq d_w \).

**Strategy sets**

Let \( y_k \) be the policy proposal made by party \( k \) if elected, \( r_k(.) \) the “yes” or “no” response to \( y_k \) by party \( k \). Let \( \pi^i_k(b_w, b_s) \) be the probability that \( i \) votes for \( k \) given policy positions \( b_w \) and \( b_s \) and \( \pi_k(b_w, b_s) \) be the probability that \( k \) is elected. A strategy for voter \( i \) is \( \pi^i_k \in \Delta(K) \), where \( \Delta(K) \) is the probability distribution over \( K \), the set of parties. A strategy for party \( k \) is the set of all \( (\Gamma_k, r_k) \) such that \( \Gamma_k : \Delta(K) \rightarrow P \) and \( r_k : \Delta(K) \times P \rightarrow [0, 1] \).

**Payoffs**

Assume voters are policy oriented and have preferences that can be represented by quadratic utility functions. Since at the voting stage the policy outcome is uncertain, voters form expectations over \( y_k \).

Let \( y \) denote the vector \( (y_w, y_s) \) and \( r = (r_w, r_s) \).

Thus, the expected utility of voter \( i \) is

\[
E_{\pi^i_k} [V_i(y, r)] = -\sum_k \pi^i_k(b_w, b_s) \left\{ \left[ y_k - \theta_i \right]^2 \cdot r_k(\cdot) + d_k(1 - r_k(\cdot)) \right\}
\]

(1)

where \( \theta_i \) is voter \( i \)'s most preferred policy; \( d_k \) is the cost imposed on voter \( i \) by party \( k \) when this party exerts its outside option.

Parties’ expected utilities depend on the difference between their policy position and the final policy outcome. The expected utility of party \( k \) is

\[
E_{\pi_k} [U_k(y, r)]
\]

\[
= -\pi_k(b_w, b_s) \left\{ \left[ (y_k - b_k)^2 \cdot r_k(\cdot) + v_k(1 - r_k(\cdot)) \right]\right\}
\]

(2)

For example, if \( w \) is elected, that is if \( \pi_w(\cdot) = 1 \), and there is no policy compromise (\( r_w(\cdot) = 0 \)), then the utility of voter \( i \) is \( V_i(\cdot) = -d_s \) and the utility of party \( w \) is \( U_w(\cdot) = -c_s \). The utility of \( s \) is given by the value of its outside option, \( -v_s \). If there is a policy compromise, that is when \( r_s(\cdot) = 1 \), and \( y_w \in [b_M, b] \), voter \( i \)'s utility is given by \( V_i(\cdot) = -(y_w - \theta_i)^2 \).
parties' utilities are, respectively $U_a(\cdot) = -(y_a - b_w)^2$ and $U_s(\cdot) = -(y_s - b_s)^2$. The payoffs in the case that $s$ wins are defined in a similar fashion.

For the sake of tractability and to facilitate the analysis of equilibrium behavior, I will make the following normalizations: $b_s = 0$, $b_w = b$, $\theta_M = 1$. In addition, $b \geq 2$. The following game tree summarize the structure of the model.

(Insert fig 1 and 2 here.)

2 Equilibrium Analysis

2.1 Complete Information

Definition 1. A political equilibrium under complete information is a profile of policy proposals, $y_k^*(\cdot)$, acceptance strategies $r_k^*(\cdot)$, and an $N$-tuple of voting strategies $\pi^*(b)$, such that,

1) $y_k^*(\cdot)$ and $r_k^*(\cdot)$ are a subgame perfect equilibrium for the bargaining game, that is,

$$r_k^*(\cdot) = 1 \text{ if } U_k(y_k^*, b_k) \geq -v_k \text{ and } 0 \text{ otherwise}. $$

Moreover,

$$y_k^* \in \arg \max U_k(y_k^*, r_k^*).$$

2) $\pi^*(b)$ is a voting equilibrium. That is,

$$E_{\pi^*} \left[ V_i(y_i^*, d_n, r_i^*) \right] \geq E_{\pi^*} \left[ V_i(y_i^*, d_n, r_i^*) \right].$$

In other words, in a political equilibrium, the winner makes the proposal which maximizes its utility and the loser accepts if its resulting utility is at least as high as the value of its outside option, and rejects anything else. Before computing the political equilibrium, I consider the following extreme cases. First, if parties have the same level of bargaining power, that is $c_w = c_s$, then the post election bargaining is irrelevant and only the parties' positions matter in determining the electoral outcome. On the

\textsuperscript{4}This implies that $p_s = \frac{\sqrt{c_s}}{b}$ and $p_w = 1 - \frac{\sqrt{c_w}}{b}$. 

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other hand, if one party has all the bargaining power and its opponent has none, that is \( v_s = 0, v_w = b^2 \) and \( c \leq c_s \) then the strong party’s position will be implemented no matter what. As a result, voting becomes irrelevant. Thus, the political equilibrium and the very structure of the game is affected by the distribution of “bargaining abilities” and the distance between parties’ positions. When parties are close to each other in terms of policy positions or evenly matched in terms of ability to riot, then the electoral process is decisive in policy formation. In contrary, when parties’ positions are “too” distant or their “outside strength” too different, then voters are powerless.

Parties’ willingness to make compromises depends on how costly are the outside options. If these costs are not high enough the winner will not make any policy concession. As a result, there will be riot in equilibrium. This is avoided only if the utility resulting from offering to the loser its reservation policy share is higher than the cost associated with this party’s outside option. Technically, this means that if \( (\sqrt{c_s} - b)^2 \leq c_k \), then there exists an equilibrium policy \( y_k^* \) such that \( r_s^* = 1 \).

**Proposition 1:** If \( (\sqrt{c_s} - b)^2 \leq c_k \), the equilibrium for this game can be characterized as follows: Party \( s \) proposes the minimum acceptable policy compromise, that is \( \lambda_s^* = p_s \), which is accepted by \( w \). Party \( w \) proposes \( \lambda_w^* = p_w \), which is accepted by \( s \). In addition, party \( s \) is elected if \[-\sqrt{c_s} + (b - 1) < |\sqrt{c_s} - 1| \] and party \( w \) is elected if \[-\sqrt{c_w} + (b - 1) > |\sqrt{c_w} - 1| \]. Otherwise, both are elected with probability one half. In other words, voters prefer \( w \) unless this party is “too extremist”.

**Proof:** (in appendix)

In other words, the winner concedes to the minimal demand of the loser and voters “prefer” the weak party or the most powerful moderactor. This equilibrium exhibits an interesting feature. The median voter does not necessarily vote for the party whose position is the closest to his ideal point. For instance, suppose \( v_v = 4, v_s = 1, b_w = b = 2.5, b_s = 0 \) and \( \theta_M = 1 \). The equilibrium policy outcome is \( y_s^* = .5 \) if \( s \) is elected and \( y_w^* = 1 \) if \( w \) is elected. As a result, party \( w \) is elected despite being positioned further away from the median voter’s ideal point. For \( s \) to win, \( v \), has to be sufficiently “high” or \( b \) to be “sufficiently” big.

The threat of disruption of the post-election environment explains why winner does not take all even in the purely majoritarian system considered in this paper. Furthermore, if it was not “too” costly for the winner to make
"too much" compromise, that is if \( C \leq c_l \), the parties might gain by losing the elections. For instance, if \( v_s = 0, v_u = 1 \), the strong party ideal point will be implemented in the case that this party loses the elections but it has to make concession in the case that it wins. So \( s \) has a higher utility by losing than winning. Then, a very uneven distribution of rioting abilities could pervert the political system in such a way that some parties might be willing to lose the elections.

2.2 Incomplete Information

As the previous analysis shows, unless the cost generated by outside options are "too" low, the political equilibrium in the previous section is "riot-free". This result relies on the assumption that the parties' characteristics are common knowledge. However, since the balance of forces in political conflicts are rarely fixed or known in advance, it is reasonable to think of parties' bargaining powers or "riot's abilities" as being private information. As a result, the winning party has to guess what its opponent really wants and then face the prospect of not making enough concessions and seeing its policy offer being rejected. If the winning party has enough at stake, it might want to make greater concessions than necessary. Moreover, since voters are more penalized when the strong party "takes to the streets", they have to weigh the gain - in policy terms - if they vote for the weak party against the prospect of a more costly conflict. If this cost is relatively high, they may simply vote for the strong party. Then, to improve its chance of being elected, the weak party will have to be positioned closer to the median voter's ideal point. Below, I derive these conjectures as equilibrium behavior by relaxing the informational assumption on the value of party \( k \)'s outside option.

We assume that parties hold private information about their bargaining power \( v_k \) and investigate how this information structure affects the policy outcome and voting behavior. For this purpose, let us assume that \( v_s \) is distributed in the interval \([1, 3]\) and \( v_w \) in \([3.5, 4]\) with cumulative \( F \) and density \( f \). The probability that an agreement is reached is then \( r_{s}(\cdot) = \Pr \left\{ (1 - \lambda_s) \cdot b \cdot b_s^2 \leq v_s \right\} = 1 - F(\lambda_s, b) \) when \( s \) is elected and \( r_{w}(\cdot) = \Pr \left\{ (1 - \lambda_w) \cdot b_w^2 \leq v_w \right\} = 1 - F(\lambda_w, b) \) when \( w \) is elected.\(^5\)

\(^5\)Consequently, if \( F \) is uniformly distributed the probability of civil unrest is \( (1 - r_s(\cdot)) \frac{|(1 - \lambda_w) \cdot b|^2 - 1}{2} \) in the case that \( w \) is elected and \( (1 - r_w(\cdot)) \)
The payoffs are given as in (1) and (2).

**Definition 2:** A political equilibrium under incomplete information is a policy proposal \( y^*_k \), a response \( r^*_k(\cdot) \), an \( X \)-tuple of voting strategies \( \pi^*(\cdot) \) such that

1) \( y^*_k \) and \( r^*_k(\cdot) \) is a Bayesian equilibrium for the post-election bargaining game; that is,

\[
v^*_k = 1 - F(y^*_k, b) \quad \text{and} \quad y^*_k = \arg \max_{y_k} E\left[ V_k \left( y_k, r^*_k \right) \right].
\]

2) \( \pi^*(\cdot) \) is a voting equilibrium. That is,

\[
E_{\pi^*} \left[ V_i \left( y^*_k, d_{u_i}, r^*_k \right) \right] \geq E_{\pi \neq \pi^*} \left[ V_i \left( y^*_k, d_{u_i}, r^*_k \right) \right].
\]

The following assumption known in the literature on Bayesian games as monotonic hazard rate condition, ensures that there is an interior solution to max \( E\left[ V_k \left( y_k, r^*_k \right) \right] \).

**Assumption 1**

\[
\frac{f^\prime(y_k, b)}{1 - F(y_k, b)} \quad \text{is increasing for all} \quad y_k \in [0, 1].
\]

This expression represents the conditional probability of acceptance when the policy offer lies in the interval \( [y_k, y_k - d y_k] \).

**Proposition 2,** (the bargaining outcome): Under Assumption 1, the equilibrium bargaining outcome is such that when the costs induced by the outside options are “too” high, parties secure acceptance by offering the minimum policy compromise acceptable by the most demanding “type” of their opponent. When the costs are “moderate” they make offers which are rejected with positive probability. They do not make any offer if the costs are “too” low.

**Proof:** (in appendix).

The bargaining outcome is clearly inefficient (in the full information sense) since in general there is a positive probability of rioting. The only
way civil unrest can be avoided is to have the winner making “too much”
compromise and this happens only when the outside options are assumed to
be “too” costly. In addition, unless \( \bar{y}_s = b - 1 \) and \( \bar{y}_w = \frac{b}{2} \), the equilibrium
policy outcome never coincides with the median voter’s ideal point.

Voting behavior, like the bargaining outcome, depends critically on the
cost structure of the outside options. When these options are too costly,
there is no riot in equilibrium and as a result, voters still prefer the most
powerful moderator. This result holds even when only one party has a
“very” costly outside option. With “moderate” costs, civil unrest breaks
out regardless of which party is in office. Voters then weigh the policy gain
when \( w \) is elected against the cost differential if civil unrest breaks out.
This situation clearly increases the likelihood of the strong party, \( s \), being elected.

In other words, voters will rather have the most “destructive” party setting
the agenda since with this party in office, conflicts are less costly and less
likely. This is summarized by the following proposition.

**Proposition 3, (the voting outcome):** If \( r_k \) is a random variable and
each is “moderate” then the range of policy positions giving electoral victories
to \( w \) shrinks. That is, \( \pi^*_w = 1 \) for all \( d_w \) only if \( b \in [1,2] \).

*Proof (in appendix).*

The following example will help to understand this statement. Suppose
\( b = 2 \), that is parties are equidistant from the median voter’s ideal point.
Without uncertainty we know from the results of Section 1 that \( w \) is the
winner no matter what. Under the new circumstances, this party wins
only if the “adjusted” cost differential is “too low”, that is \( 1 - r_n^* \cdot d_n
-(1 - r_w^*) \cdot d_w \leq r^*_s - r^*_w \). Thus, uncertainty reduces the range of policy
positions which give party \( w \) the edge over its opponent.

**Comparative Statics**

I now investigate how costs of rioting and the distance between parties’
positions affect the probability of rioting. First, we have \( \frac{\partial r_n^*}{\partial v} \geq 0 \). That
is, rejection becomes less likely when the cost of disagreement increases.
As a result, \( w \) enhances its chances of being elected. This result is a bit
depressing. It means that a peaceful transition to democracy becomes more
likely only if parties’ destruction power increases. Second, we have \( \frac{\partial r_w^*}{\partial b} \leq 0 \).
This means that as parties become more distant from each other, they are less likely to reach an agreement. As a result, voters will lean toward $s$. Thus in a country with two "extremist" parties, like in Guatemala or El Salvador, voters are more likely to prefer the party with the highest "ability to riot."

The equilibrium policy outcome of the model is in general inefficient in the sense that this policy is skewed away from the median voter's ideal point. In the following lines, I propose extensions of the model for which the median voter's most preferred policy will be implemented.

3 Endogenous Outside Options, Political Polarization and Median Voter Theorem

3.1 Outside Option as a function of vote share

In Sections 1 and 2 the ability to riot is assumed to be exogenous. The median voter could only "make the best of a bad situation" by choosing the party whose policy concession is closer to his ideal point. To introduce more flexibility into the process of policy formation, I assume that outside options depend on electoral support. This means that there is a positive correlation between electoral support and the number of days of rioting that parties' military wings can produce. This assumption can be easily supported by empirical evidence. In Pakistan and South Africa, there is an obvious connection between the intensity of political violence conducted by the Islamic Democratic Alliance and the Inkatha Freedom Party and their expected or realized electoral support. The assumption allows every voter to influence the final policy outcome, which can then be made arbitrarily close to the median voter's ideal point. The electorate will not only select a winner but also give the loser the exact voter share which will lead to the median voter ideal point as the final policy outcome.

Define by $\sigma_k$ the vote share of party $k$ during the elections. Assume that $s$ wins $\sigma_s$ of the vote and $w$ wins $\sigma_w$ of the vote. Also parties' "ability to riot" $v_k$ are now assumed to be strictly increasing mappings. In addition, as in section 1 $w$ is assumed to have a less valuable outside option: that is, $v_s(\sigma) \leq v_w(\sigma)$. The mapping $v(.)$ is common knowledge. The time sequence of the electoral competition is exactly like in Section 1. The payoffs are the same except that $v_k$ now becomes $v_k(\sigma_k)$, $d_k$ becomes $v_k(\sigma_k)$ and $c_k, c_k(\sigma_k)$.
Finally, I assume that parties are equidistant from the median voter. The equilibrium in the post-election bargaining game is exactly as in Proposition 1. The following proposition presents the equilibrium voting behavior.

**Proposition 4:** If the values of the outside options depend on vote shares, then voters still prefer \( w \). In addition, the equilibrium policy coincides with the median voter's ideal point.

**Proof:** (In appendix).

The relative military strength remains unchanged when outside options depend on electoral support. As a result, voters still prefer \( w \). However, the equilibrium policy is affected by this extension. While with exogenous outside options, voters' influence on policy outcomes was limited to the choice of the party which will set the agenda in the post-election bargaining, endogenous options allow voters to literally fine-tune the policy outcome and make it coincide with the median voter most preferred policy. Voters who are on the side of \( s \), that is those whose ideal points are between 0 and 1, would like to give this party its “right” share of vote. Otherwise, the policy outcome will be further away from their ideal point. Voters on the side of \( w \), those whose most preferred policies are between 1 and 2, will obviously always vote for this party. This result confirms the robustness of the median voter theorem even in a polity with threats of post-election “rioting.” The theorem also holds when we allow pre-electoral competition.

### 3.2 Equilibrium with Pre-electoral Competition

In the previous sections there was no pre-electoral competition. Parties were assumed to care only about policy outcomes not about winning the elections. I now examine equilibrium behavior when parties are electoralists and make campaign promises to maximize their chance of winning the election. For this purpose, they can either choose positions or control the value of their outside options.

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6. Assuming \( b \geq 2 \) will not qualitatively affect the result (see Proposition 1).

7. Note that if \( \theta_M \) were an irrational number, say \( \sqrt{3} \), then \( y_c \) could just be made arbitrarily close to \( \theta_M \). This is because \( y_c \) is a rational number.
3.2.1 Strategic Positioning

We now assume that before the election takes place, party $k$ choose a candidate $k_j$, where $j \in \{1, \ldots, J\}$, the set of possible candidates and this candidate will choose a policy position $x_{kj}$ to maximize his chances of being elected. Finally, if elected the candidate makes an offer of policy compromise $y_k$ which is accepted or rejected. Party $k$'s most preferred policy is $b_k$ but its candidate is spatially mobile and can choose any position between $b_k - 1, b_k + 1$. Denote by $x_k = (x_{k1}, \ldots, x_{kJ})$. The expected utility of candidate $k_j$ is given by:

$$E[U_{kj}(\cdot)] = \pi_k(x_{\hat{w}}, x_s) \cdot G$$

where $G$ is a strictly positive number measuring the monetary gain of being elected.

The expected utility of party $k$ becomes:

$$E[U_k(\cdot)] = \left\{ \begin{array}{ll}
- \pi_k(x_{\hat{w}}, x_s) \left\{ (y_k - b_k)^2 \cdot r_k(\cdot) + c_k (1 - r_k(\cdot)) \right\} \\
- (1 - \pi_k(x_{\hat{w}}, x_s)) \left\{ (y_k - b_k)^2 \cdot r_k(\cdot) + c_k (1 - r_k(\cdot)) \right\}
\end{array} \right.$$  

Also, the expected utility of voter $i$ becomes:

$$E[V_i(\cdot)] = - \sum_k \pi_k(x_{\hat{w}}, x_s) \left\{ (y_k - \theta_i)^2 \cdot r_k(\cdot) + d_k (1 - r_k(\cdot)) \right\}$$

In the subgame perfect equilibrium for this game, Party $k$ proposes the minimum acceptable policy compromise, which is accepted by $(-k)$. The median voter will select party $k$ if with this party in office, the policy outcome $y_k$ will be the closest to ideal point $\theta_M$. In anticipation to these outcomes, candidate will behave strategically, choosing positions to maximize their chance of winning. Formally, a symmetric equilibrium position in party $w$ for instance is a vector $x^*_w$ such that for all $j \in \{1, \ldots, J\}$,

$$x^*_w = \arg \max \pi_w(x_{w\hat{j}}, x^*_w, x^*_s) \cdot G.$$  

Before summarizing equilibrium behavior in the positioning stage game, let us define polarization. Following Esteban-Ray (1994), polarization is a distribution of attributes such there is a high degree of homogeneity within parties, a high degree of heterogeneity across parties and there is a small
number parties. It is then clear that the relevant attribute in the context of this model is the policy position and the relevant measure of its distribution is the distance between parties' positions, \( b \).

**Proposition 5:** If candidates are electoralists, then they will take polarized positions. That is, \( 0 \leq x_i^* \leq \theta_M \leq x_k^* \leq b \). A party becomes more polarized as the “rioting ability” of its opponent increases. In addition, the final equilibrium policy \( y_k \) coincides with the median voter's ideal point \( \theta_M \).

*Proof: (in appendix).*

Democratic transition provides incentives for political polarization. This comes from the fact that the policy outcome results from a compromise between the weak party and the strong party. Because parties anticipate this compromise and the “moderated” policy outcome, they have an incentive to take polarized positions. Furthermore, political polarization will be higher in countries where parties have higher “rioting abilities”. In Alesina-Rosenthal (1992) the incentives for platform divergence come from the system of checks and balances and Divided Government. The reason for the similarity of these results come from the fact that Transition to Democracy and Divided Government are both moderating institutions. In my model, the policy outcome is \( y_k \). Under complete information, the equilibrium policy is equal to \( \bar{y}_k (\sigma, k) \). In Alesina-Rosenthal (1992), the policy outcome is \( y_k = \alpha \cdot b_k + (1 - \alpha) (\sigma_k \cdot b_k + \sigma_m \cdot b_m) \), where \( \alpha \in (0, 1) \) represents the exogenous weight of the President in the policy formation. \((1 - \alpha) \) represents the weight of the Congress and \( \sigma_k \) the vote share of \( k \). For \( b_k = 0 \) and \( b_m = b \), the two institutions generate identical policy outcome if and only if \( \bar{y}_k \) have the following forms: \( \bar{y}_k (\sigma, k) = (1 - \alpha) \sigma_k b + (1 - \alpha) \sigma_m b \) and \( \bar{y}_k (\sigma, m) = \alpha \sigma_k b + (1 - \alpha) \sigma_m b \). Transition from a “winner takes all” system to a system of “checks and balances” can be realized by making parties’ weights in the process of policy formation coincide with their reservation policy share. The critical question in the design of the institutions for a successful democratic transition is then the knowledge of \( \bar{y}_k (\cdot) \) and how to reduce uncertainty with respect to this mapping.\(^8\)

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\(^8\)To reduce uncertainty with respect to parties’ bargaining power, parties in many countries have been involved in pre-election forums. That was the case in Peru with the “Conferencia Nacional de Forzas Vivas de la Nación”, in South Africa with the CODESA forums. These meetings resulted in political contracts designed to secure rat-free electoral outcomes. The enforcement of these contracts have been realized by either the Clergy or
3.2.2 Strategic “outside options”

Now assume that for a given $k$, parties can control the values of their outside options in order to maximize their probability of winning. That is, before the elections take place, parties could either increase or decrease the level of their military equipments in order to appeal to the voters. After the elections the winner makes a policy offer to the loser which is either accepted or rejected.

Again, assume for simplicity that parties are equidistant from the median voter. The expected utilities are given as in (1) and (2). The electoral or post-electoral outcomes are as in section 3.2.1. In anticipation to these outcomes, parties choose $r_t$ to maximize their chance of winning. For given policy positions, they will choose the outside options values yielding to the median voter’s ideal point as final policy outcome.

**Proposition 6**: If both parties are electoralists, then they both will choose to dissolve their military wings.

*Proof: (in appendix).*

The logic behind this equilibrium outcome is as follows: First note that the strong party $s$ will demilitarize and reach the same level of equipment as the weak party $w$. But this will not be an equilibrium since this party could increase its probability of winning by cutting its “military investment” by a small amount. The competition will drive the value of the outside options of both parties to its minimum, $b^2$. This equilibrium behavior is consistent with empirical evidence from many newly democratic countries: South Africa, Benin, Nicaragua and more recently Mozambique. The spirit of the argument remains even when parties are not equidistant from the median. In this case, $w$ will completely “demilitarize” and $s$ will keep a relatively small military equipment.

The purpose of this paper is not only to shed light on the mechanics of democratization but provide a framework for the design of democratic institutions. That is, what constitutional arrangements could increase chances for a successful transition to democracy? For instance, should parliamentarism be preferred to presidentialism in newly democratic countries.

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some foreign power (United Nations, United States or France). For instance, in Benin during the “Conference Nationale” Archbishop Emile de Souza helped to design and enforce the deal of immunity of the former President Mathieu Kerekou against his acceptance to abide by the results of the 1991 elections. This was also the case in South Africa where the electoral outcome has been partly negotiated between African National Congress, the Inkatha Freedom Party and the National Party.
4 Design of Democratic Institutions: Presidentialism versus Parlementarism

The main theoretical argument of this paper is that "winner (almost) never takes all" and there is in general no "dictatorship of the majority" even in a purely majoritarian system like a Presidential Regime. However, according to Linz (1990) "Presidentialism is ineluctably problematic because it operates according to the rule of "winner takes all" - an arrangement which tends to make democratic politics a zero sum game, with all the potential for conflict such games portend". As a proponent of parliamentary system he wrote: "While parlementarism imparts flexibility to the political process, presidentialism makes it rather rigid". These ideas are well regarded and are consistent with a variety of historical experiences reported by the author. However, the presence of outside options can introduce a great deal of flexibility into an otherwise rigid majoritarian system. For instance, assume \( w \) wins the (presidential) elections. This party's most preferred policy is \( b_w \), but it will have to implement \( y_u \) such that \( b_u < y_u < b_s \), where \( b_s \) is \( s \)'s most preferred policy. This outcome reflects \( de facto \) a balance of power between parties which prevents the tyranny of the winner and the rebellion of the loser. Such political arrangement has been associated with successful experiences of transition to democracy: South Africa, Benin, Zambia or Nicaragua.

The choice between Presidentialism and Parlementarism need not to be a "zero or one" type of decision. Voters might want to trade off the relative "stability" of the first for the relative "flexibility" of the second or vice versa. The question then becomes what combination of the two systems could maximize chances for a successful transition to democracy. Suppose we have a semi-presidential system. The party with the highest proportion of popular vote wins the presidency and some mechanism is used to transform vote shares in seats in the Assembly. Let \( \alpha \) be weight of the President in the process of policy formation and \( g_k (\alpha) \) be the equilibrium policy when \( k \) wins.

\[ p. 56 \]
\[ p. 55 \]

This confirms Kirchheimer (1969) intuition (see p. 4). Let us consider for instance the 1994 presidential elections in South Africa. The National Party, the main loser of these elections, has not been denied political status. In fact, among other concessions, this party holds key positions in the new government: a vice presidency and the ministry of Economy and Finance. The same can be said about Nicaragua where the losing party, the "Sandinistas" still controls the Army.
the Presidential race. We have $y_k(\alpha) = \alpha \cdot b_k + (1 - \alpha) [\sigma_s \cdot b_s + \sigma_n \cdot b_n]$. We know from the results of Sections 2 that the success of democratic transition become questionable only if parties abilities to “riot” are private information. The objective in designing the institutions of the newly democratic countries is then to minimize the ex-ante probability of riot. In the context of a semi-presidential regime, this probability is

$$H(\alpha) = \pi_w \cdot F(y_w(\alpha), b) + \pi_s \cdot F(y_s(\alpha), b)$$

where $F$ is the cumulative distribution function of $y_k$. As long as $H(\alpha)$ is convex in $\alpha$, the equilibrium weight is neither one nor zero. As a result, the optimal regime will be a strictly convex combination of Parliamentarism and Presidentialism and will depend on the level of polarization, measured by the distance between parties’ positions.

5 Conclusion

This paper investigated electoral incentives and outcomes when parties have outside options. Voters are found to prefer the party with the least valuable outside option (the weak party), even if this party’s position is less preferable. In the presence of uncertainty with respect to the value of the outside options, voters will weigh the policy gain by voting in the weak party against the prospect of a more costly conflict. This enhances the chances of the “strong” party being elected. We also found that in the presence of outside options winner (almost) never takes all. When outside options depend on vote share or when there is pre-electoral competition, the median voter theorem is found to hold and candidates either take polarized positions or parties dissolve their military wings. The paper finally explores the implications of the model for the design of democratic institutions. We found that an optimal institution for democratic transition will involve a trade-off between the stability of Parliamentary against the flexibility of Parliamentarism. The weight of either system will depend on the distribution of parties’ positions as well as their expected vote share.

The results of this paper can be generalized in a variety of ways. These include the assumption of uncertainty with respect to the position of the median voter or a non-uniform distribution of voters in the political spectrum. In future works we intend to provide a formal analysis of the incentives at play in pre-election bargaining and to explore the issues of commitment and
enforceability of political contracts in the context of transition to democracy.

6 Appendix

Proof of Proposition 1:

Denote by \( y_k = \lambda_k \cdot b_s + (1 - \lambda_k) \cdot b_w \) and \( \bar{y}_k = p_k \cdot b_s + (1 - p_k) \cdot b_w \).

If \( \pi_u(b) = 1 \), then the optimal strategy for \( s \) is:

\[ r^*_k(.) = 1 \text{ for } (1 - \lambda_w) \cdot b^2_s \leq r_s = (1 - p_s) \cdot b^2_s \]

or \( \lambda_u \geq p_s \cdot r^*_s = 0 \text{ if } \lambda_u < p_s \).

Given this strategy for \( s \), party \( w \) chooses \( \lambda_u \) such that:

\[ \lambda^*_u = \arg \max \left\{ - (\lambda_w \cdot b - b^2) \right\} \text{ subject to } \lambda_u \geq p_s \]

The solution to this problem is \( \lambda^*_u = p_s \).

If \( \pi(s) = 1 \), \( \lambda^*_u \) and \( r^*_u(.) \) is found in similar fashions.

In anticipation of this policy outcome, how will the median voter react?

- If \( -\sqrt{r_s} + (b - 1) < \sqrt{r_s} - 1 \), then \( \pi^*_1(s) = 1 \) and \( \pi^*(s) = 1 \).
- If \( -\sqrt{r_s} + (b - 1) > \sqrt{r_s} - 1 \), then \( \pi^*_1(w) = 1 \) and \( \pi^*(w) = 1 \).

Proof of Proposition 2

First, note that the expected payoff of party \( w \) is given by

\[
E[\mathcal{U}_w(.)] = \begin{cases} 
-(\lambda_u \cdot b - b^2) \cdot r_s(.) - c_s \cdot \{1 - r_s(.)\} & \text{if } \pi_u = 1 \\
-(1 - \lambda_s) \cdot b - b^2 \cdot r_w(.) \cdot r_u \cdot \{1 - r_u(.)\} & \text{if } \pi_u = 1
\end{cases}
\]

The expected utility of \( s \) is defined in similar fashion.

The expected utility of voter \( v \) is

\[
E[\mathcal{V}_v(.)] = \begin{cases} 
-(\lambda_u \cdot b - 1)^2 \cdot r_v - d_v \cdot (1 - r_v) & \text{if } \pi_u = 1 \\
-(\lambda_s \cdot b - 1)^2 \cdot r_v \cdot d_v \cdot (1 - r_v) & \text{if } \pi_s = 1
\end{cases}
\]

Also, note that the value of the outside option of the strongest “type” of the strong party is \( \bar{v}_s = 1 \). This means he wins the post election “fight” with probability \( \bar{p}_s = 1 - \frac{1}{b^2} \). The value for the weakest type of the strong
enforceability of political contracts in the context of transition to democracy.

6 Appendix

Proof of Proposition 1:

Denote by \( y_k = \lambda_k \cdot b_s + (1 - \lambda_k) \cdot b_v \) and \( \bar{y}_k = p_k \cdot b_s + (1 - p_k) \cdot b_v \).

If \( \pi_u(b) = 1 \), then the optimal strategy for \( s \) is:

\[
    r_s^*(.) = 1 \text{ for } [(1 - \lambda_u) \cdot b_v^2 \leq r_s = (1 - p_s) \cdot b_s^2]
\]

or \( \lambda_u \geq p_s \), \( r_s^* = 0 \) if \( \lambda_u < p_s \).

Given this strategy for \( s \), party \( w \) chooses \( \lambda_u \) such that:

\[
    \lambda_u^* = \text{arg max} \left\{ -(\lambda_u \cdot b - b)^2 \right\} \text{ subject to } \lambda_u \geq p_s
\]

The solution to this problem is \( \lambda_u^* = p_s \).

If \( \pi(s) = 1 \), \( \lambda_s^* \) and \( r_s^*(\cdot) \) is found in similar fashions.

In anticipation of this policy outcome, how will the median voter react?

- If \( -\sqrt{\bar{r}_u} + (b - 1)^2 < |\sqrt{\bar{r}_u} - 1| \), then \( \pi_u^*(w) = 1 \) and \( \pi^*(w) = 1 \).

- If \( -\sqrt{\bar{r}_u} + (b - 1)^2 > |\sqrt{\bar{r}_u} - 1| \), then \( \pi_u^*(w) = 1 \) and \( \pi^*(w) = 1 \).

Proof of Proposition 2:

First, note that the expected payoff of party \( w \) is given by

\[
    E[U_w(.)] = \begin{cases} 
        - (\lambda_u \cdot b - b)^2 \cdot r_s(\cdot) - c_s \cdot \{1 - r_s(\cdot)\} & \text{if } \bar{r}_u = 1 \\
        - (1 - \lambda_s) \cdot b - b^2 \cdot r_v(\cdot) - r_v(\cdot) \cdot \{1 - r_v(\cdot)\} & \text{if } \bar{r}_s = 1 
    \end{cases}
\]

The expected utility of \( s \) is defined in similar fashion.

The expected utility of voter \( i \) is

\[
    E[\bar{U}_i(.)] = \begin{cases} 
        - (\lambda_u \cdot b - b - 1)^2 \cdot r_s - d_s \cdot (1 - r_s) & \text{if } \bar{r}_u = 1 \\
        - (\lambda_s \cdot b - b - 1)^2 \cdot r_v - d_v \cdot (1 - r_v) & \text{if } \bar{r}_s = 1 
    \end{cases}
\]

Also, note that the value of the outside option of the strongest "type" of the strong party is \( \bar{r}_s = 1 \). This means he wins the post election "fight" with probability \( \bar{p}_s = 1 - \frac{1}{b^2} \). The value for the weakest type of the strong
party is \( \xi = 3 \) and he wins the “fight” with probability \( p_s = 1 - \frac{\sqrt{3}}{b^2} \). For the weak party it is respectively \( \bar{\eta} = 3 \) and \( \bar{w} = 5 \). In addition, \( \bar{\eta} = 1 - \frac{\sqrt{5}}{b^2} \).

The following constitute the equilibrium policy outcome under the condition that \( \pi_s = 1 \).

1. If \( p_s \geq p_s \) and \( \left( \frac{p_s}{b} - b \right)^2 \geq c_s \), then the utility resulting from making concessions, \( -\left( \frac{p_s}{b} - b \right)^2 \), is lower than the utility resulting from not making concessions \( -c_s \). Therefore we will have \( \lambda^*_s = 0 \).

2. If \( p_s \leq p_s \leq \bar{p} \) or \( \left( \frac{p_s}{b} - b \right)^2 \leq c_s \leq \left( \frac{\bar{p}}{b} - b \right)^2 \), then \( \bar{n} \) will be willing to make a compromise. This party will choose \( \lambda^*_s \) such that

\[
\lambda^*_s = \arg \max -\left( \lambda_s \cdot b - b \right)^2 \left( 1 - F(u_{\bar{w}}) \right) - c_s \cdot \int_{-3}^{u_{\bar{w}}} f(r) dr
\]

where \( u_{\bar{w}} = \left( (1 - \lambda_s) \cdot b \right)^2 \).

Denote \( u^*_w = \left( (1 - \lambda^*_s) \cdot b \right)^2 \). The first order condition of this problem is:

\[
2b^2 \left( \lambda^*_s - 1 \right) (1 - F(u^*_w)) - 2 \left( \lambda^*_s - 1 \right)^2 b^3 \int f(u^*_w) du^*_w + c_s \cdot f(u^*_w) = 0
\]

or

\[
2b^2 \left( \lambda^*_s - 1 \right) - 2 \left( \lambda^*_s - 1 \right)^2 b^3 \frac{f(u^*_w)}{(1 - F(u^*_w))} + c_s \cdot \frac{f(u^*_w)}{(1 - F(u^*_w))} = 0.
\]

Under Assumption 1, \( f(u^*_w) \) is increasing, there exists a \( \lambda^*_s \) which solves (5).

This offers is accepted with probability \( F(u^*_w) \). Assuming \( F \) is uniformly distributed, \( r^*_s = 1 - \frac{u^*_w}{b^2} \).

3. If \( p_s \geq \bar{p} \) or \( \left( \frac{\bar{p}}{b} - b \right)^2 \leq c_s \), then party \( \bar{n} \) will secure acceptance by making an offer which will be accepted by the strongest type of \( s \), that is \( \lambda^*_s = \bar{p} = 1 - \frac{1}{b^2} \) and \( r^*_s = 1 \).

When \( \pi_s = 1 \), the equilibrium is derived in a similar fashion.
Applying implicit function theorem to the first-order conditions (3) gives the desired comparative statics results.

Proof of Proposition 3

1. Suppose, $c_s \geq (\bar{p}_s b - b)^2$ and $c_w \geq (\bar{p}_w b)^2$ (i.e., both parties have “very” high cost outside options). In this case, the parties will want to secure acceptance by offering $\lambda_s^* = 1 - \frac{\sqrt{3}}{b^2}$ and $\lambda_w^* = 1 - \frac{1}{b^2}$ respectively. As a result, the median voter will select the weak party if $|b - 1 - \sqrt{3}| > 1$. He will select the strong party if $|b - 1 - \sqrt{3}| < 1$ and will be indifferent otherwise.

2. If $c_s \geq (\bar{p}_s b - b)^2$ and $(\bar{p}_w b)^2 \geq c_w \leq (\bar{p}_w b)^2$ (i.e., only the strong party has a “very” high cost outside option). In this case, the only weak party will want to secure acceptance. Then, $\pi_M^*(w) = 1$ if $-(\lambda_w \cdot b - 1)^2 \cdot r_s - y_s > .26$ and $\pi_M^*(s) = 1$ if $-(\lambda_w \cdot b - 1)^2 \cdot r_s - y_s < .26$. Otherwise, $\pi_M^*(s) = \frac{1}{2}$.

3. If $c_s \geq (\bar{p}_s b - b)^2$ and $(\bar{p}_w b - b)^2 \leq c_s \leq (\bar{p}_s b - b)^2$, then the equilibrium outcome is analogous.

4. If neither party will want to secure acceptance, then we have $(\bar{p}_s b - b)^2 \leq c_s \leq (\bar{p}_s b - b)^2$ and $(\bar{p}_w b)^2 \leq c_w \leq (\bar{p}_w b)^2$.

The median voter then selects the weak party (respectively the strong party) if the “adjusted” cost differential, that is $(1 - r_w^*) \cdot d_s - (1 - r_s^*) \cdot d_w$, is lower (respectively higher) than $r_s^* - r_w^*$. He is indifferent otherwise.

Now, the question is: does uncertainty with respect to the strength of both parties affect voting behavior? The answer is yes. To see why, suppose $b = 2$, that is parties are equidistant from the median voter’s ideal point. Without uncertainty we know from the results of Section 1 that $w$ is the winner no matter what. Under the new circumstances, this party wins only if the “adjusted” cost differential is “too low”, that is $(1 - r_w^*) \cdot d_s - (1 - r_s^*) \cdot d_w \leq r_s^* - r_w^*$. Thus, uncertainty reduces the range of policy positions which give party $w$ the edge over its opponent.

Proof of Proposition 4:

As in Proposition 1, the winner of the elections, for example $w$ will offer $\lambda_w^* = p_w(\sigma_w)$, and the loser, in this case, $s$ will accept, that is $r_s^* = 1$. The voting equilibrium is also similar to the one of proposition 1.
Let $\sigma_s$ be the vote share needed by the “strong” party to force the “weak” party to implement the median voter ideal point. Since $b = 2$, we have $2p_s(\sigma_s) = 1$ or $p_s(\sigma_s) = \frac{1}{2}$. Now let $\mu$ be the voter such that if for all $i \in \{1, \mu\}$, $\pi_s^i = 1$ then $\sum_{i=1}^{\mu} \pi_s^i = \sigma_s$. Will this voter vote for $s$. The answer is yes since by not voting for $s$, the policy outcome will be further away from his ideal point. This is true for all $i \in \{1, \mu\}$. This shows that the following strategy profile is the voting equilibrium such that $y_s^* = 1$. For all $i \in \{1, \mu\}$, $\pi_s^i = 1$. and for all $i \in \{\mu + 1, N\}$, $\pi_i^* = 1$.

Proof of proposition 5:

Each candidate’s payoff is increasing in the probability of his being elected. So, they will take a position which maximizes his respective probability. These positions will be the ones yielding policy outcomes which coincide with the median voter's ideal point. In a symmetric equilibrium $x_{jk}^* = x_{j}^*$, $x_{k}^*$ and $x_{n}^*$ are such that

$$(x_s^* - 1)^2 = b^2 - v_k$$

and

$$(x_n^* - 1)^2 = b^2 - v_n$$

A party’s utility is also increasing in $\pi_k(x_{e..e})$ and each will choose a candidate who will implement $\theta_M$. Thus, in equilibrium party $k$ chooses $j$ such $x_{jk}^* = x_{j}^*$.

Denote $d(x_i^*, 1) = (x_i^* - 1)^2$. It is clear that as $v_k$ decreases, $d(x_i^*, 1)$ increases. This means that as the ability of its opponent its opponent increases, a party will become more polarized.

Proof of Proposition 6:

This is a straightforward Bertrand argument.

References


Figure 1: Game Tree