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REVEALED PREFERENCE AND AGGREGATION

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I. INTRODUCTION

The purpose of this paper is to establish conditions under which aggregate demand behavior will have properties normally associated with individual demand, when the distribution of income remains fixed. The best known result of this type is that if each individual has a homogeneus utility function, and the distribution of income is fixed, then the aggregate demand function will be one derived from a homogeneous utility function. This was first established by Eisenberg [4], though not in the context of demand theory, who employed the duality theory of homogeneous programming. More recently Chipman [2] examined the Eisenberg results from the point of view of demand theory, and established the aggregation theorem by first showing the relationship between demand functions homogeneous of degree 1 in income and homogeneous utility functiors, and then exploiting the special properties of the Slutsky matrices of such demand functions.

In this paper utility functions and Slutsky matrices will not be employed; instead a revealed preference approach is taken. Strengthend forms of the weak and strong axioms of revealed preference are used which are preserved in aggregation, and it is shown that demand functions homogeneous of degree 1 in income which satisfy the regular revealed preference axiom will also satisfy the strengthened versions. One advantage of this approach is that it shows the Eisenberg - Chipman aggregation theorem is a purely algebraic problem and does not require continuity or differentiability assumptions. It will also be shown, by example, that there are demand functions not homogeneous of degree 1 in income which may satisfy these strengthened revealed preference axioms.

II. DEFINITIONS AND NOTATION

Throughout this paper the term <u>demand function</u> will mean a function $h:\ \text{int}\, E^{\ell}_{+} \times E^{1}_{+} \to E^{\ell}_{+}$, satisfying the budget equality $ph(p,m) = m$ and homogeneous of degree 0. $h(p,m)$ is interpreted as the ℓ-dimensional commodity vector demanded at price vector p and income m. Given n demand functions $h^1, \cdots, h^n$ and an income distribution vector $(\beta_1, \cdots \beta_n)$, where $\sum_{j=1}^{n} \beta_j = 1$ and $\beta_j \geqq 0$, the aggregrate demand function h is defined by $h(p, m) = \sum_{j=1}^{n} h^j(p, \beta_j m)$. h is clearly a demand function as defined above.

An excess demand function is defined to be a function $h:\ \text{int}\, E^{\ell}_{+} \to E^{\ell}$, satisfying Walras' law $ph(p) = 0$ and homogeneous of degree 0. In an economy with n agents, each characterized by a demand function h^j and an initial endowment vector $w_j \in E^{\ell}_{+}$, the formula $h(p) = \sum_{j=1}^{n} [h^j(p, pw_j) - w_j]$ defines an excess demand function.

Given a demand function h, restrictions will be imposed on h from the point of view of revealed preference. A set of k price-income situations $(p_i, m_i)_{i=1}^{k}$ is said to form a cycle of length k for h if:

$$p_i h(p_{i+1}, m_{i+1}) \leqq m_i \quad i=1 \ldots k \ (p_{k+1} = p_1)$$

and $h(p_i, m_i) \neq h(p_{i+1}, m_{i+1})$ for at least one i.

The weak axiom of revealed preference (WARP) is simply that h have no cycles of length 2. The strong axiom of revealed preference (SARP) is that h have no cycles of any length. In general, h will be called acyclic of degree k if h has no cycles of length k. WARP was introduced by Samuelson [10], [11], and SARP was introduced by Ville [13] and Houthakker [5]. The latter two authors showed that SARP was essentially equivilent to the hypothesis of utility maximization: the most general results can be found in Hurwicz - Richter [6]. In the case of differentiable demand functions, utility maximization, and thus SARP, is essentially equivalent to the hypothesis that the Slutsky matrix be negative semi-definite symmetric: see Hurwicz - Uzawa [7] for the most general results. WARP has been shown to be essentially equivilent to the hypothesis that the Slutsky matrix be negative semi-definite (but not necessarily symmetric): see Kihlstrom - Mas - Cosell - Sonnenschein [9].

These revealed preference properties are not, in general, a property of an aggregrate demand function even if the individual demand functions possess them. Somewhat stronger "revealed preference" properties will now be defined which are preserved in aggregation, however. To motivate the definitions, first consider a consequence of the weak axiom sometimes called the "generalized law of demand" due to Samuelson [11]. Given two price-income situations (p, m) and $(p + \Delta p, m)$, if one makes a "compensating change in income," $(\Delta m = \Delta p \cdot h(p, m))$, and calculates the corresponding change Δx^c in demand, then the inequality $\Delta p \, \Delta x^c < 0$ if $\Delta x^c \neq 0$ will hold in all such situations if and only if the demand function satisfies WARP. Our first definition will be the corresponding inequality without making the income compensation: h will be said to satisfy the generalized law of demand (uncompensated form) if for every p, q, m,

$$1) \quad (p - q)(h(p,m) - h(q, m)) < 0 \text{ if } h(p,m) \neq h(q, m).$$

Note that 1) rules out Giffen goods. In the differentiable case, 1) can easily been shown to imply the negative semi-definitness (without symmetry) of the Jacobian of h.

Using the budget constraint $ph(p, m) = m$, 1) can be rewritten

$$1') \quad ph(q,m) + qh(p,m) > 2m \qquad \text{if } h(q, m) \neq h(p, m).$$

In this form, it can readily be seen that if 1') is satisfied, then

$$ph(q, m) \leqq m$$
$$qh(p, m) \leqq m$$
$$h(p, m) \neq h(q, m)$$

cannot hold, and this together with homogeneity of degree 0 of h implies WARP. Hence 1) can be considered a strengthened version of WARP: now an analogous strengthening of SARP will be made. h will be called strongly acyclic of degree k if for any $p_1, \ldots, p_k, m$;

$$2) \quad \sum_{i=1}^{k} p_i h(p_{i+1}, m) > km \quad (p_{k+1} = p_1) \quad \text{If}$$

$$h(p_i, m) \neq h(p_{i+1}, m) \text{ for at least one } i.$$

Using the same argument as above for WARP, 2) can be shown to imply h is acyclic of degree k. The strengthened version of SARP is that h is strongly acyclic of degree k for every k. Note that strongly acyclic of degree 2 is equivalent to the uncompensated form of the generalized law of demand. Finally, note that all of the above definitions can be modified to apply to excess demand functions in a natural way.

The major results of this paper will be, that with a fixed distribution of income: i) if each individual demand function is strongly acyclic of degree k, then the aggregate demand function will have the same property; and ii) the class of demand functions for which acyclicity of degree k implies strong acyclicity of degree k includes all demand functions homogeneous of degree one in income. An example of a demand function which is strongly acyclic of degree 2 but not homogeneous of degree one in income is also given.

III. AGGREGATION THEOREMS

The first theorem supplies sufficient conditions for determining if an aggregate demand function will satisfy WARP or SARP.

Theorem 1: Let $h^1, \ldots h^n$ be demand functions, each strongly acyclic of degree k, with distribution vector $(\beta_1, \ldots \beta_n)$. Then the aggregate demand function h is strongly acyclic of degree k.

Proof: Choose $p_1, \ldots, p_k$, m with $h(p_i, m) \neq h(p_{i+1}, m)$ for at least one i. This requires $h^j(p_i, \beta_j m) \neq h^j(p_{i+1}, \beta_j m)$ for at least one j. Then

$$\sum_{i=1}^{k} p_i h(p_{i+1}, m) = \sum_{i=1}^{k} p_i \sum_{j=1}^{n} h^j(p_{i+1}, \beta_j m) \quad (p_{k+1} = p_1)$$

$$= \sum_{i=1}^{k} \sum_{j=1}^{n} p_i h^j(p_{i+1}, \beta_j m)$$

$$= \sum_{j=1}^{n} \sum_{i=1}^{k} p_i h^j(p_{i+1}, \beta_j m).$$

By hypothesis, $\sum_{i=1}^{k} p_i h^j(p_{i+1}, \beta_j m) \geqq k\beta_j m$ with strict inequality for at least one j; thus

$$\sum_{i=1}^{k} p_i h(p_{i+1}, m) = \sum_{j=1}^{n} \sum_{i=1}^{k} p_i h^j(p_{i+1}, \beta_j m)$$

$$> \sum_{j=1}^{n} k \beta_j m = k m.$$

The following corollaries are immediate consequences of Theorem 1

Corollary 1. Let $h^1,\ldots,h^n$ be demand functions each satisfying the generalized law of demand (uncompensated form), with fixed distribution of income. Then the aggregate demand function will satisfy WARP.

Corollary 2. Let $h^1,\ldots,h^n$ be demand functions, each strongly acyclic of degree k for every k, with a fixed distribution of income. Then the aggregate demand function satisfies SARP.

The next theorem shows that demand functions homogeneous of degree one in m can be expected to have the "strong" acyclity proportions needed to apply theorem 1.

Theorem 2: Let h be a demand function homogeneous of degree 1 in m and acyclic of degree k. Then h is strongly acyclic of degree k.

Proof: Given $p_1,\ldots,p_k$, m such that $h(p_i, m) \neq h(p_{i+1}, m)$ for at least one i, it must be shown that

$$3) \quad \sum_{i=1}^{k} p_i h(p_{i+1}, m) > k\,m \qquad (p_{k+1} = p_1).$$

Define numbers $\lambda_1, \lambda_2, \ldots, \lambda_k$ by the following rules:

$$4) \quad \lambda_1 = \frac{1}{m}\, p_k h(p_1, m)$$

$$5) \quad \lambda_i = \lambda_{i-1} \frac{1}{m}\, p_{i-1} h(p_1, m) \qquad i = 2,\ldots k$$

Solving 4) and 5) recursively for λ_k, one obtains

$$6)\quad \lambda_k = \prod_{i=1}^{k} \frac{1}{m} p_{i-1} h(p_i, m) \qquad (p_{1-1} = p_k)$$

$$\equiv \prod_{i=1}^{k} \frac{1}{m} p_i h(p_{i+1}, m) \qquad (p_{k+1} = p_1).$$

Equations 4) and 5) can be rewritten as follows, using the hypothesis that h is homogeneous of degree 1 in m, and thus homogeneous of degree - 1 in p:

$$4)'\quad m = p_k h(\lambda_1 p_1, m)$$

$$5)'\quad m = \lambda_{i-1} p_{i-1} h(\lambda_i p_i, m) \qquad i = 2, \ldots k$$

If $\lambda_k < 1$, then $\lambda_k p_k h(\lambda_1 p_1, m) < m$, and this with 5)' would imply the price - income situations $(\lambda_i p_i, m)$ form a cycle of length k, a contradiction. Thus $\lambda_k \geqq 1$, or

$$7)\quad \prod_{i=1}^{k} \frac{1}{m} p_i h(p_{i+1}, m) \geqq 1$$

Now the relationship between the arithmetic and geometric mean of arbitrary nonnegative numbers $x_1, \ldots x_k$ will be employed, which is

$$8)\quad \frac{1}{k} \sum_{i=1}^{k} x_i \geqq \left[\prod_{i=1}^{k} x_i \right]^{\frac{1}{k}}$$ with strict inequality if not all x_i are equal.

Let $x_i = \frac{1}{m} p_i h(p_{i+1}, m)$. If all x_i are equal, their common value must be greater than one to avoid a cycle. Thus 7) and 8) combine to yield

$$\frac{1}{k} \sum_{i=1}^{k} \frac{1}{m} p_i h(p_{i+1}, m) > 1, \qquad \text{which is 3).}$$

Theorem 1 and 2 combine to yield a result which is analogous to Chipman's [2: Theorem 4] main theorem:

Theorem 3: Let $h^1, \ldots, h^n$ be demand functions, each homogeneous of degree one in m, with a fixed distribution of income. If each h^i is acyclic of degree k, then the aggregate h is acyclic of degree k. In particular, if each h^i satisfies WARP then h will satisfy WARP, and if each h^i satisfies SARP h will satisfy SARP.

It is of interest to know if there are demand functions, not homogeneous of degree 1 in m, which may still satisfy the strong forms of the revealed preference axioms. Below an example is given of a demand function which is strongly acyclic of degree 2 but is not homogeneous of degree 1 in m. A complete characterization of the class of all such demand functions, however, is still an open problem.

Example: The demand function is the one generated by the utility function $U(x_1, x_2) = x_1 + \ln x_2$. This demand function h is given by

$$\left.\begin{aligned} x_1 &= \frac{m}{p_1} - 1 \\ x_2 &= \frac{p_1}{p_2} \end{aligned}\right\} \quad \text{if } p_1 \leqq m$$

$$x_1 = 0 \qquad \text{if } p_1 \geqq m$$
$$x_2 = \frac{m}{p_2}$$

It will be shown that $ph(q, 1) + qh(p, 1) > 2$ if $h(q,1) \neq h(p,1)$. The case $m = 1$ suffices because of homogeneity of degree 0 in (p, m). First a simple inequality will be demonstrated.

9) $\lambda^2 + \frac{2}{\lambda} > 3$ for all $\lambda > 1$:

Proof of 9): Let $f(\lambda) = \lambda^2 + \frac{2}{\lambda}$. Then $f(1) = 3$ and $f'(\lambda) = 2\lambda - \frac{2}{\lambda^2} > 0$ for $\lambda > 1$.

Case 1: $p_1 \leqq 1$, $q_1 \leqq 1$. In this case,

$$10) \quad ph(q, 1) + qh(p, 1) = \frac{q_1}{p_1} + \frac{p_1}{q_1} + p_1\left[\frac{q_2}{p_2} - 1\right] + q_1\left[\frac{p_2}{q_2} - 1\right].$$

Let $\alpha = \frac{q_1}{p_1}$, $\beta = \frac{q_2}{p_2}$. Then 10) can be written

$$11) \quad f(\alpha, \beta, p_1) = \alpha + \frac{1}{\alpha} + p_1[\beta - 1] + \alpha p_1\left[\frac{1}{\beta} - 1\right],$$

defined for $p_1 \leqq 1$ and $\alpha p_1 \leqq 1$.

The desired result will be obtained by showing, for each fixed α, β (not both $\alpha = \beta = 1$), that $f > 2$ for admissable values of p_1

($p_1 \leqq 1$ and $\alpha p_1 \leqq 1$). f, as a function of p_1, is linear, so it is sufficient to look at the values of f at $p_1 = 0$ and $p_1 = \text{Min}\left[1, \frac{1}{\alpha}\right]$. For $p_1 = 0$, $f(\alpha, \beta, 0) = \alpha + \frac{1}{\alpha} > 2$ for $\alpha \neq 1$.

Now consider $p_1 = \text{Min}\left[1, \frac{1}{\alpha}\right]$:

<u>Case 1a</u>: $\alpha > 1$: then $\text{Min}\left[1, \frac{1}{\alpha}\right] = \frac{1}{\alpha}$ and $f(\alpha, \beta, \frac{1}{\alpha}) = \alpha + \frac{1}{\alpha} + \frac{1}{\alpha}[\beta - 1] + \left[\frac{1}{\beta} - 1\right]$. Minimize f with respect to β:

$$\frac{\partial f}{\partial \beta} = \frac{1}{\alpha} - \frac{1}{\beta^2} = 0$$

$$\frac{\partial^2 f}{\partial \beta^2} = \frac{2}{\beta^3} > 0$$

Thus Min f occurs at $\hat{\beta}$ such that $\hat{\beta}^2 = \alpha$. Remember $\alpha > 1$, so $\hat{\beta} > 1$. Substituting,

$$\text{Min } f = \hat{\beta}^2 + \frac{2}{\hat{\beta}} - 1$$

Since $\hat{\beta} > 1$, Min $f > 2$ by 9).

<u>Case 1b</u>: $\alpha \leqq 1$. Then $\text{Min}\left[1, \frac{1}{\alpha}\right] = 1$ and

$$f(\alpha, \beta, 1) = \alpha + \frac{1}{\alpha} + [\beta - 1] + \alpha\left[\frac{1}{\beta} - 1\right]$$

Minimize f with respect to β:

$$\frac{\partial f}{\partial \beta} = 1 - \frac{\alpha}{\beta^2} = 0$$

$$\frac{\partial^2 f}{\partial \beta^2} = \frac{2\alpha}{\beta^3} > 0.$$

Thus Min f occurs at $\hat{\beta}$ such that $\hat{\beta}^2 = \alpha$. Remember $\alpha \leqq 1$, so $\hat{\beta} \leqq 1$. Substituting,

$$\text{Min } f = \frac{1}{\hat{\beta}^2} + 2\hat{\beta} - 1.$$

Then Min f $\geqq 2$, with strict inequality if $\alpha = \hat{\beta}^2 < 1$, by 9). (To apply 9), make the substitution $\lambda = \frac{1}{\beta}$).

<u>Case 2</u>: $p_1 \leqq 1$, $q_1 \geqq 1$. In this case

$$12) \quad ph(q, 1) + qh(p, 1) = q_1\left[\frac{1}{p_1} - 1\right] + \frac{q_2}{p_2} p_1 + \frac{p_2}{q_2}.$$

For $p_1 \leqq 1$, the expression 12) increases with q_1, so it suffices to consider the case $q_1 = 1$. But $p_1 \leqq 1$, $q_1 = 1$ is a special case of Case 1.

<u>Case 3</u>: $p_1 \geqq 1$, $q_1 \geqq 1$. In this case

$$ph(q, 1) + qh(p, 1) = \frac{p_2}{q_2} + \frac{q_2}{p_2} > 2 \quad \text{if } p_2 \neq q_2.$$

<u>Remark</u>: This example has been shown to satisfy the strengthened version of WARP, and it obviously satisfies SARP. I do not know, however, if it satisfies the strengthened form of SARP.

IV. Excess Demand.

The results of the previous section can be applied to excess demand functions provided an assumption is made to guarantee that the distribution of income is independent of prices. In the absence of income transfer schemes, this requires the severe restriction that the initial endowment vectors w_j of the agents be proportional, i.e., there must exist a distribution vector $(\beta_1, \cdots, \beta_n)$ such that $w_j = \beta_j w$ $j = 1, \cdots, n$, where $w = \sum_{j=1}^{n} w_j$. In such a case the excess demand function can be written

$$h(p) = \sum_{j=1}^{n} h^j(p, \beta_j pw) - w$$

In this case there are theorems for excess demand functions analogous to the theorems of the previous section. Of particular interest in equilibrium analysis are the results which guarantee that the excess demand function satisfies WARP. As is well know, this guarantees that the set of equilibrium price vectors is convex and that the tatonnement process is stable. Corollary 1 and Theorem 3 yield one such result:

Theorem 4: Let $\mathcal{E}$ be an exchange economy in which initial endowments are proportional. If each agent's demand function satisfies the uncompensated form of the generalized law of demand (in particular, if each demand function is homogeneous of degree 1 in income and satisfies WARP), then the excess demand function satisfies WARP.

This section is concluded with a version of a theorem of Eisenberg [4]. By showing that an economy in which each individual has a homogeneous utility function and in which income distribution is fixed acts as if it maximizes a "social welfare function." Eisenberg not only showed Theorem 3 in case of SARP but also provided an "elementary" proof of existence of equilibrium. Chipman [2] and Shafer - Sonnenschein [12] have given alternative proofs of Eisenberg's results. The techniques of this paper can be used to prove a similar result. It has already been shown that if each individual demand function satisfies SARP and is homogeneous of degree one in income, then with proportional holdings the excess demand function will satisfy SARP. Thus what is needed is an existence theorem for an excess demand function satisfying SARP. This will be, of course, essentially an existence theorem for a one person economy. This, by itself, is of no interest except for demonstrative purposes; the advantage of the proof given below is that it does not require a separation theorem, nor does it require differentials, as in the elementary existence proof of Katzner [8].

Theorem 5: Let h be an excess demand function satisfying

13) h satisfies SARP

14) h is continuous

15) for any $\hat{p} \in E^{\ell}_{+} \setminus \text{int } E^{\ell}_{+}$,

$$\lim_{p \to \hat{p}} uh(p) = +\infty \text{, where } u = (1, 1, \cdots 1) \text{ is the unit vector.}$$

Then there exists a $\bar{p} \in \text{int } E^{\ell}_{+}$ such that $h(\bar{p}) = 0$.

Remark: 15) is the boundary condition used by Arrow - Hahn [1 p. 31]

Proof of Theorem: Let $\{p_j\}_{j=1}^{\infty}$ be a dense subset of $\text{int } E^{\ell}_{+}$. First it is asserted, that for each integer $k \geqq 1$, there exists a $p_{i_k} \in \{p_1, \cdots, p_k\}$ such that $p_{i_k} h(p_j) \geqq 0 \quad j = 1, \cdots, k$. If this is false, then for each i $(1 \leqq i \leqq k)$ there exists a j $(1 \leqq j \leqq k)$ such that $p_i h(p_j) < 0$. Thus one could find $j_1, j_2, \cdots, j_{k+1}$ $(1 \leqq j_{\ell} \leqq k)$

such that $p_{j_\ell} h(p_{j_{\ell+1}}) < 0 \quad \ell = 1, \cdots, k$. But clearly some integer must be repeated in $\{j_1, \cdots, j_{k+1}\}$, and this implies a cycle, a contradiction to SARP.

By choosing $p_{i_k} \in \{p_1, \cdots, p_k\}$ so that $p_{i_k} h(p_j) \geqq 0 \quad j = 1, \cdots, k$, one obtains a bounded sequence $\left\{\frac{1}{up_{i_k}} p_{i_k}\right\}_{k=1}^{\infty}$ satisfying

$$16) \quad \frac{1}{up_{i_k}} p_{i_k} h(p_j) \geqq 0 \quad \text{for each } k \geqq j \text{ and for } j = 1, \cdots, \infty.$$

Let $\bar{p}$ be a limit point of $\left\{\frac{1}{up_{i_k}} p_{i_k}\right\}_{k=1}^{\infty}$. Then by 16), $\bar{p}\, h(p_j) \geqq 0$ for all j, and by the continuity of h and the assumption that $\{p_j\}_{j=1}^{\infty}$ is dense in int E_+^ℓ, this implies

$$17) \quad \bar{p}\, h(p) \geqq o \text{ for all } p \in \text{int } E_+^\ell.$$

It will now be shown that $\bar{p} \in$ int E_+^ℓ and that $h(\bar{p}) = 0$. For any $v \in$ int E_+^ℓ and $0 < t < 1$, $t\bar{p} + (1 - t)v \in$ int E_+^ℓ, and so $\bar{p}h(t\bar{p} + (1 - t)v) \geqq 0$ by 17) and $(t\bar{p} + (1 - t)v)\, h(t\bar{p} + (1 - t)v) = 0$. These two results together imply:

$$18) \quad vh(t\bar{p} + (1 - t)v) \leqq 0 \quad \text{for each } 0 < t < 1 \text{ and each } v \in \text{int } E_+^\ell.$$

For $v = u$, 17) becomes $uh(t\bar{p} + (1 - t)u) \leqq 0$, and by letting $t \to 1$ it can be seen that $\bar{p} \in$ int E_+^ℓ or a violation of 15) will result. Thus by taking the limit as $t \to 1$ in 18), one gets $vh(\bar{p}) \leqq 0$ for each $v \in$ int E_+^ℓ, and this clearly implies $h(\bar{p}) \leqq 0$. Then $h(\bar{p}) \leqq 0$, $\bar{p}h(\bar{p}) = 0$, and $\bar{p} > 0$ implies $h(\bar{p}) = 0$.

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