Discussion Paper No. 1068

LABOR CONTRACTS AND BUSINESS CYCLES. *

by

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and

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First Version: March 1993.

Revised: March 1994.

* The present paper benefited from comments made by V.V. Chari, Vittorio Grilli, Ashem Pesaran and seminar participants at the University of Cambridge (U.K.), Birckbek College, Seminaire Roy (Paris), Università di Venezia and Universidad Carlos III, Madrid. We are also grateful to Anna Horvath for her special assistance. The usual disclaimers apply.

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Abstract

This paper investigates the claim, often put forth by Real Business Cycle proponents (e.g Prescott (1986)), that the poor performances of their models in matching real world aggregate labor market behavior are due to the fact that observed real wage payments do not correspond to the actual marginal productivity of labor but contain an insurance component which cannot be accounted for by the Walrasian pricing mechanism.

To test this idea we dispense with the Walrasian description of the labor market and introduce contractual arrangements between employees and employers. Assuming that the former are prevented from accessing capital markets and are more risk averse than the latter we use the theory of optimal contracts to derive an equilibrium relation between aggregate states of the economy and wage-labor outcomes. This contractual arrangement is then embedded into a standard one-sector, stochastic neoclassical growth model in order to look at the business cycle implications of the contractual hypothesis. The resulting dynamic equilibrium relations are then parameterized and studied by means of standard numerical approximation techniques.

The quantitative properties of our model appear to be somewhat encouraging. We have examined different contractual environments and in all circumstances the contracts-based equilibrium performs better than standard ones with regard to the labor-market variables and at least as well with regard to the other aggregate macroeconomic variables. The present paper reports only the simulation results relative to what we consider the most empirically relevant cases. More results are available from the authors.

1. Introduction

Our point of departure is the observation that standard real business cycle (RBC) models perform poorly in mimicking the statistical properties of labor market fluctuations, factors share cyclical behavior and the comovements between capital income share and investment variations. These are not particularly new remarks. Beginning with Summers (1986), a number of different authors have either dismissed RBC models because of this feature or tried to amend them. To name but just a few: Aiyagari, Christiano and Eichenbaum (1990), Benhabib, Rogerson and Wright (1991), Blanchard and Fischer (1989), Burnside, Eichenbaum and Rebelo (1993), Christiano and Eichenbaum (1990), Dauthine and Donaldson (1992), Gomme and Greenwood (1993), Hansen (1985), Rogerson (1988), Rotemberg and Woodford (1992), Wright (1988).

While investigators have maintained very different opinions about the appropriate framework capable of modelling the labor market's cyclical oscillations, there seems to be wide aggreement on the stylized facts and on their inconsistency with the marginal productivity and intertemporal substitution models of the labor market.

Observed real wages are too smooth and estimated labor supply elasticites too low to justify the observed volatility in hours. If (as the RBC models assume) employment and real wages are generated mainly by the impact of labor demand shocks on a competitive labor market, then the data should lie close to a dynamic labor supply function. If this supply function is inelastic, the variations in real wages should be larger than the variations in employment. Reality is orthogonal to the model's predictions.

Table 1 in the next page illustrates some features of the post-second world war period U.S. economy. We have reported sample statistics for H-P filtered, log-detrended and first differenced data as the adoption of one or the other of these stationarity-inducing methods seems to make a difference with respect to the output-correlation properties of certain time series. The first method is the one most often used in the RBC literature.

A few "facts" stand out quite clearly. Real wages exhibit a weak correlation with output and about half its volatility. Sample estimates also show that while in the long-run wages and labor productivity may display a high degree of conformity, they do not exhibit much of a coherent relationship at business cycle frequencies. Furthermore real wages are highly persistent, a property which is not shared by the real wage time-series generated by the standard RBC model.

Indeed, as Table 2 shows, a high autocorrelation level is displayed by most aggregate variables. This is a crucial property of real business cycles which is seriously missed by standard RBC models.

Table 1 - Quarterly U.S. Data (1947:1-1990:4)

	H-P Filter		Log-detrend		First Diff.	
Series	St. D.	Corr.	St. D.	Corr.	St.D.	Corr.
Output	2.24	1.00	3.41	1.00	1.54	1.00
Consumption	0.86	0.75	1.53	0.64	0.95	0.63
Investment	4.40	0.81	6.90	0.80	2.91	0.68
Hours	1.88	0.88	2.59	0.86	1.05	0.77
Avg. Lab. Prod.	1.06	0.55	1.75	0.67	1.01	0.74
Real Wage	0.96	0.31	1.75	0.15	0.81	0.45
Labor Share	3.80	-0.11	3.86	-0.24	0.86	-0.46
Profits	10.49	0.81	15.16	0.75	7.05	0.64

St. D: Sample Standard Deviation of variables. Corr: Sample Correlation with Output. The HP Filter was computed for lambda = 1600. The Log Detrended data are fit to two trends, from 1947:1 to 1972:1 and from 1972:2 to 1990:4. First Differences are log first differences.

Labor hours (and employment as well) are strongly procyclical and substantially more volatile than wages. In fact, depending on sample subperiods, they may display even wider oscillations than output itself. The very high elasticity of the labor supply curve "implied" by the aggregate data is at odds with most microeconomic evidence on labor supply behavior and is the crucial reason for the rejection of the intertemporal substitution model (Altonji and Ashenfelter (1980) and Altonji (1982) contain the seminal empirical work in this direction).

Analysis of micro-level data (as reported for example in Beaudry and DeNardo (1991) and Bils (1991)) also reveal that wages depend on labor market conditions at the time workers are hired and that real wages are quite sensitive to variations in the unemployment rates that occurr during the job-tenure period.

Finally it has long been observed that a high degree of coherence exhists between most measures of profits and investment activity with the former somewhat leading the latter, (Zarnowitz (1992, chapt. 2)). Profits typically spring up at the early stage of a recovery led by strong gains in labor productivity which are not matched by raises in real wages. On the other hand, profits tend to decline in the later stages of an expansion as costs start rising faster than revenues, reducing profit margins. This is often accompanied or even caused by a tightening of labor market conditions which pushes up labor's costs, cuts down profits and as a consequence leads to a reduction of investment activity. (again see Zarnowitz (1992) for a detailed analysis).

Table 2 - First Autocorrelations (1947:1-1990:4)

Series	H-P Filter	Log-detrend	First Diff.
Output	.847	.924	.515
Consumption	.817	.916	.677
Investment	.806	.919	.279
Hours	.887	.927	.623
Avg. Lab. Prod.	.680	.865	.173
Real Wage	.803	.941	.489
Labor Share	.991	.991	.054
Profits	.786	.890	.270

It is our belief that some of these facts can be accounted for by removing the Walrasian market clearing mechanism from the labor market and by replacing it with an explicit model of labor relations. In this paper we begin to do so by assuming that contractual arrangements allocate labor resources in a manner that exploits the gains from trade that result from workers difficulty in shedding cyclical income risk and entrepreneurs (assumed) higher tolerance for such risk. Labor markets embody an insurance aspect where labor's claims on output are partially fixed prior to the realization of output while entrepreneurs bear a disproportionate share of the output uncertainty.

This approach is based on the joint hypotheses: that employees are more risk averse than employers and that they cannot access financial markets to achieve intertemporal consumption smoothing to the extent that the latter can. The first hypothesis is somewhat arbitrary, at least on strict empirical grounds. While there are well known theoretical justifications for its adoption (from Knight (1921) to Kihlstrom and Laffont (1983)) we lack hard empirical evidence to be used either against or in favor. In our research we have chosen to fix the entrepreneurs' risk aversion and to treat the workers' risk aversion as a "free parameter". The validity of this method can only be judged by the power of its predictions and by the extent to which "unreasonable" differences in risk aversion are needed to deliver interesting results. The numerical simulations presented in section 3 show we need relatively small differences in risk aversion to account for most of the empirical regularities we claim to explain.

The second hypothesis seems easier to defend. An almost endless array of studies on the distribution of wealth show a strong concentration in the upper tail of the population (*e.g.* Atkinson (1983), Champernowne and Cowell (1990), Cowell (1984), Smith (1980)). This is particularly true for financial wealth and for the ownership of equities. If one excludes pension funds (which are seldom if ever used to achieve cyclical consumption smoothing) the percentage of individuals who own and actively trade financial instruments in organized security markets is remarkably small. Mankiw and Zeldes (1991), for example, report strong evidence that no more than 25% of the households engage in this type of activities. More important for our concerns is the fact that similar figures emerge from the literature on consumption smoothing and market incompleteness (see *e.g.* Campbell and Mankiw (1989)). It seems to be a consensus view that an approximate 50-50 split occurs between households that satisfy the permanent income hypothesis and households that are constrained in their cyclical borrowing-lending possibilities.

Furthermore, daily observations suggest that a large portion of actual investment decisions is concentrated in the hands of a small fraction of agents. While this may well be the outcome of some complicated arrangement solving an economy-wide principal-agent problem, we seriously doubt the realism of such an interpretation. It seems simpler and more realistic to assume that the few agents taking responsibility for investment decisions are providing insurance services to the remaining portion of the households not by trading assets that the latter effectively own but through the employment relation.

In the model below two types of individuals meet in each period: workers (proletarians) and entrepreneurs (capitalists). Before uncertainty is realized the latter offer to the former a contract specifying the hours of work and the total payment they will receive in each possible future state of the world. Once the contract is mutually agreed upon, both agents will stick to it, thereby assuming away the ex-post recontracting and enforceability issues arising in the optimal contract literature (see Hart and Hölmstrom (1987) for a recent survey and discussion).

The workers consume in each period all of their wage payments, whereas the entrepreneur (who also supplies a portion of the total work effort) acts like the usual infinitely lived intertemporal maximizing representative agent. Capital accumulation decisions, in particular, are still modeled along the lines of Brock-Mirman (1972) as implemented in the RBC tradition of Kydland and Prescott (1982) and Long and Plosser (1983).

A typical cycle in our model consists of the following stages. Begin near the end of a recession period, when the economy has been hit by a sequence of negative shocks. Before the positive shock is realized, workers expected utility from selling their time on tomorrow's spot market is low. This induces a low reservation utility and, consequently, a contract specifying a wage-labor combination which fixes the wage in future good states well below the marginal productivity of labor. When a positive shock is realized, entrepreneurs reap most of the benefits from the higher labor productivity. The contract also specifies a relatively high supply of labor in good states and these two things jointly boost profits and therefore investments. As labor productivity increases so does workers reservation utility thereby affording them a stronger bargaining position. This generates contracts more favorable to workers that progressively erode profit margins, increase their own consumption and, as the recovery progresses, also reduce the incentive to invest in physical capital. At the end of the boom contracts reflect this tight labor market conditions and, when a negative shock arrives, will magnify its impact on the firms' profitability. In turn this induces a sharp decline in profits and investments near the peak of the cycle when the contraction occurs.

It is important to stress that the introduction of a labor contract does not alter only the cyclical pattern of wages and hours but has an impact also on the way in which investments, profits and the labor-share respond to the exogenous shocks. Basically the employees "lend" to the employers in good periods and "borrow" from them in bad ones. This increases the oscillations of profits which now bear a much larger portion of the shock in productivity. It also increases their correlation with output and it should tend to create a somewhat negative correlation between labor share and output. Furthermore profits are now the crucial source of funds for the new capital, hence one expects the volatility of investments to increase as well, which it does.

There have recently been other attempts to employ risk-sharing arguments in models seeking to explain macroeconomic fluctuations, most noticeably Danthine and Donaldson (1992) and Gomme and Greenwood (1993). A comparison between our methodology and those adopted by these authors is therefore appropriate.

The Danthine and Donaldson model is quite different from the one we use. Leisure does not enter utility functions and workers are divided into two groups (young and old) with the second only being covered by a contract. The latter guarantees full employment to the old people while the young enter and exit the employment relation according to Walrasian demand but have their income protected through a minimum wage and unemployment compensation financed by a tax on profits. It is therefore unclear what is the role played by the labor contract in generating the model's high volatility of labor as the latter comes all from the young portion of the population. Also it is unclear if workers' reservation utility vary along the cycle, or is instead specified once and for all at the beginning of time. Danthine and Donaldson are succesfull in mimicking observed volatility in hours. On the other hand they do not report wages, profits and factor shares so one cannot evaluate their model's performances along those dimensions.

The model studied by Gomme and Greenwood is closer to ours. The description of the economy, of its technology and population are quite similar. Differently from us they specify preferences with an endogenously time-varying and agent specific discount factor, whose impact on the equilibrium dynamics is hard to disentangle from that of the risk-sharing arrangement. A second, more relevant, difference is their treatment of the labor contract. Workers and entrepreneurs are both allowed to smooth consumption by holding financial securities in a complete market environment. The amount of borrowing-lending that employees carry out through securities is then included in the wage bill together with the usual marginal productivity payment. Consequently the optimal contract is not studied directly and there is no endogenous determination of the two parties' bargaining strength. More to the central point, following along the ideas of Wright (1988), Gomme and Greenwood methodology assumes that the introduction of labor contracts will only change observed factor payments but will have no impact on the real allocations. The present paper is based on the opposite assumption, i.e. that the non-walrasian features of labor markets affect not only the denomination of factors' payments but also the intertemporal behavior of most aggregate variables.

The paper is articulated in three other sections. The next one describes the theoretical model and briefly examines the qualitative intuitions underlying our approach. Here we spend some time discussing possible alternative formulations of the contractual environment which give rise to different levels of bargaining power and relatively different allocations of cyclical risk. Section three specifies the adopted functional forms, derives the equilibrium relations and illustrates the outcomes of our simulations. In each case sample statistics are reported and compared to the relevant ones for the U.S. data during the post-war period. Section 4 concludes the paper and discusses some of the issues which are still left open.

2. The Theoretical Framework.

We study the following environment. There are two kinds of infinitely lived agents: those that own some stock of capital and those that don't. For each type a continuum of identical individuals is present. We assume there are $m \geq 1$ proletarians for each capitalist. Individuals of type 1 are born without any stock of capital and are more risk averse than their type 2 capitalist counterpart. People that are not shareholders are prevented from accessing capital markets to borrow/lend out of their labor income. This constrains their consumption and wage payments to coincide in each period.

Capitalists instead can borrow and lend at will in a perfectly competitive capital market. In each period, after observing a realization of the technology shock S_t, they organize the production process, pay the workers and retain the residual output to be either consumed or invested in future capital stock.

There also exists a competitive market for θ periods ahead labor contracts ($\theta \geq 1$ with θ an integer) where, at the end of each period, shareholders hire a fraction $1/\theta$ of next period's employees by offering them a menu $\{W(S), L(S)\}_{S \in \mathcal{S}}$ of possible wage bills and hours of work. A different pair $(W(S), L(S))$ is associated to each possible realization $S \in \mathcal{S}$ of the technology shock. These contracts are assumed to be perfectly enforceable at no observable cost to either party.

The production function is written as

$$Y_t = S_t F(K_t, N_t, L_t)$$

where L_t is the labor supply of proletarians and N_t is the labor supply of the stockholders. The function F is standard: homogenoeus of degree one, concave, monotone increasing and smooth as needed. The technology shock S_t follows a stationary Markov process summarized by the transition function $P(S, S')$ with compact state space $\mathcal{S}$. Denote with $\mathcal{K}$ the real interval of feasible values of the capital stock.

Utility functions are denoted with $v(\tilde{c}, T - L)$ for agent 1 and $u(c, T - N)$ for agent 2. We want to assume that agent 1 is more averse to consumption risk than agent 2, which means

$$\frac{-v_{11}(\tilde{c}, T - L)\tilde{c}}{v_1(\tilde{c}, T - L)} > \frac{-u_{11}(c, T - N)c}{u_1(c, T - N)}$$

for $\tilde{c} = c$ and $N = L$. The common intertemporal discount factor is denoted by $\delta \in (0, 1)$.

2.1 Equilibrium without Contracts.

To compute the proletarians' reservation utility when bargaining over the labor contract, we need to look first at the competitive equilibrium when the two parties can only trade spot. In this case, after the shock S_t has been observed agent 1 sells labor on the spot market, and agent 2 buys it.

To avoid confusing individual choices with equilibrium outcomes we will use lower case letters to denote the first (i.e. ℓ for agent 1, n, k and c for agent 2) and capital letters to denote the second (L, N, K and C).

For an agent of type 1, labor supply is the solution to the simple problem:

$$\max v(\tilde{c}_t, T - \ell_t)$$

$$\text{subject to}: \tilde{c}_t \leq W_t = w_t \cdot \ell_t$$

The first order condition characterizing this choice reduces to:

$$\frac{v_2(w \cdot \ell, T - \ell)}{v_1(w \cdot \ell, T - \ell)} = w \tag{2.1}$$

which under the usual non-degeneracy conditions gives a labor supply function $\ell_t = \ell^s(w_t)$.

The stockholder solves a more complicated problem. Given a pair of initial conditions (S_0, k_0) and a sequence of wage rates $\{w_t\}_{t=0}^{\infty}$ he has to choose his own labor supply n_t, the amount of labor ℓ_t^d he demands from each of the m agents of type 1, his consumption level c_t and his investment level $i_t = k_{t+1} - (1-\mu)k_t$ for all periods $t = 0, 1, \ldots$. His stochastic optimal control problem and associated value function can then be written as:

$$W(S_0, k_0) = \max\left\{\sum_{t=0}^{\infty} \delta^t \int_{\mathcal{S}} u(c_t, T - n_t) P(S_t, dS_{t+1})\right\} \tag{2.2}$$

$$\text{subject to}: c_t + k_{t+1} = S_t F(k_t, n_t, m\ell_t) + (1-\mu)k_t - w_t \cdot m\ell_t$$

Transversality condition aside, this yields the following array of necessary and sufficient first order conditions, where λ_t denotes the Lagrange multiplier associated to the resource constraint:

$$u_1(c_t, T - n_t) = \lambda_t \tag{2.3a}$$

$$u_2(c_t, T - n_t) = \lambda_t S_t F_2(k_t, n_t, m\ell_t) \tag{2.3b}$$

$$S_t F_3(k_t, n_t, m\ell_t) = w_t \tag{2.3c}$$

$$\delta^{-1}\lambda_t = \int_S \lambda_{t+1}\{S_{t+1}F_1(k_{t+1}, n_{t+1}, m\ell_{t+1}) + (1-\mu)\}P(S_t, dS_{t+1}) \tag{2.3d}$$

A spot-equilibrium is then obtained in two steps: first substitute the labor supply function $\ell^s(w_t)$ in place of ℓ_t in (2.3) and in the resource constraint underlying (2.2) and impose market clearing in the consumption and capital good markets. Then solve the system of equations (2.3) to yield a set of functions $\{w(\cdot), L(\cdot), N(\cdot), C(\cdot), \tau(\cdot)\}$ depending on the state variables $Z_t = (K_t, S_t)$ and such that

a) $m\ell^s(w(Z_t)) = L(Z_t)$ solves (2.1) for all $t = 0, 1, \ldots$;

b) $c_t = C(Z_t)$, $n_t = N(Z_t)$, $m\ell_t = L(Z_t)$, $K_{t+1} = \tau(Z_t)$ solve the programming problem (2.2) given $w_t = w(Z_t)$.

2.2 Equilibrium with Contracts.

Begin by defining agent one's reservation utility at time t. This is the minimum total utility over the life-time of the contract he will accept at time t when signing a contract for the θ periods $t+1, \ldots, t+\theta$. It will be denoted as $\overline{v}_t$. It depends on the state of the economy at the end of period t and on the expectations this induces about future states. We can formally write it as:

$$\overline{v}_t = E_t\Big\{\sum_{i=1}^{\theta} v(\tilde{c}_{t+i}, T - \ell_{t+i})\delta^i \mid Z_t\Big\} = \tag{2.4}$$

$$\sum_{i=1}^{\theta}\delta^i \int_{\mathcal{Z}} v\Big(\tilde{w}(Z_{t+i}) \cdot \ell^s\big(\tilde{w}(Z_{t+i})\big), T - \ell^s\big(\tilde{w}(Z_{t+i})\big)\Big) Q(Z_{t+i-1}, dZ_{t-i})$$

where $\mathcal{Z} = \mathcal{S} \times \mathcal{K}$ denotes the set of feasible pairs (K_t, S_t) and $Q(Z, dZ')$ is the equilibrium transition function (see Stokey, Lucas and Prescott (1989) for the details). Furthermore, in (2.4) the notation $\tilde{w}(\cdot)$ indicates the equilibrium wage as a function of the state Z when all workers but one have entered a contractual arrangement. This is the spot-market salary that an individual worker should expect if he does not accept the employer's offer but all the other m/θ workers do. It will correspond to the marginal productivity of the input L evaluated at the level of $L(Z_{t+i})$ which is prescribed by the contract and which will be determined below. The function $\ell^s(\cdot)$ is instead the individual labor supply function derived in (2.1).

When offering a contract the stockholder must take into account the expected utility constraint induced by the workers' option of switching to the spot market and therefore obtaining at least $\overline{v}_t$. How much utility the non-stockholder should expect from the contract depends on relative bargaining powers. In this paper we take as a benchmark the case in which the proletarians have no bargaining power and all the gains from trade are collected by the capitalists. Obviously this is not completely realistic, but we believe that allowing more bargaining power to the workers would not substantially change the relative variability of wages and hours. We suspect, though that it might have non-negligible effects on the cyclical behaviors of capital and labor shares.

The stockholder decision problem can be described along the following lines. Given the state of the system at the end of period t, $Z_t = (K_t, S_t)$, and conditional on his choice of future capital stocks k_{t+i} he needs to offer a contract $\{W(Z_{t+i}), L(Z_{t+i})\}_{i=1}^{n}$ to his prospective workers and simultaneously make contingent plans as to what kind of consumption levels $c(Z_{t+i})$, labor efforts $n(Z_{t+i})$ and investment $i(Z_{t+i})$ he will carry out. While the overall equilibrium values have to be determined at once, here we can examine the two problems separately. Let us begin with the contract design problem.

The implicit contracts literature (see Rosen (1985) for a survey) teaches that the crucial properties of the optimal arrangement depend on the assumptions one is willing to make on the different degrees of risk aversion of firms and workers, on the nature of the available information (public vs. private) and, in certain circumstances, on the income-elasticity of leisure for the non-shareholder. This extreme sensitivity of the optimal contract generates a large number of outcomes which serve no purpose in the present investigation and which would be very hard to follow in any case.

From our viewpoint the salient feature of a contract is that it provides workers with an insurance mechanism during bad periods and entrepreneurs with a source of funds during good periods. This property is shared by both public and private information contracts. The latter is especially relevant only in the study of over- and under-employment of workers in (respectively) good or bad periods, a topic which does not concern us here (see Chari

(1983) and Green and Kahn (1983)). Given that the computational complexity implied by the asymmetric information model is orders of magnitude higher than the one implied by the public information setup, we have restricted our present analysis to the latter. To maintain the analytical treatment within reasonable bounds we also concentrate on the special case of one-period ahead contracts (i.e. $\theta = 1$) and leave the exploration of the impact of staggered multiperiod contracts for future work.

When the realization of the shock is public information, wages and employment can be made conditional just on S. A contract is then a pair of functions $\{W(S), L(S) = m \cdot \ell(S)\}$ maximizing the capitalist's expected utility subject to the constraint that each agent of type 1 has an expected utility no less than his reservation utility $\overline{v}_t$ as defined in (2.4). For the time being let the equilibrium values of C_{t+1}, N_{t+1}, K_{t+1} and K_{t+2} be taken parametrically by the capitalist. The optimal contract solves:

$$\max_{W(\cdot), L(\cdot)} \int_S u(c_{t+1}, T - N_{t+1}) P(S_t, dS_{t+1}) \tag{2.5}$$

$$\text{subject to}: \int_S v\big(W(S_{t+1}), T - \ell(S_{t+1})\big) P(S_t, dS_{t+1}) \geq \overline{v}_t$$

$$0 \leq c_{t+1} \leq S_{t+1} F(K_{t+1}, N_{t+1}, L(S_{t+1})) + (1-\mu) K_{t+1} - K_{t+2} - m \cdot W(S_{t+1})$$

It is well known (see e.g. Hart and Holmström (1987)), that the unique optimal contract is fully characterized by the following three conditions:

$$m \cdot u_1(C_{t+1}, T - N_{t+1}) S_{t+1} F_3(K_{t+1}, N_{t+1}, L_{t+1}) = \eta_{t+1} v_2(W_{t+1}, T - \ell_{t+1}) \tag{2.6a}$$

$$m \cdot u_1(C_{t+1}, T - N_{t+1}) = \eta_{t+1} v_1(W_{t+1}, T - \ell_{t+1}) \tag{2.6b}$$

$$\int_S v(W_{t+1}, T - \ell_{t+1}) P(S_t, dS_{t+1}) \geq \overline{v}_t \tag{2.6c}$$

where η_{t+1} is the Lagrange multiplier on the expected utility constraint and the dependence of W and ℓ on S_{t+1} has been omitted to economize on space.

The properties of the contract are straightforward and will not be repeated here. For our purposes it will suffice to stress that the risk-sharing condition (2.6b) is generally not satisfied by the spot-equilibrium allocation. The contract in fact allows the entrepreneur one extra degree of freedom: the ratio between his marginal utility of consumption and the worker's marginal utility of consumption will now be equal to the constant η_{t+1} in all states while in the spot economy that same ratio only satisfies

$$\frac{u_1(C_{t+1}, T - N_{t+1})}{v_1(W(S_{t+1}), T - \ell(S_{t+1}))} = \frac{u_2(C_{t+1}, T - N_{t+1})}{v_2(W(S_{t+1}), T - \ell(S_{t+1}))} \times \frac{F_3(K_{t+1}, N_{t+1}, m\ell(S_{t+1}))}{F_2(K_{t+1}, N_{t+1}, m\ell(S_{t+1}))}$$

which needs not be constant with respect to $S_{t+1} \in \mathcal{S}$.

A second implication of (2.6), has to do with the sensitivity of $W(\cdot)$ with respect to S_t for any given K_t. As noted in Rosen (1985) for the case in which u is linear, only when workers' preferences are completely separable in consumption and leisure the optimal contract predicts that workers' and entrepreneurs' consumptions should be perfectly correlated across states of the world, whereas a non separable $v(\cdot,\cdot)$ links consumption behavior and the employment level of workers. In our own application the utility function is not linear, and we have not observed any relevant difference in this regard between the behavior of the separable model described below and that of a non-separable version we have also simulated.

Denote with $W^*(\cdot), L^*(\cdot)$ the equilibrium solution to (2.6) as a function of the state and of the other equilibrium variables. Under the assumption that all entrepreneurs are the same, competition in the market for contracts guarantees that in equilibrium the latter will be identical across firms. The envelope theorem justifies our use of equilibrium notation when studying the dynamic programming problem of the representative capitalist:

$$U\big(S_t, k_t; W^*(\cdot), L^*(\cdot)\big) = \max_{n_t, c_t, k_{t+1}} \Big\{u(c_t, T - n_t) + + \delta \int_{\mathcal{S}} U\big(S_{t+1}, k_{t+1}; W^*(\cdot), L^*(\cdot)\big)P(S_t, dS_{t+1})\Big\} \tag{2.7}$$

$$\text{subject to}: c_t + k_{t+1} \leq S_t F(k_t, n_t, L^*(\cdot)) + (1-\mu)k_t - k_{t+1} - mW^*(\cdot)$$

Under standard restrictions (see e.g. Stokey and Lucas (1989, Chapt. 9)) (2.7) is known to possess a unique solution, summarized by the policy function $k_{t+1} = \tau(k_t; S_t, K_t)$. The latter is continuous in k_t and K_t for any given S_t. A characterization of the (interior) optimal choices of the entrepreneur can be obtained by looking at the transversality condition and at the first order conditions

$$u_1(c_t, T - n_t) = \lambda_t \tag{2.8a}$$

$$u_2(c_t, T - n_t) = \lambda_t S_t F_2(k_t, n_t, L^*) \tag{2.8b}$$

$$\delta^{-1}\lambda_t = \int_{\mathcal{S}} \lambda_{t+1}\big[S_{t+1}F_1(k_{t+1}, n_{t+1}, L^*) + (1-\mu)\big]P(S_t, dS_{t+1}) \tag{2.8c}$$

where λ_t denotes once again the lagrange multiplier associated with the technological constraint in (2.7).

A competitive equilibrium for the contract economy is then routinely defined by the existence of a set of functions $W^*(\cdot)$, $L^*(\cdot)$, $C(\cdot)$, $N(\cdot)$ and $\tau(\cdot)$ depending on the state vector $Z_t = (S_t, K_t)$ and such that:

a) $W^*(\cdot)$ and $L^*(\cdot)$ solve (2.5) for all Z_t given $C(\cdot)$, $N(\cdot)$ and $\tau(\cdot)$;

b) $C(\cdot)$, $N(\cdot)$ and $\tau(\cdot)$ solve (2.7) for all Z_t given $W^*(\cdot)$, $L^*(\cdot)$.

2.3 Bargaining Power

The formulation given in (2.5) of the way in which the contractual agreements are reached, implicitely assumes that all the bargaining power rests with the capitalists and that the proletarians walk away from the labor contract room with the same expected utility they carried when they walked in. One may indeed think of situations in which agents of type 1 have some market power and are therefore able to obtain more than their reservation utility.

This needs not destroy the efficiency properties of the optimal contract, which can be readily interpreted as the outcome of a Pareto efficient allocation where the two parties are given weights different from those implicit in (2.5). A simple way of formalizing this approach is to replace (2.5) with the following problem. Given the state vector $Z_t = (S_t, K_t)$ and the equilibrium values of N_t and K_{t+1}:

$$\max_{W,L} \int_S \left\{ \nu_t u(C_t, T - N_t) + (1-\nu_t) v(W, T - L/m) \right\} P(S, dS') \tag{2.9}$$

$$\text{subject to} : 0 \leq C_t \leq S' F(K_t, N_t, L) + (1-\mu) K_t - K_{t+1} - mW$$

The parameter $\nu_t \in [0,1]$ is chosen arbitrarily and it is a measure of the degree of market power of the entrepreneur. By varying ν_t between 0 and 1, we can trace out the whole expected utility possibility frontier. It is readily seen that by setting η_t in (2.5) equal to $(1-\nu_t)/\nu_t$ in (2.9) the two problems become identical.

It is tempting to ask if different choices of ν_t might have quantitatively relevant implications for the equilibrium behavior of the labor market variables. Taking our framework seriously yields an upper ($\overline{\nu}$) and a lower ($\underline{\nu}$) bound. The first is associated with guaranteeing that the solution to (2.9) provides the workers with the same level of expected utility they receive under the spot-equilibrium while the latter guarantees to the entrepreneurs their expected utility under the spot arrangements. An analysis along this line is not performed here. Simulations carried out by Horvath (1994b) suggest that, for reasonable values of ν, the results would be insignificantly different from those reported later in Section 3.

We have also studied the behavior of our economy in the presence of a contractual arrangement under which the proletarians are guaranteed a *constant* level of utility in each future state of the world. This constant utility level has been chosen to be equal to their expected utility in the spot-equilibrium. It is rather obvious that this contract is not optimal in the Pareto sense: both parties could be made better-off by trading some uncertainty.

Let $\overline{v}_t$ be defined as in (2.4) above. Let $g(\overline{v}_t, \ell(S_t))$ solve

$$v\big(g(\overline{v}_t, \ell(S_t)), T - \ell(S_t)\big) - \overline{v}_t = 0 \tag{2.10}$$

The function $g(\cdot)$ always exists and is well defined under standard restrictions. The contractual problem replacing (2.5) can then be written as

$$\max_{\ell(S_t)} \int_{\mathcal{S}} u(c_t, T - n_t) P(S_{t-1}, dS_t) \tag{2.11}$$

$$\text{subject to} : 0 \leq c_t \leq S_t F(k_t, n_t, m\ell(S_t)) + (1-\mu)k_t - k_{t+1} - mg(\overline{v}_t, \ell(S_t))$$

The optimal contract is fully characterized by the first order condition

$$S_t F_3(k_t, n_t, m\ell(S_t)) = g_2(\overline{v}_t, \ell(S_t)), \quad \forall S_t \in \mathcal{S} \tag{2.12}$$

With the obvious substitutions the remaining choice variables of the entrepreneur and the equilibrium functions can then be determined as in subsection 2.2.

Economic intuition and the formal results reported in Green and Kahn (1983) suggest that one should observe smaller fluctuations in L_t and larger fluctuations in W_t under the contract specified in (2.11) than under the optimal contract (2.5). As our simulations reveal this is also the case in the fully parameterized model. Given that this is, on the other hand, the only way in which the introduction of the sub-optimal contract affects the model economy we do not report the results here.

3. The Parametric Models.

In this section we introduce the specific functional forms utilized in the exercise and characterize the most intuitive properties of the equilibria.

The production function has been chosen to be Cobb-Douglas in capital (K) and total labor (E), while the latter is a CES combination of proletarians and capitalists work efforts.

$$Y_t = S_t K_t^{\alpha} E_t^{1-\alpha} \tag{3.1}$$

$$E_t = (aN_t^{\rho} + (1-a)L_t^{\rho})^{1/\rho}$$

Here $L = m\ell$ is the total amount of proletarian labor employed. The parameters α and a are in the unit interval, while ρ is assumed negative to reflect the complementarity in production between the two types of labor.

The utility functions for both agents have been chosen from the C.E.S. class, under the restriction that the worker would be more risk averse than the entrepreneur. The latter has a utility function given by

$$u_t = \frac{1}{1-\psi} c_t^{1-\psi} + \frac{\gamma}{1-\psi}(T - n_t)^{1-\psi} \tag{3.2}$$

As for the utility function of proletarians we have experimented with both separable and non-separable ones but observed very small and altogether not significative differences. We will therefore report only the results for the separable version, which is

$$v_t = \frac{1}{1-\sigma} \tilde{c}^{1-\sigma} + \frac{\theta}{1-\sigma}(T - \ell_t)^{1-\sigma} \tag{3.3}$$

Obviously $\sigma > \psi$ is to be assumed throughout the rest of the paper. The technological shock S_t follows the stochastic process

$$S_{t+1} = S_t^{\rho_s} \exp(\zeta z_t), \quad z_t \sim N(0, 1) \tag{3.4}$$

with $\rho_s \in (0, 1)$ and $\zeta > 0$.

3.1 Characterization of the Equilibrium.

The proletarians labor supply under spot market conditions is

$$\ell^s(w) = \frac{T}{1 + \tilde{\theta} w^{1-1/\sigma}} \tag{3.5}$$

where $\tilde{\theta} = \theta^{1/\sigma}$. Notice that $\sigma < 1$ is required to avoid a backward bending labor supply function. Hence we will always assume $0 < \psi < \sigma < 1$. The first order conditions characterizing the solution to (2.7) are given by

$$C_t^{-\psi} = \lambda_t \tag{3.6a}$$

$$\gamma(T - n_t)^{-\psi} = a(1-\alpha)S_t K_t^{\alpha} E_t^{1-\alpha-\rho} N_t^{\rho-1} \tag{3.6b}$$

$$\delta^{-1}\lambda_t = \int_S \lambda_{t+1}(\alpha S_{t+1} K_{t+1}^{\alpha-1} E_{t+1}^{1-\alpha} + 1 - \mu) P(S_t, dS_{t+1}) \tag{3.6c}$$

The optimal contract $\{W^*, L^*\}$ and the "bargaining power multiplier" η_t are computed by means of

$$\frac{m \cdot (1-a) MP_t \cdot L_t^{\rho-1}}{C_t^{\psi}} = \eta_t \theta (T - \ell_t)^{-\sigma} \tag{3.7a}$$

$$\frac{m}{C_t^{\psi}} = \eta_t \cdot W_t^{-\sigma} \tag{3.7b}$$

$$0 = \int_S [W_t^{1-\sigma} + \theta(T-\ell_t)^{1-\sigma} - W_{t,spot}^{1-\sigma} - \theta(T - \ell_{t,spot})^{1-\sigma}] P(S_{t-1}, dS_t) \tag{3.7c}$$

where the subscript *spot* indicates the equilibrium values associated to the labor supply function (3.5) and the notation MP_t stands for

$$MP_t = (1-\alpha) S_t K_t^{\alpha} E_t^{1-\alpha-\rho}$$

Algebraic manipulation of the systems (3.6) and (3.7) yields useful insights into some basic properties of our dynamic contract economy. The total wage payments to an individual worker are

$$W^*(Z_t) = \Big(\frac{MP_t \cdot L_t^{\rho-1}}{\theta}\Big)^{1/\sigma} (T_1 - \ell^*(Z_t)) \tag{3.8}$$

Denoting with w_{spot} the real wage of proletarians in the spot economy and with w the same real wage in the contract economy it is easy to see that

$$\frac{w_{spot}}{w} = \frac{\theta \ell}{T - \ell}$$

Hence during periods in which individual effort is higher than normal the spot wage will tend to be above the contract wage while the opposite occurs during periods in which ℓ is below average. It is apparent from (3.7) that ℓ is procyclical. A comparison of (3.7a) with the first order condition determining the spot market labor supply function (3.5) shows that in the spot economy the level of employment reacts less to variations in its marginal productivity than in the contract economy due to the presence of a wealth effect which is altogether absent in (3.7a).

3.2 Parameterization.

The system of equations we use to compute the dynamic equilibria of the model depends on a set of thirteen parameters. Four pertain to the aggregate technology (α, ρ, a, μ), two are needed to specify the stochastic process for the technological shock (ρ_s, ζ), a second group of five defines the preferences of the agents (σ, θ, ω, γ, δ) and the last two quantify the total time endowment and its distribution among capitalist and proletarians (T, m). Following along the methodology of Kydland and Prescott (1982) we will now describe the numerical values we used and the empirical support for our choices.

For some of them the restrictions imposed by our model are indistinguishable from those imposed by the standard RBC models. Finding nothing objectionable in the standard calibration procedure we have just adopted those same values. This choice sets $\delta = .993$, $\mu = .028$ and $T = 1369$ which is the total number of non-sleeping hours per average person per quarter.

The calibration of the remaining technology parameters is not a completely straightforward matter. The problem originates from our definition of the labor input E as a CES combination of the two types of time efforts, L and N. Unfortunately we lack independent observations on these two variables. We considered for a moment the hypothesis of adopting the classification supervisory vs. non-supervisory work as a possible empirical proxy. Nevertheless we chose not to consider this source of information on the ground that it provides a very bad and narrow representation of those aggregates to which our model refers. Gomme and Greenwood (1993) faced a similar problem and we share their agnostic conclusions. The most reasonable option is therefore to treat total hours as a measure of E and proceed along.

With this caveat and the chosen values of δ and μ one can proceed at estimating the technology parameter α independently from ρ and a. We have applied standard GMM procedures to the orthogonality restriction induced by the Euler condition (3.5c) which uniquely depends on α. Our point estimate $\alpha = .26$ differs substantially from the value of $\alpha = .36$ usually adopted in the RBC literature but most of the difference seems attributable to our choice of the S&P500 index as an instrument for the entrepreneurs' marginal rate of substitution in consumption. As the appropriateness of this choice is predicated on the empirical relevance of the consumption-based CAPM and the latter is at least debatable we have also simulated our model with $\alpha = .36$ and the sample statistics turn out to change only slightly. To avoid giving the impression that our results depend upon this particular estimate we have used an average between the two values, i.e. for the baseline model we have set $\alpha = .31$. To facilitate comparison we have also chosen to report the outcomes of our simulations for both $\alpha = .26$ and $\alpha = .36$ in an appendix.

As for the substitutability parameter ρ, lacking compelling empirical evidence on the matter, we have nevertheless found acceptable the idea that entrepreneurs and their employees are slightly complementary and not substitutable production factors, at least at the business cycle frequencies with which this study is concerned. The latter requires ρ to be negative but not too much so, and we have experimented with a few values in the interval $[-1.0, -.1]$, without noticing any relevant impact on the final outcomes. Very bizarre results obviously can be obtained at extreme values of ρ when the degree of complementarity between the two types of labor becomes exagerately large.

Given that T has been set equal to 1369 all that remains is to determine how many proletarians are out there for each capitalist. The theoretical underpinnings of our framework together with the empirical evidence quoted in the introduction suggest that between a quarter and a half of the population should be considered as composed of stockholders. This implies that the parameter m could be anywhere within the interval $[1, 3]$. Again we have experimented with different values and noted that, while results seem to change little as $1 \leq m \leq 2$, a number of sample statistics become very sensitive for values of $m > 2$. For this reason and also in order not to bias our calibration too heavvily toward the hypothesis that a very large portion of the population is credit-constrained we have chosen the value $m = 1.5$ for our baseline model.

Once a value of m is chosen one can use income distribution data to fix the remaining technological parameter a. The idea is that of chosing a so that the steady state portion of income going to the employees corresponds to the sample percentage of national income received by the bottom sixty percent of the population (the fraction sixty percent is implied by the choice of $m = 1.5$). Depending on measurement techinques and various possible definitions of income, the range of values we have found in the literature goes between .30 and .36. As a point estimate we have chosen .33 which is the value reported for the United States in World Bank (1993, p. 297). In our model, though, the steady state income distribution is also affected by the degrees of risk aversion of the two agents and by the intensity of their preferences for leisure. A reasonable choice of a must therefore be made jointly with that of the preferences parameters, to which we move next.

Two of them (θ and γ) can be calibrated so that the model deterministic steady state satisfies some empirical restrictions on the typical fraction of total non-sleeping hours that individuals allocate to market activities. It is customary in the business cycle literature to use point estimates between .25 and .33 which in general require values between .9 and 1.3 for the model's parameters. As for σ and ψ they are in some sense "free" in our model and are meant to capture the extent to which workers are more risk averse than entrepreneurs. After experimenting with a few non-extreme values we have observed that relatively little variations occur for σ between .3 and .9. and ψ between .2 and .6. It should be noted that in our framework a value of 1 is in any case an upper bound for both degrees of risk aversion as larger values would imply a backward bending labor supply function, hardly a realistic feature at the business cycle frequencies we are interested in studying.

Still this leaves us with a large set of parameter values from which to make our choice. To restrict it further we have concentrated on two particularly important sample statistics: the correlations between wages and output and between consumption and output. The U.S. data reported in the introduction suggest a low value for the first and a relatively high value for the second. Sensitivity analysis shows that in our model their behavior depends in a nonlinear fashion on the choice of α, σ and ψ (varying θ and γ appropriately in order to match the sample statistics on the percentage of total hours spent at work).

In order to characterize such dependence begin by considering Figures 1.1-1.3, reporting the real wage standard deviation as a fraction of the output standard deviation for different values of a, σ and $d = \sigma - \psi$. Such ratio first decreases and then increases in d, with the location of the minimum point shifting to the right as σ and a increase. This suggest that the smoothest wages occur not when the amount of insurance desired by the workers is extremely high but instead when it is moderately high. Furthermore as the workers become more risk averse the smoothest wages occur when their relative barganing position worsens (higher d). Finally, all curves shift to the right as a increases.

To explain the convex shape of the curves in Figures 1.1-1.3, consider Figures 2.1-2.3 showing that the relative volatility of hours is nearly linearly increasing in d but decreasing in σ. Recall that the contract tends to smooth out W_t, the total wage bill and that the aggregate real wage is obtained by averaging W_t/L_t with the marginal productivity of the entrepreneurs' hours. As d increases the volatility of L_t increases as it becomes more correlated with output. This tends to compensate for the correlation of W_t with output thereby reducing the volatility of w_t with respect to that of output. But as d increases the volatility of W_t also keeps increasing until it outdoes that of W_t thereby pushing up the relative volatility of w_t again. This logic implies that at low and increasing values of d the real wage should be more highly correlated with output than at very high values of d. This is confirmed in Figures 3.1-3.3 wages are less correlated with output as d increases, as σ decreases and finally as a decreases.

In Figures 3.1-3.3 and 4.1-4.3 we have reported the correlations between w_t and y_t and between c_t and y_t. The horizontal lines drawn in all figures represent the estimated values for the statistics from the U.S. data sample. The reader will note that for values of $a = .46$ one can get close to both lines for choices of $\sigma = .32$ or $.34$ and $d = .1$ or $.12$. Further simulations (not reported) show that this is the case for even lower values of a. This findings have led us to set our baseline parameter values equal to $a = .46$, $\sigma = .32$ and $\psi = .22$. As we mentioned before, lacking direct observations, the reasonableness of these choices can be judged only ex-post by the quality of the overall model's performances. On a-priori ground we find them perfectly acceptable.

Finally the two parameters of the stochastic process S_t have been estimated by constructing a "Solow residual" series in the ordinary way. The latter has been used to compute GMM estimators for the autocorrelation parameter ρ_s while ζ has been obtained by applaying GMM to the orthogonality restriction on the innovations of S_t. This procedure gives the two values $\rho_s = .968$ and $\zeta = .010$.

Table 3 - Baseline Values of Calibration Parameters.

$\delta = .993$	$\sigma = .32$	$\psi = .22$	$\gamma = 1.075$	$\theta = 1.195$	$\mu = .028$
$\alpha = .31$	$\rho = -.7$	$a = .46$	$\rho_s = .968$	$\zeta = .01$	$m = 1.5$

3.3 Simulation Results.

Using the set of parameter values listed in Table 3 we have generated 100 samples of artificial economies with 180 observations each. The data so obtained were detrended and the results were averaged over the 100 samples.

Most of the results reported below appear quite robust to parametric variations and are very indicative of the ability of the model to capture some of the business cycle puzzles we discussed in the introduction. In particular three claims we have made seems to be consistent with the behavior of this artificial economy

1. Introducing a contract increases the volatility of hours and decreases that of real wages.
2. The volatility of aggregate output is increased together with those of profits and the labor-share. The last two also display the correct sign for output correlation.
3. The correlation of wages and output can be reduced to almost zero (in fact at other acceptable parameter values it turns out be slightly negative) while hours remain strongly correlated with output.

Table 4 - Baseline Model.

	H-P Filter		Log-detrend		First Diff.	
Series	St. D.	Corr.	St. D.	Corr.	St.D.	Corr.
Output	2.82	1.00	4.99	1.00	2.06	1.00
Consumption	0.94	0.71	2.37	0.60	0.86	0.80
Investment	8.71	0.93	11.4	0.85	6.21	0.94
Hours	2.28	0.97	3.77	0.90	1.60	0.96
Avg. Lab. Prod.	0.56	0.94	1.26	0.88	0.46	0.94
Real Wage	0.31	0.41	0.50	0.40	0.46	0.64
Labor Share	0.50	-0.88	0.99	-0.88	0.30	-0.57
Profits	3.87	0.96	6.68	0.89	2.51	0.94

The performances of the model are also encouraging with respect to the first order autocorrelation of the aggregate variables, but not entirely so. As Table 5 shows, the high persistence that caracterizes the real wage in the post second world war data is not displayed by our model under H-P filtering or first differencing. The same is true for consumption and labor productivity when first differenced. If one takes into consideration the asymptotic standard errors of the sample estimates the hypothesis that these autocorrelations are actually zero cannot be rejected at conventional significance levels. The only exception is given by the real wage in the First Difference column, for which the hypothesis of zero autocorrelation can be rejected. In any case, even a zero autocorrelation remains a far cry from the empirically observed values. Similar statistics are usually not provided for standard RBC models, but simulations we have run using a standard RBC model show that these features are common to both frameworks.

On the other hand one should stress that the lack of persistence in real wages is relatively easy to eliminate in the contractual framework. It is induced by the fact that our contracts are only one-period-ahead and do not link workers and entrepreneurs for more than one quarter. This enables the two parties to quickly incorporate changes in aggregate productivity in the calculation of labor compensations.

Indeed this is a very unrealistic feature of the model, which we have chosen to maintain here only because it greatly simplifies the numerical computations. Preliminary simulations of a simplified version of the contract model allowing for staggered multi-period contracts lasting three to five quarters are presented in Horvath (1994b). They show that this modification loosens the short-run relation between changes in marginal productivity and real wages resulting in positive autocorrelation of measured real wages and, consequently, of the consumption series.

The model performs quite well in all the other dimensions and when standard errors are taken into account the empirical sample estimates (with the noted exception of the real wages autocorrelation coefficient) belong to the confidence intervals generated by the artificial economy and viceversa. This result is particularly strong in the log-detrended case.

A quantitative feeling of the way in which the optimal contract affects the performances of the artificial economy can be gauged by comparing the sample statistics for the contractual model with those of the spot-economy. This is done in Table 6 for the standard deviation and output correlation of the H-P filtered data and in Table 7 for the first order autocorrelation of the H-P filtered and log-detrendend data. All parameter values are as in Tables 4 and 5.

Table 5 - First Autocorrelations, Baseline Model

Series	H-P Filter	Log-detrend	First Diff.
Output	.725	.824	.099
Consumption	.607	.824	-.193
Investment	.728	.810	.120
Hours	.739	.827	.150
Avg. Lab. Prod.	.657	.809	-.077
Real Wage	-.084	.448	-.483
Labor Share	.798	.884	.409
Profits	.778	.838	.291

Table 6 - Contract vs. Spot Economy, H-P Filter.

	Contract Model		Spot Model	
Series	Stand. Dev.	Correlation	Stand. Dev.	Correlation
Output	2.82	1.00	2.71	1.00
Consumption	0.94	0.71	1.23	0.71
Investment	8.71	0.93	11.57	0.92
Hours	2.28	0.97	2.11	0.96
Avg. Lab. Prod.	0.56	0.94	0.63	0.94
Real Wage	0.31	0.41	0.68	0.95
Labor Share	0.50	-0.88	0.00	0.00
Profits	3.87	0.96	2.71	0.97

Table 7 - Contract vs. Spot Economy, Autocorrelations.

	Contract Model		Spot Model	
Series	H-P Filter.	Log-Detr.	H-P Filter	Log-Detr.
Output	.725	.820	.701	.812
Consumption	.607	.786	.870	.868
Investment	.728	.809	.675	.784
Hours	.739	.826	.690	.802
Avg. Lab. Prod.	.657	.800	.752	.829
Real Wage	-.084	.357	.740	.829
Labor Share	.798	.844	.000	.000
Profits	.778	.836	.701	.812

Our last comparison is between the contract model and the Hansen (1985) "indivisible labor" model, which is correctly regarded as the paradigmatic RBC model of labor market behavior (see also Rogerson (1988) for the theoretical background). Hansen did not consider factor shares, nor autocorrelation coefficients and detrended his data only with the H-P filter. Table 8 is constructed accordingly. The parameter values for our model are those of Tables 4 and 5.

Table 8 - Contract Economy and Hansen (1985) Economy.

	Contract Model		Hansen (1985) Model	
Series	Stand. Dev.	Correlation	Stand. Dev.	Correlation
Output	2.82	1.00	1.76	1.00
Consumption	0.94	0.71	0.51	0.87
Investment	8.71	0.93	5.71	0.99
Hours	2.28	0.97	1.35	0.98
Avg. Lab. Prod.	0.56	0.94	0.50	0.87
Real Wage	0.31	0.41	=	=
Labor Share	0.50	-0.88	=	=
Profits	3.87	0.96	=	=

It is fair to conclude that there is no visible dimension along which the contract model performs worse and some obvious and very important dimensions along which it performs better.

4. Conclusions

We have shown that introducing simple forms of contractual labor relationships in a standard stochastic optimal growth model makes it display more realistic properties than those that obtain when the labor market is modeled in a purely Walrasian fashion. Wages and hours oscillate at the right magnitude and in the right direction without the need of introducing an unreasonably elastic labor supply function or "unobservable" institutional mechanisms. Factor shares cyclical variability and correlation can also be accounted for by the same contractual argument which also provides an explanation for the observed behavior of profits and investments at the peak and trough of the trade cycles.

Factor shares oscillations, while going in the right direction, are still relatively small in our model. This is especially true for profits. This seems harder to capture: it may require moving away from a Cobb-Douglas specification for the aggregate technology as well as from the one-sector representation. Two-sector Cobb-Douglas models already allow for cyclical variations in factor shares; it needs to be seen if they are quantitatively relevant. Along these lines we are also considering further departures from the purely competitive framework, such as the introduction of borrowing contractual arrangements between entrepreneurs and financial institutions.

This line of research could ideally lead us to be able to dispense with the notion of large and frequent, aggregate technological shocks. They are very vague and hardly measurable entities, which can be identified only after the fact by accepting uncritically a number of simplifications on the form of the production function and on the way in which inputs are rewarded. The theory of the business cycle which stems from dynamic general equilibrium models does not require aggregate shocks, neither from a logical nor from an empirical poit of view as demonstrated in Horvath (1994a). Their current adoption seems to be motivated almost essentially by practical considerations: lacking endogenous sources of instability and built-in magnifiers one has to resort to aggregate exogenous stimuli "to get things going". Further investigation in this area may well point to other endogenous sources of business fluctuations.

Another natural extension is to look at the asset pricing implications of the contractual approach. Results obtained with a model in which non stockholders are the only suppliers of labor effort are quite promising. Intuitively this is due to a couple of factors. On one hand, as our model shows, profit earners now bear a much larger portion of the aggregate risk: return on equities is both much higher and more correlated with aggregate output.

On the other hand the equilibrium prices of assets are not evaluated by using aggregate consumption to compute the relevant rate of intertemporal substitution. Instead it is the consumption of stockholders alone that matters and the latter needs not be as stable and smooth as the economy's average consumption. In order to give operative content to this approach to asset pricing, one needs to be able to identify empirical measures of stockholder's consumption volatility.

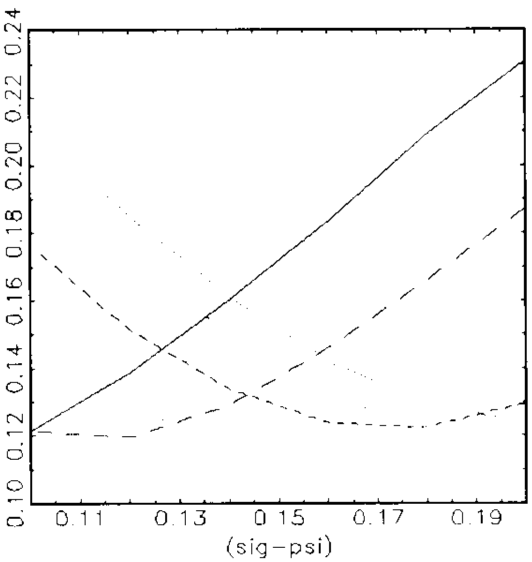


Fig. 1.1: w std./y std. for a=.46

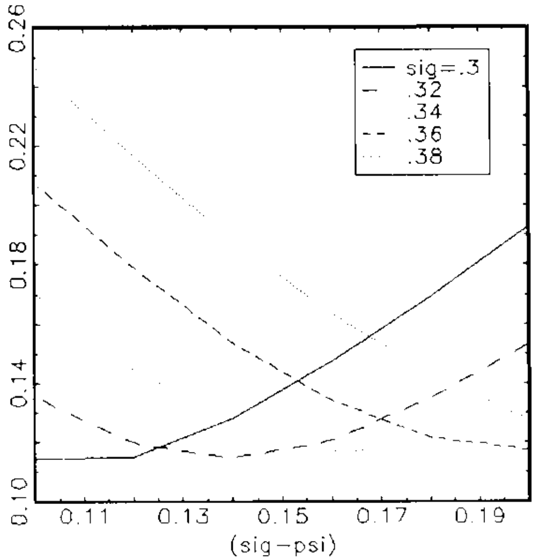


Fig 1.2: w std./y std. for a=.5

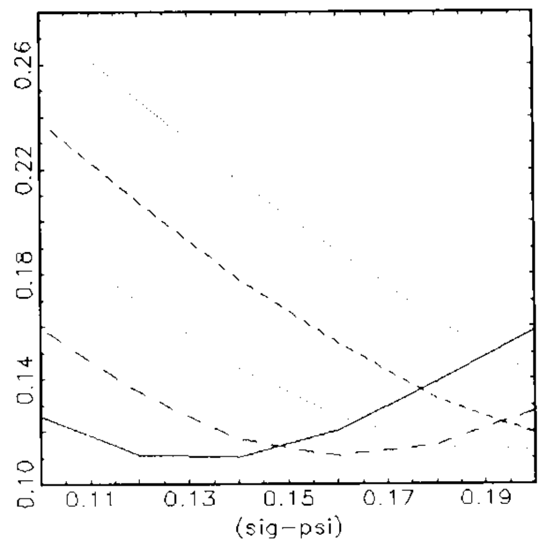


Fig 1.3: w std./y std. for a=.54

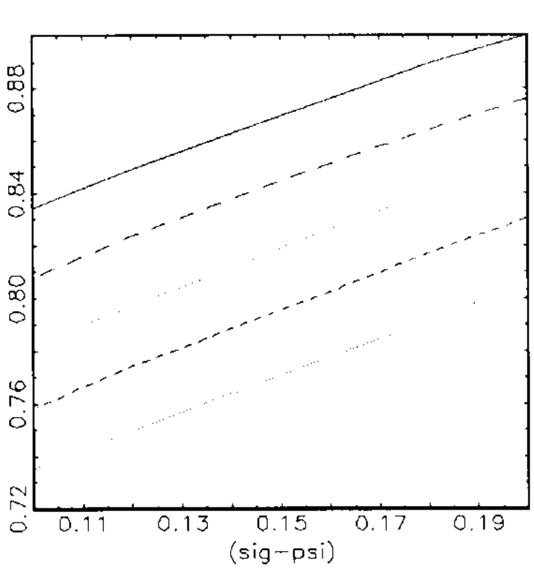


Fig 2.1: h std./y std. for a=.46

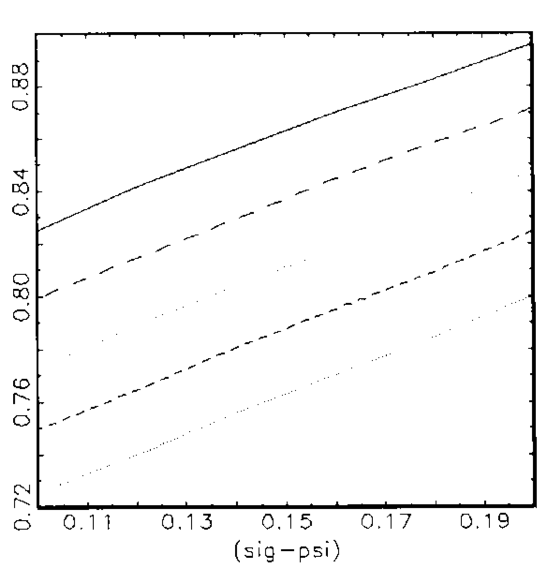


Fig 2.2: h std./y std. for a=.5

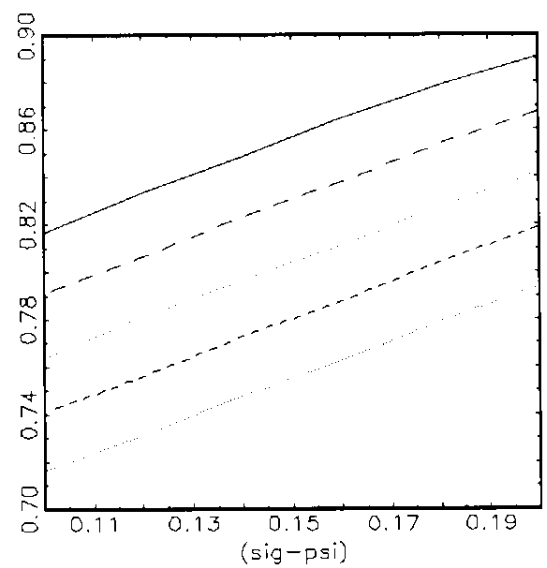


Fig 2.3: h std./y std. for a=.54

Fig 3.1: w~y corr. for a=.46

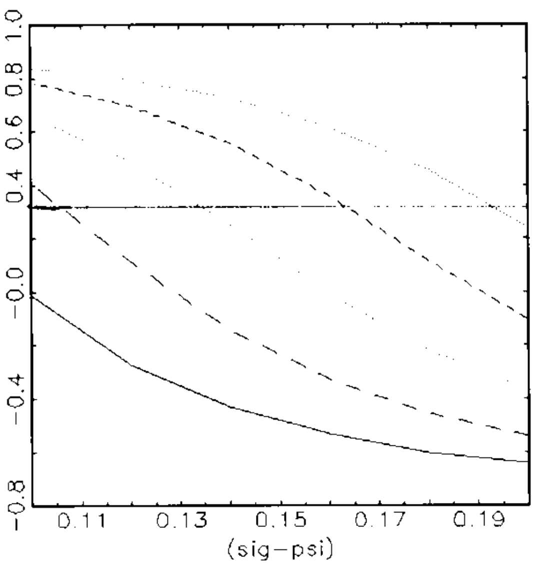


Fig 3.2: w~y corr. for a=.5

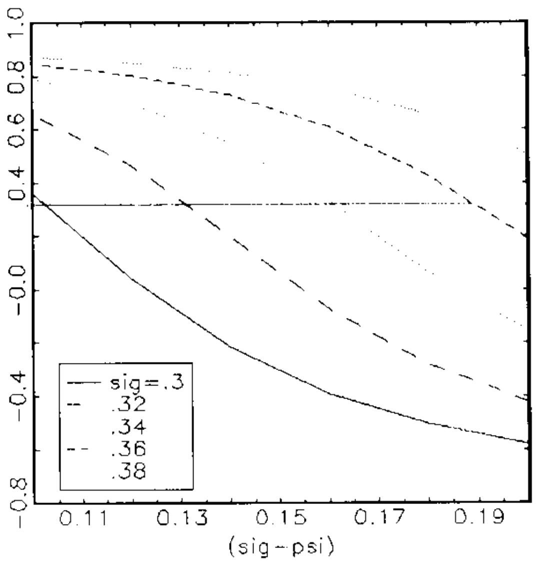


Fig 3.3: w~y corr. for a=.54

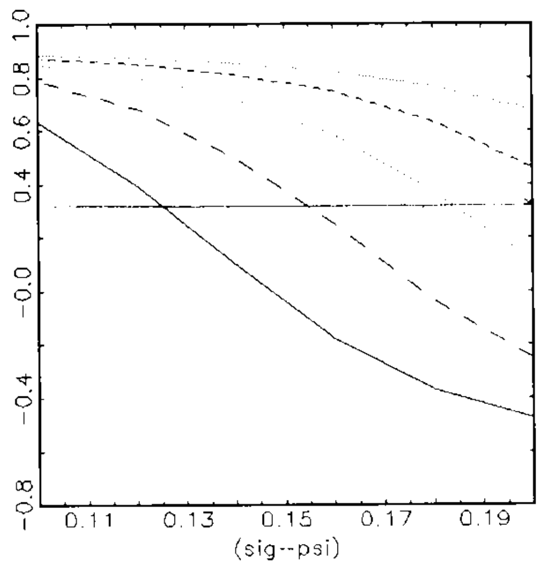


Fig 4.1: c~y corr. for a=.46

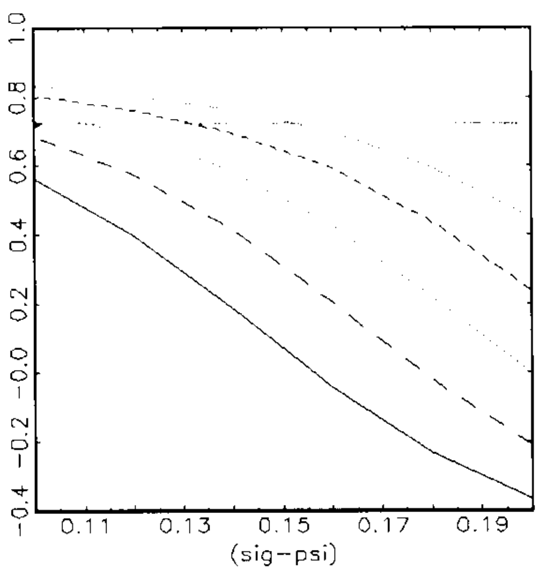


Fig 4.2: c~y corr. for a=.5

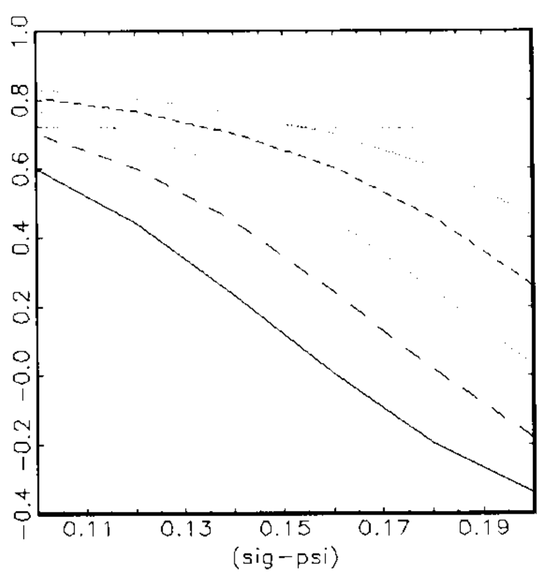
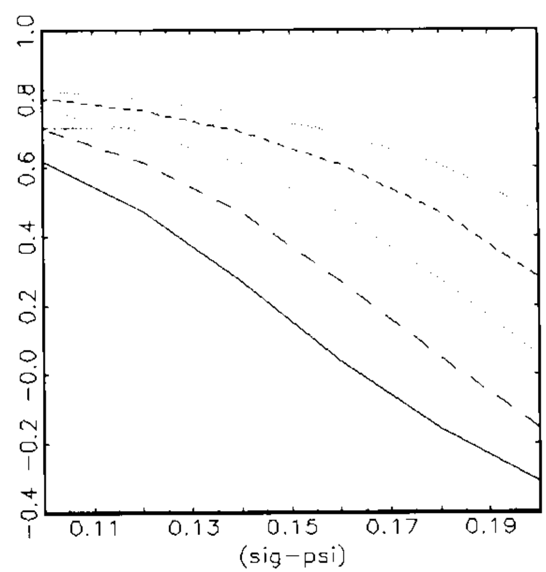


Fig 4.3: c~y corr. for a=.54



Appendix A: Data description

We have attempted to present statistics on and estimate parameters from data on private sector, non-farm, business production and factor payments. To do so, we often begin with a broader category and subtract sectors which we do not wish to include (eg., removing farm production from gross domestic product). In the list below, the series name is followed by the symbol which corresponds to the series in our model. A brief description of the data source is given and, in some cases, additional notes.

Output $= Y$. Real gross domestic product less government, housing and farm sectors, in 1987 dollars, reported quarterly in National Income and Product Accounts (NIPA).

Consumption $= C + W$, Real total consumer expenditure on non-durables and services, in 1987 dollars, reported quarterly in NIPA.

Investment $= I$. Real private sector, fixed, non-residential investment plus real expenditure on consumer durables, in 1987 dollars, reported quarterly in NIPA.

Capital Stock $= K$. Stock of investment series constructed in the usual manner by comparing net and gross investment series.

Total Hours $= L + N$. Total hours worked in non-agricultural, private business establishments, Bureau of Labor Statistics (LBMNU), reported quarterly.

Real Wages $= (W + MPN \times N)/(L + N)$. Real hourly wage of all non-agricultural, private business employees, Bureau of Labor Statistics (LBCPU7), reported in 1987 dollars, quarterly.

Profit $= Y - W - MPN \times N$. Nominal corporate profits before tax, without inventory valuation adjustment or capital consumption allowance, deflated by an implicit price deflator described below under Price. The nominal profit figures are reported quarterly in NIPA

Average Labor Productivity $= Y/(L + N)$.

Labor Share $= (W + MPN \times N)/Y$.

Price : Implicit price deflator equal to nominal output series (nominal gross domestic product minus nominal government, housing and farm sector production) divided by the real output series described above under Output.

Detrending Methods: We induce stationarity by three alternate methods: a two-trend detrending procedure on the log-levels of the data, the Hodrick-Prescott filter with λ, the cost of detrending in the filter's minimization function, set at 1600 and log first-differencing. The latter method is completely straightforward. The log-linear detrending allows for one trend in the log-levels from 1947-1975, and a potentially different trend from 1975-1990. The detrended series are spliced back together after the potentially different trends have been removed. Naturally, the log-linear detrending removes less information than the HP-filtering, however, questions remain whether the classical properties apply to the distribution of the log-linearly detrended series because they may still not be covariance stationary.

Appendix B: Alternative Parameter Values.

To complete the description of the baseline model's performances we report next the sample statistics for the case in which $\alpha = .26$ and $\alpha = .36$. All other parameter values are as in Tables 4 and 5 with the following exceptions. In both cases, γ and θ have been adjusted to keep the appropriate ratio between working and nonsleeping hours. Also, in Tables B.3 and B.4 we have chosen slightly different values for a, σ and ω (holding $d = \sigma - \omega$ constant) in order to match the sample correlations between wages and output and between consumption and output. These new values are: $a = .42$, $\sigma = .38$ and $\omega = .28$. No change of this kind was made for $\alpha = .26$ even if also in that case some very small variations of the same parameters would have allowed us to exactly match the named correlations.

Table B.1 - Contract Model ($\alpha = .26, \theta = 1.04, \gamma = .925$)

	H-P Filter		Log-detrend		First Diff.	
Series	St. D.	Corr.	St. D.	Corr.	St.D.	Corr.
Output	2.92	1.00	3.44	1.00	1.93	1.00
Consumption	1.33	0.86	2.00	0.78	0.92	0.85
Investment	10.22	0.93	10.74	0.84	6.78	0.89
Hours	2.26	0.97	2.59	0.90	1.47	0.90
Avg. Lab. Prod.	0.68	0.95	0.88	0.88	0.47	0.89
Real Wage	0.51	0.88	0.64	0.83	0.50	0.80
Labor Share	0.30	-0.83	0.34	-0.81	0.18	-0.32
Profits	3.68	0.96	4.33	0.90	2.16	0.88

Table B.2 - First Autocorrelations.

Series	H-P Filter	Log-detrend	First Diff.
Output	.601	.738	.054
Consumption	.552	.789	-.130
Investment	.587	.702	.055
Hours	.612	.736	.086
Avg. Lab. Prod.	.565	.749	-.046
Real Wage	.288	.596	-.310
Labor Share	.655	.757	.206
Profits	.661	.771	.218

Table B.3 - Contract Model ($\alpha = .36, \theta = .97, \gamma = .903$)

	H-P Filter		Log-detrend		First Diff.	
Series	St. D.	Corr.	St. D.	Corr.	St.D.	Corr.
Output	2.44	1.00	6.10	1.00	1.79	1.00
Consumption	0.71	0.76	3.77	0.82	0.74	0.82
Investment	6.03	0.95	11.18	0.87	4.31	0.95
Hours	1.91	0.97	4.34	0.96	1.14	0.96
Avg. Lab. Prod.	0.55	0.95	1.85	0.93	0.47	0.94
Real Wage	0.30	0.40	0.84	0.76	0.45	0.64
Labor Share	0.50	-0.89	1.18	-0.95	0.30	-0.62
Profits	3.29	0.96	8.20	0.97	2.18	0.95

Table B.4 - First Autocorrelations.

Series	H-P Filter	Log-detrend	First Diff.
Output	.720	.918	.091
Consumption	.489	.942	-.271
Investment	.727	.889	.115
Hours	.741	.914	.158
Avg. Lab. Prod.	.632	.927	-.103
Real Wage	-.128	.787	-.486
Labor Share	.804	.930	.443
Profits	.769	.927	.260

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