The Swing Voter's Curse*

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Abstract

One of the central puzzles within the formal literature is participation in large elections. The focus of this literature has been on the relationship between the costs to vote and the probability of being pivotal. We show that asymmetric information affects participation and vote choice independent of costs to participate or the probability of being pivotal. We use the insight that underlies the "winner's curse" in the theory of auctions to demonstrate that rational voters might abstain even if voting is costless. In this paper we consider a model in which voters are asymmetrically informed about the benefits of electing either of two candidates. We show that it is rational for less informed individuals to abstain even when voting is costless and even when they have an ex ante strict preference between the candidates. Furthermore, in contrast to the earlier literature on rational participation, participation may *increase* as the probability of being pivotal *decreases*. Finally, our model predicts that a decline in partisanship may result in both higher levels of abstention and a higher probability of electing the candidate preferred by a majority of the fully informed electorate.

Introduction

In every large scale election a substantial fraction of the electorate votes and a substantial fraction abstains. If voting is costly, then, since it is extremely unlikely that one person's vote changes the outcome, it is difficult to understand why so many people vote. Conversely, if voting is not costly, the problem is to explain why so many people abstain. The literature provides two solutions to this problem. First, perhaps voting is costly for some citizens but not for others. This solution has been proposed by Downs (1957) and by Riker and Ordeshook (1968).[1] Second, recent game-theoretic models by Ledyard (1983), Palfrey and Rosenthal (1983, 1985) and Feddersen (1992, 1993) demonstrate that significant levels of participation may be rationalizable even if voting is costly. However, as Palfrey and Rosenthal (1985) demonstrate, game-theoretic explanations of costly participation are not robust to the introduction of reasonable uncertainty.[2]

Both the decision-theoretic and the game-theoretic approaches emphasize the interaction between costs to vote and the probability of being pivotal. We agree with Aldrich (1993) that participation in elections is a low cost and low benefit activity and that a number of different factors determine participation and vote choice. Our contribution here is to demonstrate that informational asymmetries may influence both participation and vote choice independent of costs to vote and pivot probabilities.

We use the insight underlying the "winner's curse" in the theory of auctions to show that rational voters with private information may choose to abstain or even vote for a candidate that they consider inferior based on their private information. The paradigmatic example of the winner's curse is as follows.[3] A set of bidders have correlated private information about the value of an oil lease and each knows that other agents have private

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[1]More recent models of rational participation include: Ferejohn and Fiorina 1974; Morton 1991; Uhlaner 1989; Matsusaka 1992.
[2]Palfrey and Rosenthal show that, if there is sufficient uncertainty about preferences and about participation costs of voters, participation by those with strictly positive costs to vote will go to zero as the size of the population gets large.
[3]Capen, Clapp and Campbell (1971). See also Milgrom and Weber (1982).

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information as well.[4] If every bidder offers her expected evaluation determined from her private information the winning bidder has bid too much because, by virtue of winning, it follows that every other bidder's expected valuation is lower. The solution to the winner's curse is for bidders to base offers not only on private information but also on what must be true about the world if theirs is the high bid.

There is an analog to the winner's curse in two-candidate elections with asymmetric information: the swing voters curse. A swing voter is a voter whose vote determines the outcome in an election.[5] When all voters have noisy private information about the state of the world each swing voter must base her vote not only on her private information but also on what must be true about the world if her vote matters, i.e., she is pivotal.

Consider the following example. Suppose that all voters prefer candidate 0 in state 0 and candidate 1 in state 1. At least one of the voters is informed and knows with certainty the state of the world. However, voters do not know the exact number of informed voters in the electorate. All of the uninformed voters share a common knowledge prior that 90% of the time state 0 occurs and hence candidate 0 is the best candidate.

Now, suppose that all voters (informed and uninformed alike) vote only on the basis of their private information. Then all of the informed voters vote for candidate 0 in state 0 and candidate 1 in state 1 while all of the uninformed voters vote for candidate 0 in both states. The informed voters are behaving rationally while the uninformed are not. If an uninformed voter is pivotal she knows that the informed voters have voted for candidate 1 and therefore state 1 has occurred. It follows that she prefers to vote for candidate 1. On the other hand, if all uninformed voters vote for candidate 1, then each strictly prefers to vote for candidate 0. The uninformed voters suffer a swing voter's curse. It is irrational for them to vote only on the basis of their private information.[6]

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[4] We might imagine that each bidder has privately commissioned a study of the property being leased in an attempt to determine the amount of oil that may be productively exploited.

[5] Banzhaff (1966) uses the term swing voter in this fashhion.

[6] This example is not a direct analog to the winner's curse because it involved two categories of agents: more informed and less informed. In another paper, Feddersen and Pesendorfer 1994, we present a model that is a direct analog to

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In this example there is an easy way out of the swing voter's curse for the uninformed voters: abstention. Abstention is an optimal strategy because it maximizes the probability that the informed voters decide the election. If all of the uninformed abstain it follows that there are only two conditions under which an uninformed voter might be pivotal: either there are no informed voters or there is exactly one informed voter. In our example we eliminated the first possibility because we assumed that there is always at least one informed voter. In the latter case the uninformed voter strictly prefers to abstain because the only way her vote effects the outcome is if she votes for the candidate not supported by the informed voter, i.e., the wrong candidate.

Our model formalizes and extends the above example to include voters with different preferences. We assume three kinds of voters: voters who always prefer candidate 0 (0-partisans), voters who always prefer candidate 1 (1-partisans) and independents. The 0-partisans always prefer candidate 0 regardless of the state of the world while the 1-partisans always prefer candidate 1. Independent voters sometimes prefer candidate 0 and sometimes prefer candidate 1 depending on the state of the world. All voters know the expected percentage of each type within the population but not the exact numbers. Finally, we assume that with positive probability any voter knows the true state of the world.

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the winner's curse in which each agent receives a private noisy signal. In that paper we show that agents rationally vote not only on the basis of their private information but also on what must be true about the world if they cast the pivotal vote.

Our central results are as follows:

- Asymmetric information fundamentally alters the calculus of voting. It may be rational for a voter in a two candidate election to vote for the candidate she believes to be worse or to abstain even if voting is costless. Furthermore, our model predicts significant levels of abstention and participation.

- An increase in information *may* lead to a *lower* probability of being pivotal and a *higher* level of participation while a decrease in partisanship *may* lead to a *lower* probability of being pivotal and a *lower* level of participation.

- Increased abstention may be associated with greater electoral efficiency in the sense that the election is more likely to produce an outcome that a majority of the electorate would prefer if all voters were fully informed.

This paper is in three sections. In the first section, we discuss the formal literature directly related to our model. In the second section we cover the model and results. The third section is a discussion of the results, their relationship to the empirical literature in American politics and some concluding remarks. The appendix contains proofs of the results.

Related Literature

There is an extensive formal literature on participation and several recent surveys (see Aldrich 1993 or Matsusaka 1992). The effect of asymmetric information on the calculus of voting has not been analyzed in this literature. For example, Palfrey and Rosenthal (1985, p62) state that uncertainty over alternative outcomes "is of no consequence" in

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determining voting behavior; voters simply vote for the candidate associated with the most preferred expected outcome. However, if voters possess private information that might, if shared, cause other voters to change their preferences then both voting behavior and the decision to participate may be dramatically different than in models without private information.

The model we present is similar in some respects to the model developed by Austen-Smith[7] (1988) in a legislative setting and by Lohmann (1993a,b) in the context of participation in protest movements.[8] Austen-Smith showed that privately informed legislators may vote for an alternative she believes to be inferior even in a two-alternative election. Lohmann considers a model in which actors have private information about the state of the world and must decide to participate in a demonstration. A decision maker then observes the number of actions taken and determines the outcome. Our work extends Austen-Smith's insight by permitting abstention and differs from both Austen-Smith and Lohmann by considering the asymptotic properties of a model of elections with privately and asymmetrically informed voters.

Matsusaka (1992) develops a decision-theoretic informational approach to participation in which he argues that more informed voters get a higher expected benefit by voting for the candidate with the highest expected return than do less informed voters. His approach relies on the assumption that voting is costly. Voters in Matsusaka's model choose to acquire information at a cost and then choose if and for whom to vote. If voting is costless in Matsusaka's setting then all voters should vote. Our approach differs from Matsusaka's in that it is game-theoretic and uninformed voters may be strictly worse off by voting even if voting is costless. Austen-Smith's (1988) results along with our own demonstrate that voting, as modeled in Matsusaka, is unlikely to be rational. This follows

[7]See Palfrey and Srivastava (1989) for a related example.
[8]There is also a related social choice literature on Condorcet's Jury Theorem that examines majority rule elections as information aggregation devices. See for example, Ladha 1992; Grofman and Owen 1986; and Schofield 1972. In an related paper Klevorick, Rothschild and Winship (1985) show that if each juror only considers her private information then the majority rule outcome is inefficient. See also Austen-Smith and Banks (1994).

because in a decision theoretic model voters do not condition their vote on what must be true about the world when they are pivotal.

Model

There are two states, state 0 and state 1, where $Z = \{0,1\}$ denotes the set of states. There are two candidates, candidate 0 and candidate 1. The set of candidates is $X = \{0,1\}$. There are three types of agents, where $T = \{0,1,i\}$ is the set of types. Type 0 and type 1 agents are partisans: irrespective of the state type 0 agents prefer candidate 0 and type 1 agents prefer candidate 1. Type i agents are independents: given a pair (x,z), $x \in X$ and $z \in Z$, the utility of a type i agent is $U(x,z) = -(x-z)^2$. Therefore an independent prefers candidate 0 in state 0 and candidate 1 in state 1.

At the beginning of the game nature chooses a state $z \in Z$. State 0 is chosen with probability α and state 1 is chosen with probability $1-\alpha$. Without loss of generality we assume that $\alpha \leq 1/2$. The parameter α is common knowledge and hence all agents believe that state 1 is at least as likely as state 0. Nature also chooses a set of agents by taking $N \cdot 1$ independent draws. We assume that there is uncertainty both about the total number of agents and the number of agents of each type. In each draw, nature selects an agent with probability $(1 - p_\phi)$. If an agent is selected, then with probability $p_i / (1 - p_\phi)$ she is of type i, with probability $p_0 / (1 - p_\phi)$ she is type 0, and with probability $p_1 / (1 - p_\phi)$ she is type 1. The probabilities $p = (p_i, p_0, p_1, p_\phi)$ are common knowledge.[9]

After the state and the set of agents have been chosen, every agent learns her type and receives a message $m \in M$, where $M = \{0, \alpha, 1\}$. Both her type and the message are private information. If an agent receives message m then the agent knows that the state is 0 with probability m. All agents who receive a message $m \in \{0,1\}$ are informed, i.e.,

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[9] Thus, the actual number of voters n is uncertain and follows a binomial distribution $\binom{N+1}{n} p_\phi^{N+1-n} (1 - p_\phi)^n$. Similarly, the number of type j voters, $j = 0,1,i$, follows a binomial distribution with parameters $(N+1, p_j)$

they know the state with probability 1. Note that all informed agents receive the same message. The probability that an agent is informed is q. Agents who receive the message α do not learn anything about the state beyond the common knowledge prior. We refer to these agents as uninformed.

Finally, every agent chooses an action $s \in \{\phi, 0, 1\}$ where ϕ indicates abstention and 0 or 1 indicates her vote for candidate 0 or 1 respectively. The candidate that receives a majority of the votes cast will be elected. Whenever there is a tie, we assume that each candidate is chosen with equal probability.

A pure strategy for an agent is a map $s: T \times M \to \{\phi, 0, 1\}$. A mixed strategy is denoted by $\varsigma: T \times M \to [0, 1]^3$, where ς_s is the probability of taking action s.

We analyze the symmetric Nash equilibria of this game, i.e., we assume that agents who are of the same type and receive the same message choose the same strategy. Note that the number of agents is uncertain and ranges from 0 to $N \cdot 1$. Therefore, there is a strictly positive probability that an agent is pivotal. Hence all agents except the uninformed independent agents have a strictly dominant strategy. Agents of type 1 (type 0) always vote for candidate 1 (candidate 0) and all informed independent agents vote according to the signal they receive, that is if $m \in \{0, 1\}$ then $s(i, m) = m$. However, uninformed independent agents do not always have a dominant strategy.

In equilibrium agents never use a strictly dominated strategy. Therefore we can simplify our notation and specify only the behavior of the uninformed independent agents (UIAs). We denote a mixed strategy profile in the game with $N \cdot 1$ potential agents by $\varsigma^N = (\varsigma_0^N, \varsigma_1^N, \varsigma_\phi^N) \in [0, 1]^3$. Under profile ς^N all UIAs play according to the mixed strategy ς^N and all other agents choose their dominant strategies.

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ANALYSIS

In order to facilitate the exposition of our results we introduce the following notation. For a given profile $\varsigma^\vee$, define $\sigma_{zx}(\varsigma^\vee)$ to be the probability a random draw by nature results in a vote for candidate x if the state is z. The only agents who vote for x are x-partisans and independents. An informed independent agent votes for x only if z x while an UIA votes for x with probability $\varsigma_x^\vee$ in both states. Therefore the probability that a draw by nature results in a vote for candidate x in state z is defined as follows:

$$\sigma_{zx}(\varsigma^\vee) \equiv \begin{cases} p_x + p_i(1-q)\varsigma_x^\vee & \text{if } z \neq x \\ p_x + p_i(1-q)\varsigma_x^\vee + p_i q & \text{if } z = x \end{cases}$$

From the perspective of an UIA the probability that a draw by nature will result in a vote for candidate x in state x, $\sigma_{xx}(\varsigma^\vee)$, is the probability of a correct vote while $\sigma_{yx}(\varsigma^\vee)$, $y \neq x$, is the probability of a mistaken vote. Note that the probability of a draw resulting in a correct vote is always greater than the probability of a draw resulting in a mistaken vote.

Define $\sigma_{z\phi}(\varsigma^\vee)$ to be the probability that a random draw by nature does not result in a vote for either candidate in state z. This can happen either if no agent is drawn or if the agent who is drawn abstains. The only agents who abstain are UIAs. Since the probability no agent is drawn and the strategy of an UIA do not depend on the state the probability a draw does not result in a vote is independent of the state. Thus

$$\sigma_{1\phi}(\varsigma^\vee) = \sigma_{1\phi}(\varsigma^\vee) = \sigma_\phi(\varsigma^\vee) = p_i(1-q)\varsigma_\phi^\vee + p_\phi.$$

In order to determine the best responses of UIAs we must specify the conditions in which an UIA's choice changes the outcome. There are three situations in which an agent may be pivotal:

- an equal number of other agents have voted for each candidate,
- candidate 1 receives one more vote than candidate 0,
- candidate 0 receives one more vote than candidate 1.

For any agent the probabilities of each of these events given state z, N other possible agents and strategy profile ς^N are given below.

The probability an equal number of other agents have voted for each candidate, i.e., a tie is:

$$\pi_t(z, N, \varsigma^N) = \sum_{j=0}^{\frac{N}{2}} \frac{N!}{j! j! (N-2j)!} \sigma_\phi(\varsigma^N)^{N-2j} (\sigma_{z0}(\varsigma^N)\sigma_{z1}(\varsigma^N))^j .$$

The probability that candidate x receives exactly one less vote than candidate y (the probability that candidate x is down by 1 vote) is:

$$\pi_x(z, N, \varsigma^N) = \sum_{j=0}^{\frac{N-1}{2}} \frac{N!}{(j+1)! j! (N-2j-1)!} \sigma_\phi(\varsigma^N)^{N-2j-1} \sigma_{zy}(\varsigma^N)(\sigma_{zx}(\varsigma^N)\sigma_{zy}(\varsigma^N))^j$$

By $Eu(x, N, \varsigma^N)$ we denote the expected payoff to an UIA of taking action x when the strategy profile used by all other agents is ς^N and there are N other potential agents. To determine a best response by an UIA it is only necessary to consider the expected utility differences between every pair of strategies. The expected utility differentials are given below as a function of N and ς^N:

(1)
$$Eu(1, N, \varsigma^N) - Eu(\phi, N, \varsigma^N) =$$
$$\frac{1}{2}\left[(1-\alpha)[\pi_t(1, N, \varsigma^N) + \pi_1(1, N, \varsigma^N)] - \alpha[\pi_t(0, N, \varsigma^N) + \pi_1(0, N, \varsigma^N)]\right]$$

(2)
$$Eu(0, N, \varsigma^N) - Eu(\phi, N, \varsigma^N) =$$
$$\frac{1}{2}\left[\alpha[\pi_t(0, N, \varsigma^N) + \pi_0(0, N, \varsigma^N)] - (1-\alpha)[\pi_t(1, N, \varsigma^N) + \pi_0(1, N, \varsigma^N)]\right]$$

(3)
$$Eu(1, N, \varsigma^N) - Eu(0, N, \varsigma^N) =$$
$$(1-\alpha)[\pi_t(1, N, \varsigma^N) + \tfrac{1}{2}(\pi_1(1, N, \varsigma^N) + \pi_0(1, N, \varsigma^N))] -$$
$$\alpha[\pi_t(0, N, \varsigma^N) + \tfrac{1}{2}(\pi_1(0, N, \varsigma^N) + \pi_0(0, N, \varsigma^N))]$$

RESULTS

All of our results rely on the fact that an agent's vote only matters if he/she is pivotal. If agents possess private information that is useful to other agents and if they condition their voting and participation strategies on this private information then every agent must choose if or for whom to vote not only on the basis of his/her private information but also on what information other voters must have if she is pivotal. The information he/she can infer from being pivotal may even overwhelm his/her private information. This is particularly the case in large electorates (all of our results are true for large N). As we show here, optimal voting and participation strategies can change dramatically from the behavior predicted in a perfect information environment.

Lemma 1 states that for large electorates all UIAs will prefer to vote for the candidate who is correct in the state more likely to produce a mistaken vote. For example, if a mistake is more likely in state 0 then all UIAs will strictly prefer to vote for candidate 0. Conversely, to ensure that an UIA is indifferent between any two actions the probabilities of a mistake in each state must be nearly identical . All proofs are in the appendix.

Lemma 1 *Suppose $p_i q > 0$ and $0 < \alpha < 1$. Consider a sequence of strategy profiles $\{\varsigma^N\}_{N=0}^{\infty}$. Then:*

A. *if there exists an $\varepsilon > 0$ such that $\sigma_{xy}(\varsigma^N) - \sigma_{yx}(\varsigma^N) > \varepsilon$ for any $N \geq 0$ and $x \neq y$ then there exists an $\overline{N}$ such that for any $N \geq \overline{N}$*
$$Eu(x, N, \varsigma^N) > Eu(\phi, N, \varsigma^N) > Eu(y, N, \varsigma^N);$$

B. *if for all $N \geq 0$ there are two actions s, s' with $s \neq s'$ such that $Eu(s, N, \varsigma^N) = Eu(s', N, \varsigma^N)$. Then for any $\varepsilon > 0$ there is an $\overline{N}$ such that for $N \geq \overline{N}$*
$$|\sigma_{01}(\varsigma^N) - \sigma_{10}(\varsigma^N)| < \varepsilon.$$

The intuition behind Lemma 1.A can be summarized as follows: if the probability a random draw results in a mistaken vote in state 1 is larger than the probability of a mistaken vote in state 0, i.e., $\sigma_{10}(\varsigma(N)) - \sigma_{01}(\varsigma(N)) > \varepsilon$, then the conditional probability that the world is in state 1 given the agent is pivotal goes to 1 as the size of the electorate, N, increases. This follows from the fact that an agent is only pivotal if enough other agents make a mistake to compensate for the votes of the informed independent agents. If the probability of a mistake is higher in state 1 than state 0 then an UIA is much more likely to be pivotal in state 1 than in state 0 and he/she strictly prefers to vote for candidate 1 rather than abstain and would rather abstain than vote for candidate 0. Lemma 1.B follows as a corollary of part A.

Lemma 2 states that if an UIA is indifferent between voting for candidate 1 and voting for candidate 0 then he/she *strictly* prefers to abstain.

Lemma 2 *Let* $p_\phi > 0$, $q > 0$, $N \geq 2$ *and N even. For any symmetric strategy profile* ς^N, $Eu(1,N,\varsigma^N) = Eu(0,N,\varsigma^N)$ *implies* $Eu(1,N,\varsigma^N) < Eu(\phi,N,\varsigma^N)$.

If an UIA is indifferent between voting for candidate 0 and candidate 1 the utility difference between voting for either candidate and abstaining is the weighted sum of the differences between the probability of creating a tie by voting for the correct candidate minus the probability of creating a tie by voting for the incorrect candidate in each state. Each of these differences is a negative number since it is always more likely that the incorrect candidate is behind by one vote than that the correct candidate is down by one vote in each state. This is because the informed independents always vote for the correct candidate in each state.

Lemma 2 implies that there can be no mixed strategy equilibria in which UIAs mix between voting for candidate 0 and voting for candidate 1. The only possible equilibria in

our model are either pure-strategy equilibria or mixed strategy equilibria in which UIAs

mix between abstention and voting for a single candidate.

The following proposition summarizes equilibrium behavior.

Proposition 1 *Suppose* $q > 0$ *and* $p_\phi > 0$, *then for all* $\varepsilon > 0$ *there exists an* $\overline{N}$ *such that for* $N \geq \overline{N}$ *and for any equilibrium strategy profile* ς^N *the following statements are true:*

(i) *If* $p_i(1 - q) < p_0 - p_1$ *then* $\varsigma_1^N = 1$.

(ii) *If* $p_i(1 - q) < p_1 - p_0$ *then* $\varsigma_0^N = 1$.

(iii) *If* $p_i(1 - q) \geq p_0 - p_1 > 0$ *then* $\varsigma_0^N = 0$ *and* $\left| \varsigma_\phi^N - 1 + \dfrac{p_0 - p_1}{p_i(1 - q)} \right| < \varepsilon$.

(iv) *If* $p_i(1 - q) \geq p_1 - p_0 > 0$ *then* $\varsigma_1^N = 0$ *and* $\left| \varsigma_\phi^N - 1 + \dfrac{p_1 - p_0}{p_i(1 - q)} \right| < \varepsilon$.

(v) *If* $p_0 - p_1 = 0$ *then* $\varsigma_0^N = 0$ *and* $\varsigma_\phi^N > 1 - \varepsilon$.

UIAs do not know the state with certainty and therefore are unsure of the candidate

that they prefer to win. On the other hand UIAs would always prefer that informed

independent agents decide the election. The effect of equilibrium behavior of the UIAs is

to maximize the probability that the informed independent agents determine the winner.

UIAs vote to compensate for the partisans and having achieved that compensation they

abstain. In cases (i) and (ii) the expected fraction of UIAs is too small to compensate for

the partisan advantage enjoyed by candidates 0 and 1 respectively. For example, in case

(i), independent of the strategy used by the UIAs, the probability a draw results in a

mistake is higher in state 1 than in state 0 therefore by Lemma 1.A all UIAs vote for

candidate 1. In cases (iii), (iv) and (v) the expected fraction of UIAs is large enough to

fully offset the bias introduced by partisans. In these cases there are no pure strategy

equilibria. By Lemma 2 there are no equilibria in which UIAs mix between voting for each

candidate. By Lemma 1 UIAs mix between abstention and voting and exactly compensate

for the differences in partisan support.

The equilibrium compensation for the bias in the electorate caused by different proportions of partisans leads to an efficiency result. Clearly, from the point of view of a type 1 agent, candidate 1 is always the right candidate and for a type 0 agent candidate 0 is always the right choice. Hence, any election outcome is Pareto efficient. We wish to examine a stronger notion of efficiency. Consider the case in which the independent agents may be expected to decide the election, i.e., the case where $|p_0 - p_1| < p_i$. For this case we call an election outcome efficient, if the right choice from the point of view of the independent agents is made, i.e., if candidate 0 is chosen in state 0 and candidate 1 is chosen in state 1. The following result shows that the probability that an election is efficient in this case goes to one as the size of the electorate increases.

Proposition 2 *Suppose $|p_0 - p_1| < p_i$, $p_\phi > 0$ and $q > 0$. Then for every ε there exists an $\overline{N}$ such that for $N \geq \overline{N}$ the probability that in an equilibrium the election outcome is efficient is greater than $1 - \varepsilon$.*

The intuition behind Proposition 2 is that given the characteristics of the equilibrium strategies described in Proposition 1 the probability that a draw results in a mistaken vote in each state is strictly lower than the probability that a draw results in a correct vote. It follows from the law of large numbers that the probability the correct candidate receives the most votes goes to one as N gets large.

The efficiency result given here should be compared to the result that would occur if all agents voted on the basis of their private information (e.g., all UIAs vote for candidate 1 because $\alpha < 1/2$). We will call such voting sincere. In that event, for small q, candidate 1 would always win and α would be the probability of making an inefficient choice (if N is large). The election result is clearly inefficient if all agents vote sincerely.

In the following proposition we compare two electorates, electorate one has a lower expected fraction of partisans of either type than electorate two, and both electorates have

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the same bias, i.e., the same candidate has an expected advantage in partisan support. We show that the probability of choosing the wrong candidate is larger for electorate two than for electorate one. Thus, we show that a decrease in partisanship is "good for efficiency."

Proposition 3 *Let $p_\phi > 0$ and $q > 0$. Further let $p_i^1 > p_i^2$, $p_0^1 < p_0^2$, $p_1^1 < p_1^2$ and $(p_0^1 - p_1^1)(p_0^2 - p_1^2) > 0$ and assume that $p_i^j(1-q) > |p_0^j - p_1^j|$, $j = 1, 2$. Then if β^j is the probability that in an equilibrium the elected candidate and the state of nature do not coincide in electorate j 1,2 then $\beta^1 > \beta^2$ for sufficiently large N.*

Discussion

Our model generates results that are considerably different from the results obtained by standard rational models of turnout. First, the model makes predictions about the level of abstention as a function of the basic parameters. Second, our model makes predictions about the expected margin of victory. Third, we can relate the predictions about the level of abstention and the expected margin of victory to show that a parameter change that results in a larger margin of victory (and therefore a **lower** pivot probability) may actually result in **lower** abstention. We can also assess the concerns of political scientists that a decline of partisanship and an increase in abstention bode ill for American democracy.

Comparative Statics

Given that the expected fraction of UIAs is sufficiently large, $(1-q)p_i > |p_0 - p_1|$, and the uncertainty about the size of the electorate is fairly small, ($p_\phi \approx 0$), it follows from Proposition 1 that the expected fraction of agents who abstain is well approximated by the equation:

$$\sigma_\phi = p_i(1-q)\varsigma_\phi$$

In words, the probability a randomly selected agent will abstain is the probability the agent is an uninformed independent times the probability that an uniformed independent abstains. For sufficiently large N the probability an uninformed independent abstains is well approximated by the equation:

$$\varsigma_\phi = 1 - \frac{|p_0 - p_1|}{p_i(1-q)}.$$

Therefore the expected fraction of the population abstaining may be written in terms of the model parameters:

$$\sigma_\phi = p_i(1-q) - |p_0 - p_1|.$$

Thus, holding constant the difference in the expected fractions of type-0 and type-1 partisans, abstention is increasing in the expected percentage of independents (p_i). The increased abstention is due to two factors. First, as the percentage of independents increases it follows that there is an increase in the percentage of UIAs who abstain (ς_ϕ). Second, the percentage of uninformed independents also increases. Similarly, abstention decreases as the expected percentage of informed voters (q) increases.

In our model UIAs play a mixed strategy of abstention and voting for the candidate with the lower expected fraction of partisan support. Thus, an increase in the expected fraction of informed voters results in an increased probability that the uninformed independents will vote for the candidate with the lower partisan support.

Without changing either the probability of being informed or the probability that a voter is independent, a decline in the expected difference of partisan support ($|p_0 - p_1|$) results in an increase in abstention. If the expected difference in partisan support is large enough then our model predicts no abstention.

We can also make predictions about changes in the expected margin of victory (MV). If $p_i(1-q) > |p_0 - p_1|$ then in large elections MV is well approximated by the equation:

$$MV = p_i q.$$

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The expected margin of victory in large electorates is simply the percentage of informed independents. Thus, our model predicts that MV will increase with an increase either in the percentage of independents or in the probability of being informed.

In contrast to the predictions of standard models of participation (Riker and Ordeshook 1968) in our model there is no causal relationship between pivot probabilities and abstention. Changes in pivot probabilities due to dramatic changes in population size do not change the patterns of abstention and voting in our model. If we combine the comparative static results on abstention with those on the expected margin of victory we see that an increase in the expected fraction of informed voters, q, will result in both *higher margins of victory* and *lower levels of abstention*. Thus, abstention may actually increase as the probability of being pivotal increases. On the other hand, an increase in the percentage of independents will result in an increase in abstention as well as higher margins of victory.[10]

It is worth noting that our results suggest a link between several themes in the empirical literature on American politics. There is considerable evidence within the literature on congressional elections that there has been an increase over the last 30 years in the volatility of congressional elections i.e., there are larger swings in the percentage of the vote received by each party from one election to the next (see the literature on "Vanishing Marginals", e.g., Mayhew 1974; Jacobson 1987; Mann 1978; Ansolabehere, Brady and Fiorina 1987; Bauer and Hibbing 1989). There is also considerable evidence that there has been a large growth in the percentage of voters who identify themselves as independents (see for example Nie, Verba and Petrocik 1976, Crotty 1983, Niemi and Weisberg 1984, Wattenberg 1991, Burnham 1970). Conversely, there is evidence to suggest that voters are more informed (albeit slightly) about their congressmen and now

[10]There is a controversy surrounding the relationship between closeness and turnout (see for example Matsusaka 1993, and Cox and Munger 1989). Our model suggests that tests of the relationship between closeness and turnout may depend on overall levels of information as well as the percentages of independents and core supporters.

vote more on the issues (Nie, Verba and Petrocik 1976).[11] Not only are abstention rates

up, but so is the probability that an independent will abstain (Crotty 1983). All of these

facts are consistent with the predictions of our model: if the percentage of independents

were to significantly increase while the percentage of informed voters were to increase

slightly the result would be greater volatility in election results, and increased abstention.

Our model also gives results that are consistent with patterns of participation

observed by Wolfinger and Rosenstone (1980) who note that the single best predictor of

participation is education level. Clearly education level should be correlated with

information about how policy maps into outcomes. In our model it is always true that

informed voters are more likely to vote than uninformed voters.

Finally, the one parameter in our model that does not seem to play a critical role in

either the decision to participate or vote choice is the common knowledge prior belief (α)

concerning the state of the world--and therefore which candidate is ex ante believed to be

the best candidate by the UIAs. This can be seen by examining the strategy profiles

specified in Proposition 1 and noting that the parameter α does not appear. If the

population is sufficiently large, a small change in α does not cause large changes in the

voting strategies. However, for fixed population sizes and α sufficiently close to zero a

small change in α may have significant effects on the voting strategies. The intuition here

is that if the common knowledge prior is strong enough then the information gained by

being pivotal will be unable to overcome it.

Normative Implications

In our model elections are efficient means of aggregating information, i.e., the wrong

candidate from the perspective of a fully informed majority almost never wins (this follows

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[11]Some controversy surrounds the conclusion that voters have actually become more informed. However, given the increase in education levels and incomes -- parameters empirically associated with informed voters -- it would seem unlikely that voter information levels have decreased substantially.

from Proposition 2). This will come as no surprise to social choice theorists who are familiar with Condorcet's Jury Theorem (see Ladha 1992 for the most recent addition to this literature). However, our results suggest that majority rule elections may be even more efficient than the social choice literature suggests. If voters were to vote solely on their private information, they would frequently choose a candidate not preferred by a fully informed majority no matter how large the electorate. This never happens in large elections with strategic voting and abstention.

Bias in the distribution of information among preference types will not affect our results. We may assume that there is a different probability of being informed for each voter type, e.g., right wing voters are more likely to be informed than left wing voters who are more informed than independents. In this case, all of our results go through as long as the probability that UIAs are informed is strictly positive. In particular the election outcome will almost always result in the candidate preferred by the fully informed electorate.

The literature on the decline of partisanship and vanishing marginals argues that the increase in abstention, decreased competitiveness as measured by greater margins of victory and declining partisanship are unfortunate developments. Our model suggests that this is not necessarily the case. Proposition 3 states that a decline in partisanship results in greater efficiency and increased abstention. On the other hand, a decline in the probability of being informed increases abstention and results in a decreased probability that the candidate most preferred by the fully informed majority will win.

Conclusion

Our model illustrates the effects of informational asymmetries on both the decision to vote and vote choice. For example, we have demonstrated that increased informational

asymmetries may create an incentive for less informed voters to abstain even when the voter is **more** likely to be pivotal.

Many readers may remain unconvinced that there is an informational effect of the type predicted by this model and believe that costs to vote and the probability of being pivotal are much more important determinants of the choice to participate. Palfrey and Rosenthal (1985) have demonstrated that the introduction of reasonable uncertainty into participation games leads to the prediction that only those with very low cost to participate should be expected to turnout. The results of Palfrey and Rosenthal demonstrate that if significant changes take place in participation rates they must be explained by changes in participation by those with low participation costs. Standard participation models will not provide such comparative statics.

The phenomena of drop off also cannot be easily explained by the interplay of costs to vote and pivot probabilities. Drop off occurs when voters go to the polls and vote in presidential and congressional races but abstain in lower level races. In lower level races voters are more likely to be pivotal than in presidential races and voting is surely not costly since they are already in the voting booth. The standard models of participation will not be able to explain drop off whereas our model provides an explanation.

Appendix

We first state a technical fact.

Lemma 0 *Let $(a_N, b_N, c_N)_{N=1}^{\infty}$ be a sequence that satisfies $(a_N, b_N, c_N) \in [0,1]^3$, $a_N < b_N - \delta$ and $\delta < c_N$, for all N and for some $\delta > 0$. Then for $i = 0, 1$*

$$\frac{\displaystyle\sum_{j=0}^{N-i} \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N^{N-2j-i} a_N^{j}}{\displaystyle\sum_{j=0}^{N-i} \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N^{N-2j-i} b_N^{j}} \to 0 \text{ as } N \to \infty$$

Proof: The proof is in two steps. First, since $0 < a_N < b_N - \delta < 1$ choose k so that

$(a_N / b_N)^k < \varepsilon$ for any N. Choose L such that $1 / L < \varepsilon$. Given k, L and $c_N > \delta$ for any N,

we can choose $N > 2L(k+1) + 2i$ large enough so that

$$F(N, j) \equiv \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N^{N-2j-i} b_N^{j} \text{ is increasing in j for } j<(k+1)L. \text{ To see that}$$

we can choose such an N note that $F(N, j) < F(N, j+1)$ if

$$\frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N^{N-2j-i} b_N^{j} < \frac{N!}{(j+i+1)!(j+1)!(N-2j-i-2)!} c_N^{N-2j-i-2} b_N^{j+1}$$

but now by canceling terms we get

$$\frac{c_N^{2}}{b_N}(j+i+1)(j+1) < (N-2j-i)(N-2j-i)$$

Since $b_N > \delta$, $c_N \le 1$ and $j<(k+1)L$ it follows that

$$\frac{c_N^{2}}{b_N}(j+i+1)(j+1) < \frac{1}{\delta}((k+1)L+i+1)((k+1)L+1)$$

and

$$(N-2(k+1)L-i)(N-2(k+1)L-i) < (N-2j-i)(N-2j-i).$$

Now we can choose N so that

$$\frac{1}{\delta}((k+1)L+i+1)((k+1)L+1) < (N-2(k+1)L-i)(N-2(k+1)L-i)$$

so $F(N, j)$ is increasing for $j<(k+1)L$.

Step 2. Now we split the equation in the Lemma into two parts both of which are shown to be less than ε.

$$\frac{\displaystyle\sum_{j=0}^{\frac{N-i}{2}} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-1} a_N^{j}}{\displaystyle\sum_{j=0}^{\frac{N-i}{2}} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-1} b_N^{j}} =$$

$$\frac{\displaystyle\sum_{j=0}^{k} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-1} a_N^{j} + \sum_{j=k+1}^{\frac{N-i}{2}} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-1} a_N^{j}}{\displaystyle\sum_{j=0}^{\frac{N-i}{2}} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-1} b_N^{j}}.$$

We now show that the first term is less than ε.

Note that

$$\frac{\displaystyle\sum_{j=0}^{k} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-1} a_N^{j}}{\displaystyle\sum_{j=0}^{\frac{N-i}{2}} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-1} b_N^{j}} <$$

$$\frac{\displaystyle\sum_{j=0}^{k} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-1} b_N^{j}}{\displaystyle\sum_{j=0}^{\frac{N-i}{2}} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-1} b_N^{j}} =$$

$$\frac{\displaystyle\sum_{j=0}^{k} F(N,j)}{\displaystyle\sum_{j=0}^{N-i} F(N,j)} < \frac{\displaystyle\sum_{j=0}^{k} F(N,j)}{L\displaystyle\sum_{j=0}^{k} F(N,j)} = 1/L < \varepsilon.$$

(This is the case since $F(N,j)$ is increasing for j $L(k+1)$ and $N-2-i$ $L(k+1)$.)

Finally, we show that the second term is less than ε.

Note that

$$\frac{\displaystyle\sum_{j=k+1}^{N-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-i} a_N^{\,j}}{\displaystyle\sum_{j=0}^{N-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-i} b_N^{\,j}} =$$

$$(a_N/b_N)^k \; \frac{\displaystyle\sum_{j=k+1}^{N-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-i} b_N^{\,k} a_N^{\,j-k}}{\displaystyle\sum_{j=0}^{N-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-i} b_N^{\,j}} .$$

Now from $b_N > a_N$ it follows that $b_N^{\,k} a_N^{\,j-k} < b_N^{\,j}$ for j>k and therefore

$$(a_N/b_N)^k \; \frac{\displaystyle\sum_{j=k+1}^{N-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-i} b_N^{\,k} a_N^{\,j-k}}{\displaystyle\sum_{j=0}^{N-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-i} b_N^{\,j}} <$$

$$(a_N/b_N)^k \; \frac{\displaystyle\sum_{j=k+1}^{N-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-i} b_N^{\,j}}{\displaystyle\sum_{j=0}^{N-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{N-2j-i} b_N^{\,j}} <$$

$$(a_N/b_N)^k < \varepsilon .$$

□

Lemma 1 *Suppose $p_iq > 0$ and $0 < \alpha < 1$. Consider a sequence of strategy profiles $\{\varsigma^N\}_{N=0}^{\infty}$. Then:*

A. *if there exists an $\varepsilon > 0$ such that $\sigma_{xy}(\varsigma^N) - \sigma_{yx}(\varsigma^N) > \varepsilon$ for any $N \geq 0$ and $x \neq y$ then there exists an $\overline{N}$ such that for any $N \geq \overline{N}$*
$$Eu(x, N, \varsigma^N) > Eu(\phi, N, \varsigma^N) > Eu(y, N, \varsigma^N);$$

B. *if for all $N \geq 0$ there are two actions s, s' with $s \neq s'$ such that $Eu(s, N, \varsigma^N) = Eu(s', N, \varsigma^N)$. Then for any $\varepsilon \geq 0$ there is an $\overline{N}$ such that for $N \geq \overline{N}$*
$$|\sigma_{01}(\varsigma^N) - \sigma_{10}(\varsigma^N)| < \varepsilon.$$

Proof: Condition B follows as a corollary of A so we only need to show A. Suppose that there exists an $\varepsilon > 0$ such that $\sigma_{01}(\varsigma^N) - \sigma_{10}(\varsigma^N) > \varepsilon$ for any $N \geq 0$. Since $p_iq > 0$ and $\sigma_{xx}(\varsigma^N) = p_iq + \sigma_{yx}(\varsigma^N)$ for $x \neq y$ we can state the following facts:

There exists an $\eta > 0$ such that for all $N \geq 0$, $\sigma_{01}(\varsigma^N) > \eta$,

$\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N) - \sigma_{11}(\varsigma^N)\sigma_{10}(\varsigma^N) > \eta$ and $\sigma_\phi(\varsigma^N) > \eta$. (This follows since $p_\phi > 0$ and $\sigma_\phi(\varsigma^N) = p_\phi + p_i(1-q)\varsigma_\phi^N$). Furthermore $1 > \sigma_{00}(\varsigma^N) > \sigma_{10}(\varsigma^N)$ for all $N \geq 0$.

From Equation (1) it follows that $Eu(\phi, N, \varsigma^N) > Eu(1, N, \varsigma^N)$ if and only if:

$$(1-\alpha)\pi_t(1, N, \varsigma^N) - \alpha\pi_t(0, N, \varsigma^N) + (1-\alpha)\pi_1(1, N, \varsigma^N) - \alpha\pi_1(0, N, \varsigma^N) < 0.$$

From Equation (2) it follows that $Eu(0, N, \varsigma^N) > Eu(\phi, N, \varsigma^N)$ if and only if:

$$(1-\alpha)\pi_t(1, N, \varsigma^N) - \alpha\pi_t(0, N, \varsigma^N) + (1-\alpha)\pi_0(1, N, \varsigma^N) - \alpha\pi_0(0, N, \varsigma^N) < 0$$

Therefore $Eu(\phi, N, \varsigma^N) > Eu(1, N, \varsigma^N)$ and $Eu(\phi, N, \varsigma^N) > Eu(1, N, \varsigma^N)$ if the following three conditions hold:

(i) $(1-\alpha)\pi_t(1, N, \varsigma^N) - \alpha\pi_t(0, N, \varsigma^N) < 0$,

(ii) $(1-\alpha)\pi_1(1, N, \varsigma^N) - \alpha\pi_1(0, N, \varsigma^N) < 0$

(iii) $(1-\alpha)\pi_0(1, N, \varsigma^N) - \alpha\pi_0(0, N, \varsigma^N) < 0$.

Note that $0 < \alpha < 1$. Lemma 0 and the fact that $\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N) - \sigma_{11}(\varsigma^N)\sigma_{10}(\varsigma^N) > \eta$, $\sigma_\phi(\varsigma^N) > \eta$ imply that

$$\frac{\pi_t(0,N,\varsigma^N)}{\pi_t(1,N,\varsigma^N)} = \frac{\alpha\sum_{j=0}^{\frac{N}{2}}\frac{N!}{j!j!(N-2j)!}\sigma_\phi(\varsigma^N)^{N-2j}(\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N))^j}{(1-\alpha)\sum_{j=0}^{\frac{N}{2}}\frac{N!}{j!j!(N-2j)!}\sigma_\phi(\varsigma^N)^{N-2j}(\sigma_{10}(\varsigma^N)\sigma_{11}(\varsigma^N))^j} \to \infty \text{ as } N \to \infty.$$

Therefore condition *(i)* is satisfied for sufficiently large N.

Similarly, Lemma 0 and the fact that $\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N) - \sigma_{11}(\varsigma^N)\sigma_{10}(\varsigma^N) > \eta$, $\sigma_\phi(\varsigma^N) > \eta$, $\sigma_{00}(\varsigma^N) > \eta$, $\sigma_{01}(\varsigma^N) > \eta$ imply that

$$\frac{\pi_1(0,N,\varsigma^N)}{\pi_1(1,N,\varsigma^N)} = \frac{\alpha\sigma_{00}(\varsigma^N)\sum_{j=0}^{\frac{N}{2}}\frac{N!}{(j+1)!j!(N-2j-1)!}\sigma_\phi(\varsigma^N)^{N-2j-1}(\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N))^j}{(1-\alpha)\sigma_{10}(\varsigma^N)\sum_{j=0}^{\frac{N}{2}}\frac{N!}{(j+1)!j!(N-2j-1)!}\sigma_\phi(\varsigma^N)^{N-2j-1}(\sigma_{10}(\varsigma^N)\sigma_{11}(\varsigma^N))^j} \to \infty$$

as $N \to \infty$ and

$$\frac{\pi_1(0,N,\varsigma^N)}{\pi_1(1,N,\varsigma^N)} = \frac{\alpha\sigma_{01}(\varsigma^N)\sum_{j=0}^{\frac{N}{2}}\frac{N!}{(j+1)!j!(N-2j-1)!}\sigma_\phi(\varsigma^N)^{N-2j-1}(\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N))^j}{(1-\alpha)\sigma_{10}(\varsigma^N)\sum_{j=0}^{\frac{N}{2}}\frac{N!}{(j+1)!j!(N-2j-1)!}\sigma_\phi(\varsigma^N)^{N-2j-1}(\sigma_{10}(\varsigma^N)\sigma_{11}(\varsigma^N))^j} \to \infty$$

as $N \to \infty$.

Hence also conditions *(ii)* and *(iii)* are satisfied.

Using an analogous argument we can show that if there exists an $\varepsilon > 0$ such that $\sigma_{10}(\varsigma^N) - \sigma_{01}(\varsigma^N) > \varepsilon$ for any $N \geq 0$ then $Eu(1,N,\varsigma^N) > Eu(\phi,N,\varsigma^N) > Eu(0,N,\varsigma^N)$.

$\square$

Lemma 2 *Let* $p_\phi > 0$, $q > 0$, $N \geq 2$ *and* N *even. For any symmetric strategy profile* ς^N, $Eu(1,N,\varsigma^N) = Eu(0,N,\varsigma^N)$ *implies* $Eu(1,N,\varsigma^N) < Eu(\phi,N,\varsigma^N)$.

Proof: Given $Eu(1,N,\varsigma^N) = Eu(0,N,\varsigma^N)$ it follows from (3) that

$$(1-\alpha)\pi_t(1,N,\varsigma^N) + \alpha\pi_t(0,N,\varsigma^N) =$$

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$$\frac{1}{2}\Big[\alpha[\pi_1(0,N,\varsigma^N)+\pi_0(0,N,\varsigma^N)]-(1-\alpha)[\pi_1(1,N,\varsigma^N)+\pi_0(1,N,\varsigma^N)]\Big].$$

It follows from (1) that

$$(1-\alpha)\pi_i(1,N,\varsigma^N)-\alpha\pi_i(0,N,\varsigma^N)=$$

$$2[Eu(1,N,\varsigma^N)-Eu(\phi,N,\varsigma^N)]-\Big[(1-\alpha)\pi_1(1,N,\varsigma^N)]-\alpha\pi_1(0,N,\varsigma^N)\Big].$$

Combining these two expressions we get:

$$4[Eu(1,N,\varsigma^N)-Eu(\phi,N,\varsigma^N)]=$$

$$(1-\alpha)[\pi_1(1,N,\varsigma^N)-\pi_0(1,N,\varsigma^N)]+\alpha[\pi_0(0,N,\varsigma^N)-\pi_1(0,N,\varsigma^N)].$$

Thus it is sufficient to show that

(i) $\pi_1(1,N,\varsigma^N)-\pi_0(1,N,\varsigma^N)<0$ and

(ii) $\pi_0(0,N,\varsigma^N)-\pi_1(0,N,\varsigma^N)<0$

To see (i) note that

$$\pi_1(1,N,\varsigma^N)-\pi_0(1,N,\varsigma^N)=$$

$$(\sigma_{10}(\varsigma^N)-\sigma_{11}(\varsigma^N))\sum_{j=0}^{N-1}\frac{N!}{(j+1)!\,j!(N-2j-1)!}\sigma_\phi(\varsigma^N)^{N-2j-1}(\sigma_{10}(\varsigma^N)\sigma_{11}(\varsigma^N))^j.$$

Since $\sigma_\phi(\varsigma^N)=p_\phi+(1-q)p_i\varsigma_\phi^N$ and $\sigma_{11}(\varsigma^N)=p_iq+\sigma_{10}(\varsigma^N)$ (i) follows from $p_\phi>0$,

$q>0$ and $N\geq 2$. A similar argument is used for (ii). $\square$

Proposition 1 *Suppose $q>0$ and $p_\phi>0$, then for all $\varepsilon>0$ there exists an $\overline{N}$ such that*

for $N \quad \overline{N}$ and for any equilibrium strategy profile ς^N the following statements are true:

(i) *If $p_i(1-q)<p_0-p_1$ then $\varsigma_1^N=1$.*

(ii) *If $p_i(1-q)<p_1-p_0$ then $\varsigma_0^N=1$.*

(iii) *If $p_i(1-q)\geq p_0-p_1>0$ then $\varsigma_0^N=0$ and $\left|\varsigma_\phi^N-1+\dfrac{p_0-p_1}{p_i(1-q)}\right|<\varepsilon$.*

(iv) *If $p_i(1-q)\geq p_1-p_0>0$ then $\varsigma_1^N=0$ and $\left|\varsigma_\phi^N-1+\dfrac{p_1-p_0}{p_i(1-q)}\right|<\varepsilon$.*

(v) *If $p_0-p_1=0$ then $\varsigma_0^N=0$ and $\varsigma_\phi^N>1-\varepsilon$.*

Proof: *Cases (i) and (ii):* Note that $\sigma_{00}(\varsigma^N) = p_i q + \sigma_{10}(\varsigma^N)$, $\sigma_{11}(\varsigma^N) = p_i q + \sigma_{01}(\varsigma^N)$, $\sigma_{zx}(\varsigma^N) = p_i(1-q)\varsigma_x^N + p_x$ for $z \neq x$. In case (i) it follows from $p_i(1-q) < p_0 - p_1$ and $\varsigma_1^N \leq 1$ that $\sigma_{10}(\varsigma^N) + \delta > \sigma_{01}(\varsigma^N)$ for any ς^N and some $\delta > 0$. Therefore $\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N) < \sigma_{11}(\varsigma^N)\sigma_{10}(\varsigma^N) - \delta'$ for some $\delta' > 0$ where δ' is independent of N. The result follows directly from Lemma 1.A. The argument for case (ii) is analogous.

Cases (iii) and (iv) if equality holds: Consider case (iii). We can rule out equilibria in which $1 - \delta > \varsigma_1^N$ since it follows from the assumption that $p_i(1-q) = p_0 - p_1 > 0$ that $\sigma_{10}(\varsigma^N) - \eta > \sigma_{01}(\varsigma^N)$ for some $\eta > 0$. From Lemma 1.A it follows that all UIAs strictly prefer to vote for candidate 1 for N sufficiently large. It follows that $\varsigma_\phi^N < \varepsilon$. If all UIAs vote for candidate 1 then the statement of the proposition is satisfied. From Lemma 2 and $\varsigma_1^N > 0$ it follows that $\varsigma_0^N = 0$ for sufficiently large N. Case (iv) if equality holds is analogous.

Cases (iii) and (iv) with strict inequality: First, we show that for large N there are no pure strategy equilibria. We describe the argument only for case (iii). An analogous argument with all inequalities reversed holds for case (iv). Suppose $\varsigma_1^N = 0$. By the same argument as in case (i) $\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N) < \sigma_{11}(\varsigma^N)\sigma_{10}(\varsigma^N) - \delta'$ for some $\delta' > 0$. From Lemma 1.A all UIAs strictly prefer to vote for candidate 1 if N is large. It follows that $\varsigma_1^N > 0$ and, by Lemma 2, $\varsigma_0^N = 0$. Suppose $\varsigma_1^N = 1$. A simple calculation shows that this implies that $\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N) - \delta > \sigma_{11}(\varsigma^N)\sigma_{10}(\varsigma^N)$ for some $\delta' > 0$. By Lemma 1.A all UIAs strictly prefer to vote for candidate 0. Since there is always a mixed strategy equilibrium, in any equilibrium agents mix between abstention and voting for candidate 1. Now the result follows from Lemma 1.B.

Case (v): Suppose $\varsigma_\phi^N = 0$. Then it follows from Lemma 2 that, for large $N, \varsigma_1^N = 1$ or $\varsigma_0^N = 1$. But $\varsigma_0^N = 1$ implies that $\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N) < \sigma_{11}(\varsigma^N)\sigma_{10}(\varsigma^N) - \delta'$ for some $\delta' > 0$,

and by Lemma 1.A every UIA prefers to vote for candidate 1. Similarly, $\varsigma_1^N = 1$ implies that $\sigma_{00}(\varsigma^N)\sigma_{01}(\varsigma^N) - \delta > \sigma_{11}(\varsigma^N)\sigma_{10}(\varsigma^N)$ for some $\delta' > 0$, and by Lemma 1.A every UIA prefers to vote for candidate 0. Thus for large N, and for any voting equilibrium it must be true that $\varsigma_\phi^N > 0$. The result now follows from Lemma 1.B. $\square$

Proposition 2 *Suppose* $|p_0 - p_1| < p_i$, $p_\phi > 0$ *and* $q > 0$. *Then for every* ε *there exists an* $\overline{N}$ *such that for* $N \cdot \overline{N}$ *the probability that in an equilibrium the election outcome is efficient is greater than* $1 - \varepsilon$.

Proof: The proof of Proposition 2 is a straightforward consequence of Proposition 1. Note that Proposition 1 implies that if the state is 1 then the probability of any agent choosing candidate 1 is larger than the probability that any agent chooses candidate 0 by at least $\min\{qp_i, p_i - |p_0 - p_1|\} > 0$. Conversely, if the state is 0 then the probability that any agent chooses candidate 0 is larger than the probability that any agent chooses candidate 1 by at least $\min\{qp_i, p_i - |p_0 - p_1|\} > 0$. By the law of large numbers it then follows that the probability that candidate 1 wins in state 1 and candidate 0 wins in state 0 goes to one as $N \to \infty$. $\square$

Proposition 3 *Let* $p_\phi > 0$ *and* $q > 0$. *Further let* $p_i^1 > p_i^2$, $p_0^1 < p_0^2$, $p_1^1 < p_1^2$ *and* $(p_0^1 - p_1^1)(p_0^2 - p_1^2) > 0$ *and assume that* $p_i^j(1 - q) > |p_0^j - p_1^j|$, $j = 1, 2$. *Then if* β^i *is the probability that in an equilibrium the elected candidate and the state of nature do not coincide in electorate* j $1, 2$ *then* $\beta^1 > \beta^2$ *for sufficiently large* N.

Proof: Suppose that $p_1^1 - p_0^1 > 0$, $p_1^2 - p_0^2 > 0$ (an analogous argument can be applied to the case when these two inequalities are reversed). For large N we know from Proposition 1 that $\varsigma_1^{jN} = 0$ for any $j = 1, 2$. Further, note that we can express the ex ante probability that any agent chooses the "wrong" candidate as:

$$\alpha \cdot \sigma^j_{01}(\varsigma^{jN}) + (1-\alpha) \cdot \sigma^j_{10}(\varsigma^{jN}) = \sigma^j_{01}(\varsigma^{jN}) - (1-\alpha) \cdot (\sigma^j_{10}(\varsigma^{jN}) - \sigma^j_{01}(\varsigma^{jN}))$$

Note that the right hand side of this equation is approximately equal to

$\sigma^j_{01}(\varsigma^{jN}) = p^j_1 + p^j_i(1-q)\varsigma^{jN}_1 = p^j_1$ since, by Proposition 1, we know that

$|\sigma^j_{10}(\varsigma^{jN}) - \sigma^j_{01}(\varsigma^{jN})| < \varepsilon$. Thus, we know that the probability that any agent chooses the

wrong candidate is larger in the electorate 2 than in electorate 1. Since the behavior of

agents is independent of each other the Proposition follows. $\square$

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