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Is There Always a "Right" Extensive Form?

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It is well recognized that many solution concepts are not robust to changes that, a priori, appear to be strategically innocuous. The most striking such instances involve sequential equilibrium and the coalescing of moves (such as representing an agent's choice from three actions as a single choice rather than two sequential choices). Consider the classic example in Kohlberg and Mertens (1986, page 1008-9) reproduced here in Figures 1 and 2. The only difference between the two extensive forms is that in Γ^1, player I makes his choice from {T,M,B} all at once; while in Γ^2, player I's first choice is from {T,{M,B}}, with a choice of {M,B} being followed by a further choice between M and B. The outcome "2,2" is a sequential equilibrium outcome of the extensive form Γ^1 but not of Γ^2. The unique sequential equilibrium of Γ^2 is (M,L).

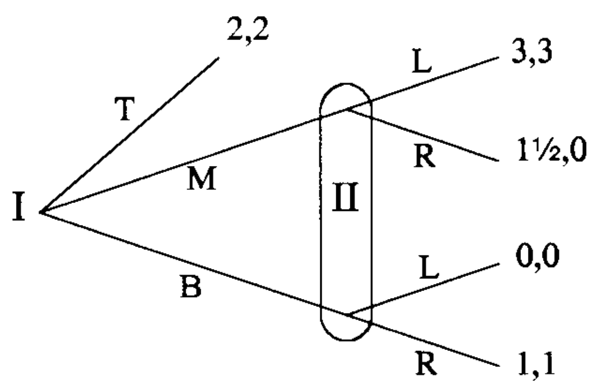

Figure 1—Γ^1.

The outcome (M,L) is also a sequential equilibrium outcome in Γ^1. In fact, (M,L) is a sequential equilibrium in *every* equivalent extensive from.[4] Further, it can be shown that regardless of what payoffs are assigned to the terminal nodes of Γ^2 (as long as these payoffs do not create ties across terminal nodes), any sequential equilibrium of Γ^2 is also a sequential equilibrium in every equivalent extensive

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[4]To verify this, we need note only that (M,L) is a normal form sequential equilibrium. See Mailath, Samuelson and Swinkels (1993).

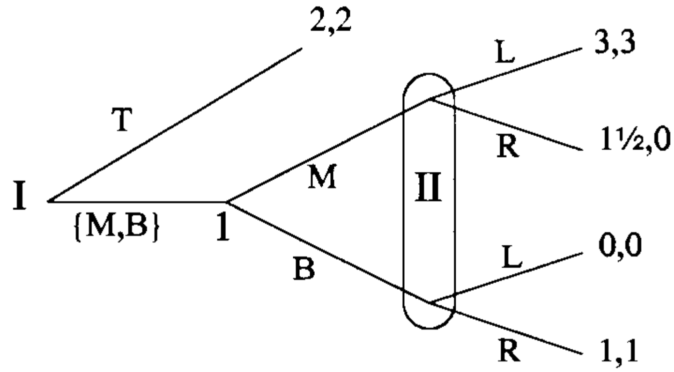

Figure 2—Γ^2.

form.[5] So, examining Γ^2 captures all restrictions that sequential equilibrium could impose in any strategically equivalent game.[6] One might then conjecture that Γ^1 is the "wrong" extensive form, while Γ^2 is the "right" one.

In general, one might hope to resolve the lack of robustness of sequential equilibrium by restricting attention to a "right" extensive form, where the right extensive form would depend only on the structure of the game and not on payoffs.[7] Can we always find an extensive form that captures only such sequential equilibrium outcomes, and further, does so for any generic assignment of payoffs to terminal nodes?

In the following example, we show that the "right" extensive form can depend on the magnitudes of payoffs. Thus, one cannot determine a right extensive form for using sequential equilibrium based on the structure of the tree alone. Note that this example does not depend on any "non-genericities" such as ties in payoffs across terminal nodes.

Consider extensive forms Γ^3, Γ^4, and Γ^5 in Figures 3, 4, and 5, where we will be assigning payoffs to y and z. The normal form of each of these games (perhaps after deletion of redundant pure strategies) is:

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[5]This follows from the observation that for any generic payoff assignment, a sequential equilibrium in Γ^2 must be a normal form sequential equilibrium.

[6]The idea that different extensive form games may be strategically equivalent is put forward by Kohlberg and Mertens, who argued forcefully that certain transformations of an extensive form, such as coalescing of moves are "strategically irrelevant." These transformations have been studied by Thompson (1952), Dalkey (1953), and Elmes and Reny (1993). Successive application of these transformations allow one to move between any two extensive forms whose normal forms differ only by the addition or removal of duplicate pure strategies.

[7]Every game has at least one outcome that is sequential in every equivalent extensive form game: in particular, a proper equilibrium of a normal form induces a sequential equilibrium in every extensive form with that normal form (see van Damme (1984) and Kohlberg and Mertens (1986); Mailath, Samuelson, and Swinkels (1993) identify a superset of proper equilibria, called normal form sequential equilibria, that also are sequential in every equivalent tree).

L (III):

I \ II	ℓ	r
T	3,2,1	5,y,1
M	1,1,z	0,0,1
B	4,1,2	4,1,2

R (III):

I \ II	ℓ	r
T	3,2,1	5,y,1
M	1,1,2	0,0,1
B	2,1,1	2,1,1

III

Suppose first that y = z = 1. It is easy to check that the unique sequential equilibrium of Γ^4 is given by (B,ℓ,L). This is also a sequential equilibrium of Γ^3 and Γ^5. However, the extensive forms Γ^3 and Γ^5 have additional sequential equilibria, in particular (T,ℓ,R). This might suggest that the "right" extensive form is Γ^4.

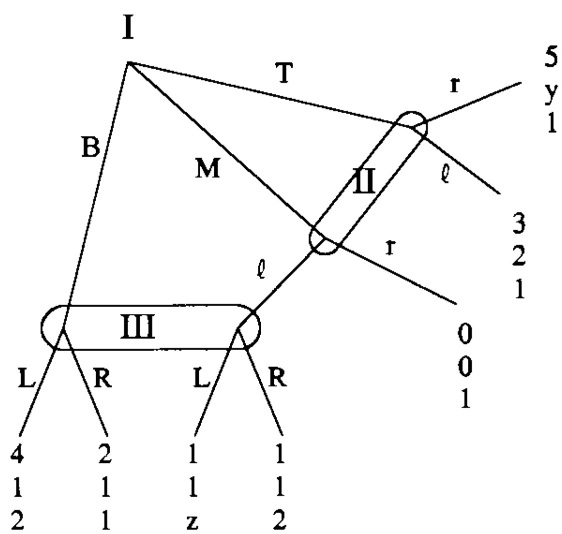

Figure 3—Γ^3.

But now suppose that y = z = 3. The extensive form Γ^5 has a unique sequential equilibrium given by (T,r,L). This is also a sequential equilibrium of Γ^3 and Γ^4. However, Γ^3 and Γ^4 have additional sequential equilibria, in particular (B,ℓ,L). This suggests that the "right" extensive form is Γ^5, not Γ^4.

This example suggests that the desirability of an extensive form, and the restrictions that sequentiality imposes in that extensive form, cannot be inferred solely from the structure of the tree. An alternative is to work with all equivalent extensive forms simultaneously, which can be done by abandoning the extensive form and conducting the analysis in the normal form. Doing so leads

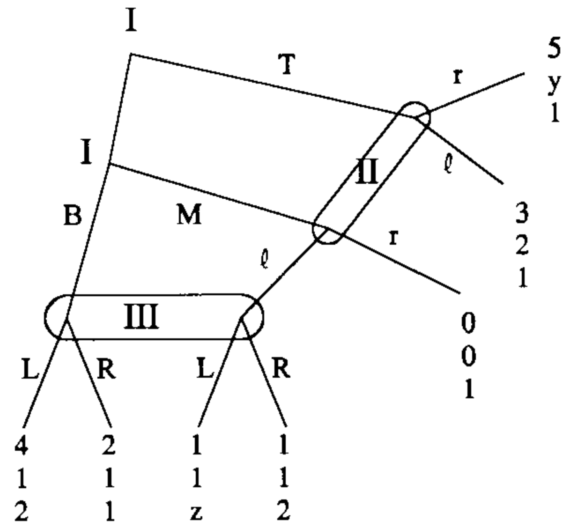

Figure 4—Γ^4.

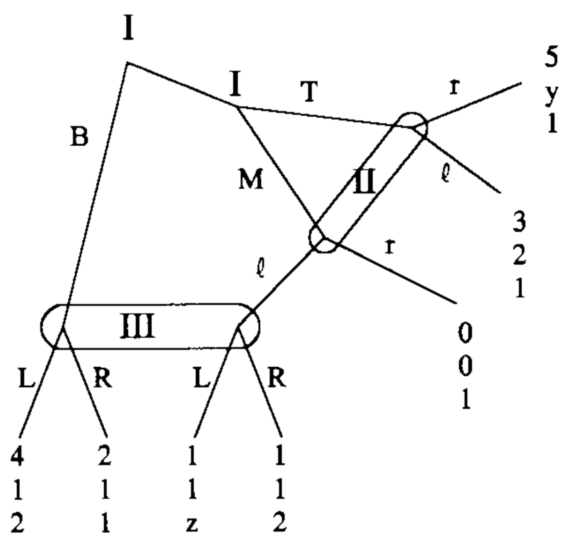

Figure 5—Γ^5.

naturally to the concept of normal form sequential equilibrium (Mailath, Samuelson, and Swinkels (1993)).

References

Dalkey, N. (1953), "Equivalence of Information Patterns and Essentially Determinate Games," in *Contributions to the Theory of Games, Volume II. Annals of Mathematical Studies* 28. ed. by H. W. Kuhn and A. W. Tucker, Princeton NJ: Princeton University Press.

Elmes S. and P. Reny (1993), "On the Strategic Equivalence of Extensive Form Games," *Journal of Economic Theory*, forthcoming.

Kohlberg, Elon and Jean-Francois Mertens (1986), "On the Strategic Stability of Equilibria," *Econometrica*, 54, 1003-1037.

Mailath, George J., Larry Samuelson, and Jeroen M. Swinkels (1993), "Extensive Form Reasoning in Normal Form Games," *Econometrica*, 61, 273-302.

Thompson, F. B. (1952), "Equivalence of Games in Extensive Form," Research Memorandum 759, The Rand Corporation.

van Damme, Eric (1984), "A Relation between Perfect Equilibria in Extensive Form Games and Proper Equilibria in Normal Form Games," *International Journal of Game Theory*, 13, 1-13.