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REGULATION OF DUOPOLY UNDER ASYMMETRIC INFORMATION: PRICES VS QUANTITIES

by

Asher Wolinsky*

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*Department of Economics, Northwestern University, Evanston, IL 60208

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ABSTRACT

This paper discusses the regulation of oligopolistic differentiated product industries under conditions of incomplete information. The regulator can control the prices, and impose quantity restrictions, but cannot control effectively the quality choices of the firms. We inquire about the optimal choice of instruments by the regulator--whether and under what conditions the regulation of prices or quantities achieves better results.

In the spatial duopoly model analyzed here uninterrupted competition will generally result in an inefficient allocation. When the regulator knows the technologies, optimal price regulation results in distortions of the quality choice, but optimal regulation of quantities achieves the first best outcome. When the regulator is uncertain about the technologies neither of these methods will yield the first best outcome. We characterize the optimal regulation problems for these two methods, and solve explicitly two specific examples. The method of price regulation tends to be more effective at extracting rents from the firms, while regulation of quantities (assignment of monopoly areas) tends to produce better quality choices. The overall comparison depends on some finer details of the environment. If in the quality competition stage the firms still do not know each other's costs, quantity regulation (assignment of local monopoly rights) performs better. If in that stage they learn each other's costs, either of the two methods might perform better. Quantity regulation will still be sometimes superior, but in contrast to the complete information environment, price regulation will also be sometimes superior. With other things equal, the latter will tend to happen when the regulator assigns relatively higher priority to rent extraction.

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1. Introduction

This paper is concerned with the theory of regulating oligopolistic industries under conditions of incomplete information. The interest in this topic is obvious since some important regulated industries feature various degrees of competition. The basic scenario that we have in mind is a differentiated product industry subject to regulation. The regulator can control the prices, and impose quantity restrictions (and collect or make payments to the firms), but cannot control effectively certain quality dimensions which are left at the discretion of the firms. The incapability of the regulator to control quality might be attributed to information or enforcement problems. Thus, in designing the regulatory measures, the regulator has to take into account not only their direct effect on prices and quantities but also their effect on the quality dimension that will be determined in the competition between the firms. The main question around which the discussion is organized concerns the optimal choice of instruments by the regulator--whether and under what conditions the regulation of prices or quantities achieves better results.

The specific model analyzed here is a spatial duopoly model where the two competing firms are located at the endpoints of a unit interval. The firms can compete in prices as well as along a vertical quality dimension. In this environment the total surplus generated by an allocation depends on two factors: the manner in which the market is divided between the firms and the quality levels. It turns out that an uninterrupted competition between the firms will generally result in an inefficient allocation which creates some scope for regulation. As mentioned above, while the quality choices are recognized by customers and may affect their choices, they cannot be controlled by the regulator. The two basic options open to the regulator are to enforce prices and let the firms compete by choosing qualities or to impose quantity restrictions (and supplement them with price regulation if needed). If the differentiation space is geographical (i.e., the firms supply the same product or service at different locations), quantity restrictions have the meaning of assignment of exclusive market areas; if the differentiation space is a more abstract product space, these restrictions have their ordinary meaning. It turns out that, when the regulator knows the cost functions of the firms, optimal regulation of the former type leads to distortions in the quality choice, but optimal regulation of the latter type achieves the first best outcome. We then introduce another imperfection into the regulatory process: the regulator is uncertain about the firms' cost functions. Provided that the regulator values rent extraction from the firms (i.e., the "cost of public funds" is not negligible), none of the methods of regulation will yield the first best outcome in this regime. To extract rents from the firms the regulator will distort the allocations away from their first best levels, even when it could implement the total surplus maximizing allocation. We characterize the optimal regulation problems for these two methods, and solve explicitly two specific examples which shed light on the tradeoffs involved. The method of price regulation, which allows quality competition over market share, tends to be more effective at extracting rents from the firms, while regulation of quantities (assignment of exclusive market areas) tends to produce more efficient quality choices. The overall comparison depends on some finer details of the environment. We distinguish two scenarios. In one scenario, after the regulator decides on policy, but before the firms choose qualities, they learn about each other's costs. In the other scenario the firms continue to be uncertain about each other's costs throughout. The former scenario intends to capture situations in which the regulator's policy is determined for a relatively longer period within which firms will learn relatively quickly about each other. It turns out that in the second scenario quantity regulation (assignment of exclusive market areas) performs better. In the first scenario either of the two methods might perform better: quantity regulation will still be sometimes superior, but in contrast to the complete information environment, price regulation will also be sometimes superior. With other things equal the latter will tend to happen when the regulator assigns relatively higher priority to rent extraction.

This paper is not aimed at discussing a specific industry, and consequently the model and analysis are not tailored to fit the actual details of any particular industry. However, the issues discussed here are of relevance for a number of regulated industries and the following concrete examples might be useful in motivating the discussion. Hospital services are differentiated by geographical location and also have important quality dimensions which are difficult to regulate. Regulation can impose direct quantity constraints (an option which is exercised in practice through restrictions on the number of beds), or exclusive market areas (which might be the case under future health care programs) and can also control prices. The choice of regulatory instrument will of course affect the unmonitored quality dimensions and raises the questions addressed here. The surface transportation industries offer another example. Competing modes such as railroad and trucking offer differentiated services with important quality dimensions which are hard or impossible to regulate effectively. Both price regulation and quantity regulation, in the form of restrictions on routes and volumes, took place in this industry. The question of whether to assign exclusive market areas or to regulate only prices while allowing competition are also relevant for the public utilities and the telecommunication industries. Note, however, that the public utility case does not fit exactly the present model, since in that case the geographical distance does not affect directly consumers' utility but rather firms' costs.

Despite the importance of regulated industries which feature some competition, the literature on regulation has focused mainly on monopolistic industries. This is particularly true for the relatively newer literature on regulation under incomplete information[1]. Two recent articles on regulation of oligopoly under complete information and in the presence of an unregulated quality dimension like that of the present model are Kamien and Vincent (1991) and Ma and burgess (1993). Some recent articles have also started to address issues in the regulation of oligopolies in uncertain environments. Anton and Yao (1989) consider split award auctions in procurement which is of course a closely related subject. McGuire and Riordan (1991) present a model of split award auction in which the regulator auctions the right of serving the market between two firms with unknown costs and has to decide whether to award the entire market to one of the firms or split it equally between them. The part of the present paper which discuses regulation of market shares resembles their work though it extends it by allowing a whole range of possible market shares and flexible prices. Auriol and Laffont (1992) discuss two other aspects: (i) the nature of the 'sampling effect' (the mere fact that the minimum of two random realizations of costs is lower than a single realization) under incomplete cost information, (ii) 'yardstick competition' which refers to the extraction of information when competitors costs have a common unknown component[2]. Both of these issues do not arise in present paper as the market is anyway served by both firms and the uncertain components of their costs are assumed independent. Biglaiser and Ma (1993) consider a spatial duopoly model with similar specifications to the one considered here. In their model the regulated firm acts as a Stackelberg leader against its unregulated competitor. Their model differs from the present one in its information structure, the nature of the competition and the main questions they address.

2. The Model

The basic model is a familiar duopoly model of spatial product differentiation to which we add a vertical quality dimension as well. A unit mass of consumers are distributed uniformly along the unit interval. Two firms, indexed by i=0,1, are located at either end. Firm i (i=0,1) sells product i of uniform quality q_i at a uniform price p_i. Each consumer is interested in getting one unit. The surplus derived by a consumer located at z from getting a unit of product i=0,1 is

$$V + q_i - t|z-i| - p_i$$

It is assumed that V>0 and $q^i \in [0,\infty)$, so that consumers' gross willingness to pay for a unit of i, $V+q_i$, is positive even if q^i is minimal. The parameter t captures the utility loss ("transportation cost") per unit distance between the brand in question and this consumer's ideal brand.

Faced with prices $p=(p_0,p_1)$ and qualities $q=(q_0,q_1)$ each consumer will demand a unit from the brand that yields him higher surplus provided it is positive. Let $x^i(p^i,p^j,q^i,q^j) \equiv x^i(p,q)$ denote the demand for product i. (Note that here and in the sequel we shall use unindexed symbols p,q and x to denote the corresponding two-component vectors). It is possible to distinguish between a "monopoly" region in which the market shares of the firms sum to less than 1 so that x^i is the solution to $V + q^i - p^i - tx^i = 0$, and a "competitive" region where the entire market is served and x^i is the solution to

$$V + q^i - tx^i - p^i = V + q^j - t(1-x^i) - p^j$$

The full description of the demand is summed up below.

$$(1) \qquad x^i(p^i,p^j,q^i,q^j) = \begin{cases} 0 & \text{if } q^i-p^i<q^j-p^j-t \\ (V+q^0-p^0)/t & \text{if } q^j-p^j-t<q^i-p^i\leq t-2V-q^j+p^j \\ (q_i-q_j+p_j-p_i+t)/2t & \text{if } t-2V-q^j+p^j<q^i-p^i\leq q^j-p^j+t \\ 1 & \text{if } q^j-p^j+t<q^i-p^i \end{cases}$$

The cost function of firm i, $c^i(x^i,q^i)$, is increasing in both arguments, convex and twice differentiable with $c^i_{12}\geq 0$. The profit of firm i is

$$(2) \qquad \pi^i(p,q) = p^i x^i(p,q) - c^i(x^i(p,q),q^i)$$

Total (consumers' plus producers') surplus is given by

$$(3) \qquad s(x,q)=\Sigma[x^i(V+q^i-tx^i/2)-c^i(x^i,q^i)],$$

where here and subsequently Σ stands for summation over $i=0,1$.

In general, the different equilibria and welfare optima in which we shall be interested below can occur in any region of the demand curves. For certain values of the parameters, these configurations might involve only one firm serving the whole market or less than the whole market being served. We choose to restrict attention to the cases in which the different equilibria and welfare optima will be 'interior' configurations in which the entire market is served and both firms are active. This will prevent some uninteresting technical complications owing to the kinked nature of the demand curve. Letting subscripts denote partial derivatives, the technical assumptions needed to achieve this are that V is sufficiently large and that $c_1^i(1,q^i)$ is sufficiently higher than $c_1^j(0,0)$. We shall impose these assumptions and consequently from now on identify the demand with the "competitive" branch of the demand curve $x^i(p,q) = (q^i-q^j+p^j-p^i+t)/2t$.

Under these assumptions the total surplus is maximized at a configuration in which $x^i+x^j=1$ and the following first order conditions hold

$$\frac{\partial s}{\partial x^i} \equiv q^i-q_j-c_1^i(x^i,q^i)+c_1^j(x^j,q^j)+t-2tx^i=0 \tag{4}$$

$$\frac{\partial s}{\partial q^i} \equiv x^i - c_2^i(x^i,q^i) = 0 \quad i=0,1. \tag{5}$$

2.B. Discussion of the Model

Although this model is rather familiar, it will be useful to discuss briefly some of the modeling decisions involved and explain their suitability for the case at hand.

The horizontal differentiation dimension: As usual, the spatial differentiation dimension can be interpreted as geographical distance or as differences in product specification in a more abstract product space. Of the industries that we have in mind hospitals and public utilities seem to fit better the geographical mold: the different firms within the industry provide the same product though possibly at different quality levels (which the model allows for) and geographical distance matters. Examples which fit the non-geographical mold include situations with different modes of transportation, such as trucking and railroads, and different regulated financial services such as banks and savings and loans associations.

The vertical quality dimension: One important assumption embodied in the description of the demand is that consumers' valuations are separable in the "transportation costs" and the quality level. This means that quality increments have the same effect on all consumers, regardless of their "distance" from the brand in question. As we shall see later in more detail, this feature implies that, when a firm is free to choose its price, its profit maximizing quality will coincide with the level that maximizes total surplus, given that firm's market share. This specification of the demand was chosen for the following reasons. First, this is the natural assumption for the important cases in which the differentiation dimension is geographical, where there is no reason to suppose that the valuation of quality of, say, medical treatment depends in a systematic way on the consumer's distance from the hospital. For other cases this assumption is not as obvious but there is no compelling reason to expect a systematic relationship between these two elements. Second, this specification simplifies the exposition and the analysis. Third, the above mentioned implication for the optimality of the quality choice by a profit maximizing firm, will allow to point out more sharply the distortions to quality provision which will be induced by the regulation considered later.

3. Competition and regulation under full information

3.A. Competition

Suppose that the two firms engage in uninterrupted competition. Simultaneously each firm chooses p_i and q_i respectively. The assumptions on the demand and cost guarantee that this interaction has a unique Nash equilibrium and that in it the entire market is served and both firms are active. The equilibrium configuration satisfies the following first order conditions.

$$(6)\quad \frac{\partial \pi^i}{\partial p^i} \equiv x^i + p^i(\partial x^i/\partial p^i) - c_1^i(\partial x^i/\partial p^i) \equiv x^i - (p_i - c_1^i)/2t = 0$$

$$(7)\quad \frac{\partial \pi^i}{\partial q^i} \equiv (p^i - c_1^i)(\partial x^i/\partial q^i) - c_2^i \equiv (p_i - c_1^i)/2t - c_2^i = 0$$

Condition (6) captures the usual mark-up of price over marginal cost in imperfect competition. It can be rewritten as $p^i - c_1^i = 2tx^i$. Substituting this into (7) yields $c_2^i = x^i$. This means that the equilibrium quality choices of the firms are socially optimal, given their equilibrium market shares. In other words, q^i maximizes $s(x,q)$ when the market shares x are fixed at their equilibrium values.

As mentioned above, the optimality of the equilibrium q^i's owes to the fact that consumers' valuations are separable in the "transportation costs" and the quality level. This means that quality increments have the same effect on all consumers, regardless of their willingness to pay. Now, since the firm can appropriate the incremental surplus of the marginal consumer through the price, it can also appropriate the increment to total surplus resulting from increments of quality. Therefore, the profit maximizing quality coincides with the surplus maximizing level, given the firm's market share. In the absence of this separability there is no such coincidence, and the equilibrium quality might be either lower or higher than the optimal level (see Spence (1975) for a discussion of these consideration for the case of monopoly).

However, despite the optimality of the equilibrium quality, the equilibrium allocation is not overall optimal. This is because, besides proper quality choice, optimality requires an appropriate allocation of the market shares, which in general will not arise under the imperfect competition assumed here.

Proposition 1: The equilibrium allocation need not be welfare maximizing. (I.e., it might achieve the first best allocation only for certain special choices of cost functions)

Proof: From (6), the competitive market shares satisfy

(8) $$2tx^i = q_i - q_j + p_j - p_i + t = p^i - c_1^i$$

In contrast, from (4), the optimal market shares satisfy

(9) $$2tx^i = q^i - q^j - c_1^i(x^i, q^i) + c_1^j(x^j, q^j) + t$$

Thus, if the equilibrium x^i's are optimal, then

$$p^i - c_1^i = p^j - c_1^j \quad \text{and hence } x^i = x^j = 1/2$$

The optimality of the equilibrium allocation therefore requires the following two conditions.

(10) $$c_2^i(1/2, q^i) = 1/2 \qquad i=0,1$$

(11) $$q^i - q_j - c_1^i(1/2, q^i) + c_1^j(1/2, q^j) = 0,$$

where (11) is obtained from plugging $x^i=1/2$ into (9). Now, since (10) uniquely determines the q^i's, equation (11) may hold only for special choice of cost functions but not in general. QED

The intuition behind this result is straightforward. Let MB^i denote the contribution of the marginal unit of product i to total surplus. I.e., $MB^i = V + q^i - tx^i - c_1^i$. Observe (e.g., from (8)) that $MB^i - MB^j = (p^i - c_1^i) - (p^j - c_1^j)$. Now, if the cost functions of the two firms are different, in equilibrium the more efficient firm--the one that has a larger market share--also features a larger mark-up. This implies immediately that the allocation of the market shares is inefficient--the market share of the more efficient firm is too small.

Note that one of the special cases for which equations (10) and (11) hold and hence the equilibrium allocation is efficient is when the two firms have identical cost functions. In a mathematical sense this case is special (non-generic), but one may object to referring to this case as "special" on the grounds that, in the symmetric model considered here, identical costs may appear natural. However, we do not attach great importance to this case for the following reasons. The symmetry is really just a simplifying assumption and does not play a substantive role in the analysis beyond reducing the complexity of the expressions. In the scenario with different products (e.g., railroading vs trucking) there is clearly no reason to suppose that the cost functions are identical. In the scenario of geographical differentiation, where the firms supply the same products, the case for identical cost functions is stronger. But then other sources of asymmetry such as uneven distribution of the consumers over the interval will achieve a similar effect in rendering the equilibrium allocation inefficient[3]. Thus, in this scenario as well efficiency of the equilibrium allocation is a special case.

3.B. Regulation

The possible suboptimality of the equilibrium allocation creates a potential role for regulation. In this part the regulator's objective is assumed to be maximization of the total surplus $s(x,q)$. Since the regulator is assumed perfectly informed about the firms' cost functions, no loss is involved in ignoring the distribution of the total surplus, for any feasible distribution of gains can presumably be achieved through non-distorting transfers.

If the regulator can fully control the firms' behavior, it will simply enforce the welfare maximizing qualities and market shares. However, here and throughout the paper we shall be interested in scenarios in which the regulator cannot directly control the quality levels. The idea is that, even if qualities are observable, they might be sufficiently difficult to verify so as to make their enforcement impractical. Therefore, the regulator's problem will be to use the instruments that it can control, such as prices and market shares, with the understanding that they will affect the quality levels which are left at the discretion of the firms.

Price regulation

Suppose that the regulator can control only the prices. The interaction unfolds in two stages. First, the regulator determines $p=(p^0,p^1)$, and then the firms simultaneously choose their quality levels, $q^i=q^i(p)$. The prices and qualities jointly determine the market shares. Thus, the regulator's problem is to choose prices so as to maximize welfare, given that the qualities will be determined in the subsequent quality choice game between the firms. That is,

(13) Choose p to maximize $s(x(p,q(p)),q(p))$

subject to

$$q^i(p) = \text{Argmax}_{q^i}\pi^i(p,q) \qquad i=0,1$$

Our assumptions on the cost functions and the parameters of the demand guarantee that the solution to this problem is such that the entire market is served and both firms have positive market shares. Now, at any such configuration, the firms' equilibrium quality choices, $q^i(p)$, are the solution to the first order conditions:

$$(14) \qquad \frac{\partial \pi^i(p,q)}{\partial q^i} = [p^i - c_1^i(x^i,q^i)]/2t - c_2^i(x^i,q^i) = 0 \qquad i=0,1$$

The first order conditions of problem (13) are thus

$$(15) \qquad \Sigma_{i=0,1}[(p^i - c_1^i)\frac{dx^i}{dp^k} + (x^i - c_2^i)\frac{\partial q^i}{\partial p^k}] = 0 \qquad k=0,1,$$

where $\frac{\partial q^i}{\partial p^k}$ and hence $\frac{dx^i}{dp^k}=\frac{dx^i(p,q(p))}{dp^k}=\frac{1}{2t}[\frac{\partial q^i}{\partial p^k}-\frac{\partial q^j}{\partial p^k}-1]$ are obtained from total differentiation of (14).

Proposition 2: Optimal price regulation need not necessarily achieve the welfare maximizing allocation. (I.e., it might achieve the first best allocation only for certain special choices of cost functions).

Proof: If this allocation were optimal, it would satisfy (5), $x^i = c^i_2$. This together with the fact that $(dx^1/dp^i)=-(dx^0/dp^i)$ imply via (15) that $p^0-c^0_1 = p^1-c^1_1$. From (14) we have $p^i-c^i_1 = 2tc^i_2$. Therefore $c^0_2 = c^1_2 = x^0 = x^1 = 1/2$. The first order conditions for the optimality of the allocation, (4) and (5), can be once again written as (10) and (11). As noted in the proof of proposition 1, condition (10) already uniquely determines the q^i's, and so equation (11) may hold only for special choice of cost functions but not in general. QED

The explanation here continues the one given after Proposition 1. In the unregulated equilibrium the more efficient firm features a larger mark-up and this implies that its market share is socially suboptimal. To improve this allocation through price regulation, the price differential between the two firms has to be widened: typically the price of the more efficient firm has to be forced downward and the price of the other firm has to be forced upwards. But since, as pointed out above, the equilibrium quality levels are optimal relative to the equilibrium shares, the regulated prices must distort the quality levels away from the efficient levels (downwards for the more efficient firm and upwards for the other one).

Regulation of market shares

Consider a scenario in which the regulator can control the quantities supplied by the firms. As we shall see momentarily, in this simple part of the model it is not important whether the regulator can also control the prices, so assume that the prices are left at the discretion of the firms. But in later parts this may matter and there we shall consider explicitly the possibility of regulating both. In the geographical differentiation scenario, the regulation of market shares takes simply the form of assignment of market areas, such as is the case with local telephone operating companies, public utilities and in some places hospitals. In the product differentiation scenario, the regulation may take the form of direct quantity restrictions such as in controlling the routes and the number of flights that an airline can operate. In this model, both of these scenarios are captured by a partition of interval between the two firms such that consumers located to the left of the dividing point are served by firm 0 and the rest by firm 1. Note that, although in the product differentiation scenario it is the quantity that is being imposed and not the specific assignment of the consumers, it is still the case that subject to that restriction the consumers served by firm i will be those who prefer its product more intensely.

The regulator's problem is then

(16) Choose $x=(x^0,x^1)$ to maximize $s(x,q)$

subject to

$$(p^i,q^i) = \text{Argmax}_{\hat{p},\hat{q}}[\hat{p}x^i - c^i(x^i,\hat{q})] \qquad i=0,1$$

$$p_i \leq V + q_i - tx^i$$

Proposition 3: Optimal regulation of market shares attains the welfare maximizing allocation.

Proof: It follows from the constraints of (16) that, for any choice of x^i by the regulator, the firm's choice of q^i is optimal, i.e., satisfies $c_2^i(x^i,q^i)=x^i$. Therefore, instructing the firms to serve the first best market share will achieve the first best allocation, since the firms will be induced to produce the first best quality as well. QED

Note that this result also owes to the fact that consumers' valuations are separable in the "transportation costs" and the quality, since this feature guarantees the optimality of the firms' quality choices.

Thus, in the presence of the unregulated quality dimension, the choice of instrument--prices or quantities--matters importantly. The regulation of quantities or market shares is unambiguously superior because it avoids the incentives to distort the provision of quality which appear under price regulation.

3.C. An example

This example illustrates the points made by propositions 1-3. Let $c^i(x^i,q^i) = \theta^i x^i + k(q^i)^2$ and assume 2kt>1. Let c, w and r index variables corresponding to the competitive, welfare maximizing and price regulated regimes respectively. From (6)-(7), the competitive magnitudes are

$$(17)\quad q_c^i = \frac{1}{4k}+\frac{\theta^j-\theta^i}{12kt-2}; \qquad p_c^i = \theta^i+t+\frac{2kt(\theta^j-\theta^i)}{6kt-1}; \qquad x_c^i = \frac{1}{2}+\frac{k(\theta^j-\theta^i)}{6kt-1}.$$

From (4)-(5) the surplus maximizing magnitudes are

$$(18)\quad q_w^i = \frac{1}{4k}+\frac{\theta^j-\theta^i}{4kt-2}; \qquad p_w^i = \theta^i; \qquad x_w^i = \frac{1}{2}+\frac{k(\theta^j-\theta^i)}{2kt-1}.$$

From (15)

$$(19)\quad q_r^i = \frac{1}{4k} + \frac{\theta^j-\theta^i}{4kt(8kt+1)-2}; \quad p_r^i = \theta^i + t + \frac{2kt(\theta^j-\theta^i)}{2kt(8kt+1)-1}; \quad x_r^i = \frac{1}{2} + \frac{k(8kt-1)(\theta^j-\theta^i)}{2kt(8kt+1)-1}.$$

If $\theta^i<\theta^j$, so that i is the more efficient firm, then $x_c^i < x_r^i < x_w^i$. That is, i's market share in the uninterrupted competition is too small from a welfare point of view. The price regulation enforces a larger price differential and so achieves a market share, x_r^i, larger than x_c^i but still smaller than x_w^i. The improvement of the allocation in this dimension comes at the expense of distorting the provision of quality. Under the competitive regime (and of course in the welfare maximizing configuration) the quality levels are optimal, given the market shares, which in terms of this example means $q^i=x^i/2k$. Under price regulation, however, the quality of the more efficient firm is suboptimal, $q_r^i<x_r^i/2k$, while the quality of the other firm is too high given its share.

As pointed out above, optimal regulation through assignment of market shares will enforce the shares x_w^i which will induce the firms to choose quality q_w^i and hence result in the welfare maximizing allocation.

4. Regulation under incomplete information: preliminaries

From this point on the discussion focuses on situations in which the regulator has to make its decision without knowing the cost functions of the firms.

Cost, profit and welfare

Let $c^i(x^i,q^i,\theta^i)$ denote the cost function of firm i, assumed increasing in all of its arguments, convex and twice differentiable, with nonnegative mixed derivatives. The parameter $\theta^i\in[0,1]$ captures firm i's private information about cost at the time in which the regulator has to choose its policy: the higher is θ^i, the less efficient is firm i. The uncertainty over

θ^i is described by a distribution function F with density f. The θ^i's are assumed independent. As is standard in the related literature, we shall assume that F/f is an increasing function.

For a given $\theta=(\theta^0,\theta^1)$, the profit functions and the welfare indicator are defined analogously to above. The direct profit of firm i as a function of its own price, quality, market share and cost parameter is

$$(20) \qquad \pi^i(x^i,p^i,q^i,\theta^i) = p^i x^i - c^i(x^i,q^i,\theta^i)$$

The total (consumers' plus producers') surplus is

$$(21) \qquad s(x,q,\theta) = \sum[x^i(V+q^i-tx^i/2)-c^i(x^i,q^i,\theta^i)]$$

In the complete information regime discussed above, we could identify the regulator's objective with maximization of the total surplus s(x,q), since any distribution of this surplus could presumably be achieved through non-distortionary transfers. In the presence of incomplete information, the regulator may not be able to fully extract the firms' profits and hence the distribution of the surplus should be explicitly accounted for in the regulator's objective. Assume that the regulator is interested in maximizing a weighted sum of consumers' surplus and firms' incomes with weights 1 and $(1-\alpha)$ respectively. Thus, α is the "cost of public funds." Let $T=(T^0,T^1)$ denote transfers from the consumers via the regulator to the firms. The regulator's objective then is to maximize

$$(22) \quad w(x,p,q,T) = \sum[x^i(V+q^i-tx^i/2)-p^ix^i-T^i]+(1-\alpha)\sum[\pi^i(x^i,p^i,q^i)+T^i] =$$
$$= s(x,q,\theta) - \alpha\sum[\pi^i(x^i,p^i,q^i,\theta^i)+T^i]$$

<u>The regulatory mechanism</u>

As before, the regulator cannot control the quality levels $q=(q^0,q^1)$, but it can enforce prices $p=(p^0,p^1)$ and market shares $x=(x^0,x^1)$. The regulatory mechanism operates as follows. First, the regulator commits to the relevant schedules. If it wants to regulate only the prices, it will commit to price schedules $p^i:[0,1]^2 \to [0,\infty)$ and transfer functions $T^i:[0,1] \to \mathbf{R}$; if it wants to regulate the market shares, it will commit to schedules $x^i:[0,1]^2 \to [0,\infty)$ instead or in addition to the schedules $p^i:[0,1]^2 \to [0,\infty)$. Then, the firms send simultaneously reports σ^i to the regulator, where σ^i is interpreted as a report on θ^i. On the basis of the reports and the pre-announced schedules the regulator determines the relevant magnitudes: prices $p(\sigma)=(p^0(\sigma),p^1(\sigma))$ and/or shares $x(\sigma)=(x^0(\sigma),x^1(\sigma))$ and transfers[4] $T(\sigma)=(T^0(\sigma^0),T^1(\sigma^1))$. Next, the two firms simultaneously choose their q^i's and the other magnitudes which were left at their discretion (i.e., the prices in case the regulator determined only the shares).

Firms' information

At the reporting stage, the firms do not know each other's θ's. But we shall consider two alternative scenarios regarding the firms' information in the subsequent quality choice stage. In one scenario the firms know each other's costs in the second stage. This scenario intends to capture a situation in which the regulatory decisions are made for a relatively long period during which firms learn each other's costs relatively quickly, and so most of their interaction will be under conditions of complete information. In the other scenario firms remain uninformed about each other's cost parameters in the quality choice stage as well.

5. Regulation under incomplete information: Price regulation

5.A. A scenario in which the firms are informed in the second stage

In this scenario, given the reports $\sigma=(\sigma^0,\sigma^1)$, the regulator determines $p(\sigma)=(p^0,p^1)$ and $T(\sigma)=(T^0,T^1)$ and leaves $q=(q^0,q^1)$ and $x=(x^0,x^1)$ to be determined in the subsequent interaction between the firms. The objective of the regulator is to choose functions p and T so as to maximize the expected value of (22). The maximization takes into account that the choice of p and T induces a subgame perfect equilibrium in the two stage game played between the firms, in which the first stage is a reporting game and the second is a quality competition game.

To express the regulator's problem formally and compactly, let $q^i(p,\theta)$ denote the equilibrium qualities arising in the competition between the firms, given prices p and cost parameters θ.

(23) $$q^i(p,\theta)=\text{Argmax}_{q^i}\pi^i[x^i(p,q^i,q^j(p,\theta)),p^i,q^i,\theta^i]$$

That is, $q(p,\theta)$ satisfy the following first order conditions for i=0,1,

(24) $$\frac{\partial\pi^i[x^i(p,q),p^i,q^i,\theta^i]}{\partial q^i}= [p^i - c^i_1(x^i,q^i,\theta^i)]/2t - c^i_2(x^i,q^i,\theta^i) = 0$$

Note that these formulae reflect the assumption that, at the quality choice stage, i knows θ^j, as q^i depends directly on θ^j. Substitute $q(p,\theta)$ into $x(p,q)$ and then both into (20)-(22) to get reduced-form profit and welfare measures embodying the second stage equilibrium quality choices. $\Pi^i(p,\theta)=\pi^i(x^i(p,q(p,\theta)),p^i,q^i(p,\theta),\theta^i)$, $S(p,\theta)=s(x(p,q(p,\theta)),q,\theta)$ and $W(p,T,\theta)=w(x(p,q(p,\theta)),p,q(p,\theta),T,\theta)$. Let $h^i(\sigma^i,\theta^i)$ denote firm i's expected income when it reports σ^i while its true parameter is θ^i, provided that firm j reports truthfully.

(25) $$h^i(\sigma^i,\theta^i) = E_{\theta^j}\{\Pi^i[p(\sigma^i,\theta^j),\theta]\} + T^i(\sigma^i)$$

The regulator's problem is

(26) Choose functions p and T to maximize $E_\theta[W(p(\theta),T(\theta),\theta)]$

Subject to

(IC) $h^i(\theta^i,\theta^i) \geq h^i(\sigma^i,\theta^i)$ all θ^i, σ^i, i=0,1.

(IR) $h^i(\theta^i,\theta^i) \geq 0$ i=0,1.

Let $H^i(\theta^i)$ denote the expected income of firm i when its cost parameter is θ^i and both firms report their true parameters, $H^i(\theta^i) = h^i(\theta^i,\theta^i)$

The following proposition will allow to reformulate problem (26).

<u>Proposition 4</u>: (i) If the functions p and T satisfy IC and if p is piecewise continuous, then H^i is differentiable almost everywhere and

$$(27) \qquad H^i(\eta) = H^i(1) + \int_\eta^1\int_0^1 [\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i} + \frac{\partial c^i}{\partial \theta^i}] dF(\theta^j)d\theta^i$$

(ii) The IC constraint in (26) is equivalent to: For all η and ν,

$$(28) \qquad \int_\nu^\eta\int_0^1 [\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i} + \frac{\partial c^i}{\partial \theta^i}] dF(\theta^j)d\theta^i \geq \int_\nu^\eta\int_0^1 [\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i} + \frac{\partial c^i}{\partial \theta^i}] |_{p=p(\eta,\theta^j)} dF(\theta^j)d\theta^i$$

(iii) The IR constraint in (26) is equivalent to $H^i(1) \geq 0$.

The proof is relegated to an appendix. Results of this nature are rather standard in the relevant literature. Note that, if we could assume that h^i is differentiable, part (i) would follow from integrating

$$(29) \qquad \frac{dH^i(\theta^i)}{d\theta^i} = h^i_1(\theta^i,\theta^i) + h^i_2(\theta^i,\theta^i) = h^i_2(\theta^i,\theta^i) =$$

$$\int_0^1 [(p^i - \frac{\partial c^i}{\partial x^i})(-\frac{1}{2t}\frac{\partial q^j}{\partial \theta^i}) - \frac{\partial c^i}{\partial \theta^i}] dF(\theta^j) = -\int_0^1 [\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i} + \frac{\partial c^i}{\partial \theta^i}] dF(\theta^j)$$

where the second equality sign follows from the IC constraint, and the last equality sign follows from the first order condition (24). The expression $\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i} + \frac{\partial c^i}{\partial \theta^i}$ which appears in (27)-(29) consists of a direct cost effect $\frac{\partial c^i}{\partial \theta^i}$ and an indirect "interactive" effect $\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i}$. The latter captures the indirect effect of θ^i on i's profit through its effect on j's choice of quality. This effect owes to the assumption that, in the quality choice stage, firm j can observe θ^i. Total differentiation of (24) yields $\frac{\partial q^j}{\partial \theta^i} \leq 0$, i.e., a higher θ^i makes firm i less competitive and hence induces j to invest less in its quality. Thus, the interactive effect moderates the rate at which i's profit falls in θ^i.

Using (27), the expected welfare measure EW can be rewritten as

(30) $$EW = E_\theta[s(p,\theta) - \alpha\sum H^i(\theta^i)] = \int_0^1\!\!\int_0^1 [s(p,\theta)-\alpha\sum H^i(\theta^i)]dF(\theta^i)dF(\theta^j) =$$

$$\int_0^1\!\!\int_0^1 [s(p,\theta)-\alpha\sum \frac{F(\theta^i)}{f(\theta^i)}[\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i}+\frac{\partial c^i}{\partial \theta^i}]]f(\theta^i)f(\theta^j)d\theta^i d\theta^j - \alpha\sum H^i(1)$$

where the last equality follows from integration by parts.

The regulator's problem, (26), can now be written as

(31) Choose functions p and T to maximize:

$$\int_0^1\!\!\int_0^1 [s(p,\theta)-\alpha\sum \frac{F(\theta^i)}{f(\theta^i)}[\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i}+\frac{\partial c^i}{\partial \theta^i}]]f(\theta^i)f(\theta^j)d\theta^i d\theta^j$$

Subject to

(IC) $$\int_\nu^\eta\!\!\int_0^1 [\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i}+\frac{\partial c^i}{\partial \theta^i}]dF(\theta^j)d\theta^i \geq \int_\nu^\eta\!\!\int_0^1 [\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i}+\frac{\partial c^i}{\partial \theta^i}]|_{p=p(\eta,\theta^j)}dF(\theta^j)d\theta^i \text{ for all } \eta \text{ and } \nu$$

It will be useful to obtain some insight into this problem by solving it explicitly for a more specific example.

<u>An Example (cost is linear in the quantity)</u>

Consider again the example discussed above where $c^i(x^i,q^i,\theta^i) = \theta^i x^i + k(q^i)^2$. System (24) applied to this case yields

(32) $q^i(p,\theta)=(p^i-\theta^i)/4kt$ and $x^i=(1/2)+(4kt-1)[(p^i-\theta^i)-(p^j-\theta^j)]/4kt$.

This implies $\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i}+\frac{\partial c^i}{\partial \theta^i}=x^i$. Therefore, problem (31) becomes:

(33) Choose p and T to maximize:

$$\int_0^1\int_0^1 \sum [x^i(V+q^i-t\frac{x^i}{2})-\theta^i x^i-k(q^i)^2-\alpha\frac{F(\theta^i)}{f(\theta^i)}x^i]f(\theta^i)f(\theta^j)d\theta^i d\theta^j$$

subject to:

$$\text{(IC)} \qquad (4kt-1)\int_v^\eta\int_0^1\{[p^i(\theta^i,\theta^j)-p^j(\theta^i,\theta^j)]-[p^i(\eta,\theta^j)-p^j(\eta,\theta^j)]\}dF(\theta^j)d\theta^i\le 0$$

where q^i and x^i are given by (32). We shall first maximize the objective of problem (33) pointwise, ignoring the (IC) constraint, and then verify that it is satisfied. Letting the subscript ur (uninformed regulation) indicate the solution in this case, we get

$$(34)\quad q^i_{ur} = \frac{1}{4k}+\frac{\theta^j-\theta^i}{4kt(8kt+1)-2}+\frac{\alpha(4kt-1)}{4kt(8kt+1)-2}[\frac{F(\theta^i)}{f(\theta^i)}-\frac{F(\theta^j)}{f(\theta^j)}]$$

$$(35)\quad p^i_{ur} = \theta^i+t+\frac{2kt(\theta^j-\theta^i)}{2kt(8kt+1)-1}+\frac{\alpha 2kt(4kt-1)}{2kt(8kt+1)-1}[\frac{F(\theta^i)}{f(\theta^i)}-\frac{F(\theta^j)}{f(\theta^j)}]$$

$$(36)\quad x^i_{ur} = \frac{1}{2}+\frac{k(8kt-1)(\theta^j-\theta^i)}{2kt(8kt+1)-1}+\frac{\alpha(4kt-1)^2}{2t[2kt(8kt+1)-1]}[\frac{F(\theta^j)}{f(\theta^j)}-\frac{F(\theta^i)}{f(\theta^i)}]$$

Note that

$$p^i_{ur}(\theta^i,\theta^j)-p^j_{ur}(\theta^i,\theta^j) = \frac{[2kt(8kt-1)](\theta^i-\theta^j)}{2kt(8kt+1)-1}+\frac{\alpha 4kt(4kt-1)}{2kt(8kt+1)-1}[\frac{F(\theta^i)}{f(\theta^i)}-\frac{F(\theta^j)}{f(\theta^j)}]$$

Since by assumption F/f is increasing and $4kt>1$, this expression is increasing in θ^i. Therefore, the IC constraint holds.

Recall from (17)-(19) the corresponding magnitudes q^i_r, p^i_r, x^i_r arising under informed regulation. Note that q^i_{ur}, p^i_{ur}, x^i_{ur} differ from q^i_r, p^i_r, x^i_r by a term which depends on $\frac{F(\theta^i)}{f(\theta^i)}-\frac{F(\theta^j)}{f(\theta^j)}$. This term reflects the additional element present under asymmetric information: the tradeoff between total surplus generation and the appropriation of rents from the firms. Since F/f is an increasing function and $4kt>1$, $\theta^i<\theta^j$ implies that $p^i_{ur} < p^i_r$ and $x^i_{ur} > x^i_r$ while $p^j_{ur} > p^j_r$ and $x^j_{ur} < x^j_r$. That is, under uninformed regulation, the price of the low cost firm is lower and its market share is larger than under informed regulation. Since x_r^i is smaller than the first best market share, x_w^i, it is possible that x_{ur}^i will be closer to x_w^i than x_r^i is. But this does not mean of course that uninformed regulation yields better allocations. On the contrary, the efficiency gains from a larger market share are outweighed by a more significant distortion of the quality: q_{ur}^i is lower than q_r^i which is already suboptimal even relative to a smaller share. The source of this added distortion is the regulator's interest in appropriating the rents of the firms. A relatively low price for the lower cost firm, reduces the profitability of the higher cost firm and hence the informational rents (i.e., the profit which has to be left to a firm to dissuade it from exaggerating its cost).

5.B. An alternative scenario: the firms are uninformed in the second stage too

Consider next the alternative scenario in which the firms continue to be uninformed about each other's costs in the quality competition stage as well. To make things simple, assume that, besides determining the price, the regulator also communicates to the firms their rivals' reports (in the examples we consider this is unnecessary since each firm will be able to infer it from the price). By simply following the preceding analysis, it is easy to verify that the only thing that changes is that now $q^i(p,\theta)$ does not depend directly on θ^j, but just on j's report. This means that (27) changes to

$$\text{(37)} \qquad H^i(\eta) = H^i(1) + \int_\eta^1\int_0^1 \frac{\partial c^i}{\partial \theta^i} dF(\theta^j)d\theta^i$$

Hence the regulator's problem now changes to

(38) Choose functions p and T to maximize:

$$\int_0^1\int_0^1 [s(p,\theta) - \alpha\sum \frac{F(\theta^i)}{f(\theta^i)}\frac{\partial c^i}{\partial \theta^i}]f(\theta^i)f(\theta^j)d\theta^i d\theta^j$$

Subject to

(IC) $$\int_{\nu}^{\eta}\int_{0}^{1}\frac{\partial c^i}{\partial \theta^i}dF(\theta^j)d\theta^i \ge \int_{\nu}^{\eta}\int_{0}^{1}\frac{\partial c^i}{\partial \theta^i}\Big|_{p=p(\eta,\theta^j)}dF(\theta^j)d\theta^i \qquad \text{for all } \eta \text{ and } \nu$$

Note that, if some price function $p(\theta)$ is implementable both in this scenario and in the previous one, then the rents accruing to the firms are higher in the present scenario. Technically, this follows immediately from comparing (27) to (37), when we take into account the following two facts. First, that with the same p function, the equilibrium q and x will be the same in both scenarios, and second that, as noted above, $\frac{\partial q^j}{\partial \theta^i}\le 0$ implying that the integrand in (27) is smaller. Intuitively, the reason is that, when firm j does not know θ^i, firm i has a more pronounced incentive to exaggerate its report of θ^i. This is because, in equilibrium, an inflated report of θ^i is assumed correct by firm j and since $\frac{\partial q^j}{\partial \theta^i}\le 0$ it induces j to choose a lower q^j than it would if it knew the true θ^i. This relaxes the competition faced by i and increases its profit. Hence the firms have to be compensated with a larger share of the rent to be kept truthful.

Since in the example $c^i(x^i,q^i,\theta^i) = \theta^i x^i + k(q^i)^2$ considered above we have $\frac{\partial q^j}{\partial \theta^i}=0$, the change of regime described here will not have any effect. Thus, the magnitudes associated with optimal price regulation remain the same as those derived above. This is however an artifact of that example. The following example, which allows for a richer interaction between the firms, captures the differences between these two regimes.

<u>An Example (the cost is quadratic in quantity)</u>

Let $c^i(x^i,q^i,\theta^i) = \theta^i x^i + c(x^i)^2 + k(q^i)^2$. Note that this example differs from the previous one in the additional quadratic term $c(x^i)^2$. System (24) applied to this case yields

(39) $q^i(p,\theta) =$

$= [p^i(4kt^2+4kct+c)-p^jc(4kt-1)-4kct^2-2c^2-(4kt^2+c)\theta^i-c\theta^j]/[t(4kt)^2+8kct]$

Consider first the scenario of 5.A where in the quality competition stage the firms know each other's costs. The solution to the optimal price regulation problem (31) is as follows. Let $A = [(4kt)^2 + 2kt - 1 + 4kc]$.

(40) $p^i_{ur} =$

$$c+\theta^i+t+[\frac{2kt^2+c}{(t+2c)A}+\frac{c}{t+2c}](\theta^j-\theta^i)+\alpha\frac{2kt^2+c}{(t+2c)A}[\frac{2kct+4kc^2}{t((4kt)^2+8kc)}+4kt-1][\frac{F(\theta^i)}{f(\theta^i)}-\frac{F(\theta^j)}{f(\theta^j)}]$$

(41) $$q^i_{ur} = \frac{1}{4k}+\frac{\theta^j-\theta^i}{2A}+\alpha[\frac{kct+2kc^2}{tA((4kt)^2+8kc)}+\frac{4kt-1}{2A}][\frac{F(\theta^i)}{f(\theta^i)}-\frac{F(\theta^j)}{f(\theta^j)}]$$

(42) $x^i_{ur} =$

$$\frac{1}{2}+[\frac{(4kt)^2-2kt+4kc}{2(t+2c)A}](\theta^j-\theta^i)+\frac{\alpha(4kt-1)}{2(t+2c)A}[\frac{2kct+4kc^2}{t((4kt)^2+8kc)}+4kt-1][\frac{F(\theta^j)}{f(\theta^j)}-\frac{F(\theta^i)}{f(\theta^i)}]$$

Next consider the scenario in which the firms do not know each other's θ's in the second stage as well. The solution to the optimal price regulation problem (38) is

(43) $$\hat{p}^i_{ur} = c+\theta^i+t+[\frac{2kt^2+c}{(t+2c)A}+\frac{c}{t+2c}](\theta^j-\theta^i)+\alpha(4kt-1)\frac{2kt^2+c}{(t+2c)A}[\frac{F(\theta^i)}{f(\theta^i)}-\frac{F(\theta^j)}{f(\theta^j)}]$$

(44) $$\hat{q}^i_{ur} = \frac{1}{4k}+\frac{\theta^j-\theta^i}{2A}+\alpha\frac{4kt-1}{2A}[\frac{F(\theta^i)}{f(\theta^i)}-\frac{F(\theta^j)}{f(\theta^j)}]$$

(45) $$\hat{x}^i_{ur} = \frac{1}{2}+[\frac{(4kt)^2-2kt+4kc}{2(t+2c)A}](\theta^j-\theta^i)+\frac{\alpha(4kt-1)^2}{2(t+2c)A}[\frac{F(\theta^j)}{f(\theta^j)}-\frac{F(\theta^i)}{f(\theta^i)}]$$

The allocation $\hat{p}^i_{ur}$, $\hat{q}^i_{ur}$ and $\hat{x}^i_{ur}$ is implementable in the scenario of 5.A. To see this it is enough to verify that these functions satisfy the IC constraint of (31). This follows from the fact that $\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i}+\frac{\partial c^i}{\partial \theta^i}$ evaluated at $\hat{q}^i_{ur}$ and $\hat{x}^i_{ur}$ is equal to $2k\hat{q}^i_{ur}\frac{\partial q^j}{\partial \theta^i}+\hat{x}^i_{ur}$ which is a decreasing function of θ^i. The welfare associated with this allocation is of course higher in the scenario of 5.A, since the rents accruing to the firms are lower in that scenario.

Therefore, maximum welfare is also higher in that scenario. Note that $x^i_{ur} \neq \hat{x}^i_{ur}$ and $q^i_{ur} \neq \hat{q}^i_{ur}$, i.e., the allocations arising under optimal price regulation in the two alternative scenarios differ not only in their rents. Because the rents tend to be higher in the scenario of 5.B the allocation is further distorted to extract some of these rents.

6. Regulation under incomplete information: Regulation of market shares

This section discusses the alternative regulatory instrument of controlling the firms' market shares. As we noted, when the products are differentiated with respect to their geographical location, the regulator's intervention amounts to assigning to the firms exclusive market areas. When the differentiation is with respect to some other product characteristics, the intervention is through restrictions on quantities. Recall that in the complete information environment, this form of regulation resulted in higher welfare. As we shall see below, with asymmetric information, this will not always be the case.

We consider two scenarios. In one of them both prices and qualities are left at the discretion of the firms. In the other scenario, prices are also determined by the regulator so that only qualities are chosen by the firms. The interaction here is conceptually simpler than under price regulation, since following the assignment of market shares, there is no further interaction between the firms. For this reason it does not matter whether or not the firms know each other's costs in the second stage and the distinction made above between the scenarios of 5.A. and 5.B need not be made here.

6.A. Regulation of market shares alone

First, the regulator commits to a mechanism consisting of $x^i:[0,1]^2 \to [0,1]$ and $T^i:[0,1] \to \mathbf{R}$, where x^i is the market share assigned to firm i, $x^0+x^1=1$, and T^i is the transfer made to firm i. Then, the firms send simultaneously reports σ^i. These reports determine the market shares $x(\sigma)$ and transfers $T(\sigma)$ through the pre-announced schedules. Finally, given x and T, the two firms choose their prices p^i and quality levels q^i.

The objective of the regulator is to choose functions x and T to maximize expected welfare, taking into account that these choices induce an equilibrium in the reporting game and profit-maximizing price-quality choices in the subsequent stage. Let $p^i(x^i,\theta^i)$ and $q^i(x^i,\theta^i)$ denote firm i's profit maximizing price-quality choice, given a market share assignment x^i and a cost parameter θ^i. Since the firm faces no competition, $p^i = V + q^i - tx^i$ (i.e., p^i is the maximal price compatible with q^i and x^i), we have

(46) $$q^i(x^i,\theta^i)=\text{Argmax}_{q^i}\{\pi^i(x^i,p^i,q^i,\theta^i) \text{ s.t. } p^i=V+q^i-tx^i\}.$$

Note that $q^i(x^i,\theta^i)$ solves the first order condition

(47) $$x^i = c_2^i(x^i,q^i,\theta^i),$$

implying that $q^i(x^i,\theta^i)$ is optimal given i's market share. As we have already noted, this observation owes to the special structure of the preferences in which quality enters independently of consumers' location in a manner that allows the firm to use the price to extract from its consumers the entire incremental surplus generated by quality increases.

Substitute $q^i(x^i,\theta^i)$ and $p^i(x^i,\theta^i)\equiv V + q^i - tx^i$ into (20)-(22) to get reduced-form profit and welfare measures which already embody the second stage optimal price-quality choices. Thus, $\Pi^i(x^i,\theta^i)=\pi^i(x^i,p^i(x^i,\theta^i),q^i(x,\theta^i),\theta^i)$, $S(x,\theta)=s(x,p(x,\theta),q(x,\theta),\theta)$ and $W(x,T,\theta)=w(x,p(x,\theta),q(x,\theta),T,\theta)$. As before, let $h^i(\sigma^i,\theta^i)$ denote firm i's expected income,

(48) $$h^i(\sigma^i,\theta^i) = E_{\theta^j}\{\Pi^i[x^i(\sigma^i,\theta^j),\theta^i]\} + T^i(\sigma^i)$$

The regulator's problem is

(49) Choose function x and T to maximize $E_\theta[W(x(\theta),T(\theta),\theta)]$

Subject to

(IC) $h^i(\theta^i,\theta^i) \geq h^i(\sigma^i,\theta^i)$ all θ^i, σ^i, i=0,1.

(IR) $h^i(\theta^i,\theta^i) \geq 0$ i=0,1.

As before, $H^i(\theta^i) = h^i(\theta^i,\theta^i)$. The counterpart of Proposition 4 above is

Proposition 5: (i) If x and T satisfy IC and if x piecewise continuous, then H^i is differentiable almost everywhere and

$$H^i(\eta) = H^i(1) + \int_\eta^1\int_0^1 \frac{\partial c^i}{\partial \theta^i} dF(\theta^j)d\theta^i \tag{50}$$

(ii) The IC constraint is equivalent to

$$\int_\nu^\eta\int_0^1 \frac{\partial c^i}{\partial \theta^i} dF(\theta^j)d\theta^i \geq \int_\nu^\eta\int_0^1 \frac{\partial c^i}{\partial \theta^i}\Big|_{x=x(\eta,\theta^j)} dF(\theta^j)d\theta^i \qquad \text{for all } \eta \text{ and } \nu. \tag{51}$$

(iii) The IR constraint is equivalent to $H^i(1)\geq 0$.

The proof is relegated to the appendix. As before, if we knew that h^i is differentiable, part (i) of the proposition would follow from integrating

$$\frac{dH^i(\theta^i)}{d\theta^i} = h^i_1(\theta^i,\theta^i) + h^i_2(\theta^i,\theta^i) = h^i_2(\theta^i,\theta^i) = -\int_0^1 \frac{\partial c^i}{\partial \theta^i} dF(\theta^j) \tag{52}$$

where $h^i_1(\theta^i,\theta^i)=0$ follows from the IC constraint.

Using the proposition and integration by parts, the expected welfare measure EW can be rewritten as

$$EW = E_\theta[S(x,\theta) - \alpha\sum H^i(\theta^i)] = \int_0^1\int_0^1 [S(x,\theta)-\alpha\sum H^i(\theta^i)]dF(\theta^i)dF(\theta^j) \tag{53}$$

$$= \int_0^1\int_0^1 [S(x,\theta)-\alpha\sum \frac{F(\theta^i)}{f(\theta^i)}\frac{\partial c^i}{\partial \theta^i}]f(\theta^i)f(\theta^j)d\theta^i d\theta^j - \alpha\sum H^i(1)$$

The regulator's problem can therefore be reformulated as:

(54) Choose x and T to maximize

$$EW = \int_0^1\int_0^1 [S(x,\theta) - \alpha\sum \frac{F(\theta^i)}{f(\theta^i)}\frac{\partial c^i}{\partial\theta^i}] f(\theta^i)f(\theta^j)d\theta^i d\theta^j$$

Subject to

$$\int_\nu^\eta\int_0^1 \frac{\partial c^i}{\partial\theta^i} dF(\theta^j)d\theta^i \geq \int_\nu^\eta\int_0^1 \frac{\partial c^i}{\partial\theta^i}|_{x=x(\eta,\theta^j)} dF(\theta^j)d\theta^i \qquad \text{for all } \eta \text{ and } \nu.$$

Example

Recall again the first example discussed above,

$c^i(x^i,q^i,\theta^i) = \theta^i x^i + k(q^i)^2$. Here, $q^i(p,\theta) = x^i/2k$ and $\frac{\partial c^i}{\partial\theta^i} = x^i$. Therefore,

problem (54) becomes:

(55) Choose x and T to maximize:

$$\int_0^1\int_0^1 \sum [x^i(V+q^i-t\frac{x^i}{2}) - \theta^i x^i - k(q^i)^2 - \alpha\frac{F(\theta^i)}{f(\theta^i)}x^i] f(\theta^i)f(\theta^j)d\theta^i d\theta^j$$

subject to:

$$\text{(IC)} \qquad \int_\nu^\eta\int_0^1 \{[x^i(\theta^i,\theta^j) - x^i(\eta,\theta^j)]\} dF(\theta^j)d\theta^i \geq 0$$

We shall first maximize the objective of problem (55) pointwise, ignoring the (IC) constraint, and then verify that it is satisfied. Letting the subscript um (uninformed market share regulation) denote the solution in this case,

$$(56) \qquad x^i_{um} = \frac{1}{2} + \frac{k(\theta^j-\theta^i)}{2kt-1} + \frac{\alpha k}{2kt-1}[\frac{F(\theta^j)}{f(\theta^j)} - \frac{F(\theta^i)}{f(\theta^i)}]$$

$$(57) \qquad q^i_{um} = \frac{1}{4k} + \frac{\theta^j-\theta^i}{4kt-2} + \frac{\alpha}{4kt-2}[\frac{F(\theta^i)}{f(\theta^i)} - \frac{F(\theta^j)}{f(\theta^j)}]$$

Since F/f is increasing, $x^i_{um}(\theta^i,\theta^j)$ is a decreasing function of θ^i. Therefore, the IC constraint holds. If $\theta^i<\theta^j$ so that firm i is the more efficient, then x^i_{um} is larger than the first best market share x^i_w. Here too the source of this distortion is the regulator's interest in appropriating the rents of the firms. A relatively larger share for the lower cost firm, reduces the inducement for a firm to exaggerate its cost and hence reduces the informational rents. Recall from (47) that, as in the complete information regime, q^i_{um} is optimal given x^i_{um}. Therefore, the welfare losses are just due to the distortion of the allocation of market shares.

6.B. Regulation of market shares and prices

Here in addition to $x^i:[0,1]^2 \to [0,1]$ and $T^i:[0,1] \to \mathbf{R}$, the regulator also determines $p^i:[0,1]^2 \to \mathbf{R}$ and, given p, x and T, the two firms choose the quality levels q^i. Recall that in the complete information environment this case was redundant: regulation of market shares alone already achieved the first best allocation. But, this need not be the case under asymmetric information, where the price might be used as an additional instrument for rent extraction.

In analogy to the previous analysis, let $q^i(x^i,p^i,\theta^i)$ denote firm i's profit maximizing quality choice. The absence of competition implies

(58) $$q^i(x^i,p^i,\theta^i) = p^i + tx^i - V$$

i.e., q^i is the minimal quality compatible with x^i and p^i. Substitute (58) into (20)-(22) to get the reduced-form functions:

$\Pi^i(x^i,p^i,\theta^i)=\pi^i(p^i,q^i(x,\theta^i),x^i,\theta^i)$, $S(x,p,\theta)=s(p,q(x,\theta),x,\theta)$ and $W(x,p,T,\theta)=w(p,q(x,p,\theta),x,T,\theta)$. Now,

(59) $$h^i(\sigma^i,\theta^i) = E_{\theta^j}\{\Pi^i[x^i(\sigma^i,\theta^j),p^i(\sigma^i,\theta^j),\theta^i]\} + T^i(\sigma^i)$$

and the regulator's problem is

(60) Choose functions x, p and T to maximize $E_\theta[W(x(\theta),p(\theta),T(\theta),\theta)]$

Subject to

(IC) $h^i(\theta^i,\theta^i) \geq h^i(\sigma^i,\theta^i)$ all θ^i, σ^i, i=0,1.

(IR) $h^i(\theta^i,\theta^i) \geq 0$ i=0,1.

As before, $H^i(\theta^i) = h^i(\theta^i,\theta^i)$. Proposition 5 remains almost unchanged, only now it starts with the words "if x, p and T...." Equation (50) remains the same and the RHS of the IC constraint (51) becomes

(61) $$\int_\nu^\eta\int_0^1 \frac{\partial c^i}{\partial \theta^i} dF(\theta^j)d\theta^i \geq \int_\nu^\eta\int_0^1 \frac{\partial c^i}{\partial \theta^i}\Big|_{x=x(\eta,\theta^j),p=p(\eta,\theta^j)} dF(\theta^j)d\theta^i \qquad \text{for all } \eta \text{ and } \nu.$$

Expected welfare EW and the regulator's problem are similar to their counterparts (59) and (60) respectively. The difference between the expressions below and their counterparts is that here q^i is a function of the regulated price as well.

(62) $$EW = E_\theta[S(x,\theta) - \alpha\sum H^i(\theta^i)] = \int_0^1\int_0^1 [S(x,\theta)-\alpha\sum H^i(\theta^i)]dF(\theta^i)dF(\theta^j)$$

$$= \int_0^1\int_0^1 [S(x,\theta)-\alpha\sum \frac{F(\theta^i)}{f(\theta^i)}\frac{\partial c^i}{\partial \theta^i}]f(\theta^i)f(\theta^j)d\theta^i d\theta^j - \alpha\sum H^i(1)$$

(63) Choose x, p and T to maximize

$$EW = \int_0^1\int_0^1 [S(x,\theta)-\alpha\sum \frac{F(\theta^i)}{f(\theta^i)}\frac{\partial c^i}{\partial \theta^i}]f(\theta^i)f(\theta^j)d\theta^i d\theta^j$$

Subject to

$$\int_\nu^\eta\int_0^1 \frac{\partial c^i}{\partial \theta^i} dF(\theta^j)d\theta^i \geq \int_\nu^\eta\int_0^1 \frac{\partial c^i}{\partial \theta^i}\Big|_{x=x(\eta,\theta^j),p=p(\eta,\theta^j)} dF(\theta^j)d\theta^i \qquad \text{for all } \eta \text{ and } \nu.$$

Direct inspection of these expressions yields:

Proposition 6: If $\frac{\partial c^i}{\partial q^i \partial \theta^i}=0$, then optimal regulation of both market shares and prices results in the same prices, qualities and shares as under regulation of market shares alone.

Proof: Since the IC constraint is stated in terms of $\frac{\partial c^i}{\partial \theta^i}$, it may depend on p only through q. But $\frac{\partial c^i}{\partial q^i \partial \theta^i}=0$ assures that it is independent of q and hence of p. Therefore, for any choice of x by the regulator, the optimal p will be selected by pointwise maximization of EW. But when $\frac{\partial c^i}{\partial q^i \partial \theta^i}=0$, then $\frac{\partial EW}{\partial p^i}=0$ is equivalent to $x^i = c_2^i(x^i, q^i, \theta^i)$ implying that, for any x, q and hence p will be the same as in the case of market share regulation. QED

Note that the functional examples considered above, where $c^i(x^i, q^i, \theta^i) = \theta^i x^i + c(x^i)^2 + k(q^i)^2$ with c=0 and c>0, satisfy the condition $\frac{\partial c^i}{\partial q^i \partial \theta^i}=0$ and hence in both of them the ability to regulate prices as well does not add to welfare when regulation of market shares is applied.

When $\frac{\partial c^i}{\partial q^i \partial \theta^i}>0$, higher equilibrium quality levels are associated with higher rents to the firms. If prices are regulated as well, the firms chosen qualities are lower and hence the rents are lower. Thus, in theses cases, supplementing the regulation of market shares with price regulation will increase welfare through more effective rent extraction.

7. Regulation of Price or Market share

Suppose that the regulator can choose between the two methods of regulation considered above: regulating prices while allowing the firms to compete through quality choice, or preventing the competition by direct regulation of the market shares. In Section 3 we compared the performance of these two methods in a perfect information environment and concluded that regulation of market shares outperforms price regulation. This section discusses the relative performance of these methods when the regulator is imperfectly informed.

What makes regulation of market shares preferable in the full information regime is that it does not distort the quality choices of the firms away from their first best levels. This is still true under uninformed regulation. But now efficient quality choice is not the only consideration in selecting a regulatory scheme--the effectiveness in rent extraction from the firms is also important. Thus, the reason that price regulation might be preferred by the regulator is that it might be more effective in extracting rents from the firms.

To simplify the comparison between these two regulatory regimes assume that $\frac{\partial c^i}{\partial q^i \partial \theta^i}=0$ so that, by Proposition 6, regulation of market shares need not be supplemented with price regulation. Consider first the scenario of Section 5.A. where in the quality competition stage the firms know each other's θ^i. From (27) and (50), when $\theta^i=\eta$, firm i's rent is $H^i(\eta)=\int_\eta^1\int_0^1[\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i}+\frac{\partial c^i}{\partial \theta^i}]dF(\theta^j)d\theta^i$ under price regulation, and it is $H^i(\eta) = \int_\eta^1\int_0^1\frac{\partial c^i}{\partial \theta^i}dF(\theta^j)d\theta^i$ under regulation of market shares. These formulae capture the tradeoff in question. If both forms of regulation result in the same market shares and the same quality levels, then the relative size of the informational rents appropriated by the firms depends on the term $\int_\eta^1\int_0^1\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i}dF(\theta^j)d\theta^i$. Since $\frac{\partial q^j}{\partial \theta^i}\leq 0$, other things equal, the informational rents of the firms under price regulation are smaller. But this is not the whole story since, if the market shares are indeed the same under both forms of regulation, the quality choices under market share regulation will be preferable. The following proposition establishes that the resolution of this tradeoff can go in either way depending on the specific parameters of the problem.

Proposition 7: Under the conditions of the scenario of 5.A, total welfare under optimal price regulation can be either higher or lower than under optimal regulation of market shares.

Proof: Recall the example $c^i(x^i,q^i,\theta^i) = \theta^i x^i + c(x^i)^2 + k(q^i)^2$. Since in this example $\frac{\partial q^i(p,\theta)}{\partial \theta^j} = - c/[t(4kt)^2+8kct]$, price regulation has the potential of reducing the firms' rents. Recall from (40)-(42) the magnitudes arising in this example under optimal price regulation (assuming scenario 5.A). For this example, the corresponding magnitudes under regulation of market shares are

$$(64)\qquad x^i_{um} = \frac{1}{2}+\frac{k(\theta^j-\theta^i)}{2kt-1+4kc}+\frac{\alpha k}{2kt-1+4kc}\left[\frac{F(\theta^j)}{f(\theta^j)}-\frac{F(\theta^i)}{f(\theta^i)}\right]$$

$$(65)\qquad q^i_{um} = \frac{1}{4k}+\frac{\theta^j-\theta^i}{4kt-2+8kc}+\frac{\alpha}{4kt-2+8kc}\left[\frac{F(\theta^i)}{f(\theta^i)}-\frac{F(\theta^j)}{f(\theta^j)}\right]$$

Now, straightforward calculations for θ^i uniform on [0,1] yield the following observations. For c=10, t=k=1 and α=0 total welfare under regulation of market shares is higher than under price regulation; for c=10, t=k=1 and α=1 the reverse is true. QED

Notice that α=1 means that the regulator values only the consumers' surplus and assigns no value to profits, while α=0 means that consumers' surplus and firms' profits are weighted equally. It is therefore not surprising that price regulation which is more effective in rent extraction performs better when α is larger. Note that the extreme values of α=0 and 1 were chosen for simplicity of the calculations. By continuity, these relations will hold for values of α between 0 and 1 as well.

It is instructive to note that in the example $c^i(x^i,q^i,\theta^i) = \theta^i x^i + k(q^i)^2$ regulation of market shares always generates higher welfare. In that example, in the price regulation regime, $q^i(p,\theta)=(p^i-\theta^i)/4kt$ so that q^i is independent of θ^j and hence price regulation is not more effective in rent extraction. Since, as we have noted, regulation of market shares produces more efficient quality choices it must generate higher surplus as well. Specifically, we have $\frac{\partial c^i}{\partial q^i}\frac{\partial q^j}{\partial \theta^i}+\frac{\partial c^i}{\partial \theta^i}=\frac{\partial c^i}{\partial \theta^i}=x^i$. Since i's market share under optimal price regulation, x^i_{ur}, is increasing in θ^i, it satisfies the IC constraint (51) and is hence implementable under market share regulation as well. With this allocation the firms will appropriate the same rent under either form of regulation: if $\theta^i=\eta$ firm i's rent will be $\int_\eta^1\int_0^1 x^i dF(\theta^j)d\theta^i$. But since under regulation of market shares the firms will make superior quality choices, this form will clearly yield higher welfare.

In the other scenario (of Section 5.B.), in which the firms do not know each other's cost functions in the quality competition stage, the competition does not have the same rent reducing effect. As was explained in Section 5.B., when firm j does not know θ^i, firm i has a more pronounced incentive to exaggerate its report of θ^i. This is because, in equilibrium, an inflated report of θ^i is assumed correct by firm j and, since $\frac{\partial q^j}{\partial \theta^i}\leq 0$, it induces j to choose a lower q^j thereby relaxing the competition faced by i. Hence, to maintain incentive compatibility, the firms have to be compensated with a larger share of the rent. Indeed, in this scenario, regulation of market shares generates higher welfare.

<u>Proposition 8</u>: In the Scenario of 5.B. in which the firms do not know each other's cost functions when they compete, and under the assumption $\frac{\partial c^i}{\partial q^i \partial \theta^i}=0$, regulation of market shares is the preferred mode.

<u>Proof</u>: Consider the allocation of market shares arising under optimal price regulation. Observe that, since it satisfies the IC constraint of (38) and since $\frac{\partial c^i}{\partial q^i \partial \theta^i}=0$, it satisfies the IC constraint of (54) as well. Therefore, it is implementable under market share regulation. Recall from (37) and (50) that, in both the price and the market share regulation regimes, firm i's rent is given by the expression $H^i(\eta) = H^i(1) + \int_\eta^1\int_0^1 \frac{\partial c^i}{\partial \theta^i} dF(\theta^j)d\theta^i$. This together with $\frac{\partial c^i}{\partial q^i \partial \theta^i}=0$ means that the aforementioned allocation of market shares will yield the firms the same rents under both regulation regimes. Since, for any given allocation of market shares, the firms make superior quality choices under regulation of market, this form of regulation will generate higher welfare

from that allocation of market shares. Therefore, the optimal regulation of market shares, which yields even higher surplus, is clearly superior to the optimal price regulation. QED

Note that both of the examples analyzed above, $c^i(x^i,q^i,\theta^i) = \theta^i x^i + k(q^i)^2$ and the example $c^i(x^i,q^i,\theta^i) = \theta^i x^i + c(x^i)^2 + k(q^i)^2$ satisfy $\frac{\partial c^i}{\partial q^i \partial \theta^i}=0$, and hence are covered by this proposition.

One point worth noting is that, in the cases discussed above, price competition might perform better only in the environment of 5.A, where in the quality competition stage firms have better information about each other's costs than the regulator has when it makes the decisions. In a sense by letting the firms compete in the later stage the regulator is able to exploit their superior information in that stage to extract rents.

8. Conclusion

In regulating oligopolies the regulator has to decide on the framework within which the interaction will take place: how to use the available regulatory measures to harness the competitive forces in a manner that promotes the social objectives. The above analysis considered two alternative regulatory instruments which seem natural in the environment we model: Price regulation and assignment of exclusive market shares. It clarified the nature of optimal regulation using these instruments. By means of examples it examined how each of these instruments functions with respect to the dual objective of inducing efficient quality provision and extracting informational rents. The analysis also identified circumstances under which one of these instruments might be superior to the other.

There are of course many important aspects that were neglected and which might be useful to pursue in further research. For instance, we did not consider a mixture of the two alternative modes. That is, allowing the regulator to select the form of regulation on the basis of the reported θ's.

Appendix

Proof of Proposition 4:

Part (i): Let $C^i(p,\theta^i,\theta^j)=c^i[x^i[p,q(p,\theta)],q(p,\theta),\theta]$. That is, $C^i(p,\theta^i,\theta^j)$ is the cost incurred by i when the prices are p and the cost parameters are $\theta=(\theta^i,\theta^j)$. Observe that

$$h^i(\eta,\eta)-h^i(\eta,\nu) =$$

$$\int_0^1\{\frac{p^i}{2t}[q^i(p,\eta,\theta^j)-q^i(p,\nu,\theta^j)-(q^j(p,\theta^j,\eta)-q^j(p,\theta^j,\nu))]-[C^i(p,\eta,\theta^j)-C^i(p,\nu,\theta^j)]\}dF(\theta^j)$$

where $p=p(\eta,\theta^j)$. Similarly, $h^i(\nu,\eta)-h^i(\nu,\nu)$ is given by the same expression but with $p=p(\nu,\theta^j)$.

It follows from the IC constraint that

$$h^i(\eta,\eta)-h^i(\eta,\nu) \geq H^i(\eta) - H^i(\nu) \geq h^i(\nu,\eta)-h^i(\nu,\nu)$$

Dividing through by $\eta-\nu$ and taking the limits of the RHS and LHS as ν approaches η we get that where p is continuous at η, these limits are equal. Therefore, H^i is differentiable at η and the derivative is equal to these limits,

$$\frac{dH^i(\eta)}{d\eta}=\int_0^1[\frac{1}{2t}[p^i-\frac{\partial c^i}{\partial x^i}][\frac{\partial q^i}{\partial\theta^i}-\frac{\partial q^j}{\partial\theta^i}]-\frac{\partial c^i}{\partial q^i}\frac{\partial q^i}{\partial\theta^i}-\frac{\partial c^i}{\partial\theta^i}]dF(\theta^j)=-\int_0^1[\frac{1}{2t}(p^i-\frac{\partial c^i}{\partial x^i})\frac{\partial q^j}{\partial\theta^i}+\frac{\partial c^i}{\partial\theta^i}]dF(\theta^j)=-\int_0^1[-$$

where $p=p(\eta,\theta^j)$, $q=q(p,\eta,\theta^j)$ and $\theta^i=\eta$. Expression (27) is now obtained by integration.

Part (ii): IC is equivalent to

$$H^i(\nu)\equiv h^i(\nu,\nu)\geq h^i(\eta,\nu)=h^i(\eta,\eta)-[h^i(\eta,\eta)-h^i(\eta,\nu)]\equiv H^i(\eta)-[h^i(\eta,\eta)-h^i(\eta,\nu)],$$

which is in turn equivalent to

$$H^i(\nu) - H^i(\eta) \geq h^i(\eta,\eta)-h^i(\eta,\nu).$$

From (27), $H^i(\nu) - H^i(\eta)$ is equal to the LHS of (28). By direct derivation, $h^i(\eta,\eta)-h^i(\eta,\nu)$ is equal to the RHS of (28).

Part (iii): $h^i(1,\theta^i) - H^i(1) = E_{\theta^j}\{\Pi^i[p(1,\theta^j),\theta] - \Pi^i[p(1,\theta^j),1,\theta^j]\} =$

$$\int_0^1 [\int_\eta^1 \frac{\partial \Pi^i(p(1,\theta^j),\theta)}{\partial \theta^i} d\theta^i] dF(\theta^j)$$

Now $\frac{\partial \Pi^i(p(1,\theta^j),\theta)}{\partial \theta^i} = [p^i(1,\theta^j) - \frac{\partial c^i}{\partial x^i}] \frac{\partial x^i}{\partial q^j} \frac{\partial q^j}{\partial \theta^i} - \frac{\partial c^i}{\partial \theta^i}$. From total differentiation of (24), $\frac{\partial q^j}{\partial \theta^i} \leq 0$. Therefore, $\frac{\partial \Pi^i(p(1,\theta^j),\theta)}{\partial \theta^i} \leq 0$ implying $h^i(1,\theta^i) \geq H^i(1)$. This together with IC yields $H^i(\theta^i) \geq H^i(1)$ and so $H^i(1) \geq 0$ is equivalent to IR. QED

<u>Proof of Proposition 5</u>:

Part (i): The proof of this part is analogous to the proof of part (i) of Proposition 4. It follows from the IC constraint that

$h^i(\eta,\eta) - h^i(\eta,\nu) \geq H^i(\eta) - H^i(\nu) \geq h^i(\nu,\eta) - h^i(\nu,\nu)$

Dividing through by $\eta - \nu$ and taking the limits of the RHS and LHS as ν approaches η we get that where x is continuous at η, these limits are equal. Therefore, H^i is differentiable at η and the derivative is equal to these limits $\frac{dH^i(\eta)}{d\eta} = -\int_0^1 [x^i - \frac{\partial c^i}{\partial q^i} + \frac{\partial c^i}{\partial \theta^i}] dF(\theta^j) = -\int_0^1 \frac{\partial c^i}{\partial \theta^i} dF(\theta^j)$, where $x = x(\eta,\theta^j)$, $q = q(x,\eta,\theta^j)$ and $\theta^i = \eta$. Expression (50) is now obtained by integration.

Part (ii): The proof of this part is identical to that of part (ii) of Proposition 4, except that the references to (27) and (28) should be replaced by references to (50) and (51).

Part (iii): Since $\frac{\partial c^i}{\partial \theta^i} \geq 0$, H^i is decreasing and hence the result.

<u>Footnotes</u>

1. The sizable literature on this topic precludes mention of arbitrary selection of articles. The reader is referred to the recent text by Laffont and Tirole (1993) and the references therein.

2. This part extends the ideas of an earlier contribution by Shleifer (1985).

3. Suppose that consumers are distributed over the interval with density h(x). Let $H^i(x)$ denote the total mass of consumers in the sub-interval whose endpoints are x and i (i.e., the integral of h(x) over this sub-interval). The first order condition for i's profit maximization are
$H^i(x^i) - [p_i - c^i_1(H^i(x^i),q^i)]h(x^i)/2t=0$ and $x^i - c^i_2(H^i(x^i),q^i) = 0$ $\quad$ i=0,1.
The first order condition for surplus maximization with respect to market shares is

$$V+q^i-tx^i - c^i_1 = 0 \qquad i=0,1.$$

Together these conditions imply that, if the allocations of profit maximum and surplus maximum coincide, then $H^i(x^i)=H^j(x^j)$. If the two cost functions are the same, these conditions further imply that $x^i=1/2$. But these two implications, that the median point and halfway point coincide, can be again regarded as special.

4. Note that T^i is a function of σ^i alone. Since T^i enters directly only the payoff of i and since the firms are expected profit maximizers, there is no loss involved in making this assumption.

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