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INTER-TEMPORAL COST ALLOCATION AND MANAGERIAL INVESTMENT INCENTIVES

by

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Inter-Temporal Cost Allocation and Managerial Investment Incentives:
A Theory Explaining the Use of Economic Value Added as a Performance Measure

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Abstract

This article considers a principal agent model of the relationship between the shareholders and manager of a firm where there are two incentive problems. Shareholders delegate an investment decision to the manager because he has better information than shareholders to base this decision on. The manager also exerts an unobservable level of effort each period which increases the firm's cash flows. The firm calculates period-by-period income for itself by allocating the investment cost to periods that benefit from the investment, and then bases the manager's wage contract on the firm's accounting income. The main result is that there exists a unique allocation rule such that it is always the case that the manager will make the efficient investment decision for any income-based wage contract that shareholders choose, so long as the wage contract is weakly increasing in income. Thus the investment delegation problem is completely solved in a robust simple way, and shareholders are left with an enormous number of degrees of freedom to attempt to deal with the moral hazard problem. The allocation rule that creates this desirable behavior turns out to create an income measure which is usually referred to as a residual income or economic value added (EVA). Therefore this paper both provides a theory of why firms may use income as a performance measure for management and how income should be calculated for this purpose.

1. Introduction

An important aspect of many mangers' jobs is making investment decisions which will affect cash flows in multiple future periods. Since managerial compensation is typically based on accounting income (Antle and Smith 1986, Lambert and Larcker 1987, Rosen 1992), managers can generally affect their future compensation by altering investment levels. The natural question which arises in this context is whether managers' private incentives to choose investment levels result in efficient investment levels from the perspective of shareholders. A frequently expressed concern is that managers may be too impatient and thus may under-invest relative to the efficient level, either because their personal cost of capital is higher than the firm's, or because they have a shorter time horizon than the firm (i.e., they plan to leave or retire before all the benefits of the investment are realized.)

One technique that firms use to help combat this potential distortion is to base managerial compensation on accounting measures of income created by allocating investment expenditures to the future periods that benefit from the investment. The intuitive justification for this procedure is that matching costs to benefits creates a more "accurate" measure of income on a period-by-period basis and thus reduces distortions caused by the fact that managers may not compare cash flows across time correctly (Dechow 1994). Although this intuition is appealing, it is obviously incomplete. The intuition does not precisely explain why such a procedure would work. Nor does it explain what allocation rule should be used and how the choice of allocation rule should depend on factors such as the time pattern of benefits from the investment, shareholders' discount rate, the manager's discount rate, or the nature of wage contracts in place across various periods. It is intuitively clear that all of these factors might enter the analysis.

Firms typically think of themselves as choosing an allocation method for investment expenditures by choosing a depreciation rule and an interest imputation rate. A depreciation rule is simply a rule which assigns a share of the original investment cost to each period of the asset's life with the property that the shares sum to the total investment cost. The share assigned to a period is referred to as that period's depreciation. The total investment cost allocated to any period is set equal to that period's depreciation plus an imputed interest cost calculated by multiplying the interest imputation rate by the remaining (non-depreciated) book value of the investment.

Many firms have traditionally used "operating income" as a performance measure for management. This amounts to using an interest imputation rate of zero, i.e., no interest is imputed. There are a small number of commonly used depreciation rules. Some firms assign an equal share of depreciation to each period of the asset's life. This is called the straight-line method. Most other commonly used depreciation rules are accelerated relative to the straight line rule, in the sense that depreciation occurs more quickly.

An alternative performance measure used by some firms is "residual income" (Horngren and Foster 1987, Kaplan 1982). This amounts to using an interest imputation rate equal to the firm's cost of capital. The depreciation rule is generally still selected from the same small group of commonly used rules used to create operating income. In the last three or four years, there has been an explosion of interest in this method and a great increase in the extent to which it is used. Management consulting companies have renamed this performance measure "economic value added" (EVA) and have quite successfully marketed it as important new way of creating better investment incentives for managers. Fortune magazine, for example, has run a cover story on EVA, extolling its virtues and listing a long string of major companies that have adopted it (Tully 1993). [1]

From the standpoint of real firms, then, these appear to be the important questions regarding the effect of investment allocation rules on investment incentives:

(1) How does the choice of depreciation rule and interest imputation rate affect managerial investment incentives?

(2) Is there an "optimal" depreciation rule and interest imputation rate, and how is this choice affected by factors such as the time pattern of benefits from the investment, shareholders' discount rate, the manager's discount rate, the manager's level of risk

aversion, etc.?

The purpose of this paper is to provide a theory which answers these questions.

This paper constructs a model where the manager is better informed than shareholders about investment opportunities, and shareholders therefore delegate the investment choice to the manager. The manager also exerts an unobservable level of effort each period which increases the firm's cash flows. The "problem" with this situation is that the two incentive problems generally interfere with one another. That is, wage contracts designed to deal with the moral hazard problem will generally distort the agent's investment decision. The main result of this paper is to show that a very large class of contracts exist which dramatically simplify this problem but which still allow shareholders to achieve a high level of expected utility. Suppose that, instead of basing the manager's wage contract on completely disaggregated accounting data, the firm calculates period-by-period income for itself by allocating the investment cost to periods that benefit from the investment, and then bases the manager's wage contract on the firm's accounting income. It is shown that a unique allocation rule exists that always induces the manager to make efficient investment decisions, so long as the wage contract is weakly increasing in income. Thus the investment delegation problem is completely solved and shareholders are left with an enormous number of degrees of freedom to attempt to deal with the moral hazard problem.

Shareholders require essentially no information about the manager's preferences in order to construct this allocation rule. In particular, they do not need to know the manager's own personal discount rate nor do they need to know anything about his attitude towards risk. The result does not depend on particular functional form assumptions or on the existence of a very structured environment where a one-dimensional "type" describes the nature of uncertainty. Therefore, unlike many theoretical agency models, this model yields a very robust result that one could imagine being used by real firms in real situations, without any further alteration or adaption.

This allocation rule is created by choosing an interest imputation rate equal to the firm's cost of capital and choosing the depreciation rule so that the <u>total</u> investment cost allocated to each period (i.e., depreciation plus imputed interest on remaining book value) remains constant across periods. Thus the current wave of enthusiasm for using an interest imputation rate equal to the firm's cost of capital seems to be justified. However, as will be shown, insufficient attention has been paid to the issue of what depreciation rule to use.

The result that the above depreciation rule induces efficient investment is derived in a model where the investment is assumed to remain equally productive over its lifetime. This assumption is also relaxed to allow the productivity of the investment to vary over its lifetime. In this case there is still a unique allocation rule that always induces the manager to make efficient investment decisions. The interest imputation rate is still set equal to the firm's cost of capital. The depreciation rule is set so that the the total cost allocated to each period is proportional to the relative productivity of the asset in each period. This rule can therefore be viewed as being consistent with a version of the "matching principle" from accounting that states that costs should be allocated across objectives in proportion to the benefits that the costs create across objectives.

The basic economic idea underlying this paper can be understood from this viewpoint. It is shown that by using the matching principle to allocate investment costs, the firm can essentially "annuitize" the manager's problem. That is, the firm can create a situation where every period creates the same investment incentive, i.e., the firm can create a situation where, even if the manager only cared about wages in a single period, he would choose the efficient investment level and this is true for every period. In such a case, the way the manager values cash flows across periods becomes completely irrelevant to determining his investment choice. In particular, the manager makes the efficient investment decision regardless of his own personal discount rate.

Therefore this paper both provides a theory of why income may be used as a performance measure for management and how income should be calculated for this purpose. Income is used as a performance measure to guarantee in a simple robust way that managers will make efficient input decisions. When the input is an investment good and benefits multiple future periods, it is important that the costs be allocated across periods in proportion to the benefits they produce and that the discounted sum of the cost allocations, using the shareholders' cost of capital, be equal to the total investment cost. This essentially "annuitizes" the problem from the manager's perspective, and creates an incentive for the manager to choose the efficient investment level, no matter how he values wage payments across periods.

The result of this paper in also very relevant to the large ongoing debate in the economics, finance, and accounting literatures on whether, from a theoretical perspective, managerial compensation ought to be tied more closely to accounting income or stock market price. A large empirical literature has documented the fact that managerial compensation is closely tied to accounting measures of income, and, in fact, that managerial compensation is probably more closely tied to accounting measures of performance than to stock market measures of performance (Antle and Smith 1986, Jensen and Murphy 1990, Lambert and Larcker 1987, Rosen 1992). This result has been viewed as somewhat puzzling and counterintuitive by many economists. One of the main reasons for this is the intuition that, by basing managerial compensation on the firm's stock market value, shareholders can clearly solve the investment incentive problem in a simple robust way. Furthermore, they should still be able to address the moral hazard problem of inducing managerial effort by basing managerial compensation on stock market value. If shareholders can completely solve the investment incentive problem in a simple robust way and are still left with a large number of degrees of freedom to address the moral hazard problem, why don't we observe compensation contracts in the real world that are much more closely tied to stock market performance than to accounting measures of performance? The answer suggested by this paper's result is that basing managerial compensation on accounting income can provide an equally simple and robust solution to the investment incentive problem. Therefore, from a theoretical perspective, consideration of the investment incentive problem does not necessarily suggest anything about the relative desirability of basing managerial compensation on stock market vs. accounting measures of performance. In particular, then, if basing managerial compensation on accounting measures of performance had some other advantage, we might expect to observe in the real world that managerial compensation is more closely tied to accounting measures of performance than to stock market measures of performance. The literature has, in fact, suggested that such advantages may exist.[2]

Section 2 of the paper introduces the basic model and section 3 analyzes it. In these early sections, the firm is viewed as directly choosing an allocation rule rather that directly choosing a depreciation rule and interest imputation rate (which in turn generate an allocation rule.) Section 4 reinterprets the results when the firm is viewed as directly choosing a depreciation rule and interest imputation rate. Section 5 generalizes the model to allow the productivity of the investment to vary over time. Section 6 relates this paper's results to other papers exploring investment incentives and the use of residual income as a performance measure. Finally, section 7 draws brief conclusions.

2. The ModelA. The Basic Model

The relationship between the owners of the firm and the manager will be modeled as a principal agent relationship. The terms "owners of the firm" and "principal" will be used interchangeably. Similarly, the terms "manager" and "agent" will be used interchangeably.

Suppose there are T+1 periods indexed by $t \in \{0, 1, \ldots, T\}$. The firm will conduct business and realize cash flows from conducting business during periods 1 through T. Let $z = (z_1, \ldots, z_n)$ denote the vector of cash flows received by the firm from conducting business. Before beginning business, the firm must choose a level of investment, x, in period 0. The level of investment chosen in period 0 will affect the value of cash flows in future periods. For example, purchasing a machine may reduce per-period expenditures on labor. The accounting system of the firm is able to directly measure x and z. It is also assumed that, in each of periods 1 through T, the agent exerts an unobservable level of effort which affects the firm's cash flow during that period. Formally, let e_t denote the agent's effort choice in period t and let $e = (e_1, \ldots, e_T)$ denote the vector of all the agent's effort choices.

Assume that the manager is potentially better informed than shareholders both about his own preferences and the marginal productivity of investment. Formally, assume that before the beginning of the relationship, that a state of nature, θ, is drawn from some set Θ according to the density $g(\theta)$. The agent directly observes θ but the principal does not. For purposes of thinking about the model, we would generally expect θ to be multidimensional. It contains information about both the marginal productivity of capital and the agent's preferences, and information about either of these could be very complex.

Period t cash flow is therefore affected by the state of nature, the investment level, and the agent's effort choice in period t. It will be initially assumed that the investment lasts all T periods and remains equally productive over its entire lifetime.[3] Formally, assume that period t cash flow is determined by

$$z_t = \delta(x, \theta) + \varepsilon_t \quad (2.1)$$

where $\delta(x, \theta)$ is an increasing function of x for every θ and ε_t is a random variable affected by effort according to the density f_t (ε_t / e_t).

Assume that the principal is risk neutral and has a cost of capital of $r^* \in [0, \infty)$. Since ε_t and x are additively separable in (2.1), the investment level that maximizes expected discounted cash flows can be determined independently of effort levels. The investment level that maximizes expected discounted cash flows for the firm is the level that maximizes

$$\text{(2.2)} \qquad \alpha(r^*)\ \delta(x,\theta) - x$$

where $\alpha(r)$ denotes the discounted value of receiving one dollar per period over periods 1 through

$$\text{(2.3)} \qquad \alpha(r) = \sum_{t=1}^{T} 1/(1+r)^t$$

T, using the interest rate r and is given by
Assume that for every $\theta \in \Theta$, that $\delta(x,\theta)$ is continuously differentiable in x, strictly increasing in x, strictly concave in x, and that its first derivative with respect to x assumes all values in the range $(0, \infty)$ as x varies over $[0, \infty)$. These assumptions are sufficient to guarantee that for every θ there exists a unique value of x that maximizes (2.2) and that it is determined by the first-order condition

$$(2.4) \qquad \delta_x(x,\theta) = 1/\alpha(r^*).$$

This will be called the efficient investment level given θ and will be denoted by $x^*(\theta)$.

Let $w = (w_1, \ldots, w_T)$ denote a vector of wage payments the agent receives in periods 1 through T. Let $u(w, e, \theta)$ and $\underline{u}$ denote the agents expected utility function and reservation utility.

The principal hires the agent at the beginning of period 0 to choose a level of investment in period 0 and then exert effort in each of periods 1 through T. The principal delegates the investment decision to the agent because the agent has better information regarding the level of investment that would be efficient. A contract specifies the wage the agent will be paid in each of periods 1 through T as a function of the agent's investment choice in period 0 and all cash flows that have been observed through the end of each period. Such a contract will be called a disaggregated data based (dd-based) contract, to connote the fact that it is based on all available accounting data in a disaggregated form. Therefore, a period t dd-based contract is a real valued function $\phi_t(x, z_1, \ldots, z_t)$, giving the wage the agent will be paid conditional on observing $(x, z_1, \ldots, z_t)$. A dd-based contract is a function from (x,z) to R^T denoted by $\phi(x, z) = (\phi_1, \ldots, \phi_T)$.[4]

To complete the description of the basic model, the order of play and the information of each player at each stage will now be briefly reviewed. At the beginning of period 0, nature has already drawn θ. Only the agent is able to directly observe θ. The principal and agent both know the entire structure of the model, including $\delta(x, \theta)$, f_t (ε/e), $u(q,e, \theta)$, $\underline{u}$, and $g(\theta)$. The principal offers the agent a dd-based contract $\phi(x,z)$. If the agent rejects the contract, the relationship is over and the agent receives $\underline{u}$ utils. If the agent accepts the contract, the agent then chooses x in period 0. For each of periods 1 through T, the agent chooses the effort level that he will exert at the beginning of the period and then that period's cash flow is determined according to (2.1). The agent is paid a wage at the end of each period as specified by the contract.

An optimal contract is defined as follows. For every contract, the principal is able to predict the agent's behavior conditional on θ (where "behavior" includes whether the agent will accept the contract or not, as well as the agent's investment and effort decisions conditional on accepting the contract.) Therefore, for any contract, the principal can calculate his expected discounted cash flow, taking the agent's predicted behavior into account. The optimal contract is the contract that maximizes the principal's expected discounted cash flow.

B. Income-Based Contracts and Allocation Rules

In the above model, a contract can be made a function of completely disaggregated accounting data. That is, the agent's wage payment each period can be made to depend on the agent's investment choice and all cash flows observed up until the end of that period, and each of these variables can enter as a separate argument in the wage function. In reality, firms do not typically consider such a broad class of contracts. Typically, they will aggregate accounting data to calculate income on a period-by-period basis. Then wage payments are based only on current and possibly historic income levels. In the context of this model, completely disaggregated accounting data is a vector (x, z) in R^{T+1}. An income measure would be a vector $y = (y_1, \ldots, y_T)$ in R^T where y_t denotes the firm's income in period t. The T+1 dimensions of information are aggregated into T dimensions by allocating the investment cost to the periods that benefit from it. Notation to formally describe this will now be introduced.

Define an allocation rule to be a vector of real numbers $a = (a_1, \ldots, a_T)$ where a_t denotes the investment cost allocated to period t for every dollar of investment. That is, if x dollars are invested, then a cost of a_t x dollars is allocated to period t.[5] Formally, let I_t (x, z, a) denote the function determining the firm's period t accounting income conditional on the investment level x, cash flows z, and allocation rule a. It is given by

$$(2.5) \qquad I_t\ (x,\ z,\ a) = z_t - a_t\ x.$$

Let $I(x,\ z,\ a) = (I_1, \ldots, I_t)$ denote the function determining the entire vector of incomes.

An income-based contract specifies the wage the agent will be paid each period as a function of current and possibly historic accounting incomes. Therefore a period t income-based contract is a function $\psi_t\ (y_1, \ldots, y_t)$ giving the wage the agent will be paid if the firm's income in periods 1 through t is given by $(y_1, \ldots, y_t)$. An income-based contract is then a function from R^T to R^T, denoted by $\psi(y) = (\psi_1, \ldots, \psi_T)$.

It is clear that the principal can create a dd-based contract by choosing an allocation rule to define income and an income-based contract. Formally, a dd-based contract will be said to be created by $(a,\ \psi)$ if

$$(2.6) \qquad \phi(x,y) = \psi(\ I(x,y,a)\ ).$$

In the context of this paper's model, the practice of real firms is to choose an allocation rule for investment expenditures based on observable characteristics of the investment such as its useful lifetime, and to restrict themselves to choosing a dd-based contract that can be induced by this allocation rule and some income based contract. The goal of this paper is to explain why such a practice might be desirable, and to explain how the allocation rule should be chosen. This will provide a theory of why income is used as a performance measure and explain how the allocation rule should be selected to calculate income.

Before beginning the analysis, it will be useful to describe one property of allocation rules. In the single period context, where costs are allocated between multiple products produced in the same period, we usually think of an allocation rule as "completely" allocating a cost if the allocation shares sum to one. In the multi-period case considered by this paper, the natural analogue is having the discounted allocation shares sum to one. For an interest rate, r, an allocation rule will be said to be complete with respect to r, if the discounted value of the allocation shares sum to one. Formally, a is complete with respect to r if

$$(2.7) \qquad \sum_{t=1}^{T} \frac{a_t}{(1+r)^t} = 1 .$$

Straightforward algebra shows that, for every r, there is a unique allocation rule such that the allocation rule is complete with respect to r and the allocation share remains constant across periods. This will be called the r-annuity allocation rule and will be denoted by $a^r = (a_1^r, \ldots, a_T^r)$. Formally, the r-annuity allocation rule is given by

$$(2.8) \qquad a_t^r = 1/\alpha(r)$$

where, recall that $\alpha(r)$ is defined by (2.3).

3. Income Based Contracts and Inducing Efficient Investment

The standard approach of the formal incentives literature to analyzing the problem described above is to calculate the optimal contract and then attempt to say something interesting about it. The major problem that economists have experienced when employing this approach is that, from an applied standpoint at least, it is often the case that nothing of much interest can be said. In problems such as this where the agent makes decisions based upon private information, in order to be able to analytically solve the problem with existing methods, it must generally be assumed that the agent's private information is one-dimensional and that some type of single crossing condition is satisfied so that the various "types" can be induced to sort themselves. Even given all the structure that is generally assumed, the nature of the calculations is still extremely complex. Furthermore, the nature of the optimal contract is highly dependent on the particular functional form assumptions made about preferences, the nature of uncertainty, etc. Small changes in any of these assumptions might cause quite large changes in the nature of the optimal contract. In the real world, where the principal's information is always somewhat "fuzzy," uncertainty occurs over more than a single dimension of information, and the principal has limited computational abilities, the type of solution provided by the standard approach therefore often does not seem to shed much light on real behavior and practices.

This paper will adopt a different approach to analyzing this incentive contracting problem that is more consistent with the view that simplicity, robustness, and ease of calculation play an important role in determining the types of contracts that principals and agents actually use. The nature of asymmetric information will be left perfectly general. In particular, then, the possibility that the principal has poor information about the agents preferences as well as poor information about the marginal productivity of investment will be allowed for. Rather than calculate a single optimal contract, the main result will be to show that there exists a large class of very simple contracts that always induce the agent to choose the efficient investment level. Thus by restricting himself to this set of contracts, the principal can guarantee in a simple robust way that the investment problem is completely solved and he is still left with an enormous number of degrees of freedom to attempt to solve the moral hazard problem.

More specifically, the main result is to show that if the principal uses the r*-annuity allocation rule to define income, that the agent will <u>always</u> (i.e., for every state of nature θ) choose the efficient investment level for any income-based wage contract such that wages are weakly increasing in income. Furthermore, the r*-annuity allocation rule is the unique allocation rule with that exhibits this property.

The result therefore can be interpreted as providing a theory that explains why firms might choose to base managerial compensation on an accounting measure of income and how the allocation rule should be chosen for calculating accounting income. By restricting himself to considering contracts that can be implemented by using the r*-annuity allocation rule and a weakly monotone income-based contract, the principal automatically guarantees in a very robust way with no further calculations that the investment decision problem is completely solved. Then he can select an income-based wage contract from the entire class of monotone income-based contracts to find the best possible solution to the moral hazard problem. Such an approach might be particularly natural in a world of bounded rationality, where the principal was constantly monitoring the performance of existing contractual arrangements and then attempting to make incremental improvements. It might be very sensible for such a principal to restrict himself to using the r*-annuity allocation rule and income-based wage contracts. He would always induce the agent to invest efficiently and could adjust the sharing ratio of the wage contract over time to induce more or less effort as seemed appropriate.

Of course simplicity, robustness, and ease of calculation are not sufficient ends in and of themselves. It is also important to consider the level of welfare that the principal can achieve by restricting himself to this set of contracts. On an intuitive level, it seems that the main constraint placed on the principal by restricting himself to this set of contracts, is that the principal must select a contract that induces the agent to select the efficient investment level. (Recall that all contracts in this set induce the agent to select the efficient investment level.) In simplified versions of this model where the agent is assumed to have a one-dimensional type and the standard techniques can be used to solve for the optimal contract, in general, it may be optimal for the principal to induce at least some distortion in the agent's investment decision in order to gain extra leverage on the moral hazard problem. However, this is precisely the type of contract that is non-robust to slight changes in the environment and that is extremely complex. In a wide variety of circumstances, it may be that "settling" for inducing the efficient investment choice may be a relatively small price to pay for achieving a robust simple solution to the investment incentive problem. Whether it can be formally shown in some sense that the optimal contract lies within this set of simple contracts if the principal's information is imprecise enough is an interesting question for future research that is beyond the scope of this paper.[6] This paper will restrict itself to simply showing that all contracts within this set induce the efficient investment level.

Notation and definitions necessary to formally state the main result will now be introduced. Recall that an income-based contract, ψ, determines each period's wage as a function of current and historic periods' accounting incomes. An income-based contract ψ will be called weakly increasing if every period's wage is weakly increasing in current and historic accounting incomes. Formally, ψ is weakly increasing if ψ_t $(y_1, \ldots, y_t)$ is weakly increasing in y_i for every $t \in \{1, \ldots, T\}$ and $i \in \{1, \ldots, t\}$.

For any dd-based contract, ϕ, it will be said that ϕ induces efficient investment if for every possible θ and every possible strategy for effort choice, the agent maximizes his expected utility by choosing the efficient investment level. Some extra notation needs to be introduced to formally define this concept. The effort level chosen by the agent in any period can be made a function of observed cash flows in all preceding periods. Thus an effort strategy for the agent can be denoted by a vector of functions $s = (s_1, \ldots, s_T)$ where s_1 is a constant function and s_t is a function of $(z_1, \ldots, z_{t-1})$ for every $t \in \{2, \ldots, T\}$. Let $U(x, s, \theta, \phi)$ denote the agents expected utility given that nature has drawn θ, the agent accepts the contract ϕ, the agent chooses the investment level x, and makes the effort decision s.[7] Formally, a contract ϕ will be said to induce efficient investment if

(3.1) $$x^*(\theta) \in \underset{x}{\operatorname{argmax}}\, U(x, s, \theta, \phi) \qquad \text{for every } s \text{ and } \theta.$$

Now the main property of interest will be defined. An allocation rule will be said to induce efficient investment if the dd-based contract created by using any weakly increasing income-based contract induces efficient investment. Formally, an allocation rule, a, induces efficient investment if, for every weakly increasing income-based contract ψ, the dd-based contract created by (a, ψ) induces efficient investment.

Proposition 3.1 now presents the result that the r*-annuity allocation rule is the unique allocation rule that always induces efficient investment.

Proposition 3.1:

(i) Suppose that u(w, e) is weakly increasing in w_t for every $t \in \{1, . . . , T\}$. Then the r*-annuity allocation rule induces efficient investment.

(ii) The r*-annuity allocation rule is the only allocation rule that induces efficient investment for every u(w, e) such that u is weakly increasing in w_t for every $t \in \{1, . . . , T\}$.

proof:

First, (i) will be proven. For any allocation rule, a, period t accounting income for the firm is defined by

$$(3.2) \qquad y_t = \delta(x, \theta) - a_t x + \varepsilon_t.$$

The r*-annuity allocation is defined by (2.8). Substitution of (2.8) into (3.2) yields

$$(3.3) \qquad y_t = \{ \delta(x, \theta) - x/\alpha(r^*)\} + \varepsilon_t .$$

Note that the choice of x has exactly the same effect on each period's accounting income which is given by the bracketed term in (3.3). Recall that $x^*(\theta)$ is the unique maximizer of the bracketed term. This means that, for every θ, the distribution over y induced by $x^*(\theta)$ first-order stochastic dominates the distribution over y induced by any other choice of x. By assumption, the agent's expected utility function is weakly increasing in wage income. Because the wage function is assumed to be weakly increasing, this means that the agent will always prefer one distribution of y over another if the former first-order stochastic dominates the latter. In particular, then, the agent weakly prefers the income distribution generated by choosing $x^*(\theta)$ to the income distribution generated by any other choice of x.

Now (ii) will be proven. Suppose that an allocation rule, a, induces efficient investment for every utility function that is weakly increasing in wage income. In particular, choose a utility function which is strictly increasing in every period's wage income. Now choose any $t \in \{1, . . . , T\}$. Consider an income-based wage contract such that the wage in all periods except period t is constant and the period t wage depends only on the firm's period t accounting income and is strictly increasing in period t accounting income. Period t accounting income for the firm is given by (3.2). Just as argued above, the investment choice given by

(3.4) $\delta_x (x, \theta) - a_t = 0$

produces a distribution over y_t that first-order stochastic dominates the distribution over y_t produced by any other x. Therefore, the unique optimal investment choice for the agent is given by (3.4). However, by assumption, the allocation rule a induces the efficient investment choice given by (2.4). Equations (2.4) and (3.4) imply that a must be the r*-annuity allocation rule.QED

The intuition for this proposition is very simple. The r*-annuity allocation rule essentially "annuitizes" the problem from the agent's perspective. By matching costs to benefits, the effect of investment on each period's income is the same. For any period, the agent causes the distribution of income to shift maximally to the right by choosing the efficient investment level. Since this is true for any period, it is optimal for the agent to choose the efficient investment level regardless of how he compares cash flows across periods.

It is straightforward to show that the uniqueness result in part (ii) of Proposition 3.1 can be considerably strengthened. Part (ii) states that the r*-annuity allocation rule is the unique allocation rule that induces efficient investment over the set of all managerial utility functions such that utility is weakly increasing in each period's wage. This is very large set. As formally stated, part (ii) of Propostion 3.1 therefore leaves open the possibility that there are other allocation rules that induce efficient investment over sets of managerial utility functions which are still quite large. In fact, the uniqueness result can be proven for much smaller sets of managerial utility functions. An earlier version of this paper (Rogerson 1993), defines a property called the "full spanning condition" and shows that the r*-annuity allocation rule is the unique allocation rule that induces efficient investment over any set of managerial utility functions satisfying the "full spanning condition." The "full spanning condition" is very weak. For example, it is satisfied by the set of utility functions corresponding to the case where the manager is risk-neutral and his discount rate is known to be chosen from some interval. Thus, in real situations, we would generally expect that the set of possible utility functions for the agent is large enough that the r*-annuity allocation rule is the only allocation rule that induces efficient investment for every possible utility function.

4. Depreciation Rules and Interest Imputation Rates

As previously mentioned, firms typically think of themselves as choosing an allocation rule by directly choosing a depreciation rule and interest imputation rate. This section will now reinterpret the paper's results from this viewpoint. Part A will introduce notation to describe depreciation rules and interest imputation rates. Part B will then identify the unique depreciation rule and interest imputation rate that generate the r^*-annuity allocation rule. Part C will compare this solution to actual practices of firms. Part D will report comparative statics results that show whether current practices generally induce over-investment or under-investment.

A. Notation

A depreciation rule is defined to be a vector $d = (d_1, \ldots, d_T) \in D$ where D is the set of all vectors in R^T that sum to one. Interpret d_t as the share of depreciation allocated to period t. That is, if x dollars are invested in period 0, then $d_t x$ dollars of depreciation are assigned to period t.

Let $r \in [0, \infty)$ denote the interest imputation rate. Therefore the set $D \times [0, \infty)$ is the set of all depreciation rule interest rate pairs. Let $\eta(d, r) = (\eta_1(d, r), \ldots, \eta_T(d, r))$ denote the allocation rule generated by (d,r). This is formally given by

$$(4.1) \qquad \eta_t = d_t + r\, b_t$$

where

$$(4.2) \qquad b_1 = 1$$

$$(4.3) \qquad b_t = 1 - \sum_{j=1}^{t-1} d_j, \qquad t>1$$

According to (4.2) and (4.3), b_t can be interpreted as the book value in period t. Therefore (4.1) states that the cost allocated to any period equals depreciation plus interest on book value.

For any $r \in [0, \infty)$, let A^r denote the set of allocation rules that are complete with respect to r.[8] It is straightforward to see that, for any fixed $r \in [0, \infty)$, that $\eta(d, r)$ is a one-to-one onto mapping between D and A^r. That is, the allocation rule generated by (d, r) is always r-complete and every r-complete allocation rule can be generated by a unique (d,r) pair for some $d \in D$. Let $\lambda(a, r) = (\lambda_1(a, r), \ldots, \lambda_T(a, r))$ denote the inverse of η. That is, for any $r \in [0, \infty)$ and $a \in A^r$, the unique d such that (d, r) generates a is given by $\lambda(a, r)$. It is also straightforward to see that $\lambda(a, r)$ is given by

$$\lambda_1(a, r) = a_1 - r \tag{4.4}$$

and

$$\lambda_t(a,r) = a_t - (1+r)^{t-1}\, r\, \Big(1 - \sum_{i=1}^{t-1} \frac{a_i}{(1+r)^i}\Big), \qquad t > 1 \tag{4.5}$$

This result is summarized as Proposition 4.1.

Proposition 4.1

For any fixed $r \in [0, \infty)$, the function $\eta(d, r)$ is a one-to-one onto mapping between D and A^r. The inverse of η is given by $\lambda(a, r)$ defined by (4.4)-(4.5).

proof:

Straightforward algebra. QED

Proposition 4.1 states that every allocation rule in A^r can be generated by one and only one depreciation rule using the interest rate r. We would normally expect an allocation rule to be complete with respect to only one non-negative interest rate. For example, this is true if all the allocation shares are non-negative. In this case it is straightforward to see that Proposition 4.1 implies that an allocation rule in A^r cannot be generated using any other interest rate. Therefore, for the normal case of an allocation rule that is complete with respect to a single interest rate there is a unique depreciation rule interest rate pair that generates the allocation rule. This result is formally presented as Corollary 4.1.

Corollary 4.1:
Suppose that a is an allocation rule that is complete with respect to the non-negative interest rate r**. Suppose that a is not complete with respect to any other non-negative interest rate. Let d** denote the depreciation rule λ(a, r**). Then (d**, r**) is the unique (d, r) pair that generates a.
proof:

Proposition 4.1 directly shows that (d**, r**) generates a. It remains to show that (d**, r**) is the unique such pair. This will now be done. Suppose that (d, r) is a depreciation rule interest rate pair that generates a. It will be shown that (d, r) must be equal to (d**, r**). First suppose that r equals r**.

Then Proposition 4.1 directly shows that d must be equal to d**. Now suppose that r is unequal to r*. Proposition 4.1 states that the allocation rule generated by (d, r) must be complete with respect to r. Therefore a is complete with respect to r. If r is unequal to r**, this contradicts the assumption that a is not complete with respect to any non-negative interest rate other than r**. QED

B. Generating the r*-Annuity Allocation Rule

For any $r \in [0, \infty)$, the only interest rate that the r-annuity allocation rule is complete with respect to is r. Therefore, by Corollary 4.1, there is a unique depreciation rule interest rate pair that generates the r-annuity allocation rule for any $r \in [0, \infty)$. The interest imputation rate must be set equal to to r. Then the depreciation rule is given by $\lambda(a^r, r)$. This depreciation rule will be called the r-annuity depreciation rule and be denoted by d^r. Substitution of (2.8) into (4.4)-(4.5) yields

$$(4.6) \qquad d^r_t = \frac{(1+r)^{t-1}}{\alpha(r)\ (1+r)\mathrm{sup}T}$$

This result is formally stated as Proposition 4.2:

Proposition 4.2:

There is a unique (d,r) pair that generates the r*-annuity allocation rule. It is given by $(d^{r*}, r*)$.

proof:

As above. QED

Note that the r-annuity depreciation rule is strictly increasing over time so long as r is positive. That is, more depreciation is assigned to later periods than early periods. Under the r-annuity allocation rule, the total cost allocated to each period remains constant. Total cost equals depreciation plus imputed interest cost. Imputed interest is calculated by multiplying the interest imputation rate by book value. Since the book value of the investment falls over time, this causes the imputed interest cost to fall over time. Therefore the only way to make total cost constant over time is to have depreciation rise over time.

C. Implications For Real Practices

The unique way to generate the r*-annuity allocation rule is to set the interest imputation rate equal to r* and to use the r*-annuity depreciation rule as defined above. This has two implications for the real practices of firms. First, it is correct to impute interest costs at the firm's cost of capital when using income as a performance measure for management. Therefore the current wave of enthusiasm for residual income and EVA measures seems justified. Second, the depreciation rules typically used by firms are not correct. Firms typically use either the straight line method or methods which are more accelerated than this in the sense that more depreciation is recorded in earlier periods so that depreciation falls over time. Under the correct rule, precisely the reverse should occur. More depreciation should be allocated to later periods than to early periods so that depreciation rises over time.

D. Comparative Statics

In an earlier version of this paper (Rogerson 1993), more structure is added to the model to develop comparative statics results to predict how variations in the interest imputation rate and depreciation rule will affect the agent's investment choice. Since the results are very intuitive but formally presenting them requires a significant amount of extra notation, the results will only be briefly summarized here.

Increasing the interest imputation rate, quite intuitively, causes the agent to invest less, i.e., the agent invests less when he is told that capital has a higher cost. The effect of using a more accelerated depreciation rule depends on whether the agent is more or less patient than the principal. The most plausible case is where the agent is less patient than the principal, either because the agent expects to leave the firm before the full effects of the investment are realized or because the agent has a higher personal cost of capital than does the firm. In this case, using a more accelerated depreciation rule causes the manager to invest less. This result is also quite intuitive. Using a more accelerated depreciation rule pushes more costs into the early periods that the manager cares too much about. This causes him to view investment as being more costly and thus causes him to invest less.

For tangible assets, the traditional practice used by firms is to depreciate the asset over time but impute no interest costs. Since no interest is imputed, this causes the manager to over-invest. However, since the depreciation rule used is too accelerated, this cause the manager to under-invest. Since the two effects work in opposite directions, no unambiguous prediction is possible. Firms that currently use residual income or EVA as a performance measure only create the latter effect. Therefore the manager should under-invest relative to the efficient level. It is also unambiguously the case that if a firm switches from using operating income as a performance measure, to using residual income, that the manager should respond by investing less than before. This latter prediction is consistent with the stylized facts reported in the practitioner-oriented literature (Sheehan 1993, Stern and Stewart 1993, Stewart 1993).

Firms generally expense intangible assets such as R&D expenditures. In terms of the formal model, this is simply an extremely accelerated form of depreciation. (Since the expense is charged to the period that it is incurred in, no interest needs to be imputed.) Therefore the above comparative statics results predict that current practice creates incentives for managers to under-invest in intangible assets relative to what would be efficient.

5. Variable Productivity Across Periods

Instead of assuming that cash flows across periods are determined by (2.1), assume that cash flows are determined by

$$(5.1) \qquad z_t = p_t\,\delta(x,\ \theta) + \varepsilon_t$$

where $p = (p_1, \ldots, p_T)$ is a vector of non-negative numbers which will be called the relative productivity profile of the investment. Without loss of generality, assume that $p_1 = 1$. The variable p_t will be called the relative productivity of the asset in period t. Assume that the principal and agent both know p. The model is unchanged in all other respects.

Some examples may be helpful. In the basic model, it was assumed that $p_t = 1$ for every t. If the asset's usefulness declined at some rate ß per period then p would be given by

$$(5.2) \qquad p_t = (1 - ß)^{t-1}\ .$$

If there was an inflation rate of γ and the real productivity of the asset remained constant, then p would be given by

$$(5.3) \qquad p_t = (1 + \gamma)^{t-1}\ .$$

Two remarks should be noted about assumption (5.1). First, even though the principal knows the time pattern of the investment's relative productivity as given by p, he does not know enough to calculate the optimal level of investment himself. For this, he would have to know the absolute level of productivity given by the function $\delta(x, \theta)$. Second, the formulation in (5.1) has the productivity parameter enter in a multiplicatively separable way. The consequence of this is that the relative marginal productivity of investment across periods I and j is simply given by the ratio p_i / p_j and is <u>not</u> affected by the level of investment. The assumption that the relative marginal productivity of the investment is not affected by the level of investment is important for the derivation of the result. In an earlier version of this paper (Rogerson 1993), it is shown that this is a necessary condition for there to exist an allocation rule that always induces efficient investment. For the generalized problem, the efficient level of investment is the level which maximizes

$$(5.4) \qquad \sum_{t=1}^{T} \frac{\rho_t \delta(x,\theta)}{(1+r^*)^t} - x$$

Given the regularity assumptions made about $\delta(x, \theta)$, there is a unique level of efficient investment characterized by the first order condition

As before, let $x^*(\theta)$ denote the efficient investment level.

Consider any productivity profile, p. An allocation rule, a, allocates costs in proportion to benefits as defined by p if

$$(5.6) \qquad \frac{a_i}{a_j} = \frac{\rho_i}{\rho_j} \quad \text{for every } i,j.$$

Recall that an allocation rule is r-complete if it satisfies (2.7). Simple algebra shows that for every p and r, there is a unique allocation rule that satisfies (5.6) and (2.7). This will be called the relative marginal benefits(RMB) allocation rule given (p, r) and be denoted by

$a^{p,r} = (a_1^{p,r}, \ldots, a_T^{p,r})$. It is given by

$$(5.7) \qquad a_t^{\rho,r} = \frac{\rho_t}{\sum_{i=1}^{T} \frac{\rho_i}{(1+r)^{i}}}$$

For the basic model, where p_t equals 1 for every t, then $a^{p,r}$ is simply the r-annuity allocation rule. In the basic model it was shown that the r^*-annuity allocation rule was the unique rule that induces efficient investment. Therefore, the generalization of this result would be to show that the (p, r^*)-RMB allocation rule is the unique allocation rule that induces efficient investment. This is stated and proven as Proposition 5.1.

Proposition 5.1:

$$(5.5) \qquad \delta_x(x,\theta) - \frac{x}{\sum_{i=1}^{T} \frac{\rho_i}{(i+r^*)^i}} = 0$$

Suppose that the relative productivity profile of the investment is p and the principal's cost of capital is r^*.

(i) Suppose that u(q, e) is weakly increasing in q_t for every t ∈ {1, . . . , T}. Then the (p, r*)-RMB allocation rule induces efficient investment.

(ii) The (p, r*)-RMB allocation rule is the only allocation rule that induces efficient investment for every u(q, e) such that u is weakly increasing in q_t for every t.

proof:

First (i) will be proven. For any allocation rule, a, period t accounting income for the firm is defined by (2.5).

$$(5.8) \qquad y_t = \rho_t \left[\delta(x,\theta) - \frac{x}{\sum_{i=1}^{t} \frac{\rho_i}{(1+r^*)^i}}\right] + \varepsilon_t$$

Substitution of (5.7) and (5.1) into (3.2) yields

Note that the term in square brackets is the same for every t. Furthermore x*(θ) maximizes this term. The proof now follows the same steps as the proof of (I) for proposition 3.1.

The proof of (ii) parallels the proof of (ii) for proposition 3.1, with the same type of modification as above, so it will not be presented. QED

Therefore there is a unique allocation rule that always induces efficient investment and it is the unique rule that satisfies two properties. The first property is that it is r*-complete. The second property is that it allocates costs across periods in proportion to the benefits the asset creates across periods. For the special case where benefits across periods are constant the allocation rule is the r^*-annuity allocation rule. For any profile of relative benefits across periods given by p, the allocation rule is the (p, r^*)-RMB allocation rule.

This result can be interpreted as being consistent with a version of the "matching principle" from accounting which states that the correct way to allocate a joint cost across objectives is to allocate the cost in proportion to the benefits it creates across objectives. The basic idea explaining this paper's result is that the effect of matching costs to benefits is to "annuitize" the problem from the agent's point of view, in the sense that every period creates the same incentive for the agent. In particular, the agent has the incentive to choose the efficient investment level, if he considers any single period's results. Therefore, the manner in which the agent compares cash flows across periods becomes irrelevant to predicting the agent's behavior.

Proposition 4.1 and Corollary 4.1 show that for every $r \in [0, \infty)$, there is a unique depreciation rule interest imputation rate pair which generates the (p, r)-RMB allocation rule. The interest imputation rate must be set equal to r. The depreciation rule is given by $\lambda(a^{p,r}, r)$. This depreciation rule will be called the (p, r) RMB depreciation rule and be denoted by $d^{p,r}$. This result is formally stated as Proposition 5.2:

Proposition 5.2:

There is a unique (d, r) pair that generates the (p, r*) RMB allocation rule. It is given by (d^{p,r^*}, r*).

proof:

As above. QED

6. Relationship to the Literature

This paper's results are most closely related to two previous papers by Ramakrishnan(1988) and Rogerson (1992). They both consider principal agent models where an investment decision must be made at the start of the relationship and the agent has better information than the principal about the productivity of the investment. Both papers assume that the productivity of the investment remains constant across future periods, and show that the r*-annuity allocation rule (where r* is the principal's discount rate) induces the agent to make an efficient investment decision. Thus one difference between this paper and these two previous papers is that the case of variable productivity across time periods is considered in this paper. The extension of the result to the variable productivity case makes it clear that the result is a version of the matching principle. However, the major difference is that both previous papers make significant special assumptions, so that the generality of the results is not apparent. Ramakrishnan (1988) assumes that the agent is risk neutral and that the principal restricts himself to using wage contracts where each period's wage is a linear function of that period's income and the same linear function is used every period. It is also assumed that the stochastic effect of investment on future cash flows is determined by a simple two-point distribution function (i.e., investment will reduce cash flows by one of two amounts, a high amount or a low amount). Rogerson (1992) makes similar types of special assumptions. The structure of the model in Rogerson(1992) also differs because Rogerson (1992) analyzes a public utility regulation problem, where the principal is the regulator and the agent is the regulated firm. (The incentive effects created by the wage function are replaced by incentive effects due to the existence of regulatory lag.)

Reichelstein (1996) has recently extended the results of this paper to show that they also hold when there is a series of overlapping investment problems. Reichelstein also considers a slightly different formulation of the investment problem. Recall that this paper creates a continuum of possible investment choices for the manager by assuming that the manager chooses a scalar level of investment. Reichelstein (1996) assumes that the manager makes a simple yes/no decision regarding whether or not to invest but that, ex ante, there is a continuum of possible investment projects. He requires that the allocation rule induce the efficient choice for all possible projects. This alternate formulation turns out to generate the same mathematical structure.

A series of papers by Anctil(1966), Jordan(1990), and

Jordan, Anctil and Mukherji (1995) have shown that, in a dynamic model, a firm attempting, each period, to myopically maximize next period's residual income will in the limit converge to the optimal investment path. Although these models are extremely different than that of this paper, they yield the same general type of conclusion that residual income is a desirable performance measure.

7. Conclusion

This paper considers a principal agent model of the relationship between the shareholders and manager of a firm where there are two incentive problems. Shareholders delegate an investment decision to the manager, because he has better information than shareholders to base this decision on. The manager also exerts an unobservable level of effort each period which increase the firm's cash flows. The "problem" with this situation is that the two incentive problems generally interfere with one another. That is, wage contracts designed to deal with the moral hazard problem will generally distort the agent's investment decision.

The main result of this paper is to show that a very large class of contracts exist which dramatically simplify this problem but which still allow shareholders to achieve a high level of expected utility. Suppose that, instead of basing the manager's wage contract on completely disaggregated accounting data, the firm calculates period-by-period income for itself by allocating the investment cost to periods that benefit from the investment, and then bases the manager's wage contract on the firm's accounting income. The result is that there exists a unique allocation rule such that it is always the case that the manager will make the efficient investment decision for any income-based wage contract that shareholders choose so long as the wage contract is weakly increasing in income. Thus the investment delegation problem is completely solved in a very robust and simple manner and shareholders are left with an enormous number of degrees of freedom to attempt to deal with the moral hazard problem..

Therefore this paper both provides a theory of why income may be used as a performance measure for management and how income should be calculated for this purpose. Income is used as a performance measure to guarantee in a simple robust way that managers will make efficient input decisions. When the input is an investment good and benefits multiple future periods, it is important that the costs be allocated across periods in proportion to the benefits they produce and that the discounted sum of the cost allocations, using the shareholders' cost of capital, be equal to the total investment cost. This essentially "annuitizes" the problem from the manager's perspective, and creates an incentive for the manager to choose the efficient investment level, no matter how he values wage payments across periods.

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<u>Endnotes</u>

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1. See the round table discussion in the <u>Journal of Applied Corporate Finance</u> (Stern and Stewart 1993) and the associated articles (Stewart 1993, Sheehan, 1993) for more complete descriptions and discussions and further references.

2. A number of recent papers have suggested that basing managerial compensation contracts on accounting income may provide a superior solution to the moral hazard problem of inducing effort, in some cases (Bushman and Indjejikian 1993, Kim and Suh 1993, Lambert 1993, Paul 1992, Sloan 1993). The basic idea is that accounting income may be a less noisy signal of managerial effort than is stock market value.

3.Section 5 will generalize the model to allow the productivity of the asset to vary over time.

4.It would be potentially more general to allow the principal to choose a mechanism where the agent announces his observation of θ and where the investment level and the wage contract are functions of the agent's announcement of θ. This is potentially more general than simply allowing delegation of the investment choice, because θ may be multi-dimensional and it may be that it is possible for the principal to obtain incentive compatible revelation of more than one dimension of information. However, such contracts would be extremely complex, especially when the agent is risk averse, and theorists have not been able to make much progress in describing the nature of the optimal contract in such a case (Melumad and Reichelstein 1989). It seems unlikely that such contracts would ever be seen in practice.

5. Firms do not typically think of themselves as directly choosing an allocation rule. Rather, they think of themselves as directly choosing a depreciation rule and an interest imputation rate. These in turn generate a cost allocation rule. (The cost allocated to any period equals depreciation plus imputed interest charges on the remaining book value of the investment.) To keep the analysis as clear and simple as possible, the principal will initially be modeled as directly choosing an allocation rule. Consideration of depreciation rules and interest imputation rates will be delayed until section 4.

6. An earlier version of this paper (Rogerson 1996) begins to develop the idea that a sufficient condition for this set of contracts to "perform well" is that observation of the efficient investment level be uninformative about the firm's future cash flows.

7. Formally, $U(x, c, \theta, \phi) = \int u(\phi(x, z), e, \theta) f(\varepsilon/e) d\varepsilon$ where z is defined by (2.1).

8.Recall, that an allocation rule is r-complete is the discounted sum of the allocation shares (using the interest rate r) equals 1.

Inter-Temporal Cost Allocation and Managerial Investment Incentives

by

William P. Rogerson

Abstract

This paper provides a formal analysis of how managerial investment incentives are affected by alternative allocation rules when managerial compensation is based on accounting measures of income which include allocations for investment expenditures. The major result is that a unique allocation rule exists, called the relative marginal benefits (RMB) rule, which always induces the manager to choose the efficient investment, no matter how the manager values his own personal cash flows from wage compensation over time and no matter what wage schedule is in place (so long as wages are weakly increasing each period's income). That is, the *same* allocation rule works for every possible managerial utility function over wage payments and every monotone wage function over accounting incomes. Thus the firm can choose an allocation rule which induces efficient investment choices without knowing the manager's preferences. Furthermore, since the same rule works for every monotone wage schedule, the firm is left a "degree of freedom" to choose the wage schedule to solve some other incentive problem. In addition to demonstrating the optimality of the RMB rule and describing its properties, the paper considers how currently used allocation rules qualitatively affect managers' investment incentives. It is shown that the practice of expensing intangible assets (i.e., allocating 100 percent of the cost to the current period) causes managers to underinvest relative to the efficient level. The case of tangible assets is more complicated. It appears that current practices may cause either underinvestment or overinvestment, depending on various factors described in more detail in the text.

I
Introduction

An important aspect of many managers' jobs is making investment decisions which will affect cash flows in multiple future periods. Since managerial compensation is typically based on firm profits, managers can generally affect the net present value of their compensation by altering investment levels. The natural question which arises in this context is whether managers' private incentives to choose investment levels result in efficient investment levels from the perspective of shareholders. A frequently expressed concern is that managers may be too impatient and thus may underinvest relative to the efficient level, either because their personal cost of capital is higher than the firm's, or because they have a shorter time horizon than the firm (i.e., they plan to leave or retire before all the benefits of the investment are realized).

One technique that firms use to help combat this potential distortion is to base managerial compensation on accounting measures of income created by allocating investment expenditures to the future periods that benefit from the investment. The intuitive justification for this procedure is that matching costs to benefits creates a more "accurate" measure of income on a period-by-period basis and thus reduces distortions caused by the fact that managers may not compare cash flows across time correctly. Although this intuition is appealing, it is obviously incomplete. The intuition does not precisely explain why such a procedure would work. Nor does it explain what allocation rule should be used and how the choice of allocation rule should depend on factors such as the time pattern of benefits from the investment, shareholders' discount rate, the manager's discount rate, or the nature of wage contracts in place across various periods. It is intuitively clear that all of these factors might enter the analysis.

This paper provides a formal analysis of how managerial investment incentives are affected by alternative allocation rules when managerial compensation is based on accounting measures of income which include allocations for investment expenditures. The major result is that a unique allocation rule exists, called the relative marginal benefits (RMB) rule, which always induces the manager to choose the efficient investment, no matter how the manager values his own personal cash flows from wage compensation over time and no matter what wage schedule is in place (so long as wages are weakly increasing each period's income). That is, the *same* allocation rule works for every possible managerial utility function over wage payments and every monotone wage function over accounting incomes. Thus the firm can choose an allocation rule which induces efficient investment choices without knowing the manager's preferences. Furthermore, since the same rule works for every monotone wage schedule, the firm is left a "degree of freedom" to choose the wage schedule to solve some other incentive problem.

The RMB allocation rule depends on some characteristics of the production technology, i.e., the function determining how investment affects future periods' cash flows. However, in a broad class of plausible cases, the information requirements for calculating the optimal rule are minimal and it is plausible that firms will generally possess the required information. In this broad class of cases, the firm does not have enough information to directly calculate the efficient investment level itself. However, it possesses enough information to calculate the RMB allocation rule which in turn induces the manager select the efficient investment level.

The optimal cost allocation rule is completely characterized by the following two properties.

(i) The cost allocated to each period is proportional to the relative marginal benefit of investment to that period (i.e., a period experiencing twice the marginal benefit of some other period would be allocated twice the cost.

(i) The discounted sum of cost allocations (using the firm's discount rate) equals the investment cost.

Two interesting conclusions follow from this. First, according to property (i), the RMB rule can be viewed as implementing a (somewhat unusual) version of the "matching principal" from accounting which states that costs should be allocated in proportion to benefits. This paper can thus be viewed as formally deriving the optimality of a version of the matching principal in an incentive-based model. Second, firms typically think of themselves as choosing a depreciation schedule and an interest rate rather than directly choosing a vector of cost allocations. Each period's cost allocation equals that period's depreciation plus interest on the remaining (nondepreciated) book value. Under this interpretation, property (ii) states that the RMB rule is created by choosing an interest rate equal to the firm's cost of capital. Of course, ordinary accounting income is created by using an interest rate equal to zero (i.e., depreciation, but no interest cost, is subtracted). Thus, use of ordinary accounting income as a performance measure for management cannot yield correct investment incentives. Many firms use ordinary accounting income as a performance measure. However, for purposes of evaluating managerial performance, a substantial fraction of firms actually create a special income measure calculated by subtracting an imputed cost of capital on remaining book value from ordinary accounting income.[1] Such a measure is called a residual income measure. The RMB rule, of course, is a residual income measure. Thus, this paper's result provides some analytic support for the practice of using residual income as a performance measure.

In addition to demonstrating the optimality of the RMB rule and describing its properties, the paper considers how currently used allocation rules qualitatively affect managers' investment incentives. It is shown that the practice of expensing intangible assets (i.e., allocating 100 percent of the cost to the current period) causes managers to underinvest relative to the efficient level. The case of tangible assets is more complicated. It appears that current practices may cause either underinvestment or overinvestment, depending on various factors described in more detail in the text.

The model and results of this paper can also be interpreted as applying to situations of cost-based regulation and taxation. Under cost-based regulation, the RMB allocation rule is the unique allocation rule which induces the utility to make efficient investment decisions regardless of how regulatory procedures determine its revenues as a function of current and historic accounting costs.

Under taxation, the RMB allocation rule is the unique rule which induces private firms to make efficient investment decisions (i.e., decisions which maximize before-tax discounted profit) regardless of how their marginal tax rate changes over time. The model can also be interpreted as applying to situations where the investment is a joint cost which affects multiple products within the same time period. This interpretation is particularly interesting for the applications to regulation and taxation. In the regulation context, the RMB rule is the unique allocation rule which induces efficient investment choices when the utility has regulated and unregulated business. In the taxation context, the RMB rule is the unique allocation rule which induces efficient investment choices when a firm has multinational business subject to different tax jurisdictions. Since these alternate interpretations require some extra notation and alterations of the model, they are considered in a separate, companion, paper (Rogerson 1993).

Accounting textbooks often consider an incentive problem which is closely related, in spirit, to the problem considered by this paper.[2] They define the compound depreciation rule for an investment to be the rule which causes the accounting return on investment (ROI) measured in each period to remain constant and equal to the internal rate of return (IRR) of the investment. If a manager's wage each period is a function of that period's ROI, the compound depreciation rule will obviously induce the manager to choose the efficient investment. Thus, just as in this paper, this approach identifies a depreciation rule which induces efficient investment behavior given a plausible assumption about how managerial wages are based on accounting data. The major problem with this approach is informational. In order to calculate the compound depreciation rule, the firm must already know the IRR of the investment. Thus, in order to calculate the compound depreciation rule, the firm must have sufficient information to directly calculate the efficient investment level itself. Of course, if the firm has enough information to directly calculate the efficient investment itself, it is not clear why it needs to use any incentive mechanism at all. It could simply order the manager to choose the desired investment level. In contrast, as explained above, in this paper's model, the firm does *not* possess sufficient information to directly calculate the efficient investment level. However it does possess enough information to calculate the RMB allocation rule, which in turn induces the manager to select the efficient investment level.

This paper is most closely related to Ramakrishnan (1988) and Rogerson (1992a). Both of these papers derive the RMB allocation rule for special cases. Ramakrishnan (1988) considers a managerial compensation model where the manager is offered the same linear wage contract every period, the investment choice is a simple discrete invest/do not invest decision, and investment affects all future periods' costs in the same way. Rogerson (1992a) considers a regulation model where a particular type of regulatory lag is assumed to exist and with a simple type of investment decision similar to that of Ramakrishnan (1988).

The principal-agent model of this paper is a model of technology choice. By this, it is meant that the agent simply makes an input choice and no unobservable effort or unobservable benefit on the agent's part is involved. Three groups of papers, primarily focussed on the regulation problem, have considered models of technology choice and are thus somewhat related to this paper. The first group, consisting of a single paper,[3] analyzes how accounting treatment of the sale and purchase of used assets affects regulated firms' incentives to substitute efficiently between used and new assets. This paper considers a single time period and does not analyze allocation rules. The second group[4] analyzes how some existing rules for allocating overhead across products within periods distort regulated firms' decision making. They do not attempt to identify optimal allocation rules. The third group of papers[5] considers incentive mechanism models which Sappington and Sibley (1988) have termed anonymous mechanisms. These mechanisms are basically models of technology choice, i.e., the agent simply makes input and output decisions and the mechanisms depend solely on observable accounting data.[6] The primary focus of most of these papers is on decisions which affect only the current period (price in the current period and/or an input decision which only affects current period costs). Sappington and Sibley (1988) consider an investment decision but simply assume that all investment costs are expensed (i.e., charged to the current period). This paper can be viewed as expanding upon the anonymous mechanism literature to consider the effect of cost allocation rules on investment decisions. The detailed connection of this paper's results to those of the above three groups of papers is explained in more detail in the companion piece to this paper (Rogerson 1993) which considers the regulation application of this paper's model.

2
The Model

Part A of this section will present the basic model. Then part B will provide extra intuition regarding the nature of the model by analyzing the special case of the model where there is only a single period. Finally, part C will discuss two special aspects of the model in more detail.

A. The Basic Model

The relationship between the firm and manager is modelled as a principal agent relationship. The terms principal and firm will be used interchangeably. Similarly the terms agent and manager will be used interchangeably.

Suppose that there are T + 1 periods indexed by $t \in \{0,\ldots,T\}$ where T is a finite positive integer.[7] The agent chooses a level of investment, x, at the end of period 0 where x is a non-negative real number. The principal experiences a cash flow equal to minus the investment cost, -x, at the end of period 0. In addition to the investment cost, at the end of every period, the principal receives a cash flow from the business it conducted during that period. Let $B_t(x)$ denote this cash flow for t = 0,..., T and let B(x) denote the vector of benefits $(B_0(x),\ldots,B_T(x))$.

Let r_p denote the principal's cost of capital and let α_p denote the associated discount rate

$$\alpha_p = \frac{1}{1 + r_p} \tag{2.1}$$

Let b(x) denote the discounted present value of cash flows to the principal. This is defined by

$$b(x) = \sum_{t=0}^{T} B_t(x)\alpha_p^t - x \quad . \tag{2.2}$$

This model can be interpreted as applying to many different types of investments. One example is a machine which reduces labor expenditures in future periods. Another example is process R&D which reduces production costs in future periods. A final example is design R&D or advertising which increases demand for the firm's product in future periods.

The following six assumptions will be made.

(a.1) $B_t(x)$ is twice continuously differentiable.

(a.2) $B_t^{'} \geq 0$ for every t.

(a.3) $B_t^{''}(x) \leq 0$ for every t.

(a.4) $B_0^{'}(x) = 0$ (i.e., $B_0(x)$ does not depend on x).

(a.5) A unique interior maximum exists to $b(x)$ at x^*.

(a.6) $0 < \alpha_p \leq 1$ (or, equivalently, $r_p \geq 0$).

All of the above are natural, standard assumptions. Assumption (a.4) states that investment does not affect period 0 cash flows. This is because the investment occurs at the very end of period 0. Assumptions (a.2), (a.4), and (a.6) are not used to prove the basic existence and uniqueness results. Rather, they are used to show that the RMB allocation rule satisfies some natural properties that we would intuitively expect it to satisfy.

An allocation rule is defined to be a vector in R^{T+1} denoted by $s = (s_0,\ldots,s_T)$. Interpret s_t as the *share* of cost allocated to period t. Therefore, if an investment of x dollars is made in period

0, then $s_t x$ is the investment cost allocated to period t.[8] The accounting benefits to period t, denoted by $A_t(x,s)$, equals the direct cash flow minus the allocated investment cost.

$$A_t(x,s) = B_t(s) - s_t x \quad . \tag{2.3}$$

Let A(x,x) denote the vector of accounting benefits, i.e., $A(x,s) = (A_0(x,s),\ldots,A_T(x,s))$.

When allocating investment costs over time, firms generally think of themselves as choosing a depreciation rule and on interest rate rather than directly choosing an allocation rule. Derivation of the basic existence and uniqueness result in section 3 is most naturally conducted by directly considering allocation rules. Therefore consideration of depreciation rules and interest rates will be delayed until section 4.

It will be useful to define two properties of allocation rules. For any interest rate $r \geq 0$, an allocation rule s will be said to be r-complete if the discounted shares sum to 1, i.e., if

$$\sum_{t=0}^{T} s_t/(1+r)^t = 1 \quad . \tag{2.4}$$

An allocation rule will be said to be proper if $s_0 = 0$. Both of the properties will be discussed in more detail in section 4.

The agent has a utility function directly defined over his wage income. Let $U(w_0,\ldots, w_T)$ denote this function where w_t denotes period t wage income. The agent's period t wages, w_t, are determined by some wage function $w_t(A_0,\ldots, A_t)$ depending on current and possibly historic accounting incomes. Given these two functions, the agent can be viewed as possessing an indirect utility function V(A) defined over the firm's accounting income. Formally,

$$V(A) = U\big(w_0(A_0), w_1(A_0,A_1),\ldots, w_T(A)\big) \tag{2.5}$$

For the purposes of this model, the indirect utility function, V(A), is the relevant description of the agent's preferences. Therefore, most of the analysis will be conducted using V(A) rather than the underlying wage or direct utility functions.

Let $x^E(s,V)$ denote the (possibly empty) set of utility maximizing investment choices for the agent given the allocation rule s and the indirect utility function V. This is defined by

$$(2.6) \qquad x^E(s,V) = \underset{x \geq 0}{argmax}\ V(A(x,s)) \ .$$

It will be said that s induces efficient investment for V if

$$(2.7) \qquad x^* \in x^E(s,V) \ .$$

In the analysis of this paper, the indirect utility function of the agent (and thus, implicitly, the wage function) will be viewed as exogenously specified. The goal will be to show that an allocation rule exists which induces efficient investment for a broad class of indirect utility functions. The class of indirect utility functions that this paper will primarily focus on is the class exhibiting what will be termed weak efficiency incentives (WEI).

It will be said that V exhibits weak efficiency incentives (WEI) if V is weakly increasing in all periods' accounting incomes. Formally, V exhibits WEI if

$$(2.8) \qquad A_t \geq \hat{A}_t \text{ for every t} \Rightarrow V(A) \geq V(\hat{A}) \ .$$

The term "weak efficiency incentives" is used because the agent has a weak incentive to be efficient in the sense that he has no incentive to "throw away" the firm's income in any period, if such a thing were possible. Thus the property of WEI can be interpreted as meaning the absence of extremely perverse incentives and we would generally expect to see it satisfied. In terms of direct utility functions and wage functions, a sufficient condition for WEI (that is typically satisfied) is that each period's wage be a weakly increasing function of that period's and historic periods' accounting incomes, and that the agent's direct utility be increasing in each period's wage income.

Let Ω^{WEI} denote the set of all indirect utility functions exhibiting WEI. If an allocation rule induces efficient investment for every $V \in \Omega^{WEI}$, it will be said to be distortion free. Since we would generally expect managers' indirect utility functions to exhibit WEI, a distortion free rule will generally induce efficient investment choices. The major result of this paper is that a unique distortion free allocation rule, call the relative marginal benefits (RMB) rule, exists.

This completes the basic description of the model except for the specification of what information the principal has available to him. If the principal knew everything, and could directly calculate x^*, the problem would clearly be uninteresting. There would be no need for delegation, and furthermore, "forcing contracts" which gave the agent a massive penalty unless he chose x^* would be possible. The interesting question is whether plausible informational environments exist such that the principal cannot calculate x^* but can calculate a distortion free allocation rule. This paper will show that such informational environments exist. The strategy for showing this will be to first simply show that a unique distortion free allocation rule exists without considering the informational requirements to calculate the rule. Then, the question of what information is needed to calculate the rule will be considered. Therefore, the question of what information the principal has available will be delayed until the rule is derived in section 3.

B. The Single Period Case

One way to understand the result of this paper is to view it as a natural extension of a well-recognized result for the single period case where no allocation is required. In this paper's model, the single period case occurs if $T = 1$. In this case, investment occurs at the end of period 0 and all benefits occur in period 1. (This is the single period case in the sense that all benefits occur in a single period.) Suppose that no investment cost is allocated to period 0 and that period 1 accounting income is defined to be

(2.9) $$A_1(x) = B_1(x) - (1+r_p)x \ .$$

That is, all investment costs, including the time cost of money, are allocated to the period that benefits from the investment. It is clear that expression (2.9) is the discounted value of the investment to the principal (calculated in period 1 dollars). Therefore, if the agent is paid a wage which increases in accounting income, the agent will choose an investment that maximizes the net present value of the investment.

It is clear, then, for the single period case that

(2.10) $$s_t = \begin{cases} 0 \quad , & t = 0 \\ (1+r_p) \ , & t = 1 \end{cases}$$

is a distortion free allocation rule. Furthermore, it is also straightforward to show that it is the unique such rule.

Four properties of this result are worth noting. First, the same allocation rule induces efficient investment choices for any monotone wage function. Thus the principal is left a "degree of freedom" to choose the wage function to address some other nonmodelled problem, such as, the perhaps, a moral hazard problem. Second, the principal needs to know very little to calculate the allocation rule. In particular he does not need to know the agent's preferences or the benefit function $B_1(x)$. He only needs to know his own discount rate. Third, there is a trivial sense in which investment cost is allocated across periods in proportion to the benefits the investment creates. Fourth, the allocation rule "fully allocates" the investment cost in the sense that the discounted value of the cost allocations (using the principal's discount rate) sum to the investment cost.

The basic result of this paper is to show that a unique distortion free allocation rule continues to exist for the case of multiple periods and that generalized versions of the above four properties continue to hold.

The basic intuition for the single period case is trivial. The principal agent problem of this paper can be termed a technology choice problem. By this, it is meant that the agent's only job is to make an input decision and the agent is not required to exert any unobservable effort, nor does he receive any unobservable benefits. In the single period case, so long as the wage is an increasing function of the principal's income, the agent maximizes his wage by maximizing the principal's income. Thus weak efficiency incentives are completely sufficient to solve the technology choice problem in the single period case.

In the multiple period case, there is an extra problem. Weak efficiency incentives guarantee that the agent would like to increase any period's income, *ceteris paribus*. However, calculation of the optimal investment now requires the comparison of cash flows across periods and weak efficiency incentives do not guarantee that the agent will make this comparison correctly. Two factors exist which might distort the agent's choice. First, the agent's own rate of time preference affects his decisions. Second, unless the wage functions are the same linear function for every period, the wage functions themselves might distort the way the agent compares income streams across time periods.

The result of this paper will be to show that an appropriate choice of allocation rule can neutralize both these factors and restore the property that weak efficiency incentives are sufficient to solve the technology choice problem.

C. Discussion

The purpose of this part is to discuss two aspects of the model in more detail. The first aspect is that the wage function is treated as being exogenously given rather than viewed as a choice variable for the principal. In the formal confines of this paper's model, a first-best solution exists to the investment incentive problem by paying the agent a constant wage each period. (Since the agent is then weakly indifferent between all outcomes, he is weakly willing to choose the efficient investment.) In reality, wages depend on income in order to solve some other incentive problem. For example, there may be a moral hazard problem on a period-by-period basis, so that wages must be made an increasing function of income in order to induce effort on the agent's part. Rather than formally model this extra incentive problem, this paper has adopted the simpler approach of assuming that wages are some fixed function of income but not formally modelling the reasons for this. Three rationales for this approach exist.

First, since the result of this paper is that the same allocation rule works for any wage function which induces WEI, it may be that the result will be a useful building block in models which simultaneously consider investment incentives and some other incentive problem such as a moral hazard problem.

Second, in general it seems that the moral hazard problem and investment problem will not always be "separable" in the sense that the allocation rule calculated by this paper continues to be optimal when one expands the model to formally include a moral hazard problem. In some circumstances, it may be optimal to alter the allocation rule to help solve the moral hazard problem. Nonetheless, the result of this paper may still be of great practical value. As will be seen, the informational requirements for calculating the RMB allocation rule are extremely minimal in a broad class of plausible cases. Firms would generally be able to estimate the required information quite reliably. Further, the simplicity of the result and the underlying logic suggest that the result may be quite robust. In contrast, calculating the optimal solution to moral hazard problems generally requires information about the agent's preferences which the principal is quite unlikely to possess. Furthermore, the nature of the optimal solution often depends quite critically on the difficult-to-estimate information. Given these informational limitations, it may be that this paper's allocation rule would be a desirable choice for firms. It completely solves the investment incentive problem. Although more refined solutions exist in theory, their calculation requires information that may generally not be available. Formally modelling this issue in an expended model which added a moral hazard problem and considered the reliability of various types of information is beyond the scope of this paper.

Third, it may be in some situations that the principal finds it much easier to change the allocation rule than to change the wage function. This may be especially true in the regulation and taxation interpretations of this model where the wage function corresponds, respectively, to the regulatory process determining revenues based on accounting costs and the marginal tax rate.

The second aspect of this paper's model is that the objective function which the agent is induced to maximize by the RMB rule is not precisely the principal's objective function. The agent is induced to maximize gross discounted cash flows. The principal's objective function is actually gross discounted cash flows minus the discounted value of wage payments to the agent. Therefore, the result of this paper is most relevant for cases where the wage payments to the manager are small relative to the overall cash flows of the firm. In this case, the RMB rule will be approximately optimal for the principal. Furthermore, it may be in a more complete model which simultaneously considered wage choice and allocation rule choice, that compensating *ex ante* wage payments would be possible. In this case, the goal of the principal would be to choose an allocation rule which maximized gross discounted cash flows since *ex ante* wage payments could "divide the pie" any way that was desired.

3
The RMB Allocation Rule

Part A will define the RMB allocation rule and prove that it is the unique distortion free allocation rule. Then part B will consider the informational requirements to calculate the RMB rule.

A. The Basic Result

Since the $B_t(x)$ functions are concave and the efficient investment choice is interior, the first-order condition

(3.1) $$\sum_{t=0}^{T} B_t'(x)\alpha_p^t - 1 = 0$$

characterizes the efficient investment choice. That is, $x = x^*$ if and only if (3.1) is satisfied.

The relative marginal benefits (RMB) allocation rule, denoted by s^{RMB}, is defined as follows.

(3.2) $$s_t^{RMB} = B_t'(x^*) \quad .$$

That is, the cost share allocated to period t equals the marginal benefit of investment in period t evaluated at the efficient investment choice.

Proposition 3.1 states, for future reference, some properties of the RMB allocation rule that follow immediately from the definition.[9]

Proposition 3.1

(i) s^{RMB} is r_p-complete, i.e.,

(3.3) $$\sum_{t=0}^{T} s_t^{RMB}\alpha_p^t = 1$$

(ii) s^{RMB} is proper, i.e.,

(3.4) $$s_0^{RMB} = 0$$

(iii) every element of s^{RMB} is non-negative, i.e.,

(3.5) $$s_t^{RMB} \geq 0 \text{ for every } t = 0,\ldots,T$$

Recall that an allocation rule is defined to be distortion free if it induces efficient investment for every indirect utility function exhibiting weak efficiency incentives. It will be useful to define one more related notion in order to prove that s^{RMB} is the unique distortion free allocation rule. Consider some period t, standing alone. Suppose that x^* maximizes period t accounting income. Suppose that this is true for every t. Then the allocation rule will be said to be stand-alone distortion free. Formally, s will be said to be stand-alone distortion free if it induces efficient investment over the set Ω^{SA} where Ω^{SA} is defined by

$$\Omega^{SA} = \{V: V(A) = A_t \text{ for some } t = 0, \dots, T\} \quad (3.6)$$

If an allocation rule is distortion free it is clear that the allocation rule is also stand-alone distortion free, because Ω^{SA} is a subset of Ω^{WEI}. Proposition 3.2 shows that, in fact, the reverse is also true, so the two concepts are equivalent. The reason for this is very intuitive. The agent chooses x to maximize $V(A_0, \dots, A_T)$ where the choice of x affects each period's accounting income. If s is stand-alone distortion free, then x^* maximizes A_t for every t. If V exhibits WEI, it is increasing in each A_t. Obviously, then, x^* also maximizes $V(A_0, \dots, A_T)$.

Proposition 3.2

The following two statements are equivalent.

(i) s is stand-alone distortion free.

(ii) s is distortion free.

The main conclusion from proposition 3.2 is that one can search for a distortion free allocation rule by searching for a stand-alone distortion free allocation rule. This is useful because the property of being stand-alone distortion free is much simpler and easier to characterize. Proposition 3.3 shows that the first-order conditions trivially imply that s^{RMB} is the unique stand-alone distortion free rule.

Proposition 3.3

s^{RMB} is the unique stand-alone distortion free allocation rule.

It is, of course, immediate from Propositions 3.2 and 3.3 that s^{RMB} is the unique distortion free allocation rule. Since this is the paper's main conclusion, it is formally stated as proposition 3.4.

Proposition 3.4

s^{RMB} is the unique distortion free allocation rule.

Thus the basic explanation for the result is that when accounting income is defined using the RMB rule, each period's accounting income is maximized by the efficient investment choice, x^*. So long as the agent's utility is increasing in each period's accounting income, the agent will then be induced to choose x^*. The agent's relative valuation of income streams across periods is irrelevant because accounting income for *every* period is maximized by x^*. Thus, the RMB allocation rule succeeds in erasing distortions due to the way the agent compares income streams across time by defining accounting income in such a way that this comparison is irrelevant to the agent's choice.

Proposition 3.4 can be interpreted as stating an existence and a uniqueness result.

[1] Existence: An allocation rule exists which induces efficient investment for every $V \in \Omega^{WEI}$.

[2] Uniqueness: The allocation rule which does this is unique.

As discussed in section 2, Ω^{WEI} is a "large" set in the sense that we would generally expect a manager's indirect utility function to exhibit WEI. This means that the existence result is quite strong, i.e., s^{RMB} will induce efficient investment for most indirect utility functions which occur in practice. However, this also means that the uniqueness result is, in a sense, weak. That is, it may be in some cases that the principal knows that the agent's indirect utility function belongs to some set Ω which is a subset of Ω^{WEI}. Proposition 3.4 implies that s^{RMB} induces efficient investment choice over Ω. However, it does not rule out the possibility that other allocation rules exist which work equally well. It is possible to strengthen the uniqueness result by showing that s^{RMB} is the unique allocation rule which induces efficient investment over many subsets of Ω^{WEI}. Since this result is tangential to the main thrust of the paper, it is stated and proven in Appendix B.

B. Informational Requirements

In (3.2), s_t^{RMB} is defined to be the marginal benefit of investment in period t, evaluated at the efficient investment, x^*. Thus, in general, it appears that the principal must know x^* in order to calculate the RMB rule. When this is true, the result of this paper is not very useful. If the principal is able to calculate x^*, there is no need to delegate the decision of x^* to the agent. The principal could simply order the agent to choose x^*. The purpose of this part is to show that a broad class of plausible cases exist where the principal does not need to calculate x^* in order to calculate the RMB rule. The information the principal needs to calculate the RMB rule is quite minimal and firms would generally be able to estimate the required information quite reliably. Thus in this class of cases, the principal does not have enough information to calculate x^*. However he has enough information to calculate the RMB rule, which, in turn, induces the agent to choose x^*.

The situation in which this result occurs is when the benefit function, B(t), satisfies a property which will be termed relative benefit invariance (RBI). Formally B will be said to exhibit RBI if the ratio $B_i'(x)/B_j'(x)$ does not depend on x for every i and j. That is, B exhibits RBI if the relative marginal benefits to various periods do not depend on the level of investment, x.

If B(x) can be written in the form

$$B_t(x) = g(t) + f(t)\delta(x) \quad , \tag{3.7}$$

it obviously satisfies RBI because

$$\frac{B_i'(x)}{B_j'(x)} = \frac{f(i)\delta'(x)}{f(j)\delta'(x)} = \frac{f(i)}{f(j)} \quad . \tag{3.8}$$

Proposition 3.5 states that all benefit functions satisfying RBI can be written in the form (3.7).[10]

Proposition 3.5

B satisfies RBI if and only if it can be written in the form (3.7).

By (3.1), the definition of the RMB rule in (3.2) can be rewritten as

$$(3.9) \qquad s_t^{RMB} = \frac{B_t'(x^*)}{\sum_{i=0}^{T} B_i'(x^*)\alpha_p^i}$$

From (3.9) it is clear that s_t^{RMB} only depends on values of relative marginal benefits, i.e., on terms of the form $B_t'(x^*)/B_i'(x^*)$. Therefore, if B satisfies RBI, s_t^{RMB} does not depend on x^*. Substitution of (3.7) into (3.9) yields

$$(3.10) \qquad s_t^{RMB} = \frac{f(t)}{\sum_{i=0}^{T} f(i)\alpha_p^i} \quad .$$

Therefore, in order to calculate the RMB rule, the principal needs to know only the function f(t) and not the function $\delta(x)$. Of course calculation of x^* requires the principal to know $\delta(x)$.

When B is of the form in (3.7), the function $\delta(x)$ can be interpreted as determining some "absolute" level of benefit to investment while f(t) determines the pattern of relative benefits over time. (The function g(t) is additively separable from the investment problem and can be ignored.) That is, if the principal knows f(t), he can predict how the relative benefits of investment will change over time. However, to specify the actual magnitude of each period's benefit, the principal must also know $\delta(x)$. Some examples will now be given to illustrate that the RBI property is a natural condition in many circumstances, and that in these circumstances, it is often natural to expect the principal to be well-informed about the relative benefits of investment even if he is unable to estimate the absolute benefits.

First, consider the case where the real benefit of the investment remains constant over time but inflation may cause the nominal benefit to grow. This will be called the case of real stationary benefits. This is a plausible case. For example, it may be that the purchase of a machine will reduce the per-period labor requirements (measured in hours) by a given amount and that this reduction will be permanent for the life of the machine. Purchase of a more expensive machine will reduce labor requirements by a greater amount, but not change the fact that the machine will be equally useful over its life.

Formally, suppose that[11]

$$B_t(x) = \begin{cases} 0 & , \quad t = 0 \\ (1+z)^t \, \delta(x) \, , & \quad t > 0 \end{cases} \tag{3.11}$$

where $\delta(x)$ is some increasing, concave function and z is a non-negative real number. Interpret $\delta(x)$ as the per-period benefit of the investment measured in period 0 dollars and z as the inflation rate. In this case, it is clear that B(x) satisfies RBI and is of the form (3.7) where

$$f(t) = \begin{cases} 0 & , \quad t = 0 \\ (1+z)^t \, , & \quad t > 0 \end{cases} \tag{3.12}$$

Therefore, the principal only needs to know the inflation rate to calculate the RMB allocation rule. It is given by substituting (3.12) into (3.10) and equals

$$s_t^{RMB} = \begin{cases} 0 & , \quad t = 0 \\ \dfrac{(1+z)^t}{\sum_{i=1}^{T} \left[\dfrac{1+z}{1+r_p} \right]^i} & , \quad t > 0 \end{cases} \tag{3.13}$$

Therefore, in the case of real stationary benefits, the principal only needs to know that benefits are stationary and the inflation rate in order to calculate the RMB rule. In particular he does not need to know the real usefulness of the investment given by $\delta(x)$. It is natural to expect that the principal may know that a machine will remain equally useful over its lifetime even if he does not know precisely how useful it will be. It is also natural to expect the principal to be as well informed about future inflation rates as is the agent.

It is clearly possible to generalize the above example in a variety of ways. For example, suppose that production occurs at a constant marginal cost and the effect of investment is to lower this cost. Suppose, as well, that the usefulness of the investment decays over time. Formally let the marginal cost calculated in period 0 dollars be

$$c_t = (1-\theta)^t c \quad , \tag{3.14}$$

where θ is interpreted as the rate of decay in usefulness. Also suppose that output grows at the rate g per year. Finally, let γ denote the inflation rate. It is clear that (3.11) still describes $B_t(x)$ where the term $(1+z)$ is defined to be

$$(1+z) = (1+g)(1-\theta)(1+\gamma) \quad . \tag{3.15}$$

That is, now z is interpreted as the exponential growth rate of nominal benefits reflecting the combined effects of inflation, market growth and decay in usefulness of the asset. Once again, in order to calculate the RMB rule, the principal only needs to know the factors which affect relative benefits such as the rate of market growth or the rate of decay of usefulness of the machine and not the absolute productivity of the investment given by $\delta(x)$.

In summary, it is often natural to expect the time pattern of the relative benefits of investment to be independent of the amount invested. In such a situation, it is also natural to expect the principal to be aware of the time pattern of relative benefits even if the absolute value of the investment is not known to him. Because the principal does not know the absolute benefit of investment, he is unable to calculate the efficient investment level. However, he has sufficient information to calculate the RMB rule. By using the RMB rule, the principal can induce the agent to choose the efficient investment level.[12]

4
Allocation Rules and Depreciation Schedules

When allocating investment costs over time, most firms think of themselves as directly choosing a depreciation rule and interest rate instead of directly choosing an allocation rule. The cost allocated to a period then equals that period's depreciation plus interest on the nondepreciated book value. The formal analysis in section 3 which established the basic existence and uniqueness result was most naturally conducted by directly considering allocation rules. However, it will be useful to translate between allocation rules and depreciation rule/interest rate pairs in order to apply this paper's results to actual firm practices. This section will introduce notation to formally describe this correspondence. It will also define some useful properties of allocation rules and derive the equivalent properties expressed in terms of depreciation rules and interest rates.

A depreciation rule, d, is simply a vector in R^{T+1}.

$$d = (d_0,\ldots,d_T) \tag{4.1}$$

Interpret d_t as the share of depreciation allocated to period t. Thus, if x dollars of investment occurs, $d_t x$ dollars of depreciation are allocated to period t.

Period zero depreciation deserves special comment. Recall that the investment is assumed to be made at the very end of period 0. If the firm's accounting practice was to capitalize this expenditure, it would typically charge no depreciation to period 0, i.e., d_0 would be chosen to be 0. However, if the firm's accounting practice was to not capitalize this expenditure, but to instead expense it, it would typically change 100 percent of the cost to period 0 since this is the period the expenditure occurred in, i.e., d_0 would be chosen to be 1. Therefore, d can be interpreted as a generalized depreciation rule which allows as special cases, normal depreciation rules (where $d_0 = 0$) and the normal expensing rule (where $d_0 = 1$ and $d_t = 0$ for every $t > 1$). It also allows "partial expensing" (which would occur if d_0 were chosen to be a fraction between 0 and 1) although this is not seen in practice. It turns out to be analytically convenient to model generalized depreciation rules of this sort in order to compare the results of expensing investment expenditures with capitalizing and depreciating them.

Firms normally only speak of themselves as choosing a depreciation rule when they capitalize an asset. They normally do not speak of themselves as choosing a depreciation rule when they expense an investment. Consistent with this, a depreciation rule, d, will be said to be proper if $d_0 = 0$, i.e., a proper depreciation rule is what firms normally call a depreciation rule.

Let r denote an interest rate and let $\alpha = 1/(1+r)$ denote the corresponding discount rate. It will always be assumed that $r \geq 0$, or, equivalently, that $0 < \alpha \leq 1$. An ordered pair (d,r) will be referred to as a depreciation rule/interest rate pair.

Let ϕ(d,r) denote the allocation rule generated by (d,r). If $s = \phi(d,r)$, s is given by

$$s_t = d_t + r\,b_t \tag{4.2}$$

where

$$b_0 = 0 \tag{4.3}$$

$$b_t = 1 - \sum_{i=0}^{t-1} d_i \ . \tag{4.4}$$

Interpret b_t as the book value of the investment employed during period t calculated as a share of the initial purchase price. Since the investment was made at the very end of period 0, $b_0 = 0$. The book value in all subsequent periods equals 1 minus total depreciation shares recorded in all previous periods. These definitions are given by (4.3) and (4.4). Equation (4.2) states that the cost allocated to period t equals the period t depreciation plus interest on the nondepreciated book value of the investment employed during period t.

For any $r \geq 0$ and $d \in R^{T+1}$, it is clear that (4.2)-(4.4) define a unique $s \in R^{T+1}$. It is also straightforward to see that the mapping is linear and invertible so that it is one-to-one and onto. This is recorded as proposition 4.1.

Proposition 4.1

For any fixed $r \geq 0$, the function ϕ is a one-to-one function mapping R^{T+1} onto R^{T+1}. That is, for every $d \in R^{T+1}$ there exists a unique $s \in R^{T+1}$ such that $s = \phi(d,r)$. Similarly, for every $s \in R^{T+1}$ there exists a unique $d \in R^{T+1}$ such that $s = \phi(d,r)$.

Recall that an allocation rule was defined to be proper if $s_0 = 0$ and a depreciation rule was defined to be proper if $d_0 = 0$. The same word is used for both properties because they are equivalent. This is stated as proposition 4.2.

Proposition 4.2

Suppose that $s = \phi(d,r)$. Then s is proper if and only if d is proper.

For any allocation rule, s, let $s^t(r)$ denote the discounted sum of allocation shares through period t.

(4.5) $$s^t(r) = \sum_{i=0}^{t} s_i \alpha^i$$

Similarly, for any depreciation rule, d, let d^t denote the undiscounted sum of depreciation payments through period t.

(4.6) $$d^t = \sum_{i=0}^{t} d_i$$

Recall that an allocation rule, s, is said to be r-complete if $s^T(r) = 1$. Similarly, a depreciation rule, d, will be said to be complete if $d^T = 1$.

Proposition 4.3 describes the relationship between these two concepts.

Proposition 4.3

Suppose that $s = \phi(d,r)$. Then s is r-complete if 0, and only if d is complete.

An immediate corollary of propositions 4.1 and 4.3 is that any r-complete allocation rule can be generated by a (d,r) pair where d is complete. Firms typically restrict themselves to using complete depreciation rules for the case considered by this paper where the investment has no salvage value. Propositions 4.1 and 4.3 show that this restriction essentially does not limit the firms' choices. They can still generate any r-complete allocation rule for any $r \geq 0$.

If will be useful to define notions of speed of repayment for allocation rules and depreciation rules and derive the relationship between them. Consider two allocation rules $s = (s_0,\ldots,s_T)$ and $\hat{s} = (\hat{s}_0,\ldots,\hat{s}_T)$. It will be said that s is r-accelerated relative to $\hat{s}$ (or, equivalently, that $\hat{s}$ is r-decelerated relative to s) if

(4.7) $$s^T(r) = \hat{s}^T(r)$$

(4.8) $$s^t(r) \geq \hat{s}^t(r) \text{ for every } t = 0,\ldots,T-1 \quad .$$

(4.9) $$s^t(r) > \hat{s}^t(r) \text{ for some } t = 0,\ldots,T-1 \quad .$$

That is, s is r-accelerated relative to $\hat{s}$ if both rules eventually yield the same discounted cost allocation but at any point before period T, the sum of discounted allocations is weakly greater under s than $\hat{s}$ (and strict inequality holds for some t).

Similarly, consider two depreciation rules, $d = (d_0,\ldots,d_T)$ and $\hat{d} = (\hat{d}_0,\ldots,\hat{d}_T)$. It will be said that d is accelerated relative to $\hat{d}$ (or, equivalently, that $\hat{d}$ is decelerated relative to d) if

(4.10) $$d^T = \hat{d}^T$$

(4.11) $$d^t \geq \hat{d}^t \text{ for every } t = 0, \ldots, T-1 \quad .$$

(4.12) $$d^t > \hat{d}^t \text{ for some } t = 0, \ldots, T-1 \quad .$$

That is, d is accelerated relative to $\hat{d}$ if both rules eventually yield the same total depreciation, but, at any point before period T, the sum of depreciation is weakly greater under d than $\hat{d}$ (with strict inequality holding for some t).

Proposition 4.4 now states the relationship between these two notions of speed of repayment.

Proposition 4.4

Suppose that s and $\hat{s}$ are two allocation rules and d and $\hat{d}$ are two depreciation rules such that $s = \phi(d,r)$ and $\hat{s} = \phi(\hat{d},r)$ for some interest rate r. Then s is r-accelerated relative to s if and only if d is accelerated relative to $\hat{d}$.

It will be useful to define some commonly used depreciation rules and describe some of their properties. Let d^E denote the depreciation rule which expenses 100 percent of the asset, i.e.,

(4.13) $$d_t^E = \begin{cases} 1 \,, & t = 0 \\ 0 \,, & t > 1 \end{cases}$$

Firms normally either fully expense an investment or fully capitalize it. Thus, other than d^E, most typically used depreciation rules are proper. Let d^{SL} denote the straight line depreciation rule defined by

(4.14) $$d_t^{SL} = \begin{cases} 0 \,, & t = 0 \\ \frac{1}{T} \,, & t = 1, \ldots, T \end{cases}$$

The straight line depreciation rule is a commonly used rule. Most other commonly used depreciation rules are accelerated relative to the straight line method.

It is straightforward to see that for any positive interest rate, the straight line depreciation rule generates an allocation rule with strictly decreasing allocation shares for $t \geq 1$ (i.e., $s_{t+1} < s_t$ for $t \geq 1$).[13] This observation suggests that, in order to generate an allocation rule with constant or increasing allocation shares over time, one must choose a depreciation rule which charges more costs to later periods than the straight line rule and is thus less accelerated than the straight line rule. This result is true and is stated as proposition 4.5.[14]

Proposition 4.5

Suppose that $T \geq 2$. Let s be an allocation rule such that $s = \phi(d,r)$ for some positive r. Suppose that s is proper, s is r-complete, and

$$s_t \leq s_{t+1} \tag{4.15}$$

for $t \geq 1$. Then d^{SL} is accelerated relative to d.

Let $d^{RA(r,z)}$ denote what will be called the real annuity depreciation rule given the interest rate r and inflation rate z. The depreciation rule $d^{RA(r,z)}$ is defined to be the unique d such that $s^{RA(r,z)} = \phi(d,r)$ where $s^{RA(r,z)}$ is defined by

$$s_t^{RA(r,z)} = \begin{cases} 0 & , \quad t = 0 \\ \dfrac{(1+z)^t}{\sum_{i=1}^{T} \left[\dfrac{(1+z)}{(1+r)}\right]^i} & , \quad t > 0 \end{cases} \tag{4.16}$$

The allocation rule $s^{RA(r,z)}$ is characterized by the properties that

(i) $s^{RA(r,z)}$ is r-complete

(ii) $s^{RA(r,z)}$ is proper

(iii) the real value of the allocation shares for $t \geq 1$ stays constant, i.e.,

$$(4.17) \qquad s_t^{RA(r,z)} = \frac{s_{t+1}^{RA(r,z)}}{(1+z)}$$

for every $t = 1,\ldots,T-1$.

Accounting textbooks often define $d^{RA(r,z)}$ for $z = 0$ and simply call it the annuity depreciation rule. It can be interpreted as the depreciation rule which keeps nominal payments constant. It is convenient for this paper's purposes to generalize this definition to consider depreciation rules which keep real payments constant for some inflation rate, z.

Note that, so long as $z \geq 0$, that $s_t^{RA(r,z)}$ is weakly increasing in t. Therefore, by proposition 4.5, $d_t^{RA(r,z)}$ is decelerated relative to d^{SL} for for $r > 0$ and $z \geq 0$.

5
The Depreciation Rule/Interest Rate Pairs That Generate the RMB Allocation Rule

Let D denote the set of all (d,r) pairs such that d is complete and $r \geq 0$.

$$(5.1) \qquad D = \{(d,r) : d \text{ is complete and } r \geq 0\} \ .$$

Firms, of course, typically restrict themselves to choosing a (d,r) from the set D. Furthermore, explained in section 4, this restriction essentially does not limit firms' choices because they can still generate any r-complete allocation rule for any $r \geq 0$. Therefore, in this section, attention will be restricted to complete depreciation rules.

From proposition 4.1, there exists a unique d such that $s^{RMB} = \phi(d,r_p)$. Let d^{RMB} denote this unique depreciation rule. Proposition 5.1 states that d^{RMB} is the unique complete depreciation rule which can be used to generate s^{RMB}.

Proposition 5.1

The depreciation rule/interest rates pair (d^{RMB}, r_p) is the unique $(d,r) \in D$ such that $s^{RMB} = \phi(d,r)$.

This section will conclude by describing the nature of d^{RMB} for the case of real stationary benefits introduced in section 3. Recall that for this case $B_t(x)$ is defined by (3.11) and the RMB allocation rule is defined by (3.13). Referring back to the definition of the real annuity depreciation rule in (4.16), it is clear that d^{RMB} is equal to $d^{RA(r_p,z)}$. That is, for the case of real stationary benefits, the real cost share allocated to each period under the RMB rule is constant. This allocation rule, by definition, is generated by $d^{RA(r_p,z)}$. Recall from section 3 that z can also be interpreted as an exponential growth rate reflecting the combined effects of market growth, inflation, and decay in usefulness of the asset as it ages. Therefore the RMB depreciation rule is of the real annuity form for this case, as well.

6
The Effect of Interest Rates and Depreciation Schedules on Investment

This section will adopt the viewpoint of most real firms and view the principal as choosing an allocation rule by choosing a depreciation rule and an interest rate. Comparative statics results will be derived which show how the level of investment the agent is induced to choose is affected by the changes in the depreciation schedule or interest rate. These results will be useful in section 7 which describes the differences between currently used (d,r) pairs and (d^{RMB}, r_p) and analyzes the nature of the investment distortions that currently used (d,r) pairs create. Part A will describe a special assumption that is required to derive the formal results. Then part B will consider the effect of changes in the interest rate and part C will consider the effect of changes in the depreciation schedule.

A. The Specialized Model

Although the economic intuitions underlying the results are quite general and seem likely to play an important role in the general case, it appears to be necessary to employ a more specialized model to formally prove the results of parts B and C. It will be assumed that the agent's indirect utility function over accounting income is of the form

$$(6.1) \qquad V(A,\theta) = \sum_{t=0}^{T} \theta_t A_t \alpha_p^t + k$$

where

$$(6.2) \qquad \theta_t > 0 \text{ for every } t$$

and k is any constant. We can interpret (6.1) as stating that the agent's utility over accounting income is identical to the principal's except for the fact that period t is weighted by θ_t. Assumption (6.2) guarantees that $V(A,\theta)$ satisfies WEI.[15]

Let $x^E(s,\theta)$ denote the investment choice of the agent, i.e.,

$$(6.3) \qquad x^E(s,\theta) = \underset{x \geq 0}{\operatorname{argmax}}\, V\big(A(x,s),\theta\big) \quad .$$

It will be assumed that $x^E(s,\theta)$ exists, is unique and is positive for every s that is r-complete for some $r \geq 0$.

A large class of examples that satisfy (6.1) can be created by assuming that the agent's wages are linear in accounting income, the agent's goal is to maximize discounted wage income for some discount rate, and there is a fixed vector of probabilities describing the probability that the agent will continue to be employed each period. Then (6.2) is satisfied by simply assuming that wages increase in accounting income. This class of examples will now be formally described. Suppose that the agent's direct utility over wage income equals discounted wage income using the interest rate r_a. Let $\alpha_a = 1/(1+r_a)$ denote the associated discount rate.

(6.4) $$U(w_o,\ldots,w_T) = \sum_{t=0}^{T} w_t \alpha_a^t \quad .$$

Suppose that the period t wage, w_t, is a linear function of current and historic accounting incomes, i.e.,

(6.5) $$w_t = \sum_{i=0}^{t} w_{ti} A_i + k_t \quad ,$$

where w_{ti} and k_t are constants. Finally, suppose that the agent's probability of actually being employed in period t by the principal is p_t. Assume that the agent is paid w_t if he is employed and 0 if he is not employed. Under these assumptions the agent's indirect utility over accounting income is given by

(6.6) $$\sum_{t=0}^{T} \left\{ \sum_{i=0}^{t} w_{ti} A_i + k_t \right\} p_t \alpha_a^t \quad ,$$

which can be rewritten in the form (6.1) where

(6.7) $$\theta_t = \left[\sum_{i=t}^{T} w_{it} p_i \alpha_a^i \right] / \alpha_p^t$$

and

(6.8) $$k = \sum_{i=0}^{T} p_i \alpha_a^i k_i \quad .$$

It is clear that if

(6.9) $w_{it} \geq 0$ for every i,t

(6.10) $w_{it} > 0$ for every i

(6.11) $p_i > 0$ for every i

(6.12) $0 < \alpha_a \leq 1$

then (6.2) is satisfied.

There is a natural way to define a notion of the agent's patience in this specialized model. If θ_t is constant, the agent has the same preferences over accounting income as the principal. If the θ_t's are decreasing in t, it is natural to interpret the agent as being too impatient relative to the principal in the sense that he values earlier accounting incomes relatively more than the principal. Similarly, if the θ_t's are increasing in t, it is natural to interpret the agent as being too patient relative to the principal. These interpretations will be formalized in a definition. It will be said that the agent is

	too impatient		strictly decreasing in t
(6.13)	of correct patience	if θ is	constant in t
	too patient		strictly increasing in t

Intuitively, two important factors which should affect the agent's patience with respect to his investment decision are the agent's personal rate of time preference and the agent's expected length of employment with the principal. If the agent grows more impatient to receive wage income, his preferences over accounting income will also exhibit most impatience because shifting more accounting income to earlier periods will shift wage payments to earlier periods. If the agent expects to leave the firm sooner, his preferences over accounting income will exhibit more impatience because he wants to generate higher wages in earlier periods when he is more likely to still be employed.

These effects can be clearly illustrated in a simple version of the above example. Suppose that the period t wage is simply equal to the constant fraction, w, of period t accounting income.

(6.14) $$w_t = wA_t \ .$$

Suppose that each period the agent has a constant probability, e, of continuing employment for that period so that

(6.15) $$p_t = e^t \ .$$

Then (6.7) becomes

$$\theta_t = w\left\{\frac{e(1+r_p)}{(1+r_a)}\right\}^t \quad . \tag{6.16}$$

Of course θ_t will be increasing, constant, or decreasing depending upon whether the term in brackets is greater than, equal to, or less than 1. In particular, the agent is more likely to be impatient if the probability of continuing employment, e, is small or if the agent's personal cost of capital, r_a is high relative to the principal's cost of capital, r_p. If $e = 1$, so severance of employment is not an issue, then the agent is too impatient (of correct patience, too patient) if and only if $r_a > (=,<) r_p$. If $r_a = r_p$, so divergence in costs of capital is not an issue, then the agent is too impatient (of correct patience) if and only if $e < (=) 1$.

It is generally believed that managers exhibit impatience in their preferences over accounting incomes. This is due to both of the above two described factors. For purposes of interpretation, the case where the agent is impatient will be viewed as the typical case. However, all three cases will be analyzed.

B. The Interest Rate

The effect of changing the interest rate used to determine the allocation rule is extremely simple and intuitive. Suppose that a depreciation schedule is being used such that book value is non-negative for every period and strictly positive for at least one period, i,e.,

$$d^t \leq 1 \text{ for every } t = 0,\ldots,T \tag{6.17}$$

and

$$d^t < 1 \text{ for some } t = 0,\ldots,T \tag{6.18}$$

This condition is, of course, typically satisfied in practice. It is also satisfied by d^{RMB} by proposition 3.1 and lemma A.3.

In this case, increasing the interest rate will weakly increase the cost allocated to each period and strictly increase the cost allocated to at least one period. From the agent's perspective, investment is, therefore unambiguously more costly. Therefore, the agent invests less. Proposition 6.1 formally states the result.

Proposition 6.1

Suppose that d is a depreciation rule satisfying (6.17)-(6.18). Then the agent's investment choice, $x^E(\phi(d,r),\theta)$ is strictly decreasing in r.

C. Depreciation Rules

This part will explain how acceleration of the depreciation rule affects the agent's investment choice. The basic result is as follows. Fix the interest rate at the principal's interest rate, r_p. Then choosing a more accelerated depreciation rule will cause the agent to invest less (the same, more) depending upon whether the agent is too impatient (of correct patience, too patient).

Before formally stating and proving the result, the intuition for it will be explained. Suppose that d and $\hat{d}$ are two depreciation rules and that d is accelerated relative to $\hat{d}$. Let $s = \phi(d,r_p)$ and $\hat{s} = \phi(\hat{d},r_p)$ be the two allocation rules generated by d and $\hat{d}$ when the interest rate r_p is used. Because d is accelerated relative to $\hat{d}$, the total discounted cost of investment using r_p is the same under either allocation rule (i.e., $s^T(r_p) = \hat{s}^T(r_p)$), but s allocates more of these costs to early periods than does $\hat{s}$. Now suppose that the agent is of the correct patience. Then the agent attempts to maximize discounted accounting income using the interest rate r_p. Since total discounted investment cost using r_p is the same under s and $\hat{s}$, the agent views investment as equally costly under either allocation rule and therefore makes the same decision under s and $\hat{s}$. Now suppose that the agent is too impatient. Now the agent is overly concerned about near-term costs. Since s allocates more costs to early periods than does $\hat{s}$, the agent views investment as more costly under s and thus invests less under s. The same intuition predicts the reverse result when the agent is too patient.

Proposition 6.2 now states the formal result.

Proposition 6.2

Suppose that d and $\hat{d}$ are two depreciation rules such that d is accelerated relative to $\hat{d}$. Let $s = \phi(d,r_p)$ and $\hat{s} = \phi(\hat{d},r_p)$ be the allocation rules generated by d and $\hat{d}$ and the interest rate r_p. Then

$$\text{(6.19)} \qquad \text{the agent is} \begin{array}{l} \text{too impatient} \\ \text{of correct patience} \\ \text{too patient} \end{array} \Rightarrow x^E(s,\theta) \begin{array}{c} < \\ = \\ > \end{array} x^E(\hat{s},\theta)$$

QED

7
Analysis of Current Practices

This section will compare the RMB allocation rule to typical allocation rules used by firms and apply the results of section 6 to investigate how currently used rules distort managers' investment decisions. Part A will describe the comparative statics results that will be assumed to hold. Then parts B through D will consider different types of allocation rules used by firms.

A. Assumptions

Section 5 demonstrated the following two comparative statics results for the special case where V is of the form (6.1)–(6.2).

Result #1

$x^E(\phi(d,r),V)$ is strictly decreasing in r.

Result #2

$x^E(\phi(d,r_p),V)$ is strictly decreasing as d grows more accelerated.

Result #1 requires the extra (mild) assumption (6.17)-(6.18) that book values are non-negative over the life of the asset and strictly positive for some period. Result #2, as stated above, is true for the case where the agent is impatient. This is probably the typical case in the real world so, for purposes of applying the results to real practices, it is natural to restrict attention to this case. (In the following discussion the affects ascribed to accelerated depreciation rules are reversed for the case of managers that are too patient.)

The approach adopted in this section will be to consider the general model and additionally assume that results #1 and #2, above, are true. The main reason for doing this, as opposed to simply making the assumptions of the special case from section 6 sufficient to generate these results, is to emphasize the fact that the *only* role of these special assumptions in generating the conclusions of this section is to guarantee that results #1 and #2, above, are true. They are not required, for example, to generate any extra conclusions about the nature of the RMB rule. This is an important point because the economic intuitions underlying both results #1 and #2 are clear and plausible. Thus it is likely that results #1 and #2 will continue to hold true in a range of circumstances beyond the special case of section 6. To the extent this is true, the same point applies to the conclusions of this section because they only depend on the special case to generate results #1 and #2.

Thus, in addition to (a.1)-(a.6) as stated in section 2, it will be assumed that

(a.7) Results #1 and #2 as stated above are true.

B. Intangible Assets

Firms typically expense investment costs in intangible assets such as advertising and R&D. That is, 100 percent of the cost is charged to the period the cost is incurred in and none of it is capitalized. In terms of this paper's model, firms use the expense depreciation rule, d^E, defined by (4.13). Since 100 percent of the investment cost is allocated to the current period, the book value of the asset is always equal to 0 and the interest rate chosen by the firm is irrelevant. In particular, we can view the firm as choosing the pair (d^E, r_p).

Therefore, the only difference between the pair used by firms, (d^E, r_p), and the pair which generates the RMB rule, (d^{RMB}, r_p), is the depreciation rule. Therefore, the investment induced by current practices can be compared to the efficient level if d^E and d^{RMB} can be ranked according to the acceleration criterion. It is straightforward to show that d^{RMB} satisfies them. Therefore, d^{RMB} is decelerated relative to d^E. By (a.7) the agent therefore invests less under (d^E, r_p) than under (d^{RMB}, r_p). But, of course, the agent chooses the efficient investment x^*, under (d^{RMB}, r_p). Therefore, the agent invests less than the efficient amount under d^E.

This conclusion is summarized by proposition 7.1.

Proposition 7.1

Suppose the principal chooses the depreciation rule/interest rate pair (d^E, r) where r is any non-negative interest rate. Then the agent invests strictly less than the efficient level, x^*.

Therefore, the current practice of expensing investments in intangible assets causes managers to underinvest in these types of assets. This is because the depreciation rule is more accelerated than the correct rule and impatient managers thus underinvest.

C. Tangible Assets: Residual Income

Some firms create a special income measure to base managerial compensation on by subtracting imputed interest (using the firm's cost of capital) on nondepreciated book value from ordinary accounting income. Such an income measure is called a residual income measure. In terms of this paper's model, using a residual income measure corresponds to choosing a (d,r) pair where $r = r_p$.

This paper's major result is that (d^{RMB}, r_p) is the unique (d,r) which generates a distortion free allocation rule. Thus, this paper's result provides support for the use of residual income to base managerial compensation on. Of course, a continuum of residual income measures can be created by choosing different depreciation rules and this paper's result is that only one of these measures, (d^{RMB}, r_p) generates a distortion free allocation rule. In general, firms do not use the correct residual measure identified by this paper because they generally do not choose the depreciation rule d^{RMB}. In particular, it seems likely that in the majority of cases, they use depreciation rules which are accelerated relative to d^{RMB}. Thus by (a.7), they induce managers to underinvest relative to the efficient level.

The explanation for this is as follows. Proposition 4.5 shows that in order to generate an allocation rule which is constant or increasing in t, that one must choose a depreciation rule which is less accelerated than the straight line rule, d^{SL}. Of course, firms typically choose either the straight line rule or depreciation rules which are even more accelerated than d^{SL}, such as sum-of-the-years-digits, etc. Therefore, whenever s^{RMB} is constant or increasing, d^{RMB} will be less accelerated than all typically used depreciation rules.[16]

It seems likely that in most circumstances, s^{RMB} will be constant or increasing in t. As discussed in section 3, if real benefits are stationary and there is no inflation rate, then s^{RMB} is constant. If there is a positive inflation rate then s^{RMB} will increase in t. Thus, for this plausible "base case," s^{RMB} is increasing in t. Of course, if the real benefit of investment decays over time at a sufficiently fast rate, this may cause s^{RMB} to decline over time. However, it seems likely that, at least in a broad class of cases, that s^{RMB} will be constant or increasing in t.

D. Tangible Assets: Ordinary Accounting Income

Many firms base managerial compensation on ordinary accounting income. Under this income measure, the investment cost allocated to any period equals that period's investment with no imputed interest cost. In terms of this paper's model, use of ordinary accounting income therefore, corresponds to using a pair (d,r) where r = 0. By proposition 5.1, the resulting allocation rule cannot be distortion free.

Therefore, basing managerial compensation on ordinary accounting income will generally distort managerial investment decisions. Unfortunately it is not possible to prove any general qualitative result regarding the effects of typically used depreciation schedules (i.e., to prove that the manager always overinvests or always underinvests). The reason for this is that there are generally two counteracting effects. The first effect is that the value of r that is used is too low. By comparative statics result #1, this causes the manager to overinvest. However, as explained in the previous part, it will often be the case that typically used depreciation rules are accelerated relative to the d^{RMB} rule. By comparative statics result #2, this causes the manager to underinvest.

Appendix A

Proposition 3.1

Property (i) follows immediately from the first order condition, (3.1). Property (ii) follows from (a.4) and property (iii) follows from (a.2).

QED

Propositon 3.2

It is clear that (ii) implies (i) because $\Omega^{SA} \subseteq \Omega^{WEI}$. Now it will be shown that (i) implies (ii). Suppose that s is stand-alone distortion free. Consider some $x \neq x^*$. Then, by assumption

(A.1) $$A_t(x,s) \leq A_t(x^*,s)$$

for every t. If V exhibits WEI this implies that

(A.2) $$V\big(A(x,s)\big) \leq V\big(A(x^*,s)\big)$$

which means that

(A.3) $$x^* \in \underset{x}{argmax}\ V\big(A(x,s)\big)$$

for every $V \in \Omega^{WEI}$.

QED

Proposition 3.3

The allocation rule s is stand-alone distortion free if and only if

(A.4) $$x^* \in \underset{x \geq 0}{argmax}\ B_t(x) - s_t x$$

for every t. Since B_t is concave and x^* is interior, this is true if and only if

(A.5) $$B_t'(x^*) - s_t = 0$$

for every t. This is the definition of s^{RMB}.

QED

Proposition 3.4

Immediate from propositions 3.2 and 3.3

QED

In order to prove the propositions in section 4, a useful lemma will be proven. This lemma might be viewed as the fundamental accounting identity of allocation/depreciation rules.

Lemma A.1

Suppose that $s = \phi(d,r)$. Then for every $t = 0,\ldots,T$,

$$s^t(r) + b_{t+1}\alpha^t = 1 \tag{A.6}$$

where b_t is defined by (4.3)-(4.4).

proof: The proof is by induction on t. For the basis of induction, consider $t = 0$. In this case, the LHS of (A.6) is given by

$$s_0 + b_1 \tag{A.7}$$

Substitution of (4.2)-(4.4) into (A.7) shows that (A.7) equals 1.

Now for the induction step, suppose that

$$s^{t-1}(r) + b_t\alpha^{t-1} = 1 \quad . \tag{A.8}$$

Rewrite the LHS of (A.6) as

$$s^{t-1}(r) + s_t\alpha^t + (b_t - d_t)\alpha^t \quad . \tag{A.9}$$

Substitution of (A.8) and (4.2) into (A.9) and algebraic reorganization shows that (A.9) equals 1.

QED

Proposition 4.1

Straightforward

QED

Proposition 4.2

Straightforward

QED

Proposition 4.3

By lemma A.1,

$$s^T(r) + b_{T+1}\alpha^T = 1 \quad . \tag{A.10}$$

Therefore s is r-complete if and only if $b_{T+1} = 0$. (Since $r \geq 0$, this implies that $\alpha > 0$.) But by (4.4), b_{T+1} is defined by

$$b_{T+1} = 1 - d^T \quad . \tag{A.11}$$

Therefore, s is r-complete if and only if $d^T = 0$.

QED

Proposition 4.4

Suppose that s is r-accelerated relative to $\hat{s}$. By definition (4.7)-(4.9) are true. By lemma A.1, these are equivalent to

$$1 - b_{T+1}\alpha^t = 1 - \hat{b}_{T+1}\alpha^T \tag{A.12}$$

and

$$1 - b_{t+1}\alpha^t \geq 1 - \hat{b}_{t+1}\alpha^t \tag{A.13}$$

for every $t = 0, \ldots, T - 1$ with strict inequality holding for some t. Substitution of (4.3)-(4.4) into (A.12)-(A.13) and algebraic reorganization yields (4.10)-(4.12).

QED

Before proving proposition 4.5 it will be useful to introduce a definition and state and prove a lemma related to it.

Definition: A depreciation rule will be said to be increasing when positive (IWP) if

$$d_t > 0 \Rightarrow d_{t+1} > d_t \tag{A.14}$$

for every $t = 0, \ldots, T - 1$.

Lemma A.2

Suppose that d is proper, complete, and IWP and that $T \geq 2$. Then d^{SL} is accelerated relative to d.

proof: It must be shown that

$$d^{SL,t} \geq d^t \tag{A.15}$$

for every $t = 0, \ldots, T$ with strict inequality holding for some t. Since d is complete, proper, and IWP there must exist a $t^* \in \{1, \ldots, T\}$ such that

$$d_t > 0 \Leftrightarrow t \geq t^* \quad . \tag{A.16}$$

Now it will be shown that (A.15) holds for every t and strictly for some t by considering four cases for t.

Case #1 $t = 0$

In this case (A.15) holds with equality since both depreciation rules are proper.

Case #2 $t = T$

In this case (A.15) holds with equality since both depreciation rules are complete.

Case #3 $0 < t < t^*$

In this case $d^t \leq 0$. However, $d^{SL,t} > 0$ so (A.15) holds with strict inequality.

Case #4 $t^* \leq t < T$

It will be shown that (A.15) holds with strict inequality. Suppose for contradiction that

(A.17) $$d^{SL,t} \leq d^t$$

Recall that

(A.18) $$d_{SL,t} = \begin{cases} 0 & , \quad t = 0 \\ \frac{1}{T} & , \quad t > 0 \end{cases} .$$

Since d is IWP, (A.17)-(A.18) imply that

(A.19) $$d_t \geq d_t^{SL} .$$

However, this implies that

(A.20) $$d_i > 1/T$$

for every $i > t$, which implies that

(A.21) $$d^T > d^{SL,T} .$$

However, this contradicts the assumption that d is complete (i.e., that $d^T = d^{SL,T} = 1$).

Since $T \geq 2$, cases #3 and #4 cannot both be vacuous. Therefore, strict inequality holds for some t.

QED

Proposition 4.5

By lemma A.2, it is sufficient to show that d is IWP. Consider any $t \in \{1,\ldots,T\text{-}1\}$. Suppose that

(A.22) $$d_t > 0 .$$

By (4.2)-(4.4),

(A.23) $$s_t = d_t + r\, b_t$$

and

(A.24) $$s_{t+1} = d_{t+1} + r(b_t - d_t)$$

By (4.15), $s_t \leq s_{t+1}$. Therefore, by (A.23)-(A.24),

(A.25) $$d_t + r b_t \leq d_{t+1} + r(b_t - d_t)$$

which is equivalent to

(A.26) $$d_t(1+r) \leq d_{t+1} \quad .$$

Since $r > 0$, this implies that

(A.27) $$d_t < d_{t+1} \quad .$$

QED

Proposition 5.1

By proposition 3.1, s^{RMB} is r_p-complete. Therefore, by proposition 4.3, d^{RMB} is complete. Therefore $(d^{RMB},r_p) \in D$ and generates s^{RMB}. Since every element of s^{RMB} is non-negative by proposition 3.1, s^{RMB} cannot be r-proper for any $r \neq r_p$. Therefore, by proposition 4.3, (d^{RMB},r_p) is the unique element of D that generates s^{RMB}.

QED

Proposition 6.1

The first-order condition determining x^E is

(A.28) $$\sum_{t=0}^{T} \frac{\theta_t}{(1+r_p)^t} \left(B_t'(x) - s_t\right) = 0$$

where s_t equals $\phi_t(d,r)$. Total differentiation yields

(A.29) $$\frac{dx}{dr} = \frac{\sum_{t=0}^{T} \frac{\theta_t}{(1+r_p)^t} \frac{\partial \phi_t}{\partial r}(d,r)}{\sum_{t=0}^{T} \frac{\theta_t}{(1+r_p)^t} B_t''(x)} \quad .$$

The denominator of (A.29) is negative by the second order condition. By (6.17)-(6.18) and (4.2)-(4.4).

(A.30) $$\frac{\partial \phi_t}{\partial r}(d,r) \geq 0$$

for every t with strict inequality holding for at least one t. Therefore, by (6.2), the numerator of (A.29) is positive.

QED

Proposition 6.2

From (6.1), for any allocation rule $\hat{\hat{s}}$,

$$\frac{d}{dx} V(A(x,\hat{\hat{s}}),\theta) = \sum_{t=0}^{T} \theta_t (B_t'(x) - \hat{\hat{s}}_t)\alpha_p^t \quad . \tag{A.31}$$

Since B_t is concave, (A.31) is decreasing in x. Therefore, it is clearly sufficient to show that

$$\text{the agent is} \begin{array}{l} \text{too impatient} \\ \text{of correct patience} \\ \text{too patient} \end{array} \tag{A.32}$$

implies that

$$\frac{d}{dx} V\big(A(x,s),\theta\big) - \frac{d}{dx} V\big(A(x,\hat{s}),\theta\big) \begin{array}{c} < \\ = \\ > \end{array} 0 \quad . \tag{A.33}$$

Substitute (A.31) into the LHS of (A.33) to yield

$$\sum_{t=0}^{T} \theta_t(\hat{s}_t - s_t)\alpha_p^t \quad . \tag{A.34}$$

Define the first differences of θ_t by

$$\Delta_t = \begin{cases} \theta_t & , \quad t = 0 \\ \theta_t - \theta_{t-1} & , \quad t = 1,\ldots,T \end{cases} \tag{A.35}$$

Substitute (A.35) into (A.34) and reverse the order of summation to yield[17]

$$\theta_T\big(\hat{s}^T - s^T\big) + \sum_{t=0}^{T-1} \Delta_{t+1}\big(s^t - \hat{s}^t\big) \tag{A.36}$$

From proposition 4.3, s is r_p-accelerated relative to $\hat{s}$. Therefore

$$s^T = \hat{s}^T \tag{A.37}$$

and

$$s^t \geq \hat{s}^t \tag{A.38}$$

for every t=0,...,T-1 with strict inequality holding for some t. Substitute (A.37) into (A.36) to yield

$$\sum_{t=0}^{T-1} \Delta_{t+1}\left(s^{t}-\hat{s}^{t}\right) \quad . \tag{A.39}$$

By (A.38), each term in brackets in (A.39) is non-negative and one is positive.

Now consider (A.32). This implies that

$$\Delta_{t} \begin{matrix} < \\ = \\ > \end{matrix} 0 \text{ for every } t = 1,\dots,T \quad . \tag{A.40}$$

which in turn implies that

$$\sum_{t=0}^{T-1} \Delta_{t+1}\left(s^{t}-\hat{s}^{t}\right) \begin{matrix} < \\ = \\ > \end{matrix} 0 \tag{A.41}$$

which in turn implies (A.33).

QED

In order to prove proposition 7 it will be useful to first prove a lemma.

Lemma A.3

Suppose that $s = \phi(d,r)$ and that

(i) s is r-complete

(ii) $s_t \geq 0$ for every t

(iii) $s_t > 0$ for some $t > 0$

Then d^E is accelerated relative to d.

proof:

By definition, d^E is accelerated relative to some other depreciation rule, d, if

$$d^{T} = 1 \tag{A.42}$$

$$d^{t} \leq 1 \text{ for every } t = 0,\dots,T-1 \quad . \tag{A.43}$$

$$d^{t} < 1 \text{ for some } t = 0,\dots,T-1 \quad . \tag{A.44}$$

It is sufficient to show that d satisfies (A.42)-(A.44). Condition (i) implies (A.42). By lemma A.1

(A.45) $$b_{t+1}\alpha^t = 1 - s^t(r) \ .$$

Substitute (4.4) into (A.45) and reorganize to yield

(A.46) $$d^t = 1 - \frac{\left(1 - s^t(r)\right)}{\alpha^t} \ .$$

Since $0 < \alpha \leq 1$, in order to prove (A.43)-(A.44) it is sufficient to show that

(A.47) $$s^t(r) \leq 1 \quad \text{for every } t = 0,\ldots,T-1$$

and

(A.48) $$s^t(r) < 1 \quad \text{for some } t = 0,\ldots,T-1 \ .$$

Condition (ii) implies that

(A.49) $s^t(r)$ is weakly increasing in t.

Condition (i) implies that

(A.50) $$s^T(r) = 1 \ .$$

Condition (iii) implies that

(A.51) $$s^0(r) < s^T(r)$$

Conditions (A.49)-(A.51) imply (A.47)-(A.48).

Proposition 7.1

By the discussion in the text preceeding proposition 7.1, it is sufficient to show that d^E is accelerated relative to d^{RMB}. This follows immediately from lemma A.3 and proposiiton 3.1.

Appendix B

The purpose of this appendix is to state and prove a result which strengthens the uniqueness result of proposition 3.4. The last paragraph of part A of section 3 provides the rationale for investigating this issue.

It will be said that a set of utility functions Ω satisfies the full spanning condition if for every $A \in R^{T+1}$, there exists T+1 utility functions $V^0,\ldots,V^T \in \Omega$ such that

(i) V^t is continuously differentiable at A

(ii) The set of T+1 vectors[18]

$$\left\{ \frac{\partial V^0}{\partial A}(A), \ldots, \frac{\partial V^T}{\partial A}(A) \right\}$$

is linearly independent and thus spans R^{T+1}.

Proposition B.1 now states the result.

Proposition B.1

Suppose then s induces efficient investment choice over Ω and Ω satisfies the full spanning condition Then $s = s^{RMB}$.

proof: Suppose that s induces efficient investment choice over Ω and Ω satisfies the full spanning condition. Let A^* denote $A(x^*,s)$. By the full spanning condition, there exist T+1 utility functions in $\Omega, V^0,\ldots,V^T$, such that the T+1 vectors of marginal utility at A^* are linearly independent. Since s induces efficient investment choice for every $V \in \Omega$, the first order conditions

$$\sum_{i=0}^{T} V_i^t(A^*) \frac{\partial A_i}{\partial x}(x^*,s) = 0 \tag{B.1}$$

must be satisfied for every t. By the full spanning condition, these conditions can only be satisfied if

$$\frac{\partial A_i}{\partial x}(x^*,s) = 0 \tag{B.2}$$

for every $i = 0,\ldots,T$. Of course (B.2) is simply the definition of being stand-alone distortion free.

QED

It may be in some circumstances that the principal knows that the domain of possible indirect utility functions for the agent is some subset Ω of Ω^{WEI}. Based on proposition B.1, the question of whether s^{RMB} is likely to be the unique allocation rule that induces efficient investment for every $V \in \Omega$, depends on whether Ω is likely to satisfy the full spanning condition or not. Some intuition for the answer to this question can be obtained by considering a simple example.[19]

Suppose that the agent is paid a wage each period equal to a constant fraction of that period's accounting income, i.e.,

(B.3) $$w_t = wA_t$$

for some positive number w. Suppose also that the agent's direct utility over wage income equals the discounted value of income using a discount rate of α_a. Thus the agent's indirect utility over accounting income is given by

(B.4) $$V(A) = w\sum_{t=0}^{T} A_t \alpha_p^t$$

Let Ω denote a set of $T+1$ indirect utility functions corresponding to $T+1$ different positive values for α_p. It is straightforward to see that Ω satisfies the full spanning condition. Obviously Ω is also a subset of Ω^{WEI}. Therefore s^{RMB} induces efficient investment choice over Ω (by proposition 3.4) and is the unique allocation rate to do so (by proposition B.1).

Based on this example, it seems intuitively clear that any uncertainty about the agent's discount rate is likely to generate a domain of possible indirect utility functions satisfying the full spanning condition. It is also intuitively clear that nonlinearities or nonstationarities in the wage would not change this conclusion except by pure coincidence in unusual cases. The same comment holds true for nonlinearities in the agent's utility function. Thus, it seems reasonable to conclude that s^{RMB} will generally be the unique rule which induces efficient investment over the entire domain of possible indirect utility functions for the agent.

Notes

1. See Horngren and Foster (1987), page 874, for statistics describing firm practices.

2. See Horngren and Foster (1987), pages 889-891.

3. Rogerson (1992c).

4. Braeutigam and Panzar (1989), Brennan (1980), Rogerson (1992b), Sweeny (1982).

5. Braeutigam (1993), Finsinger and Vogelsang (1985), Loeb and Magat (1979), Sappington (1980), Sappington and Sibley (1988), Sibley (1989), Vogelsang and Finsinger (1979).

6. These papers sometimes also consider a moral hazard problem and investigate how the mechanism deals with it. However, the answer is always "not very well." The primary focus of these papers is on the efficiency of the agent's decisions in cases where there is no unobservable effort.

7. The analysis is essentially unchanged if $T = \infty$. Attention is restricted to the case where T is finite to avoid the expositional complexity of adding assumptions to guarantee that various sequences converge, etc.

8. Note that this definition has implicitly restricted consideration to linear allocation rules. That is, a more general definition of an allocation rule would be a function $a(x) = (a_0(x),\ldots, a_T(x))$ where $a_t(x)$ is the cost allocated to period t and $a_t(x)$ is not necessarily of the form $s_t x$ for some constant s_t. This restriction is basically for expositional convenience. It will be shown that a unique optimal allocation rule exists within the class of linear allocation rules. Furthermore, it will be clear from the proof that the principal does not generally have enough information to attempt to use nonlinear rules. Since firms actually use linear rules and since restricting attention to linear rules simplifies the notation and presentation somewhat, the formal model will therefore consider only linear rules. Footnote 12 will explain why the principal does not generally have enough information to use nonlinear rules.

9. All proofs in this paper are contained in Appendix A.

10. Since the proof involves straightforward integration it is not provided.

11. Recall that $B_0(x) = 0$ because the investment is made at the end of period 0.

12. As promised in footnote 8, this footnote will address the issue of nonlinear allocation rules. Let $a(x) = (a_0(x),\ldots,a_t(x))$ denote a (possibly nonlinear) allocation rule where $a_t(x)$ is the cost allocated to period t. Proposition 3.2 is still true. The argument of proposition 3.3 now shows that any stand-alone distortion free allocation rule must satisfy

$$a_t'(x^*) = B_t'(x^*) \quad .$$

That is, the requirement that an allocation rule be stand-alone distortion free determines the first derivatives of $a_t(x)$ at x^*. Of course if $a_t(x)$ is linear, as assumed in the main body of the text, this determines the entire function. There are thus a continuum of nonlinear distortion free allocation rules. However, if we assume that the principal does not know x^*, then the only one that the principal has enough information to calculate is the linear one. (Of course, B must satisfy RBI for the principal to be able to calculate the linear rule.) That is, since the principal does not know x^*, the only way for the principal to guarantee that the derivative of $a_t(x)$ equals $B_t'(x^*)$ is to choose $a_t(x) = B_t'(x^*)x$ so that this condition is satisfied at every x. Therefore the linear distortion free rule described in the text is the unique distortion free rule that the principal can calculate without knowing x^*.

13. Under straight line depreciation, d_t is constant and book value is strictly decreasing. Therefore, the sum, $d_t + rb_t$, is strictly decreasing.

14. The proposition requires the assumption that $T \geq 2$. If $T = 1$, the only proper complete depreciation rule is the straight line rule so there are no other proper complete depreciation rules to compare d^{SL} to.

15. V satisfies WEI if (6.2) is satisfied with weak inequality. It is convenient to strengthen the assumption to strict inequality in order to sharpen the comparative statics results. (It can be shown that various variables are strictly increasing instead of weakly increasing under this strengthened assumption.)

16. Furthermore, this is a sufficient condition but not a necessary condition. Even when s^{RMB} is decreasing at a "slow" rate, d^{RMB} will still be accelerated relative to d^{SL} and thus to all typically used methods.

17. For the remainder of this proof, s^t will denote $s^t(r_p)$ and $\hat{s}^t$ will denote $\hat{s}^t(r_p)$. The interest rate is supressed to simplify notation.

18. Let $\partial V/\partial A$ denote the vector $(\partial V/\partial A_0,\ldots,\partial V/\partial A_T)$.

19. This example is used by Ramakrishnan (1988).

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