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Renegotiation of Sales Contracts*

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Abstract:

Contracts adopted with later renegotiation in mind may take simple forms. In a principal-agent model, if renegotiation may occur after the agent chooses effort, the principal protects against unfavorable renegotiation by "selling the project" to the agent via a sales contract. If only singleton (single-scheme) contracts are feasible, the equilibrium initial contract must be a sales contract if the principal's renegotiation position will be inherently inferior in the sense that (a) the agent will have the bargaining power; (b) the principal will not observe the agent's effort, and (c) the agent has the talent, i.e. a rich set of feasible efforts, to exploit contractual nuances. Renegotiation necessarily occurs, and it yields (second-best) efficient allocations. Even when menu (multiple-scheme) contracts are available, if the selection of a scheme from a menu entails any cost, then the final contract is a singleton and equilibrium renegotiation occurs. If there is any complexity cost to specifying a menu, the initial contract must also be a singleton; it is necessarily a sales contract if the agent has talent. A weak forward induction refinement criterion is used to obtain these results.

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1. Introduction

Optimal contracts in many environments tend to be unrealistically complicated. The discrepancy is often attributed to the cost of writing complicated contracts, or the presence of asymmetric or unverifiable information. These explanations, which maintain that observed contracts are (constrained) optimal, have well-known difficulties. For example, it is often implausible that the cost of writing an obviously missing clause should be greater than its value. And the presence of asymmetric information generally leads to more, not fewer, complications, since then an optimal contract should specify an elaborate revelation mechanism.

However, a contract may not be optimal in any straightforward way if its designers expect it to be renegotiated. Some contracts are indeed expected to be renegotiated. For example, the financial contracts between a start-up firm and its initial backers are expected, by all parties, to be renegotiated: either the firm will be successful and exchange the initial contracts for more favorable securities, or they will be renegotiated in a bankruptcy proceeding. This paper's theme is that sometimes contracts designed to be renegotiated will be simple. In particular, when the renegotiation process and information structure inherently favor one party, the other party may obtain the best protection against unfavorable renegotiation by insisting on a simple contract.

The idea is explored in a moral hazard, principal-agent model along the lines of Holmström (1979) or Grossman and Hart (1983). The difference is that contract renegotiation is possible after the agent chooses effort, but before its consequence is realized. The main result is that if the principal's position in the renegotiation stage is inherently weak, then ex ante, she should insist on a *sales contract*, the familiar contract which transfers the project's random profit to the agent for a price. A sales contract makes the principal's income a constant function of all contractible signals, and the agent's income independent of any contractible signal except the project's profit. This, together with its requirement that the agent receive all profit in all events, rather than a possibly varying share, make a sales contract simple to write and enforce.

The principal's renegotiating position in the model is inherently weak in two ways. The first is informational, due to her inability to observe the agent's effort. The extent to which this is important depends on the richness of the set of possible efforts. The second inherent weakness of the principal's renegotiation position is its lack of bargaining power, due to a renegotiation process in which the agent makes ultimatum offers (subsequent to, or at the same time as, he takes his unobservable effort). Thus, the principal is especially vulnerable to renegotiation, and this causes her to find an initial sales contract attractive. By giving her an income independent of the agent's effort, a sales contract gives her a solid base from which to bargain, an unswayable knowledge of her payoff if she refuses to renegotiate.

In the first version of the model, a contract is a single scheme specifying an income for each party as a function of the contractible signal. In the equilibrium of central interest, the parties agree initially to a sales contract. The agent then takes an effort and proposes a scheme which together form an allocation that is efficient in the (second-best) sense of, e.g., Holmström (1979) or Grossman and Hart (1983). (Henceforth, an unmodified "efficient" is meant in this second-best sense.) The principal infers from the agent's proposal that he took the efficient effort, and agrees to substitute the proposed scheme for the sales contract. Any scheme the agent might prefer, such as a first-best scheme, would be rejected by the principal because she would infer from its proposal that the agent shirked, i.e., chose an effort which makes his proposal worse for the principal than the initial sales contract.

This equilibrium has three interesting features: (1) its efficient outcome, (2) the equilibrium-path occurrence of renegotiation, and (3) the initial adoption of a sales contract. The same features are exhibited by other plausible equilibria, ones which satisfy a certain belief-based refinement criterion. The criterion requires the principal to believe that the agent has avoided playing a dominated strategy that requires him to take an effort and propose a contract that would give him less than his reservation utility if it were to be accepted. Attention is restricted to equilibria satisfying this relatively weak, "forward-induction" criterion.

All equilibria (thus refined) exhibit efficiency and actual renegotiation, features (1) and (2). All equilibria also exhibit (3), the initial adoption of a sales contract, if the principal's informational handicap is sufficiently severe. It is so severe if the set of possible efforts is of maximal dimension, so that the agent has the "talent" to freely and minutely control the probabilities of contractible events. Such talent, together with the ability to influence (via the forward-induction refinement) the principal's beliefs by making appropriate proposals, enables the agent to exploit nuances in the initial contract. The principal is then especially vulnerable to renegotiation. She obtains maximal protection only by insisting initially on a simple sales contract, the profitability of which is unaffected by how the agent might influence her beliefs about his effort.

A second version of the model is also studied, one in which the definition of a contract is broadened to include "menu contracts." A menu contract specifies a set of schemes from which the agent selects one that will determine the actual payments; the agent selects a scheme after he takes his effort and any renegotiation occurs, but before a contractible signal is realized. Menu contracts are equivalent in this setting to revelation mechanisms for the agent, and are used in studies of informed principals (Myerson, 1983; Maskin and Tirole, 1992), and some studies of incentive contract renegotiation (see below). A menu is sufficiently complicated that renegotiation can be built-in. In a menu contracts game much like that of this paper, Ma (1993) exhibits an efficient equilibrium in which the initial contract, because it is a particular menu that contains a distinct scheme for each possible effort, is not renegotiated.

Menus are more complicated than single schemes, and casual empiricism does not confirm their prevalence. Though this suggests that menus be assumed infeasible (e.g., Huberman and Kahn, 1988; Hermalin and Katz, 1991), doing so does not confront the issue of why menus with at least a few distinct schemes would not be used. Such menus should be only slightly more difficult to specify and enact than single schemes, and their screening benefits might be substantial. Menus are thus considered here (Section 5). However, to reflect their additional intricacy, the use of a menu is assumed to entail a small extra cost. The main question is then whether relatively simple singleton contracts, and very simple sales contracts, still necessarily arise.

Two kinds of menu costs are considered. The first is a "selection cost," incurred only when a menu contract is enacted, which is due to the agent having to decide and verify his selection of a scheme from a menu. The second kind of menu cost is a sunk "complexity cost," incurred at the time a menu is agreed, which is due to the cost of specifying more than one scheme. Both kinds of cost are assumed to be arbitrarily small. Nonetheless, all equilibria (again, refined) are efficient. Both kinds of menu cost imply that renegotiation must occur, and that the final contract is a singleton. Complexity costs further imply that the initial contract is a singleton, and that it is a sales contract if the agent has talent.

That completes the summary of results. Most of them depend on a renegotiation process in which only the agent can propose a new contract. Only then is the principal's inherent bargaining position so poor that she should initially insist on a sales contract. Besides its role in making this theoretical point, the assumption that only the agent can make renegotiation proposals is plausible in some applications. It can either be the exogenously given renegotiation process, or one that is determined by law. For example, in a chapter 11 bankruptcy proceeding used to renegotiate a firm's financial contracts, only the firm, and not its creditors, is allowed to make proposals.[1] A related possibility is that the renegotiation process may be specified in the initial contract. If this is feasible, the model indicates conditions under which the equilibrium initial contract will specify both a sales agreement, and a requirement that the agent make any renegotiation proposal (see Section 6).

Related Literature

A small literature studies renegotiation in moral hazard settings. Hermalin and Katz (1991) consider a model in which the principal observes the agent's effort, but the effort choice is unverifiable to a contract enforcer. Since information is perfect in the renegotiation subgame played after the agent takes his effort, its outcome is the first-best efficient scheme given the effort. The game in which the agent makes ultimatum

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[1] E.g., White (1989). Management has the sole right to file reorganization plans in the first 120 days, and extensions are common.

renegotiation offers has an equilibrium in which a sales contract is adopted and renegotiated, and the resulting allocation is first-best efficient.[2] The difference in this paper is that the agent's effort is unobservable; this makes the first-best unattainable, the analysis more intricate, and some results stronger (e.g. the necessity of a sales contract if the agent has talent).

In Fudenberg and Tirole (1990) and Ma (1991), the principal also does not observe the agent's effort. However, the principal is not at a bargaining disadvantage — she is the one who makes a renegotiation proposal. She finds a menu useful for screening the different "types" of agent (the possible efforts he might have chosen). Menus are used, and would be used even if they entailed a small extra cost. Equilibrium initial contracts which are renegotiation-proof menus exist; in all equilibria, the agent uses a mixed effort strategy, and the outcome is not (second-best) efficient. These models do not address the central issues of this paper, the appearance of simple contracts and equilibrium-path renegotiation.

Ma (1993) is more related, as it too concerns an agent who makes ultimatum renegotiation proposals, and a principal who does not observe the agent's effort. Contracts are menus, and they are no more costly than singletons. The main result is that all equilibria satisfying a refinement criterion are efficient, which is analogous to the efficiency result of this paper.[3] However, the analysis focuses on a renegotiation-proof menu; no reference is made to sales or other singleton contracts, nor to equilibrium renegotiation. Thus, Ma (1993) too does not address the prevalence of simple contracts and actual renegotiation.

Two other papers argue, along different lines and in very different models, that contractual simplicity may result from the possibility of renegotiation. First, Spier (1992) shows that if one party must pay the court costs of enforcing the initial contract if renegotiations break down, then that party may offer a contract that is non-contingent

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[2] This is Proposition 3 in Hermalin and Katz (1991); the sales contract appears in its proof. Huberman and Kahn (1988) and Demski and Sappington (1991) obtain similar results.

[3] The two efficiency results are not identical because the model here is somewhat more general and, more importantly, the refinement criteria differ. See Section 5.

and, consequently, will not be challenged in court.[4] Second, Dewatripont and Maskin (1992) show that the possibility of renegotiation may cause the parties to not collect the information that would make certain events verifiable, thus leading to an endogenously small number of contractible contingencies.

Outline of the Paper

The environment is described in Section 2. The game without menu contracts is described in Section 3, together with the equilibrium concept (perfect Bayesian refined by a weak forward induction criterion). In Section 4, equilibria are shown to be efficient, to entail renegotiation, and to require, if the agent is talented, the initial adoption of a sales contract. In Section 5, menu contracts with small selection or complexity costs are introduced. Selection costs are shown to imply that the final contract is a singleton and renegotiation occurs; complexity costs are shown to imply further that the initial contract is a singleton, and it is a sales contract if the agent has talent. Extensions are discussed in Section 6. Technical details and some proofs are in the Appendices.

2. Preliminaries

The Environment

A principal may hire an agent to make her enterprise productive. The agent's effort is unobservable to the principal and to any enforcer of contracts. The contractible consequence of his effort is a *signal*, which has a finite number of realizations, $i = 1,\ldots, n$. With no loss of generality, the agent's effort is taken to be the probability distribution of the signal. Thus, an effort is denoted as $e = (e_1,\ldots, e_n)$, where e_i is the chosen probability that the signal will take on value i. The set of possible efforts, E, is a subset of Δ, the set of probability vectors in $\Re^n$. We assume E holds at least two efforts,

[4] Another argument in Spier (1992) is that simple contracts may arise not because of renegotiation, but because a privately informed party who makes the initial offer can signal that he is a high type by offering a non-contingent (no-insurance) contract. Allen and Gale (1992) also give a signaling explanation for contractual simplicity.

is compact, and each $e \in E$ is strictly positive. Thus, E is finite or infinite, and the set of possible signal probabilities is bounded below by a positive number.

The signal is sufficient for performance. Conditional on i and e, profit (expected, gross of the agent's compensation) is independent of e, and is denoted as π_i. Expected profit given e is the product $\pi \cdot e$, where $\pi \equiv (\pi_1, \ldots, \pi_n)$.

A *(compensation) scheme* is a vector, $s = (s_1, \ldots, s_n)$, which specifies for each signal i the principal's payment s_i to the agent. The set of schemes is S, and $S = \Re^n$ is assumed; thus, liability constraints are assumed slack (this is discussed in Section 6).

An *allocation* is a scheme and an effort, (s, e). The principal is assumed, for the sake of simplicity, to be risk neutral. Her expected utility is the expected profit of the enterprise less the expected payment to the agent:

$$P(s, e) \equiv (\pi - s) \cdot e. \tag{2.1}$$

The agent's utility is $u(y) - c(e)$ if he is paid y and chooses e. Function u is strictly increasing on $\Re$. It is also strictly concave; the agent is risk averse with respect to money. Function c measures the utility cost of effort; it has domain E and is continuous. The agent's reservation utility is $\bar{A}$. Large payments are assumed to adequately compensate him for any effort: $\bar{y} \in \Re$ exists such that $u(\bar{y}) - c(e) > \bar{A}$ for each $e \in E$.

Scheme s generates a vector $u(s) \equiv (u(s_1), \ldots, u(s_n))$ of possible utilities. Allocation (s, e) gives the agent the expected utility

$$A(s, e) \equiv u(s) \cdot e - c(e). \tag{2.2}$$

His optimal expected utility from scheme s is

$$A^*(s) \equiv \max_{e \in E} \; A(s, e). \tag{2.3}$$

The optimal effort correspondence is

$$E(s) \equiv \operatorname*{argmax}_{e \in E} \; A(s, e), \tag{2.4}$$

and it is nonempty because E is compact and $A(s, \cdot)$ is continuous.

The Principal-Agent Problem

The standard principal-agent problem arises when the parties can commit to a scheme before the agent chooses effort. In the usual treatment, the principal offers the scheme to the agent as an ultimatum offer. An allocation (s, e) is then an equilibrium outcome if and only if it gives the agent at least his reservation utility, and e is his optimal effort given s. These allocations maximize the principal's payoff subject to the *individual rationality constraint* (IR) and the *incentive constraint* (IC):

$$\text{(P)} \quad P^* = \underset{s \in S,\, e \in E}{\text{maximize}}\, P(s, e) \text{ subject to}$$

$$\text{(IR)} \quad A(s, e) \geq \overline{A} \quad \text{and}$$

$$\text{(IC)} \quad e \in E(s).$$

Program (P) has a solution, and every solution satisfies IR with equality (e.g., Grossman and Hart, 1983). To avoid triviality, assume $P^* > \overline{P}$, where $\overline{P}$ is the principal's utility if the agent rejects the scheme.

Solutions (s^*, e^*) of (P) are Pareto optimal in the set of allocations defined by S, E, and IC; such allocations will be referred to as *(second-best) efficient*. A scheme or an effort that is part of an efficient allocation will also be called *efficient*.

Lemma 1 shows that "ex post random schemes" are of no value in (P). These schemes are random vectors $\tilde{s}$ that take on realizations, after the agent has choosen effort, in the set of schemes S. (In the lemma, $\mathcal{E}$ is the expectation operator for $\tilde{s}$.)

<u>LEMMA 1</u>: *If $\tilde{s}$ is a nondegenerate ex post random scheme, and*

$$e \in \underset{d \in E}{\text{argmax}}\ \mathcal{E}A(\tilde{s}, d),$$

then $t \in S$ exists such that $e \in E(t)$, $A(t, e) > \mathcal{E}A(\tilde{s}, e)$, and $P(t, e) > \mathcal{E}P(\tilde{s}, e)$.

<u>PROOF</u>: Let t be an upward perturbation of the certainty equivalent, $\hat{t} \in \Re^n$, defined by $u(\hat{t}) = \mathcal{E}u(\tilde{s})$. See Grossman and Hart (1983) for details. ■

Strict Inducibility

Say that s *induces* e if $e \in E(s)$, and that s *strictly induces* e if $E(s) = \{e\}$. For each (strictly) inducible effort e, a scheme can be found with respect to which e is (uniquely) optimal for the agent. The following regularity assumption will be useful.

ASSUMPTION I: Every inducible effort is strictly inducible.

This assumption holds if $c(e)$ is strictly convex, since then $A(s, e) = u(s) \cdot e - c(e)$ is strictly concave in e, and each inducible effort is strictly induced by every scheme that induces it.

Assumption I generally holds even if c is not convex. Intuitively, inducibility but not strict inducibility is a knife-edge property of an effort e, destroyed by perturbing the cost $c(e)$ either up or down. Proposition 0(*iv*) in Appendix A shows that Assumption I is satisfied generically if E is a finite set.

The consequence of Assumption I that will be useful is the following.

LEMMA 2: *For any scheme $s \in S$ and effort $e \in E(s)$, every neighborhood of s contains some t such that $E(t) = \{e\}$ and $A(t, e) = A^*(s)$.*

PROOF: Scheme s and effort e satisfy

$$A^*(s) \;=\; A(s, e) \;\geq\; A(s, d) \;\text{ for all } d \neq e. \tag{2.5}$$

From Assumption I, scheme $\hat{s}$ exists such that

$$A^*(\hat{s}) \;=\; A(\hat{s}, e) \;>\; A(\hat{s}, d) \;\text{ for all } d \neq e. \tag{2.6}$$

For any number k, and a sufficiently small $\delta > 0$, a scheme t is defined by

$$u(t_i) \;\equiv\; \delta(k + u(\hat{s}_i)) + (1 - \delta)u(s_i).$$

Note that $t \to s$ as $\delta \to 0$. Hence, δ can be chosen small enough to fit t into any given neighborhood of s. For $d \neq e$, (2.5) and (2.6) imply,

$$A(t, e) - A(t, d) \;=\; \delta[A^*(\hat{s}) - A(\hat{s}, d)] + (1-\delta)[A^*(s) - A(s, d)] \;>\; 0.$$

Thus, $E(t) = \{e\}$. Setting $k = A^*(s) - A^*(\hat{s})$ yields $A(t, e) = A^*(s)$. ■

Contracts

Two kinds of contract are considered. A *singleton contract* specifies just one scheme, $s \in S$. A *menu contract* specifies a nonempty, compact set of schemes, $m \subset S$. If a menu contract is adopted, the agent must select one scheme from the menu to be the actual scheme that will be used — he makes the selection after he has taken his action and any renegotiation has occurred. Menus are considered in Fudenberg and Tirole (1990), Ma (1991, 1993), Maskin and Tirole (1992), and Section 5 of this paper.

Two kinds of singleton contract (scheme) are of special interest. The first is a *wage contract* , which pays the agent the same amount regardless of the realized signal: $s_i = s_1$ for all i. Because the principal is risk neutral, wage contracts are first-best efficient. We assume that first-best efficiency is unattainable, i.e., that wage schemes do not solve (P). It follows that efforts which minimize c are also not efficient.

The second special kind of singleton contract is a sales contract. The *sales contract with price p*, denoted as $r(p) = (r_1(p),\ldots, r_n(p))$, transfers the profit of the enterprise to the agent for price p:

$$r_i(p) \equiv \pi_i - p \quad \text{for each } i = 1,\ldots n. \tag{2.7}$$

A sales contract gives the principal an income, $\pi_i - r_i(p) = p$, that does not depend on the signal i. It gives the agent an income that is a linear function of profit π_i, and does not depend on other information the signal might contain: $r_i(p) = r_j(p)$ if $\pi_i = \pi_j$. This is what makes a sales contract relatively simple. The crucial feature of a sales contract is that it makes the principal's payoff independent of her beliefs about the agent's effort: $P(r(p), e) = p$ for all e. The *optimal sales contract* has price P^*, the principal's payoff in the principal-agent problem:

$$r^* \equiv r(P^*). \tag{2.8}$$

An Example

The following example will be useful. In this example the signal is equal to profit. It can be good, π_g, or bad, $\pi_b < \pi_g$. In this example only, "effort" refers to the probability of π_g (rather than a probability vector). It can be low, e_ℓ, or high, e_h, where $0 < e_\ell < e_h < 1$ and $c(e_\ell) < c(e_h)$. A scheme takes the form $s = (s_g, s_b)$. Hence,

$$P(s, e) = e(\pi_g - s_g) + (1 - e)(\pi_b - s_b),$$

$$A(s, e) = eu(s_g) + (1-e)u(s_b) - c(e).$$

The efficient allocation, (s^*, e^*), is assumed unique, with $e^* = e_h$.

Figure 1 illustrates. Points in the box are schemes (s_g, s_b). The agent's expected utility increases to the northeast, and the principal's to the southwest. The locus of wage contracts is the agent's 45° line. The locus of sales contracts is the principal's 45° line.

The indicated IC curve is the locus of schemes at which the incentive constraint binds, i.e., at which $A(s, e_h) = A(s, e_\ell)$. Schemes below the IC curve induce the agent to work hard by rewarding him with a large bonus if the observed profit is π_g. Schemes above the IC curve, such as the wage contracts, induce low effort.

The steeper indifference curve in Figure 1 gives the agent his reservation utility when he chooses high effort; it is defined by $e_h u(s_g) + (1-e_h)u(s_b) = \bar{A} + c(e_h)$. The flatter indifference curve gives him $\bar{A}$ when he chooses low effort.

The principal must offer the agent a scheme below the IC curve if she wants him to choose high effort. Given that the offered scheme induces high effort, the agent accepts it only if it is above the steeper indifference curve. The shaded area is thus the set of schemes which are acceptable to the agent and induce him to take high effort.

Assuming the agent takes high effort, the principal's isoprofit lines are steep, with slope $-e_h/(1-e_h)$, and increase to the southwest. Her best scheme in the shaded area when the agent takes high effort is therefore s^*. The optimal sales contract, r^*, is the sales contract on the same steep isoprofit line as s^*; its price P^* is indicated on the right axis. Both s^* and r^* give the principal payoff P^*, assuming the agent takes high effort. This payoff should be compared to that obtained by inducing low effort, which is achieved by offering the indicated wage contract $\hat{s}$. The corresponding payoff to the principal is the indicated $\hat{P}$. Since $\hat{P} < P^*$, inducing high effort is optimal.

3. The Renegotiation Game

Program (P) relies on the the parties being able to commit to not renegotiate the agreed scheme later, after the effort is chosen and before the signal is realized. In order to study what may happen when this commitment is impossible, an explicit renegotiation process is defined. Two will be considered, differing in whether menu contracts are feasible. The description in this section is of the game in which they are not feasible; menu contracts are deferred to Section 5.

The game starts when the principal offers a contract to the agent. If the agent accepts it, play moves into a *renegotiation subgame*. This subgame starts with the agent choosing an effort and proposing a new contract. The principal, unaware of the agent's effort, chooses between the agent's proposal and the initial contract. Given that only singleton contracts are feasible, a contract can be identified with the scheme it specifies. The scheme specified in the principal's offer is denoted r, and the ensuing renegotiation game is $\Gamma(r)$. The scheme specified in the agent's renegotiation proposal is denoted s. The extensive form is shown in Figure 2. As will be clear, its most important feature is that the agent, not the principal, makes the renegotiation offer.

A strategy for the principal is an offer r, and an acceptance rule $a_P(r, s)$ giving her probability of accepting the agent's proposal s when the initial contract is r. The agent's strategy is an acceptance rule $a_A(r)$ giving his probability of accepting offer r, and a proposal $s(r)$ and an effort $e(r)$ in each subgame $\Gamma(r)$. The principal chooses between r and s according to her beliefs about the agent's effort. Her belief function, $\beta(e \mid r, s)$, is a probability measure on E conditional on the agent accepting r and proposing s.

A *perfect Bayesian equilibrium* (PBE) is a profile of mixed strategies and a belief function satisfying three conditions. First, the strategies must induce a Nash equilibrium on each subgame. Second, the belief function must be consistent with Bayes' rule and the agent's strategy, whenever possible. Third, the principal's acceptance rule must maximize her expected utility according to her beliefs. Given a PBE, let A^{eq} and P^{eq} denote the equilibrium payoffs, and $A^{eq}(r)$ and $P^{eq}(r)$ the equilibrium payoffs of subgame $\Gamma(r)$. A PBE is *efficient* if the equilibrium payoffs correspond to solutions of program (P), i.e., are $\overline{A}$ and P^*. Lemma 3, proved in Appendix B, shows that a PBE with these payoffs is in fact Pareto optimal within the set of perfect Bayesian equilibria.

<u>LEMMA 3</u>: *PBE payoffs satisfy* $P^{eq} \le P^*$, $A^{eq} \ge \overline{A}$, *and* $A^{eq} = \overline{A}$ *whenever* $P^{eq} = P^*$. *For any r accepted by the agent with positive probability,* $P^{eq}(r) \le P^*$ *and* $A^{eq}(r) \ge \overline{A}$. *For any r satisfying* $P^{eq}(r) = P^*$, $A^{eq}(r) \le \overline{A}$.

The next lemma shows that wage contracts cannot be renegotiated. Any contract the agent might propose which he prefers to a wage contract must give him an expected payment greater than the wage. The risk neutral principal therefore prefers paying the wage to accepting the proposal if she has 'rational expectations' about the agent's effort, which she does in equilibrium.[5]

<u>LEMMA 4</u>: *Wage contracts are not renegotiated: if r is a wage contract and* $s \ne r$, *then in every PBE the principal surely rejects s in favor of r* (*i.e.* $a_P(r, s) = 0$).

<u>PROOF</u>: Let $r_i = w$ for all i. Consider a perfect Bayesian equilibrium of subgame $\Gamma(r)$. Let s be an equilibrium offer of the agent in this subgame, and let $a \equiv a_P(r,s)$. Suppose $a > 0$. Because the principal finds s acceptable, effort e exists such that $s \cdot e \le w$, and (s, e) is an equilibrium choice of the agent. The agent could have offered r instead of s, and so,

$$a(u(s) \cdot e) + (1-a)u(w) - c(e) \;\ge\; u(w) - c(e).$$

Since u is strictly concave and a is positive, this implies that $s \cdot e \ge w$, with the inequality strict unless $s = r$. Hence, $s = r$. ■

A Refinement Criterion

The subgames $\Gamma(r)$ are essentially signaling games in which the agent's proposal signals his effort. Equilibria therefore abound. Some, however, require the principal to believe, off the equilibrium path, that the agent has played a dominated strategy. These

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[5] This "no-trade theorem" resembles that of Milgrom and Stokey (1982). Since a wage contract efficiently shares risk, it is common knowledge that one party must lose by renegotiating a wage contract (even though e is not common knowledge).

equilibria would not survive a refinement criterion based on twice removing dominated strategies, first for the agent and then for the principal. A weaker criterion is formulated here, one which requires the principal to not believe the agent has chosen a certain kind of dominated strategy. The criterion embodies a weak notion of forward induction.[6]

At issue is an agent strategy which accepts some r with positive probability, and in $\Gamma(r)$ plays a pair (s, e) such that $A(s, e) < \bar{A}$. Such a strategy is dominated. The dominating strategy depends on whether the agent can obtain $\bar{A}$ with r. If he cannot, i.e. if $A^*(r) < \bar{A}$, then a dominating strategy rejects r. Alternatively, if $A^*(r) \geq \bar{A}$, a dominating strategy accepts r, but then proposes r instead of s, and chooses an effort in $E(r)$. If the principal believes, at the information set indexed by (r, s), that the agent has not played such a dominated strategy, she must assign all probability to the set

$$\bar{E}(s) \equiv \{e \in E \mid A(s, e) \geq \bar{A}\}. \tag{3.1}$$

This motivates Criterion C (for want of a better name).[7]

<u>CRITERION C</u>: For all contracts r and s, $\beta(\bar{E}(s) \mid r, s) = 1$ whenever $\bar{E}(s) \neq \emptyset$.

The unmodified term *equilibrium* henceforth refers to a PBE satisfying Criterion C. The existence of equilibria is shown in Section 5.

This criterion is illustrated by the following strategies and beliefs in the two-by-two example. According to them, the principal always believes the agent has shirked, and the agent does in fact shirk:

> the principal believes the effort is e_ℓ after any (r, s), and accepts s if and only if $P(s, e_\ell) \geq P(r, e_\ell)$; (3.2*a*)
>
> in each $\Gamma(r)$, the agent chooses e_ℓ and proposes the wage contract s_r that pays him $w(r) \equiv e_\ell r_g + (1 - e_\ell) r_b$; (3.2*b*)

---

[6] Forward induction is discussed, e.g., in Kohlberg and Mertens (1986), Cho and Kreps (1987), van Damme (1992), and Dekel and Ben-Porath (1992).

[7] The criterion is most sensible at the (r,s) pairs where it will be used. At these pairs, $P(s, e) > P(r, e)$ for all $e \in \bar{E}(s)$, and so it is consistent for the agent to believe the principal will accept s.

the agent accepts r if and only if $u(w(r)) - c(e_\ell) \geq \bar{A}$; and (3.2*c*)

the initial offer of the principal is the wage contract $\hat{s}$ (shown in Figure 3) that gives the agent $\bar{A}$ if he chooses e_ℓ. (3.2*d*)

For some parameters, (3.2) defines a PBE.[8] Yet, the beliefs in (3.2*a*) violate Criterion C. Suppose the agent proposes the s in Figure 3. The principal is supposed to infer that the agent chose e_ℓ. However, $\bar{E}(s) = \{e_h\}$, since $A(s, e_h) > \bar{A} > A(s, e_\ell)$. Criterion C therefore requires the principal to believe the agent chose e_h if he proposes s. Believing the agent chose e_ℓ is tantamount to believing that he played a dominated strategy. If the initial r cannot give the agent his reservation utility, then accepting r, proposing s, and choosing e_ℓ is dominated by simply rejecting r. If r can give the agent his reservation utility, a dominating strategy accepts r and does not propose a new contract (or re-proposes r).

4. Singleton Game Results

The results of this section are that when only singleton contracts are feasible, then (1) all equilibria are efficient; (2) renegotiation must occur; and (3) equilibrium initial contracts are necessarily sales contracts if the agent has "talent."

Equilibria Are Efficient

Equilibria are efficient because Criterion C implies that the the principal can make profitable initial offers which the agent cannot refuse. In particular, the agent must accept certain sales contracts. Referring again to Figure 3, consider sales contract $r(p)$. Suppose that in subgame $\Gamma(r(p))$, the agent proposes the indicated s. As we have seen, Criterion C requires the principal to believe the agent chose high effort if he proposes s. The principal consequently accepts s, as it is below the steep isoprofit line through $r(p)$. Thus,

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[8] A PBE is defined by (3.2) if for each r, the agent prefers $(w(r), e_\ell)$ to (s, e_h), where s is any contract the principal would accept in $\Gamma(r)$, i.e., any s for which $e_\ell s_g + (1-e_\ell)s_b \leq e_\ell r_g + (1-e_\ell)r_b$. This is the case, for example, if $u(y) = \ln(y)$, and

$$\frac{c(e_h) - c(e_\ell)}{e_h - e_\ell} > \ln\left(\frac{e_h(1-e_\ell)}{e_\ell(1-e_h)}\right)$$

Even with this inequality, e_h is efficient if π_g is large.

proposing s and choosing e_h gives the agent more than $\bar{A}$; thus, he accepts $r(p)$. The principal's equilibrium payoff hence exceeds p, as she can obtain that by offering $r(p)$ and not renegotiating. This is true of any $p < P^*$, and so her equilibrium payoff is P^*.

The next lemma (proved in Appendix B) is used to generalize this argument. It gives a way of finding proposals for the agent which the principal cannot refuse.

LEMMA 5: *Any neighborhood of a non-wage contract t contains an s such that*

(*i*) $A^*(s) > A^*(t)$, *and*

(*ii*) $P(s, e) > P(t, e)$ *for all e satisfying* $A(s, e) \geq A^*(t)$.

A consequence of Lemma 5 is that a scheme having the properties of the s shown in Figure 3 exists. Lemma 6 proves this, and the efficiency proposition follows.

LEMMA 6: *Let* (s^*, e^*) *be efficient, and consider any* $p < P^*$. *Every neighborhood of* s^* *contains some s such that* (*i*) $A^*(s) > \bar{A}$, *and* (*ii*) $P(s, e) > p$ *for all* $e \in \bar{E}(s)$.

PROOF: As the first-best is assumed unachievable, s^* is not a wage scheme. Lemma 2 then implies that near s^* exists another non-wage scheme, t, such that $E(t) = \{e^*\}$ and $A(t, e^*) = A(s^*, e^*) = \bar{A}$. This t can be choosen so close to s^* that, for some $\varepsilon > 0$,

$$P(t, e^*) > p + \varepsilon. \tag{4.1}$$

Correspondence $\bar{E}(\cdot)$ is u.h.c. Thus, using (4.1) and $\bar{E}(t) = \{e^*\}$, a neighborhood N of t exists such that for all $s \in N$,

$$P(t, e) > p \text{ for all } e \in \bar{E}(s). \tag{4.2}$$

From Lemma 5, $s \in N$ exists such that $A^*(s) > A^*(t)$, and $P(s, e) > P(t, e)$ for all e satisfying $A(s, e) \geq A^*(t)$. This is the desired s, using (4.2) and $A^*(t) = \bar{A}$. ■

PROPOSITION 1: *Every equilibrium is efficient.*

PROOF: Fix an equilibrium, and let $p < P^*$. Let s be as in Lemma 6. By (*i*), $\bar{E}(s) \neq \varnothing$. Criterion C and (*ii*) imply that the principal will accept s over $r(p)$. Thus, by (*i*), the agent has a way of obtaining more than $\bar{A}$ in $\Gamma(r(p))$ (proposing s and choosing $e \in E(s)$). He therefore accepts $r(p)$. Hence, $P^{eq} \geq p$. This is true for all $p < P^*$, and so $P^{eq} \geq P^*$. Lemma 3 now implies the result, $P^{eq} = P^*$ and $A^{eq} = \bar{A}$. ■

Renegotiation Occurs

The second set of results concern the necessity of renegotiation. The first is that the principal cannot simply offer an efficient s^* initially, because efficient schemes are renegotiated to inefficient ones. This is true of any PBE in the two-by-two example, as Figure 4 shows. If the initial contract is the indicated s^*, then *regardless of her beliefs about the agent's effort*,[9] the principal will agree to renegotiate into the shaded lens. This is because her expected payment to the agent is larger with s^* than it is with any s in the lens, holding the agent's effort fixed.[10] Thus, by proposing a scheme in the lens and shirking, the agent can obtain more than $\bar{A}$. This observation implies that in any PBE, the agent accepts s^* if it is initially offered. But after accepting it, the agent shirks and proposes the indicated wage contract t, which the principal accepts.

In more general environments, Criterion C is needed to show that the principal obtains less than P^* by offering an efficient scheme:

<u>PROPOSITION 2</u>: *In any equilibrium, $P^{eq}(s^*) < P^*$ and $A^{eq}(s^*) > \bar{A}$ if s^* is efficient.*

<u>PROOF</u>: Since IR binds in (P), $A^*(s^*) = \bar{A}$. Thus, using Lemma 5 (with $t = s^*$), scheme s exists such that $A^*(s) > \bar{A}$, and $P(s, e) > P(s^*, e)$ for all e satisfying $A(s, e) \geq \bar{A}$. By Criterion C, the principal agrees to renegotiate s^* to s. In this way the agent can obtain more than $\bar{A}$ in $\Gamma(s^*)$, and so $A^{eq}(s^*) > \bar{A}$. This and Lemma 3 imply $P^{eq}(s^*) < P^*$. ■

An implication of Propositions 1 and 2 together is that in any equilibrium, initial contracts that are efficient would be renegotiated if they were to be offered, but they are not offered on the equilibrium path.

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[9] I thank Albert Ma for bringing this property of the two-effort model to my attention.

[10] The fact that the principal prefers (s, e_h) to (s^*, e_h) does not contradict the latter's efficiency. Allocation (s, e_h) is infeasible for program (P) because it violates IC.

Proposition 3 below characterizes equilibrium initial contracts, shows that they too are renegotiated, and characterizes equilibrium final contracts. The characterization is in terms of the following program, parameterized by schemes r.

$$(\mathrm{A}\text{ - }r)\quad A^r \equiv \underset{s\in S,\, e\in E}{\text{maximize}}\, A(s,e) \ \text{ such that}$$

$$(\mathrm{IC})\quad e \in E(s) \ \text{ and}$$

$$(\mathrm{AC})\quad P(s,e) \geq P(r,e).$$

This program has a solution for all r, as is shown in in the next section. Its objective is to maximize the agent's utility subject to the incentive constraint IC, and to the principal's *acceptance constraint*, AC. The value A^r is an upper bound on the agent's set of Nash equilibrium payoffs in subgame $\Gamma(r)$.[11] If r is an equilibrium initial contract, an equilibrium final contract is a solution of (A - r). This is part of the following proposition, proved in Appendix B.

<u>PROPOSITION 3:</u> *Given an equilibrium, let r be an equilibrium offer of the principal. Then,*

(*i*) *r is not a wage contract;*

(*ii*) $A^*(r) < \bar{A}$, *but the agent surely accepts r;*

(*iii*) *the principal surely accepts any equilibrium proposal of the agent in $\Gamma(r)$; and*

(*iv*) *the equilibrium allocations of $\Gamma(r)$ are efficient and solve* (A - r).

Talented Agents Receive Sales Contracts

Sales contracts have so far appeared only as hypothetical initial offers used to prove the efficiency of equilibria in Proposition 1. We now show that if the agent has talent in the sense of being able to perturb the signal probabilities in any possible way, then the optimal sales contract is the only equilibrium initial contract.

Without the talent assumption, equilibrium initial contracts need not be sales contracts. For example, in Figure 4 any scheme on the isoprofit line through r^* and s^*

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[11] The proof of this is easy for a pure strategy Nash equilibrium, since its allocation satisfies IC and AC. The general proof is involved and not needed here.

that is below the indicated $\hat{r}$ is an equilibrium initial contract. In the corresponding equilibrium, the agent accepts such a contract and then renegotiates it to s^*.

However, a scheme on the r^*-s^* isoprofit line in Figure 4 that is above $\hat{r}$ is not an equilibrium initial contract. In any equilibrium, such a scheme would be renegotiated to a wage contract above $\hat{s}$ (the argument is the same as for why s^* would be renegotiated to a wage contract. If an inefficient third effort e_H greater than $e^* = e_h$ exists, schemes too far southeast of r^* also are not equilibrium initial contracts.[12] Given this third effort, the equilibrium initial contracts can be shown to form an interval of the r^*-s^* isoprofit line that shrinks to r^* as the interval $[e_\ell, e_H]$ shrinks to e_h.

Instead of taking limits, it is easier to work in the limiting case of a continuum of efforts. First, define the principal's cost function for inducing effort:

$$C(e) \equiv \min_{s \in S} s \cdot e \text{ subject to IC and IR.}^{13} \tag{4.3}$$

Say that the agent "has talent" if the following condition is satisfied:

<u>CONDITION T</u>: (*i*) The interior of E relative to the simplex Δ is nonempty;

(*ii*) Each efficient effort is in the relative interior of E; and

(*iii*) C is smooth at each efficient effort.[14]

Given Condition T, the agent can perturb any efficient effort, i.e. signal probability vector, a small amount in any direction. The principal can induce these perturbations at no first-order loss, since an efficient e^* maximizes the differentiable function $\pi \cdot e - C(e)$.

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[12] An s exists to the southeast of s^* such that $A(s, e_H) > \bar{A} > A(s, e_h) > A(s, e_\ell)$. If r is a scheme far to the southeast of r^*, then $s \cdot e_H < r \cdot e_H$. In this case, by Criterion C, the principal agrees to renegotiate r to s. Thus, in any equilibrium $A^{eq}(r) > \bar{A}$. As equilibria are efficient, r cannot be an equilibrium initial contract.

[13] Grossman and Hart (1983) show that this program has a solution if the constraint set is nonempty, which it is for inducible efforts. Let $C(e) = \infty$ if e is not inducible.

[14] Point e is in the interior of E relative to Δ if an open set $U \subset \Re^n$ exists such that $e \in U \cap \Delta \subset E$. Any f: $E \rightarrow \Re$ is *smooth* at a relatively interior e if it has a continuously differentiable extension to some such U. We may assume f is actually defined on U, so that the partial derivatives $f_i(e)$ and gradient $\nabla f(e)$ can be written without ambiguity; the upcoming arguments do not depend on the extension used (Milnor, 1965, pp. 1–7).

Parts (*i*) and (*iii*) of Condition T are satisfied, for example, if $E = \Delta$, the agent's cost and utility functions are twice continuously differentiable, and c is convex. Then, a strictly positive e is induced by s if and only if multiplier ψ exists such that

$$u(s_i) = \psi + c_i(e) \text{ for each } i,$$

where $c_i(e)$ is the partial derivative. The agent's resulting expected utility is

$$u(s)\cdot e - c(e) = \psi + \nabla c(e)\cdot e - c(e).$$

If this expected utility is $\bar{A}$, as is true when s solves (4.3) (as IR binds), then $s_i = u^{-1}[\bar{A} + c(e) - \nabla c(e)\cdot e + c_i(e)]$. Therefore,

$$C(e) = \sum_{i=1}^{n} e_i u^{-1}[\bar{A} + c(e) - \nabla c(e)\cdot e + c_i(e)].$$

Note that this C is smooth at each inducible, relative interior effort.

PROPOSITION 4: *Under Condition T, the only equilibrium initial contract is the optimal sales contract* r^*.

PROOF: Let r be an equilibrium initial contract. Let (s^*, e^*) be an equilibrium allocation of $\Gamma(r)$. By Proposition 3(*iv*), it solves (P) and (A - r). The former implies that e^* solves

$$(\text{P} - \pi) \quad \max_{e\in E} \ \pi\cdot e - C(e).$$

Because (s^*, e^*) is efficient and solves (A - r), $A^r = \bar{A}$. Hence, dual to (A - r) is,

$$(\text{DA} - r) \quad \underset{s\in S,\, e\in E}{\text{maximize}} \ P(s, e) - P(r, e) \text{ subject to IC and}$$

$$(\text{IR}) \ A(s, e) \geq \bar{A}.$$

This program is like (P) except that r replaces π, since $P(s, e) - P(r, e) = r\cdot e - s\cdot e$. Thus, any allocation solving (DA - r) has an effort solving the program,

$$(\text{P} - r) \quad \max_{e\in E} \ r\cdot e - C(e).$$

A standard exercise shows that (s^*, e^*) solves (DA - r) becauses it solves (A - r). We conclude that e^* solves (P - r) as well as (P - π).

Under Condition T, interior first order conditions for both programs are satisfied at e^*. Since the constraint $\Sigma e_i = 1$ is linear, constraint qualification holds. Thus, multipliers λ and μ exist such that for each i,

$$\pi_i - C_i(e^*) - \lambda = 0 \quad \text{and} \quad r_i - C_i(e^*) - \mu = 0.$$

Letting $p = \lambda - \mu$, we see that $r_i = \pi_i - p$; r is the sales contract with price p. Since AC binds, $P(s^*, e^*) = P(r, e^*) = p$. Since (s^*, e^*) is efficient, $P(s^*, e^*) = P^*$. Thus $p = P^*$, and r is the optimal sales contract. ■

5. Menu Game Results

Menu contracts are now introduced. The game starts with the principal offering a menu m^o. Acceptance of m^o by the agent leads to a subgame $\Gamma(m^o)$ in which the agent takes an effort and proposes a menu m. The principal chooses between m and m^o, after which — this is the new stage — the agent selects a scheme from the final contract to determine actual payments.

The central questions are whether (a) equilibrium-path renegotiation, (b) relatively simple singleton contracts, or (c) quite simple sales contracts necessarily arise when menus are available. Affirmative answers require that some extra cost be borne if a multiple-scheme menu is used. This follows from Ma's (1993) construction, for the game in which menus are freely available, of an efficient equilibrium in which a renegotiation-proof, multiple-scheme menu is adopted. This menu is $m^* \equiv \{s(e) \mid e \in E_I\}$, where E_I is the set of inducible efforts, and $s(e)$ solves the cost minimization (4.3). The agent's strategy in this equilibrium is to accept m^* initially, to not renegotiate m^*, to take an efficient e^*, and to select from m^* the efficient scheme $s^* = s(e^*)$.

However, as shall be shown, even small menu costs imply that a menu will not be used. Two kinds of menu costs are considered. The first is a *selection cost*, borne when the agent selects a scheme from a menu. This cost could be the transportation or opportunity cost required to make the selection, or the cost of hiring a third-party to verify the selection, or the agent's computational cost of finding his optimal selection. Selection costs are avoidable by renegotiating a menu contract to a singleton, since they are only incurred if the agent actually selects from a menu. The second kind of menu cost is a *complexity cost* of formulating, writing, reading, and agreeing to a multiple-scheme menu. Complexity costs, unlike selection costs, are sunk when a menu is agreed, even if it is later renegotiated to a singleton. Both kinds of menu cost will be shown to imply that renegotiation occurs and the final contract is a singleton, even if the costs are arbitrarily small. With menu complexity costs, the initial contract is necessarily a singleton, and it must be the optimal sales contract if the agent is talented.

These results are obtained using the previous section's demonstration that efficiency is achievable without menus. However, the derivations are not trivial; in principle, even if menus are costly, they could be adopted in equilibrium. In fact, without a refinement criterion, menus that have only small costs are sustained by perfect Bayesian equilibria in which the principal believes the agent will reject initial contracts that are singletons, and will have shirked whenever he proposes to renegotiate to a singleton.

Selection Menu Costs

A simple model of small selection costs suffices. Accordingly, the cost of selecting from any multiple-scheme menu is assumed to be a constant $\varepsilon_A \geq 0$ to the agent, and $\varepsilon_P \geq 0$ to the principal. If the agent selects scheme s from menu m and takes effort e, the payoffs are

$$A(s, m, e) = A(s, e) - \varepsilon_A g(m), \text{ and}$$

$$P(s, m, e) = P(s, e) - \varepsilon_P g(m),$$

where $g(m) \equiv 1$ if m contains multiple schemes, and $g(m) \equiv 0$ otherwise. If the final contract is m and the agent choose e, his payoff is

$$A(m, e) \equiv \max_{s \in m} A(s, m, e).$$

Given menu m, the agent's set of optimal efforts is $E(m) \equiv \operatorname{argmax}_{e \in E} A(m, e) \neq \emptyset$. Each scheme in $E(m)$ gives him utility $A^*(m)$.

Although Criterion C could be extended to apply to menus, doing so is unnecessary.[15,16] The results below rely on those of Section 4, where Criterion C is applied only to singleton offers. Thus, a belief restriction is needed only at singleton offers. Say that a PBE in this game is an *equilibrium* if it satisfies Criterion C exactly as it is defined — for singletons m^o and m — in Section 3.

The existence of equilibrium is an issue, here and in the menuless game of Section 4, since (1) strategy sets are infinite, (2) payoffs are discontinuous (with menu costs), and (3) Criterion C must be met. The proof of Proposition 5 below specifies beliefs and strategies that are an equilibrium regardless of whether menus are availiable.

First, a generalization of program (A - r) is needed. For any menu m, let $S(m, e)$ be the subset of schemes in m that maximize $A(\cdot, e)$. Note that $S(m, \cdot)$ is a nonempty, compact-valued, u.h.c. correspondence. Thus, the set $S(m, e)$ contains a worst scheme for the principal, $s^{w}(m, e) \in \underset{t \in S(m, e)}{\operatorname{argmin}} P(t, e)$. The desired program is:

$$(\text{A - } m^o) \quad A^{m^o} \equiv \underset{s \in S,\, e \in E}{\text{maximize}}\, A(s, e) \ \text{ such that}$$

$$(\text{IC}) \quad e \in E(s) \ \text{ and}$$

$$(\text{AC}) \quad P(s, e) \geq P(s^{w}(m^o, e), m^o, e).$$

The next lemma (proved in Appendix C) shows that (A - m^o) has a solution, despite its unbounded set of feasible schemes, and the discontinuous right side of AC.

<u>LEMMA 7</u>: *For any menu* m^o, (A - m^o) *has a solution.*

In the equilibrium constructed to prove Proposition 5 , in subgame $\Gamma(m^o)$ the agent selects $s^{w}(m^o, e)$ if he chose e and m^o is not renegotiated. His equilibrium renegotiation

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15 For any m^o, a strategy of the agent is dominated if it requires him to accept m^o, but then to choose e and propose m such that $A(m, e) < \overline{A}$. Thus, Criteron C can be extended to menus merely by replacing s by m: let $\overline{E}(m) \equiv \{e \in E \mid A(m, e) \geq \overline{A}\}$, and require $\beta(\overline{E}(m) \mid m^o, m) = 1$ when $\overline{E}(m) \neq \varnothing$.

16 One could worry whether the equilibrium constructed to prove existence, in Proposition 5 below, satisfies the previous footnote's extension of Criterion C to cover menus. It does, as its belief function satisfies $\beta(E(m) \mid m^o, m) = 1$.

proposal and effort is a singleton and effort that jointly solve (A - m^0), and the principal accepts the proposal. The equilibrium is efficient (as before, an equilibrium is declared efficient if the payoffs are P^* and $\bar{A}$). The full proof is in Appendix C.

PROPOSITION 5: *The game with nonnegative selection menu costs has an equilibrium. Furthermore, it has an equilibrium that is efficient and entails the adoption and renegotiation of the optimal sales contract. The game of Section 4 in which menu contracts are not feasible has an equilibrium with the same equilibrium path.*

The next proposition shows that all equilibria are efficient, and that if selection costs are positive, renegotiation occurs and the final contract is a singleton.

PROPOSITION 6: *All equilibria of the selection menu cost game are efficient. If $\varepsilon_A + \varepsilon_P > 0$, then renegotiation occurs, and the final contract is a singleton, in all equilibria.*

PROOF: The proof of Lemma 3 is easily modified to show that $P^{eq} \leq P^*$ and $A^{eq} = \bar{A}$ if $P^{eq} = P^*$, and the argument of Proposition 1 still holds to show that the agent accepts sales contracts with prices less than P^*. Hence, Proposition 1 still proves efficiency. If menu costs are positive, only a singleton can achieve efficiency. The equilibrium final contract is thus a singleton. If renegotiation does not occur, the initial contract must be this efficient singleton. That is impossible, by Proposition 2. Hence, renegotiation, to a singleton, must occur if total menu costs are positive. ■

REMARK: Ma (1993) also shows (when the set of efforts is finite, and menus are costless) that any PBE of this game which satisfies a refinement criterion is efficient. Ma's criterion restricts the principal's beliefs at information sets (m^0, m) for which $E(m^0) = E(m) = \{e\}$ for some e, i.e., for which the agent has a unique optimal effort regardless of whether the final contract is m^0 or m. At such an information set, the criterion requires the principal to believe the agent took effort e. This, like Criterion C, requires the principal to believe the agent is not playing a certain kind of dominated strategy — the difference is in the kind of dominated strategy. The two criteria are not nested; Criterion C does not depend on $E(m^0)$, and Ma's criterion does not depend on $\bar{A}$.

Interestingly, selection costs do not imply that the initial contract is a singleton. One might conjecture that the principal would not offer a menu initially, because if she did, the agent would later extract as rent the principal's cost ε_P by proposing a non-refusable (by Criterion C) singleton, in which case the principal's payoff could be no greater than $P^* - \varepsilon_P$. This conjecture is false.[17]

Complexity Menu Costs

Complexity menu costs are conceptually straightforward, but a formal treatment at this point would be tedious. Thus, the analysis here is informal.

Complexity costs are modeled most simply as constants, $\delta_P \geq 0$ for the principal and $\delta_A \geq 0$ for the agent. These costs are borne immediately once a new multiple-scheme menu contract is agreed, regardless of the number and nature of its schemes, and regardless of whether it is later renegotiated. For example, if the initial contract m^0 is a multiple-scheme menu and is renegotiated to another multiple-scheme menu m, and the agent takes effort e and selects $s \in m$, the payoffs are $A(s, e) - 2\varepsilon_A$ and $P(s, e) - 2\varepsilon_P$, respectively. Both δ_P and δ_A are viewed as small.

As with selection costs, Criterion C is still a reasonable restriction of the principal's beliefs at information sets following an initial singleton contract and a singleton renegotiation proposal. Refer to any PBE satisfying the criterion at such information sets as an equilibrium. A construction as in Proposition 5 proves the existence of equilibrium. The inclusion of complexity costs does not affect the arguments used in Section 4. In particular, the logic of Proposition 1 still implies that the agent will accept a sales contract with price less than P^*. Thus, all equilibria are efficient, with payoffs P^* and $\bar{A}$.

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[17] In the two-by-two example, an equilibrium exists in which the initial contract is a menu $m^0 = \{r^1, r^2\}$, where r^1 (r^2) is on the type-e_h agent's reservation indifference curve slightly above (below) s^*. The agent selects r^1 if m^0 is not renegotiated, except when he takes e_h and his rejected proposal is s^*, in which case he selects r^2. If she believes the effort is e_h, the principal prefers r^1 to s^* to r^2, and so she will agree to renegotiate if and only if the proposal is s^*. In equilibrium the agent chooses e_h and proposes s^*, which the principal accepts. If $\varepsilon_A \geq A(r^1, e_\ell) - \bar{A}$ (which is the case when $\varepsilon_A > 0$ and r^1 is close to s^*), these are equilibrium strategies.

These payoffs are not attainable, when δ_A or δ_P is positive, if a multiple-scheme menu is used in equilibrium. Thus, $\delta_A + \delta_P > 0$ implies that only singleton contracts occur on the equilibrium path, and the results of Section 4 hold *a fortiori*.

PROPOSITION 7: *All equilibria of the complexity menu cost game are efficient. If* $\delta_A + \delta_P > 0$, *all equilibria entail the renegotiation of one singleton contract to another. If the agent has talent, the only equilibrium initial contract is the optimal sales contract* r^*.

6. Remarks on Extensions

Initial Offer by the Agent

The analysis extends to situations in which the agent makes the initial, as well as the renegotiation, offer. The relevant efficiency benchmark is then set by maximizing the agent's expected utility subject to the incentive constraint IC, and to a lower bound on the principal's expected profit. An efficient allocation is defined as a solution to this problem; it generally gives the agent a utility A^* greater than his reservation utility $\bar{A}$.

Efficient perfect Bayesian equilibria in which the agent offers a sales contract, and later renegotiates it, still exist. But efficiency is not implied by Criterion C; its focus on contracts which give the agent $\bar{A}$ is not useful if $A^* > \bar{A}$. However, efficiency is implied by similar, but stronger criteria. One of them requires the principal to sometimes believe the agent chose his effort under the assumption that his renegotiation offer would be accepted. The principal should have such beliefs if the agent's renegotiation offer, for example, is preferred by both parties to the initial contract, given that the agent's effort is optimal for it. In this case the agent can reasonably expect the principal to accept the offer.[18] Formally, this criterion (which implies Criterion C) is the following.

CRITERION CC: For all contracts r and s, $\beta(E(s) \mid r, s) = 1$ if $A^*(s) > A(r)$ and $P(s,e) > P(r,e)$ for all $e \in E(s)$.

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[18] This criterion is thus similar to that of Farrell (1990) and Grossman and Perry (1986).

If this criterion is substituted for Criterion C, the results extend to the game in which the agent makes both offers. The proofs are nearly the same.

Endogenous Bargaining Power

The model also extends to situations in which the initial contract can specify a bargaining process. For example, suppose it can specify both a menu, and a party who will be able to make ultimatum renegotiation proposals. The results of Fudenberg and Tirole (1990) imply that in any PBE, the best initial contracts that the principal can offer which give her the right to make renegotiation proposals lead to inefficient allocations; they induce the agent to play a mixed effort strategy, and the principal's payoff will be less than P^*. However, the argument in Proposition 1 shows that in PBE satisfying Criterion C, the principal can offer and the agent will accept an initial contract that specifies a sales scheme with a price less than P^*, if the contract lets the agent lead the renegotiation. Thus, every equilibrium is efficient. All equilibrium initial contracts give the agent the sole right to make renegotiation proposals. One equilibrium initial contract specifies the optimal sales scheme r^*, and it is renegotiated to an efficient scheme. This is the only equilibrium outcome if the agent has talent and menus have complexity costs.

This extension requires that a renegotiation/revision process specified in the initial contract be enforceable, as is assumed, e.g., in Hart and Moore (1988), Chung (1991), Rubinstein and Wolinsky (1992), and Aghion *et. al.* (forthcoming). Seemingly, the steps of a bargaining process, its sequence of written proposals and acceptances, could be made verifiable and hence contractible. However, the issue is problematic. Methods for enforcing an agreement to the effect that one party will not make renegotiation offers may also be methods for enforcing an agreement that neither party will make renegotiation offers, in which case the parties can commit to not renegotiate.[19,20]

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[19] I thank a referee for bringing this possibility to my attention.

[20] Aghion *et. al.* (forthcoming) present a way, using a hostage, by which bargaining power can be credibly specified in a contract. But the method sometimes relies on a court to enforce outcomes that are commonly known to be inefficient, contrary to the motivation that renegotiation is always possible. Further remarks on contractual commitment are in Moore (1992) and Hart and Moore (1988, especially fn. 19 and 20).

Limited Wealth

The optimal sales contract has been assumed feasible. However, in some applications the price it requires, P^*, may be greater than the agent's wealth, and greater than the firm's proceeds when they are low. In this case the optimal sales contract would not be feasible. An important question for future inquiry is whether another kind of simple contract, such as a debt contract, then arises.

Appendix A: Inducibility and Strict Inducibility

Proposition 0 below reveals the geometry of inducibility, and proves that generically, any inducible effort is strictly inducible if the set of efforts E is finite.

As a preliminary, say that e is *redundant* if for a finite set of strictly inducible efforts, $\{e^1,\ldots, e^m\}$, the point $(e, c(e)) \in \Re^{n+1}$ is a convex combination of the points $(e^k, c(e^k))$. Thus, e is payoff-equivalent to a mixed strategy on $\{e^1,\ldots, e^m\}$. This set does not include e, since strictly inducible efforts are not redundant (part (*ii*) of Proposition 0). Redundant efforts can be deleted in any order to yield the same reduced environment, and this new environment will be payoff-equivalent to the original.

Let $\bar{c}$ be the convex hull of c, the greatest convex function majorized by c: for any $e \in E$, $\bar{c}(e)$ is the infimum of the set of finite convex combinations of the form $\Sigma\lambda_k c(e^k)$ for which $e = \Sigma\lambda_k e^k$. Similarly, let $\bar{c}(\cdot|e)$ be the convex hull of the restriction of c to $E\backslash\{e\}$.[22] Then $c(e) \geq \bar{c}(e)$, but $c(e)$ and $\bar{c}(e|e)$ have no necessary relationship.

PROPOSITION 0: *Assume E is a finite set.*[23]

(*i*) *If* $e \in E$ *is inducible,* $c(e) = \bar{c}(e)$; *the converse holds if* u *is unbounded above.*[24]

(*ii*) *If* $e \in E$ *is redundant, it is not strictly inducible.*

(*iii*) *If* $e \in E$ *is inducible but not strictly inducible, it is redundant and* $c(e) = \bar{c}(e|e)$.

(*iv*) *Assumption I holds for a generic set of cost functions c.*

PROOF: (*i*) Assume e is inducible, say by s, and equal to a convex combination of (possibly) other efforts, $e = \Sigma\lambda_k e^k$. Then $c(e) \leq c(e^k) + u(s)\cdot(e - e^k)$ for each k. Hence, $c(e) \leq \Sigma\lambda_k c(e^k)$. As $\bar{c}(e)$ is the infimum of such convex combinations, $c(e) = \bar{c}(e)$. Conversely now, assume $c(e) = \bar{c}(e)$ and u is unbounded above. Since E is finite, $\bar{c}$ is a polyhedral convex function, and so has a subgradient everywhere in conv E (Rockafellar, 1970, p226). Let $v \in \Re^n$ be a subgradient of $\bar{c}$ at e. Then for each $d \in E$,

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[22] If $e \notin \text{conv}E\backslash\{e\}$, then $\bar{c}(e|e) \equiv \infty$.

[23] Even if E is infinite, the first part of (*i*), the second part of (*i*) if $e \in \text{ri}\,E$, and (*ii*) are still true. I conjecture that an analog of (*iv*), the genericity result, holds if E is infinite.

[24] Proposition 2 in Hermalin and Katz (1991) is similar to (*i*).

$$c(d) \geq \bar{c}(d) \geq \bar{c}(e) + v \cdot (d - e) = c(e) + v \cdot (d - e).$$

Hence, an s defined by $s_i = u^{-1}(v_i)$ induces e (note that u: $\Re \rightarrow \Re$ is invertable because it is unbounded above by hypothesis, and is increasing and concave).

(ii) We prove that if $(e, c(e))$ is a convex combination $\Sigma\lambda_k(e^k, c(e^k))$, then any s inducing e also induces each e^k. *A fortiori*, no redundant effort is strictly inducible. So assume s induces e, and set $v = u(s)$. Then $(v, -1) \cdot (e, c(e)) \geq (v, -1) \cdot (e^k, c(e^k))$ for each k. Multiply by λ_k and add over k to obtain $(v, -1) \cdot (e, c(e)) \geq (v, -1) \cdot \Sigma\lambda_k(e^k, c(e^k))$. This inequality is not strict, as $(e, c(e)) = \Sigma\lambda_k(e^k, c(e^k))$. The previous inequality for e^k is hence not strict, and so s induces e^k.

(iii) We first show the following:

> If $\hat{e}$ is inducible and $z \in \Re^{n+1}$ exists such that $z \cdot (\hat{e}, c(\hat{e})) > z \cdot (d, c(d))$ for all $d \in E \backslash \{\hat{e}\}$, then $\hat{e}$ is strictly inducible. (a1)

Given $\hat{e}$ and z, let s induce $\hat{e}$. Then $(u(s), -1) \cdot (\hat{e}, c(\hat{e})) \geq (u(s), -1) \cdot (d, c(d))$ for $d \in E \backslash \{\hat{e}\}$. Choose $\varepsilon > 0$ so small that $1 > \varepsilon z_{n+1}$ and $(u(s_i) + \varepsilon z_i)/(1 - \varepsilon z_{n+1}) < \lim_{y \to \infty} u(y)$ for each $i \leq n$. Then define $t \in \Re^n$ by $u(t_i) = (u(s_i) + \varepsilon z_i)/(1 - \varepsilon z_{n+1})$. Scheme t strictly induces $\hat{e}$.

Now assume e is inducible but not strictly inducible. By (a1), $(e, c(e))$ cannot be strictly separated from $B \equiv \text{conv}\{ (d, c(d)) \mid d \in E, d \neq e\}$. As E is finite, B is a polytope and hence closed. Thus, $(e, c(e)) \in B$, and so is equal to a convex combination of extreme points $(e^k, c(e^k))$ of B. By the proof of *(ii)* above, each e^k is inducible (by any s that induce e). Since B is a polytope, each extreme point $(e^k, c(e^k))$ can be strictly separated from the rest of B. Hence, (a1) implies e^k is strictly inducible. This proves e is redundant. Since $(e, c(e)) \in B$, $c(e) \geq \bar{c}(e \mid e)$. The argument used to prove the first part of *(i)* shows that $c(e) \leq \bar{c}(e \mid e)$. Hence, $c(e) = \bar{c}(e \mid e)$.

(iv) Enumerate E as $e^1, \ldots, e^m$. View c as a point $(c_1, \ldots, c_m) \in \Re^m$. Let $D^k \subset \Re^m$ be the set of c at which e^k is either strictly inducible or not inducible. We show that D^k contains a set that is open, dense, and has null complement. This shows that the set of c at which Assumption I is satisfied, $\bigcap_{k=1,m} D^k$, is similarly generic.

View $\bar{c}(\cdot \mid e^k)$ as a function f of c_{-k}. That is, for $K \equiv \{1, \ldots, k-1, k+1, \ldots, m\}$,

$$f(c_{-k}) \equiv \min_{(\lambda_j)_{j\in K}} \sum_{j\in K} \lambda_j c_j \quad \text{such that } \lambda_j \geq 0 \text{ for } j \in K,$$
$$\sum_{j\in K} \lambda_j = 1, \text{ and } \sum_{j\in K} \lambda_j e^j = e^k \tag{a2}$$

if $e^k \in \text{conv } E \backslash \{e^k\}$, and $f(\cdot) \equiv \infty$ otherwise. Now, if $c \notin D^k$, then (*iii*) implies $c_k = f(c_{-k})$. Hence, D^k contains the complement of the graph of f. The graph of f is closed because f is continuous, and so has measure zero (Fubini Theorem for measure zero, Guillemin and Pollack, 1974, p204). Thus, the complement of the graph of f is dense, open, and has a null complement. It is the desired generic subset of D^k. ■

Appendix B: Proofs for Sections 3 and 4

<u>PROOF OF LEMMA 3</u>: Fix a PBE. As the agent can reject any initial offer, $A^{eq} \geq \bar{A}$ is obvious, and so is $A^{eq}(r) \geq \bar{A}$ for any r the agent accepts with positive probability. Consider a scheme r. In $\Gamma(r)$, with positive probability the agent's choice gives the principal at least her equilibrium payoff. Hence, for some equilibrium choice (s, e) of the agent, i.e. some (s, e) in the support of his (possibly mixed) strategy in $\Gamma(r)$,

$$\hat{P} \equiv aP(s,e) + (1-a)P(r,e) \geq P^{eq}(r),$$

where a is the principal's equilibrium probability of accepting s ($a = a_P(r, s)$). Because (s, e) is an equilibrium choice of the agent,

$$\begin{aligned} A^{eq}(r) &= aA(s,e) + (1-a)A(r,e) \\ &= \max_{d\in E} \{aA(s,d) + (1-a)A(r,d)\}. \end{aligned}$$

Lemma 1 now implies the existence of scheme t such that $e \in E(t)$, $A(t, e) \geq A^{eq}(r)$, and $P(t, e) \geq \hat{P} \geq P^{eq}(r)$.

Suppose the agent accepts r with positive probability. Then $A^{eq}(r) \geq \bar{A}$, and hence $A(t, e) \geq \bar{A}$. So (t, e) is feasible for (P), implying that $P(t, e) \leq P^*$. Thus, $P^{eq}(r) \leq P^*$. As this is true for all r accepted with positive probability, $P^{eq} \leq P^*$.

Suppose now that r is any scheme satisfying $P^{eq}(r) = P^*$. Then $P(t, e) \geq P^*$. If $A(t, e) > \bar{A}$, then (t, e) is feasible for (P), and hence solves it. This is impossible, since IR binds in (P). Thus $A(t, e) \leq \bar{A}$, and so $A^{eq}(r) \leq \bar{A}$.

If $P^{eq} = P^*$, then $P^{eq}(r) = P^*$ for almost all r the agent accepts with positive probability. The previous two paragraphs imply $A^{eq}(r) = \overline{A}$ for such r. Hence $A^{eq} = \overline{A}$. ■

<u>PROOF OF LEMMA 5</u>: Let $d \in E(t)$ be an effort which minimizes $t \cdot e$ on $E(t)$. For small $\varepsilon > 0$ and $\delta > 0$, define scheme $s^{\varepsilon,\delta}$ by,

$$\begin{aligned} u(s_i^{\varepsilon,\delta}) &= \delta(\varepsilon + u(t)\cdot d) + (1-\delta)u(t_i) \\ &= \delta(\varepsilon + A^*(t) + c(d)) + (1-\delta)u(t_i). \end{aligned}$$

The s we seek is $s^{\varepsilon,\delta}$ for sufficiently small ε and δ. Observe that

$$\begin{aligned} A(s^{\varepsilon,\delta}, e) &= u(s^{\varepsilon,\delta})\cdot e - c(e) \\ &= \delta\varepsilon + A^*(t) - \delta\{c(e) - c(d)\} - (1-\delta)\{A^*(t) - \mathrm{A}(t, e)\}. \end{aligned}$$

Thus, $A^*(s^{\varepsilon,\delta}) \geq A(s^{\varepsilon,\delta}, d) = \delta\varepsilon + A^*(t)$. This proves (i).

Let $E(s, t) \equiv \{e \in E \mid A(s, e) \geq A^*(t)\}$. To prove (ii), we must show

$$(s^{\varepsilon,\delta} - t)\cdot e < 0 \quad \text{for } e \in E(s^{\varepsilon,\delta}, t). \tag{a5}$$

The t_i's vary, the distribution d has full support, and u is strictly concave. Thus, $k > 0$ exists such that, for sufficiently small ε,

$$\varepsilon + u(t)\cdot d \leq u(t\cdot d - k). \tag{a6}$$

Note that $s^{\varepsilon,\delta} \to t$, and $E(t, t) = E(t)$. Also, $E(\cdot, t)$ is u.h.c., and the set E of possible efforts is compact. Thus, as d minimizes $t\cdot e$ on $E(t)$, we can choose (ε, δ) sufficiently small and positive so that both (a6) and the following hold:

$$t\cdot d < t\cdot e + k \quad \text{for } e \in E(s^{\varepsilon,\delta}, t). \tag{a7}$$

By the convexity of u^{-1}, the fact that $A^*(t) + c(d) = u(t)\cdot d$, and (a6), we have

$$\begin{aligned} s_i^{\varepsilon,\delta} &= u^{-1}\{\delta(\varepsilon + A^*(t) + c(d)) + (1-\delta)u(t_i)\} \\ &\leq \delta u^{-1}(\varepsilon + u(t)\cdot d) + (1-\delta)u^{-1}(u(t_i)) \\ &\leq \delta(t\cdot d - k) + (1-\delta)t_i. \end{aligned}$$

Consequently, $(s^{\varepsilon,\delta} - t)\cdot e \leq \delta(t\cdot d - k - t\cdot e)$, which by (a7) is negative for $e \in E(s^{\varepsilon,\delta}, t)$. This proves (a5), and hence (ii). ■

PROOF OF PROPOSITION 3:

Step 1: Prove $a_A(r) = 1$ (the agent surely accepts r).

Because the equilibrium is efficient,

$$P^* = P^{eq} = a_A(r)P^{eq}(r) + (1-a_A(r))\overline{P}. \tag{a10}$$

Thus $a_A(r) > 0$, as $P^* > \overline{P}$. By Lemma 3 and $a_A(r) > 0$, $P^{eq}(r) \leq P^*$; (a10) now implies $a_A(r) = 1$.

Step 2: Prove $P^{eq}(r) = P^*$ and $A^{eq}(r) = \overline{A}$.

From Step 1 and (a10), $P^{eq}(r) = P^*$. Lemma 3 now implies $A^{eq}(r) = \overline{A}$.

Step 3: Prove (*i*) (r is not a wage contract).

If r were to be a wage contract, it would not be renegotiated (Lemma 4), and $A^*(r) = A^{eq}(r) = \overline{A}$. Hence, equilibrium allocations (r, e) of $\Gamma(r)$ are feasible for (P) and, since $P^{eq}(r) = P^*$, solve it. This contradicts the presumed inefficiency of wage contracts.

Step 4: Finish proving (*ii*) (i.e., prove $A^*(r) < \overline{A}$).

By Step 1, we need only show $A^*(r) < \overline{A}$. As the agent could propose r, $A^*(r) \leq A^{eq}(r)$. Hence, by Step 2, $A^*(r) \leq \overline{A}$. Assume equality. Then (Lemma 5) s exists such that $A^*(s) > \overline{A}$, and $P(s, e) > P(r, e)$ for all $e \in \overline{E}(s)$. By Criterion C, the principal accepts s in $\Gamma(r)$. Thus $A^{eq}(r) \geq A^*(s) > \overline{A}$, contrary to $A^{eq}(r) = \overline{A}$. Hence, $A^*(r) < \overline{A}$.

Step 5: Prove $A^r \leq \overline{A}$.

Assume otherwise. Then $(\hat{t}, e)$ exists such that $A(\hat{t}, e) > \overline{A}$, $P(\hat{t}, e) \geq P(r, e)$, and $e \in E(\hat{t})$. Define t by $u(t_i) = u(\hat{t}_i) - \varepsilon$, where $0 < \varepsilon < A(\hat{t}, e) - \overline{A}$. Then $A(t, e) > \overline{A}$, $P(t, e) > P(r, e)$, and $e \in E(t)$. By Lemma 2, $\{e\} = E(t)$ can be assumed. Further, we can assume t is not a wage contract.[25] Define $\hat{s}$ by

$$u(\hat{s}_i) = u(t_i) + \overline{A} - A(t, e).$$

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[25] Since $E(\cdot)$ is upper hemicontinuous and $\{e\} = E(t)$, a neighborhood N of t exists such that for all $s \in N$ and $d \in E(s)$, $A(s, d) > \overline{A}$ and $P(s, d) > P(r, d)$. Let s be a non-wage contract in N, and let $d \in E(s)$, and replace (t, e) by (s, d). Then $\{d\} = E(s)$ can be assumed by Lemma 2.

Then $E(\hat{s}) = \{e\}$ and $A^*(\hat{s}) = \bar{A}$. Hence, $\bar{E}(\hat{s}) = \{e\}$. Also, $P(\hat{s}, e) > P(r, e)$ since each $\hat{s}_i < t_i$. As $\bar{E}(\cdot)$ is u.h.c., a neighborhood N of $\hat{s}$ exists such that if $s \in N$,

$$P(\hat{s}, d) > P(r, d) \text{ for all } d \in \bar{E}(s). \tag{a11}$$

As t is non-wage, $\hat{s}$ is non-wage. So by Lemma 5, given that $A^*(\hat{s}) = \bar{A}$, $s \in N$ exists such that $A^*(s) > \bar{A}$, and $P(s, d) > P(\hat{s}, d)$ for all $d \in \bar{E}(s)$. The latter and (a11) imply

$$P(s, d) > P(r, d) \text{ for all } d \in \bar{E}(s).$$

Criterion C thus requires the principal to accept s. This yields a contradiction, $A^{eq}(r) \geq A^*(s) > \bar{A}$. Hence $A^r \leq \bar{A}$.

Step 6: Prove (*iii*) (the principal surely accepts equilibrium offers in $\Gamma(r)$).

Let s be an equilibrium offer of the agent in $\Gamma(r)$, and let $a = a_P(r, s)$ be the probability with which the principal accepts s. Because $A^{eq}(r) > A^*(r)$, $s \neq r$ and $a > 0$. The principal's beliefs about the agent's effort conditional on (r,s) are correct, since (r, s) is on the equilibrium path. Thus, as the principal finds s acceptable, e exists such that (s, e) is an equilibrium choice of the agent, and $P(s, e) \geq P(r, e)$.

Assume $a < 1$. Since e maximizes $aA(s,\cdot) + (1-a)A(r,\cdot)$ on E, Lemma 1 implies the existence of t such that $e \in E(t)$,

$$A(t, e) > aA(s, e) + (1-a)A(r, e), \text{ and}$$
$$P(t, e) > aP(s, e) + (1-a)P(r, e).$$

The right side of the first inequality is $A^{eq}(r) = \bar{A}$, and so $A(t, e) > \bar{A}$. Since $P(s, e) \geq P(r, e)$, the second inequality implies $P(t, e) > P(r, e)$. Thus, (t, e) is feasible for (A - r). But Step 5 and $A(t, e) > \bar{A}$ imply $A(t, e) > A^r$. This contradiction proves $a = 1$.

Step 7: Prove $A^r = \bar{A}$.

Let (s, e) be the equilibrium allocation of $\Gamma(r)$ identified in Step 6. Since $P(s, e) \geq P(r, e)$, (s, e) satisfies AC. Since the agent knows when he chooses effort that s will be the final contract, (s, e) satisfies IC. Thus, (s, e) is feasible for (A - r). Therefore $A^r \geq A(s, e) = A^{eq}(r) = \bar{A}$. Hence, by Step 5, $A^r = \bar{A}$.

Step 8: Prove (*iv*) (equilibrium allocations of $\Gamma(r)$ are efficient and solve (A - r).)

Let μ denote the agent's mixed strategy in $\Gamma(r)$, with support Supp(μ) $\subset S \times E$. By (*iii*), the principal accepts the agent's offers. Hence, μ is the equilibrium distribution of allocations in $\Gamma(r)$, and allocations in Supp(μ) give the agent $A^{eq}(r)$. Define $g(s, e) \equiv P(s, e) - P(r, e)$. Consider some $(s, e) \in$ Supp(μ).

By Steps 2 and 7, $A(s, e) = A^r$. Since the agent knows when he chooses effort that s will be accepted, (s, e) satisfies IC. Thus, (s, e) solves (A - r) if and only if it satisfies AC. It cannot satisfy AC with slack, as AC binds in (A - r).[26] Thus, $g \leq 0$ on Supp(μ). But the expectation of g according to μ is nonnegative, since the principal accepts the agent's offers. Because g is continuous, this implies that $g = 0$ on Supp(μ). Thus, (s, e) satisfies AC, and so solves (A - r). This shows that all allocations in Supp(μ) solve (A - r).

Because $A(s, e) = A^{eq}(r) = \overline{A}$, (s, e) satisfies IR as well as IC. Allocations in Supp(μ) are therefore feasible for (P), and so $P \leq P^*$ on Supp(μ). The expectation of P according to μ is $P^{eq}(r) = P^*$. Thus $P = P^*$ on Supp(μ), since P is continuous. This proves that allocations in Supp(μ) are efficient. ■

Appendix C: Proofs for Section 5 (Existence Proofs)

PROOF OF LEMMA 7: Let $X \subset \Re^{2n}$ be the set of pairs (s, e) satisfying the constraints of (A - m^{o}). We first show that $X \neq \emptyset$. Let (s, e) maximize $A(\cdot, \cdot)$ on the (compact) set $m^{\mathrm{o}} \times E$. Then $e \in E(s)$ and $s \in S(m^{\mathrm{o}}, e)$. The former is IC, and the latter implies $P(s, e) \geq P(s^w(m^{\mathrm{o}}, e), e) \geq P(s^w(m^{\mathrm{o}}, e), m^{\mathrm{o}}, e)$, which is AC. Thus, $(s, e) \in X$.

For this same (s, e), let $\alpha' \equiv A(s, e)$. Let Y be the subset of pairs $(t, d) \in X$ that satisfy $A(t, d) \geq \alpha'$. Then $Y \neq \emptyset$, and any solution of (A - m^{o}) belongs to Y. To show the existence of a solution, it suffices to show Y is closed and bounded.

To show that Y is closed, let $\{(s^k, e^k)\}$ be a sequence in Y converging to (s, e). Because each (s^k, e^k) satisfies IC and $E(\cdot)$ has closed graph, (s, e) satisfies IC. Because $S(m^{\mathrm{o}}, \cdot)$ is u.h.c. with values in the compact m^{o}, the definition of $s^w(m^{\mathrm{o}}, \cdot)$, implies that

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[26] Proof: Let (s, e) satisfy IC and, with slack, AC. Define t by $u(t_i) = u(s_i) + \varepsilon$. For $\varepsilon > 0$ small, (t, e) satisfies IC and AC, and $A(t, e) > A(s, e)$. Thus, (s, e) cannot solve (A - r).

$P(s^w(m^o, e^k), m^o, e^k)$ can only jump down as $e^k \to e$, that is, $P(s^w(m^o, \cdot), m^o, \cdot)$ is a lower semicontinuous function of effort. Hence, as each (s^k, e^k) satisfies AC, so does (s, e). Therefore $(s, e) \in Y$. This proves Y is closed.

To show Y is bounded, we show an arbitrary sequence $\{(s^k, e^k)\}$ in Y is bounded. Because $u(s^k) \cdot e^k - c(e^k) = A(s^k, e^k) \geq \alpha'$, a constant α ($\equiv \alpha' + \min_{e \in E} c(e)$) exists such that

$$u(s^k) \cdot e^k \geq \alpha. \tag{a12}$$

Rewritten and applied to (s^k, e^k), constraint AC is $s^k \cdot e^k \leq \varepsilon_P g(m^o) + s^w(m^o, e^k) \cdot e^k$. Thus,

$$s^k \cdot e^k \leq \gamma \tag{a13}$$

for some number γ. We now show that (a12) and (a13) imply $\{(s^k, e^k)\}$ is bounded.

As u is concave, nonlinear and increasing, there exists $x < 0$, $b = u'(x)$, $z > 0$, and $c = u'(z)$ such that $0 < c < b$. Let a be a number greater than both $u(x) - bx$ and $u(z) - cz$. The concavity of u implies it is bounded above by a function defined by $v(y) = a + by$ if $y \leq 0$, and $v(y) = a + cy$ if $y \geq 0$. From (a12),

$$\alpha \leq v(s^k) \cdot e^k. \tag{a14}$$

For each k, let $I(k) = \{i \mid s_i^k > 0\}$. Using (a13),

$$\begin{aligned} v(s^k) \cdot e^k &= a + b \sum_{i \notin I(k)} e^k s_i^k + c \sum_{i \in I(k)} e^k s_i^k \\ &\leq a + b\gamma + (c - b) \sum_{i \in I(k)} e^k s_i^k. \end{aligned} \tag{a15}$$

Let $M^k = \max_i s_i^k$. As E is compact, we may assume $\{e^k\}$ converges to some $e \in E$. Since every effort in E is strictly positive, a number $\delta > 0$ exists such that $e_i^k > \delta$ for all i and large k. Since $s_i^k > 0$ for $i \in I(k)$,

$$\sum_{i \in I(k)} e^k s_i^k \geq \delta \sum_{i \in I(k)} s_i^k \geq \delta M^k. \tag{a16}$$

Combine (a14) - (a16), recalling that $c < b$, to obtain

$$\alpha \leq a + b\gamma + (c - b)\delta M^k.$$

This proves that $\{M^k\}$ is bounded above, say by M. Let $m^k = \min_i s_i^k$. Then,

$$\alpha \leq v(s^k)\cdot e^k \leq \delta v(m^k) + (1-\delta)v(M).$$

Since $v(y) \to -\infty$ as $y \to -\infty$, $\{m^k\}$ is bounded below. ■

PROOF OF PROPOSITION 5: Items (a) - (e) below specify strategies and beliefs that form an equilibrium of both the singleton game, and the menu game with selection costs.

For any feasible (m, e), let $s(m, e)$ be a scheme in $S(m, e)$, i.e., a best scheme for the type-e agent in menu m. Recall that $s^w(m^0, e)$ is the worst scheme for the principal in $S(m, e)$, given that the agent's effort is e. The following is an optimal selection rule.

(a) Agent's selection rule: If the agent chose e and the final contract is m^0, he selects $s^w(m^0, e)$ from m^0. If the final contract is m, he selects $s(m, e)$.

For any feasible m^0, let $(s(m^0), e(m^0))$ be a scheme-effort pair that solves $(A - m^0)$. Recall that $a_P(m^0, m)$ is the probability with which the principal accepts the agent's proposed m when the initial contract is m^0.

(b) Principal's beliefs and acceptance rule: If $m = s(m^0)$,[27] then $\beta(e(m^0) \mid m^0, m) = a_P(m^0, m) = 1$. If $m \neq s(m^0)$ and $e \in E(m)$ exists such that $P(s(m, e), m, e) \geq P(s^w(m^0, e), m^0, e)$, then $\beta(e \mid m^0, m) = a_P(m^0, m) = 1$. Finally, if $m \neq s(m^0)$ and no such e exists, then $a_P(m^0, m) = 0$ and $\beta(\cdot \mid m^0, m)$ is any probability measure with support in $E(m)$.

The acceptance strategy in (b) is sequentially rational, given (a). It is only worth observing that because $(s(m^0), e(m^0))$ satisfies AC, and $s(m, e) = s(m^0)$ when $m = s(m^0)$, accepting $m = s(m^0)$ is optimal for the principal when she believes the effort is $e(m^0)$. Criterion C holds because for each m, the support of the beliefs is in $E(m)$, which is a subset of $\bar{E}(m) \equiv \{e \in E \mid A(m, e) \geq \bar{A}\}$, if the latter is nonempty. (Note that $\bar{E}(m) = \bar{E}(s)$ if $m = \{s\}$.) Beliefs are correct on the equilibrium path because of the following.

(c) In subgame $\Gamma(m^0)$, the agent proposes $s(m^0)$ and takes effort $e(m^0)$.

To see that (c) specifies a best reply, note first that according to (a) – (c), the agent's payoff is A^{m^0} in $\Gamma(m^0)$. Suppose he plays some (m, e) in $\Gamma(m^0)$. Assume m is accepted.

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[27] Singleton menus are written as $m = s$, rather than the more proper $m = \{s\}$.

Then (b) implies that for some $d \in E(m)$, $P(s(m,d),d) \geq P(s^{w}(m^{o},d), m^{o}, d) + \varepsilon_P g(m)$. This shows that $(s(m,d),d)$ satisfies AC. It also satisfies IC, as $d \in E(m)$ implies that

$$\begin{aligned} A(s(m,d),d) &= A(m,d) \geq A(m,e) \\ &= A(s(m,e),e) \geq A(s(m,d),e). \end{aligned}$$

Hence, $(s(m,d),d)$ is feasible for (A-m^{o}). This implies $A(m,e) \leq A(s(m,d),d) \leq A^{m^{o}}$. The agent therefore gets no less from $(s(m^{o}), e(m^{o}))$ than from (m,e). Now assume m is rejected. Then the agent's payoff is at most $A^{*}(m^{o})$, which is $A(s(m^{o},d),d)$ for some $d \in E(m^{o})$. This $(s(m^{o},d),d)$ is feasible for (A-m^{o}), by an argument similar to the preceeding one. Hence, $A^{*}(m^{o}) \leq A^{m^{o}}$. This completes the proof that (a) and (b) imply that the agent can get no more than $A^{m^{o}}$ in $\Gamma(m^{o})$, and hence that (c) defines a best reply.

(d) The agent accepts m^{o} if and only if $A^{m^{o}} \geq \bar{A}$.

This is optimal for the agent because (a) – (c) imply his payoff is $A^{m^{o}}$ in $\Gamma(m^{o})$.

(e) The principal's initial offer is the optimal sales contract: $m^{o} = r^{*}$.

The solution $(s(r^{*}), e(r^{*}))$ of (A-r^{*}) gives the agent a payoff of $A^{r^{*}} = \bar{A}$. So by (d), the agent accepts r^{*} and the principal gets P^{*}. She gets no more than P^{*} by offering some other m^{o}, since it is either accepted and so results in an allocation $(s(m^{o}), e(m^{o}))$ feasible for (P), or it is rejected and so gives the principal $\bar{P} < P^{*}$. This shows that (e) defines a best reply for the principal. ■

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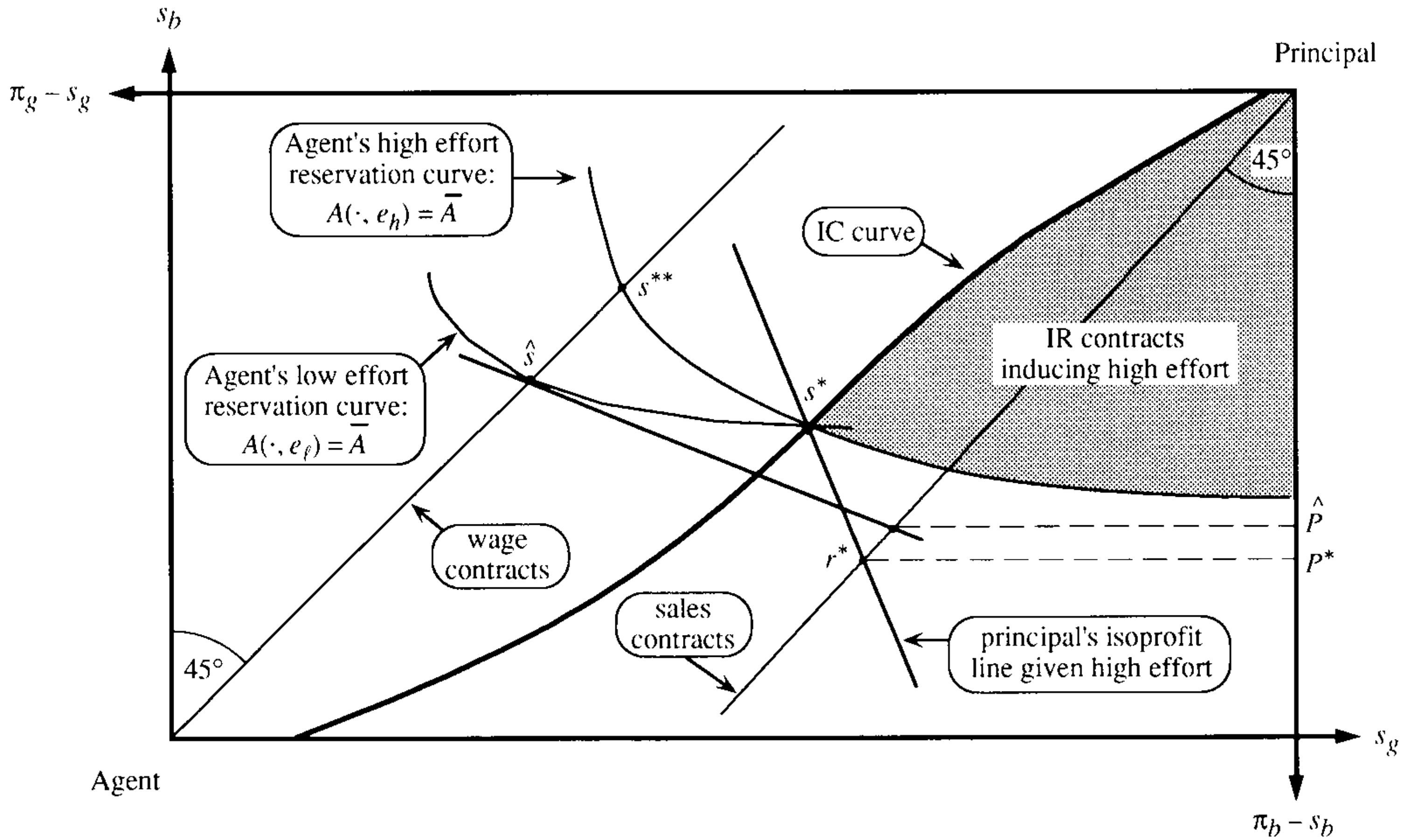

Figure 1

Scheme s^* maximizes the principal's expected profit subject to IR, IC, and $e = e_h$. The optimal sales contract r^* lies below the sales contract $\hat{r}$ that corresponds to $\hat{s}$. Hence, (s^*, e_h), rather than $(\hat{s}, e_\ell)$, is the (second-best) efficient allocation.

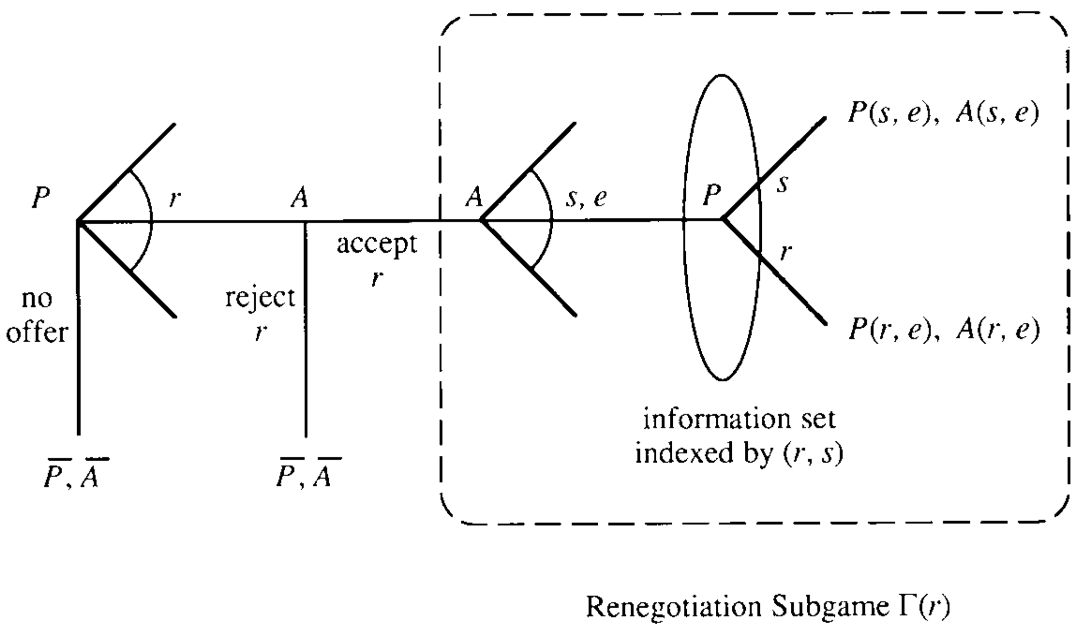

Figure 2

The extensive form of the game in which all contracts are singletons.

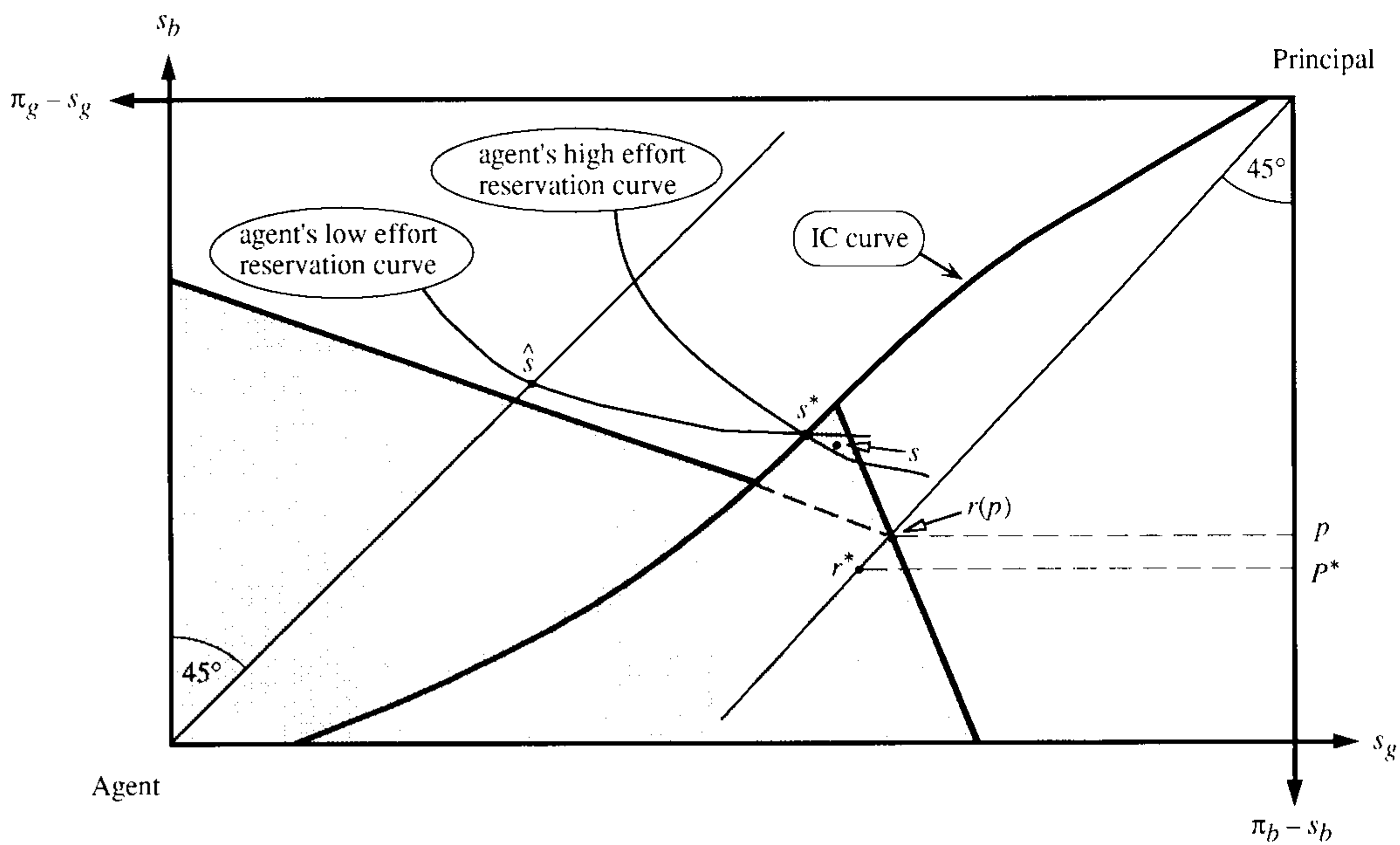

Figure 3

Criterion C implies that the principal will believe the agent has taken high effort if he proposes to renegotiate to s. She will therefore agree to renegotiate $r(p)$ to s.

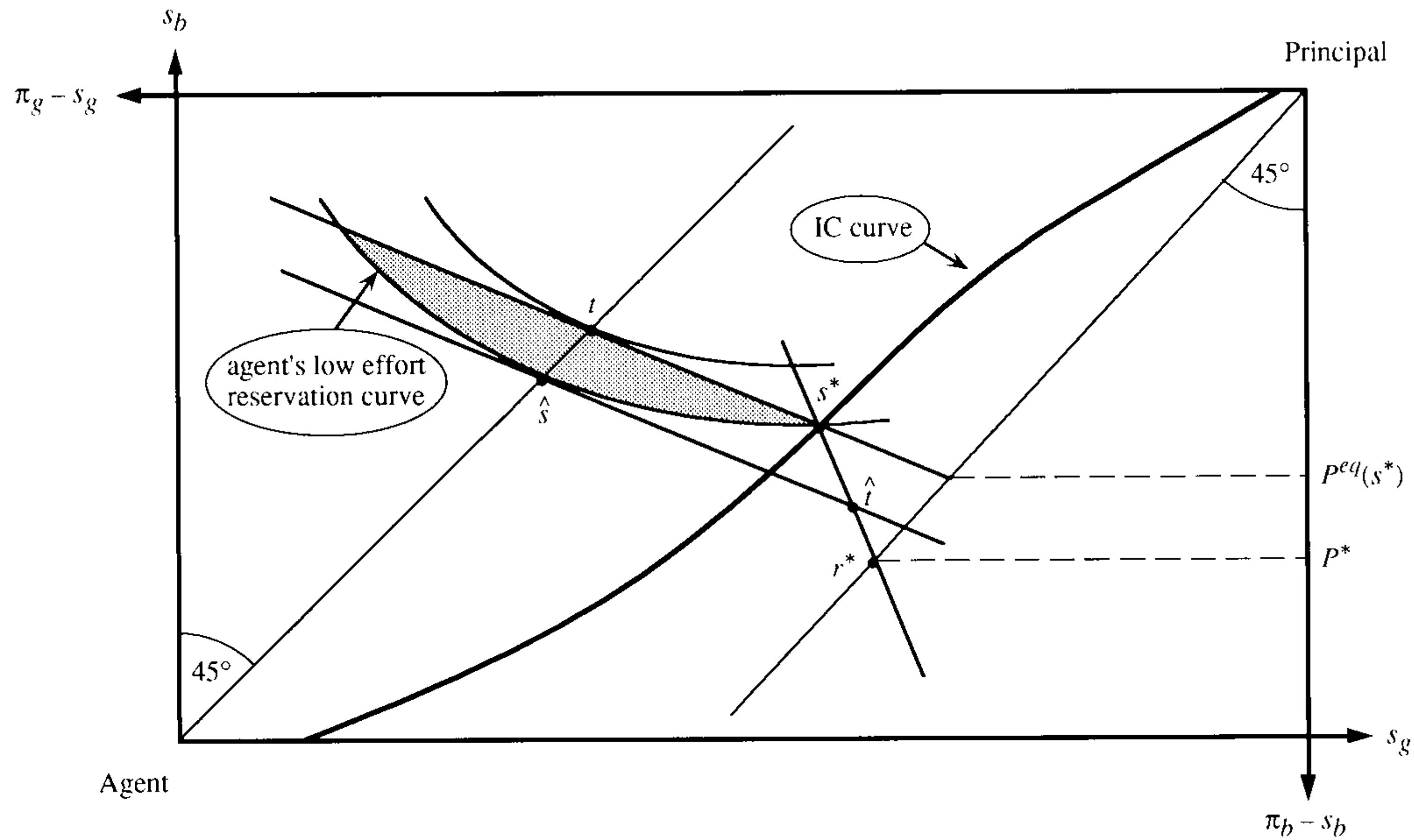

Figure 4

In any SPE, the agent would accept an initial offer of s^*, but then choose low effort and propose to renegotiate to the wage contract t, which the principal would accept.