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Self-Defeating Regional Concentration

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Abstract

We present a simple model of a two-region economy in which undesirable concentration may occur. With freedom to choose where to live, individuals in this economy concentrate into one region in their pursuit of better life, and end up becoming worse off. We characterize the conditions under which such self-defeating concentration occurs in terms of a few key parameters, such as economies of scale in nontradeable service sectors, regional differences in labor productivity in tradeable goods sectors, and substitutability of tradeable goods in consumption.

Keywords: Labor Migration, Stability of Population Distribution, Undesirable Regional Concentration, Essentiality in Consumption

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1. Introduction

One of the most salient features of the geography of economic activity is concentration. Over the course of economic development, the patterns of regional agglomeration become more pronounced: different regions grow at differential rates, and a few metropolitan areas continue to attract people while others remain stagnant or even witness a decline in their population figures. Many recent studies have argued that geographic concentration and uneven regional growth can be explained by some sorts of agglomeration economies, such as external economies in production and increasing product variety in the presence of economies of scale (Krugman 1991c; for related literature in urban economics, see Fujita (1989) and Henderson (1988)).

Policymakers appear to think that they need to do something to correct uneven patterns of regional development. In their attempts to restore the stability of balanced growth and to control the flow of people across regions, many national governments set up special agencies overseeing poor regions, redirect public investment in infrastructure, and administer income transfer programs, the sort of policies that northern Italians love to hate.

Despite the prevalence of regional policies, it is very difficult to defend such government interventions, at least on efficiency ground. Regional concentration may be efficiency-enhancing to the extent that it is necessary to take advantage of economies of scale and other external economies. And it is precisely the presence of such advantages that lead people and businesses to cluster together. Nor does merely pointing out that growing cities are already overcrowded and have pollution problems provide the answer. As long as people are fully aware of the costs and benefits of living in different areas, the very fact that people, despite many problems with big cities, continue to arrive suggests that they may be better off by moving in. Of course, there is an issue of equity; there are always some people in poor regions who, for some reasons, cannot afford to move.[1] But,

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[1]For example, Krugman (1991c) show that, in his two-region model, concentration of manufacturing activities in one region may make immobile agricultural workers in the other region worse off.

then the root of the problem is the lack of concentration, rather than concentration itself.

Undoubtedly some sorts of government intervention should be able to enhance efficiency in the presence of externalities and economies of scale. Yet this does not necessarily imply that there is too much concentration in the observed patterns of regional distribution of economic activity. For example, if external economies are the source of agglomeration, one should expect too little concentration, and hence policies need to be designed to promote, rather than discourage, more regional concentration. The point is that there is no presumption about the direction in which governments should intervene in their regional policies.

What is missing is then a conceptual framework in which one could judge the desirability of regional policies. As a step toward this end, we present in this paper a two-region model where undesirable concentration occurs in some cases and desirable concentration may fail to materialize in others. Our model is not meant to be realistic: nor does it incorporate all the important factors in this complex problem.[2] Rather, its main purpose is to provide a simple intuition-building device on seemingly intractable issues.

We should emphasize that it is not difficult to write down a model in which desirable concentration occurs, or a model in which concentration fails to occur, whether it is desirable or not. The main contribution of this paper is to demonstrate the possibility that undesirable concentration does occur, the possibility that many of regional policies take for granted. We hence entitle this paper, "Self-Defeating Regional Concentration." It is self-defeating in the sense that individuals in this economy are free to move, and in their pursuit of better life, concentrate in one region. But, as a result, they end up becoming worse off: they would be better off if their freedom to move were

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[2]The model does not deal with any technological externalities in production or other external diseconomies such as congestion and pollution. This omission is intentional because, as a first step, we would like to see whether equilibrium interactions alone can make undesirable concentration happen; it also helps to keep the model as simple as possible and to maintain a reasonable distance between assumptions and conclusions.

taken away!

To grasp the intuition behind the result, imagine an island economy, surrounded by the ocean. The eastern part of the island is mountainous and endowed with natural harbors. Hence, it is very difficult to raise cattle but easy to catch fish there. On the other hand, the western part is a flat plateau, which make it suitable for raising cattle but not for fishing. Thus, East has both a comparative and absolute advantage in fish and West has both a comparative and absolute advantage in beef. There is also a nontradeable service sector in each region. East, the more populous of the two, offers a much wider range of restaurants, theaters, schools, and so on. In their pursuit of better life, people would migrate from West to East, leading to a concentration. However, as more people leave West, beef production declines, leading to lower beef consumption. If beef is an essential consumption good, this also leads to a lower standard-of-living in East. Although beef can be produced more efficiently in West, a high price of beef does not provide a sufficient inducement for people to go back to West, as it is in the middle of nowhere.

Demonstrating self-defeating concentration is important enough, providing a possible justification for many regional policies. What is more important is that concentration need not always be self-defeating: whether concentration occurs as well as whether concentration is desirable or not depends on a few key parameters in an interesting way. Among them are the share of service sectors in expenditure, economies of scale in service sectors, regional differences in labor productivity in beef and fish, and substitutability of beef and fish (and a closely related, but distinct, concept, the essentiality of beef and fish).

In addition to the traditional literature in regional economies, a large number of studies have recently addressed to regional issues in the context of economic growth and development (e.g., Barro and Sala-i-Martin 1991; Blanchard and Katz 1992; Glaeser et. al. 1992). We comment on a few that are closely related. In a series of articles, Krugman (1991a,b,c) has forcefully demonstrated the importance of analyzing regional concentration in a formal model. He showed, among others, that a reduction in transportation costs of differentiated manufacturing products can cause a sudden concentration of manufacturing activities. He did not address the efficiency implications of concentration in these studies, given the complexity of his models. Matsuyama (1992c) has a model of regional concentration based on nontradeable differentiated services; the model is so simple that one could address the efficiency problem. It turns out, however, that regional concentration in that model could be inefficient only when a wrong region has been chosen: that is, only when there is another concentration equilibrium that gives higher welfare. The inefficiency problem is purely a matter of coordination; there is nothing wrong with concentration per se in that model.

The rest of the paper is organized as follows. In the next section, we present our model and characterize equilibrium allocation for a given population distribution across regions. In section 3, the main section, we analyze the stability of population distributions and its efficiency implications. In this section, we discuss the stability in an informal way, as the main results do not depend on the details of labor migration dynamics. For the sake of completeness, however, we present a formal model of migration processes in section 4. We conclude in section 5.

2. A Two-Region Model.

We now describe our model of a two-region economy and characterize equilibrium allocation for any given population distribution. Our economy is a standard Ricardian model of trade with two tradeable goods, augmented by a Dixit-Stiglitz (1977) type monopolistically competitive nontradeable service sector. Since the building blocks of our model are well-known, we refrain from commenting on the specification issues in detail; the main goal of this section is to establish the notation and to highlight some features of the model that would be crucial for the ensuing analysis.

2.A. Consumers.

Consider an economy with two regions, East and West. The population consists of a continuum of individuals with unit mass. All individuals are identical, except their location. Let L^E and $L^W = 1 - L^E$ be the fraction of the population living in East and West, respectively. (Throughout the paper, we use superscripts for regions and subscripts for goods.) There is one factor of production, labor; it is assumed that each individual supplies one unit of labor inelastically in his or her region. Thus, L^E and L^W also represent labor supply in the two regions.

Individuals consume three classes of goods: E, W, and N. Both E and W are homogeneous tradeable goods, which can be transported at zero cost across the regions. On the other hand, N consists of differentiated nontradeable services. More specifically, all the individuals share the following preferences:

$$V^i = V(c_E^i, c_W^i, C_N^i) = \{U(c_E^i, c_W^i)\}^{\mu}\{C_N^i\}^{1-\mu}, \qquad (i = E, W) \tag{1}$$

and

$$C_N^i = \left[\int_0^{n^i} [c_N^i(z)]^{1-\frac{1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1, \qquad (i = E, W) \tag{2}$$

where z is an index of a differentiated service, and n^E and n^W represent the ranges of nontradeable services available in East and West. (We take the space of services to be continuous and ignores the integer constraint.) The low-tier utility function, U, is assumed to be symmetric, quasi-concave and linearly homogeneous. The upper-tier utility function is a Cobb-Douglas, with μ being the share of the tradeable goods in expenditure.[3] Note that the linear homogeneity of U implies that preferences are homothetic.

Let p_E and p_W be the prices of E and W. In order to describe consumer demand for E and W, it is convenient to define the price index of the tradeable goods, $e(p_E,p_W)$, that is, the unit expenditure function corresponding to U, by

$$e(p_E, p_W) \equiv Min_{(c_E,c_W)}\{p_E c_E + p_W c_W \mid U(c_E, c_W) \geq 1\}.$$

This function is increasing in each argument; it is symmetric, quasi-concave, and linearly homogeneous. Let the derivative of e with respect to p_j be denoted by e_j. These derivatives are homogeneous of degree zero. From the standard result of the duality theory, demand for E and W can then be expressed as

$$\frac{e_j(p_E,p_W)}{e(p_E,p_W)}\mu(Y^E+Y^W) \qquad (j = E, W)$$

where Y^i represents aggregate income in region i. The relative demand for E and W is thus equal to

$$\frac{e_E(p_E,p_W)}{e_W(p_E,p_W)},$$

which depends solely and negatively on p_E/p_W and the elasticity of substitution between E and W can be defined by

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[3]Assuming a Cobb-Douglas upper-tier utility function is useful for three reasons. First, it helps to simplify the analysis considerably and to keep the number of parameters at a minimum level. Second, it means that standard-of-living in an unsettled region is zero, not an unrealistic feature of the model. This makes concentration outcomes always stable, and hence allows us to focus on the stability of the outcome in which the population is equally divided. Third, if the elasticity of substitution between the tradeable goods and nontradeable services is too big, then a large regional population does not necessarily lead to a wider range of local services available. Such a possibility itself is interesting and has been explored by Matsuyama (1992c), but it would be an unnecessary complication for the purpose of the present analysis. The Cobb-Douglas specification can rule out this possibility.

$$\Theta(p_E/p_W) = -\frac{d\log[e_E(p_E,p_W)/e_W(p_E,p_W)]}{d\log(p_E/p_W)}$$

where the symmetry implies that $\Theta(u) = \Theta(1/u)$.

Aggregate demand for a service z is, on the other hand, given by

$$\left[\frac{p_N^i(z)}{P_N^i}\right]^{-\sigma}\frac{(1-\mu)Y^i}{P_N^i}, \qquad (i = E, W)$$

where $p_N^i(z)$ is the price of service z in region i and

$$P_N^i = \left[\int_0^{n^i}[p_N^i(z)]^{1-\sigma}dz\right]^{\frac{1}{1-\sigma}}, \qquad (i = E, W)$$

is the price index of nontradeable services in region i.

2.B. Tradeable Goods Sectors.

We now turn to the production side of this economy. Tradeable goods sectors are competitive, and both E and W can be produced by labor with constant returns to scale technologies. Labor productivity differs across the regions, perhaps due to some natural factors. In East, it takes one unit of labor to produce one unit of E and Ω units of labor to produce one unit of W, where $\Omega > 1$. For the sake of symmetry, it is assumed that, in West, one unit of labor is required to produce one unit of W and Ω units of labor for one unit of E. The tradeable goods sectors thus have the familiar structure of the Ricardian model of trade. The assumption, $\Omega > 1$, implies that East has a comparative advantage in E and West has a comparative advantage in W. As a result, East always produces E and West always produces W. Hence,

$$p_E = w^E, \qquad p_W = w^W, \tag{3}$$

where w^i is the wage rate in region i. On the other hand,

$$p_W \le \Omega w^E , \quad X_W^E \ge 0 , \quad (p_W - \Omega w^E) X_W^E = 0 ,$$
$$p_E \le \Omega w^W , \quad X_E^W \ge 0 , \quad (p_E - \Omega w^W) X_E^W = 0 , \tag{4}$$

where X_j^i denote the output of j in region i. The market clearing conditions for E and W become

$$X_j^E + X_j^W = \frac{e_j(p_E, p_W)}{e(p_E, p_W)} \mu (Y^E + Y^W) . \qquad (j = E , W) \tag{5}$$

The parameter Ω, which can be interpreted as a measure of regional differences, will play an important role in the analysis below. It should be noted that the two regions are different but symmetric. In particular, each region has an absolute advantage in one tradeable good: East in E and West in W. This means that, if individuals prefer consuming at least a little bit of both E and W, it may be desirable for the population not to concentrate into one region, particularly when Ω is high.

2.C. Nontradeable Service Sectors.

The market structure in the nontradeable service sector is monopolistically competitive. Each service is supplied by a single, atomistic firm; n^i thus represents not only the range of available services but also the "number" of specialist firms that operate in region i. Supplying x units of any service requires h(x) units of labor. Then, marginal cost equals to $w^i h'(x)$ and average cost $w^i h(x)/x$. It is assumed that there are economies of scale at least at a low level of operation so that the ratio of average cost to marginal cost, $h(x)/xh'(x)$, is strictly decreasing in x. (This is the case, for example, if $h(x) = ax + F$.) As each firm faces a demand curve of price elasticity σ, the pricing rule is $p_N^i(1-\sigma^{-1}) = w^i h'(x)$. Furthermore, profits will be competed away in the absence of any restrictions on entry and exit, implying a price equal to average cost: $p_N^i = w^i h(x)/x$. Combining the pricing rule and the zero profit condition yields

$$\frac{h(x)}{h'(x)x} = \frac{\sigma}{\sigma - 1} .$$

As the left hand side of this equation is decreasing, this condition uniquely determines the output of each firm; it implies that every firm operates at the same scale, independent of the regional population. This implication is entirely due to the Dixit-Stiglitz formulation, but it helps to simplify the analysis considerably and is not crucial for our result.

What is crucial is that the number of firms, and hence the variety of services available, in each region increases with the regional population. To see this, first recall that the budget share of nontradeable services is $1 - \mu$, from which $(1 - \mu)Y^i = n^i p_N^i x = n^i w^i h(x)$. Second, since profits will be zero in equilibrium, aggregate income consists only of wage income:

$$Y^i = w^i L^i . \qquad (i = E , W) \tag{6}$$

Together, we have

$$n^i = \frac{1 - \mu}{h(x)} L^i . \qquad (i = E , W) \tag{7}$$

That is, the range of services available in each region is proportional to the population. In the presence of economies of scale, a large population is necessary to support more firms and a wider range of services. This gives an incentive for the population to concentrate into one region in this model.

2.D. Labor Markets.

Finally, the labor market needs to clear in each region. Note that (7) implies that the level of employment in the nontradeable service sector is equal to $n^i h(x) = (1 - \mu)L^i$. Hence, the labor market clears in East if

$$\mu L^E = X_E^E + \Omega X_W^E \ . \tag{8}$$

Similarly, the labor market clearing condition in West is

$$\mu L^W = \Omega X_E^W + X_W^W \ . \tag{9}$$

2.E. Equilibrium Terms of Trade.

We now solve for the equilibrium terms of trade between the two regions for a given population distribution. From equations (3) and (4), one can determine the upper and lower bounds of the terms of trade: $\Omega^{-1} \leq p_E/p_W \leq \Omega$. If $\Omega^{-1} < p_E/p_W < \Omega$, then both regions specialize: $X_E{}^W = X_W{}^E = 0$. Hence, from (8) and (9), $X_E{}^E = \mu L^E$ and $X_W{}^W = \mu L^W$. The relative supply of E and W is thus equal to the ratio of the population and, from the equilibrium conditions for the tradeable goods (5),

$$\frac{L^E}{L^W} = \frac{e_E(p_E, p_W)}{e_W(p_E, p_W)} \ ,$$

or

$$\frac{p_E}{p_W} = \Psi\left(\frac{L^E}{L^W}\right), \qquad \frac{d \log \Psi\,(L^E/L^W)}{d \log\,(L^E/L^W)} = -\frac{1}{\Theta} < 0 \ , \tag{10}$$

where Ψ is an inverse relative demand function. The condition $\Omega^{-1} < p_E/p_W < \Omega$ then simply becomes $\Omega^{-1} < \Psi(L^E/L^W) < \Omega$. This shows that, when the population distribution is not too lopsided, p_E/p_W declines as L^E/L^W goes up: one percentage change in L^E/L^W reduces p_E/p_W by $1/\Theta$ percents. If $\Psi(L^E/L^W) \leq \Omega^{-1}$, on the other hand, $p_E/p_W = \Omega^{-1}$ and East produces both E and W, while West specializes.[4] Similarly, if $\Psi(L^E/L^W) \geq \Omega$, then $p_E/p_W = \Omega$ and West produces both E and W, while East specializes.[5] To summarize,

$$\frac{p_E}{p_W} = \begin{cases} \Omega & \text{if} \quad \Psi(L^E/L^W) \geq \Omega , \\ \Psi(L^E/L^W) & \text{if} \quad \Omega^{-1} < \Psi(L^E/L^W) < \Omega , \\ \Omega^{-1} & \text{if} \quad \Psi(L^E/L^W) \leq \Omega^{-1} . \end{cases} \tag{11}$$

3. Regional Concentration.

We are now ready to analyze labor migration and its efficiency implications. In order to see incentives for migration, it is necessary to derive the standard-of-living index for each region. First, each individual in region i earns w^i, and hence enjoys the utility equal to $U(c_E^i, c_W^i) = \mu w^i/e(p_E, p_W)$ from consumption of the tradeable goods. Second, s/he consumes x/L^i units of all services available in region i. By inserting these expressions back to the preferences (1) and (2), and using (3), the standard-of-living index in region i can be expressed as

[4]In this case, by solving (3) through (9), one can show that outputs of E and W in the two regions are

$$X_E^E = \frac{e_E(1,\Omega)}{e(1,\Omega)}\mu(L^E + \Omega L^W) , \qquad X_E^W = 0 ,$$

$$X_W^E = \frac{\mu}{e(1,\Omega)}\left[e_W(1,\Omega)L^E - e_E(1,\Omega)L^W\right] \geq 0 , \quad X_W^W = \mu L^W .$$

[5]In this case, from (3) through (9), the outputs of E and W in the two regions are

$$X_E^E = \mu L^E , \quad X_E^W = \frac{\mu}{e(\Omega,1)}\left[e_E(\Omega,1)L^W - e_W(\Omega,1)L^E\right] \geq 0 ,$$

$$X_W^E = 0 , \qquad X_W^W = \frac{e_W(\Omega,1)}{e(\Omega,1)}\mu[\Omega L^E + L^W] .$$

$$V^i = K\left[\frac{p_i}{e(p_E, p_W)}\right]^{\mu}[n^i]^{\frac{1-\mu}{\sigma-1}} = K'\left[\frac{p_i}{e(p_E, p_W)}\right]^{\mu}[L^i]^{\frac{1-\mu}{\sigma-1}}, \tag{12}$$

where K and K' are positive constants. In East, standard-of-living increases with p_E/p_W, and n^E. In West, it is increasing in p_W/p_E and n^W. The relative standard-of-living index is, on the other hand,

$$\frac{V^E}{V^W} = \left[\frac{p_E}{p_W}\right]^{\mu}\left[\frac{n^E}{n^W}\right]^{\frac{1-\mu}{\sigma-1}} = \left[\frac{p_E}{p_W}\right]^{\mu}\left[\frac{L^E}{L^W}\right]^{\frac{1-\mu}{\sigma-1}}. \tag{13}$$

This formula indicates that the population distribution affects the relative attractiveness of the two regions through two mechanisms. First, a high L^E/L^W makes East more attractive as a wider range of services will be available in East than in West. On the other hand, it makes East less attractive to the extent that it moves the terms of trade against East and for West.

Inserting the equilibrium terms of trade (11) into (12), the standard-of-living index in each region can be expressed as a function of L^E only. Four typical cases are illustrated in Figure 1A through 1D; because of the symmetry of the two regions, the standard-of-living index in West can be obtained by flipping the index function of East around $L^E = 1/2$. When L^E is sufficiently large or sufficiently small, one region produces both E and W and the terms of trade is independent of the population distribution. As a result, standard-of-living in each region increases monotonically with its population. On the other hand, in the middle range, an increase in population leads to a deterioration of the terms of trade, and consequently, standard-of-living may decline. From (10) and (12), some algebra shows that

$$\frac{L^E}{V^E}\frac{dV^E}{dL^E} = \frac{1-\mu}{\sigma-1} - \frac{\mu}{\theta}\frac{\alpha_W}{L^W}, \qquad \frac{L^W}{V^W}\frac{dV^W}{dL^W} = \frac{1-\mu}{\sigma-1} - \frac{\mu}{\theta}\frac{\alpha_E}{L^E}, \tag{14}$$

where $\alpha_j \equiv p_j e_j/e$ is the share of good j in expenditure on the tradeable goods. On the other hand, from (10) and (13), the relative standard-of-living index can be shown to depend on the population ratio in the following way:

$$\frac{L^E/L^W}{V^E/V^W}\frac{d(V^E/V^W)}{d(L^E/L^W)} = \frac{1-\mu}{\sigma-1} - \frac{\mu}{\theta} \ . \tag{15}$$

The first terms in these expressions represent the scale economy effect that pulls population into concentration, or, as Krugman (1991c) put it, "centripetal forces." The second terms represent the terms-of-trade effect, "centrifugal forces" that tend to deter such concentration.

Let us now give individuals freedom to choose where to live; they are allowed to migrate to the region that offers a higher standard-of-living. We will then explore the stability of population distribution and its efficiency implications. A complete treatment of this issue would need to specify the process of labor migration. However, what we present below would not depend on the details of the processes. Hence, for the rest of this section, we will discuss the consequences of labor migration heuristically, and postpone our formal presentation of labor migration dynamics until the next section.

There are always at least three population distributions for which an individual has no incentive to migrate. The first two are the cases in which all individuals concentrate into one region, either $L^E = 0$ or $L^E = 1$; they all share the same level of standard-of-living. By setting $L^E = 1$ and $p_E/p_W = \Omega^{-1}$ for V^E, or by setting $L^W = 1$ and $p_E/p_W = \Omega$ for V^W in (12), standard-of-living is equal to

$$V = \frac{K'}{[e(1,\Omega)]^{\mu}} > 0 \ . \tag{16}$$

The third one is $L^E = 1/2$, the case in which each region has half of the population. Standard-of-living is then equalized across the regions and, from (12), it is equal to

$$V^E = V^W = \frac{2^{\frac{1-\mu}{1-\sigma}}K'}{[e(1,1)]^{\mu}} \ . \tag{17}$$

These outcomes may differ in stability. When the entire population concentrates in one region, all individuals enjoy a positive standard-of-living, and hence any individual or even a small coalition of individuals, whose measure is positive but arbitrary small, has no incentive to migrate to the other region, as it would lead to a strictly lower standard-of-living. In this sense, both $L^E = 0$ and $L^E = 1$ are stable outcomes. On the other hand, when the two regions are equally populated, no individual incurs a loss from migration, and an even arbitrary small coalition of individuals would gain from migration if the standard-of-living in each region is increasing in its population at $L^E = 1/2$. From either (14) or (15), the equal distribution outcome is hence unstable if

$$\Lambda \equiv \frac{1-\mu}{\mu(\sigma-1)} > \frac{1}{\theta(1)} . \tag{18}$$

When this condition is met, as in Figure 1C or Figure 1D, a slight perturbation leads to an incentive to migrate toward one region. Intuition behind this condition is easy to grasp. The left hand side of this inequality, Λ, is related to benefits of concentration due to economies of scale in nontradeable service sectors, which is large when services are more differentiated and account for a larger share in expenditure. Working against concentration is the terms of trade change that would follow a change in the relative supply of E and W caused by migration. When E and W are less substitutable, the terms of trade change would be large enough to ensure the stability, as shown in Figure 1A and Figure 1B.

The stability property of the fifty-fifty division provides neither a sufficient or necessary condition for its desirability in this economy, however. From (16) and (17), this outcome is better than the concentration outcome if and only if

$$\Lambda < \frac{\log[e(1,\Omega)/e(1,1)]}{\log 2} . \tag{19}$$

This condition shows that the equal distribution outcome is more desirable with a small Λ. A high Ω also makes this condition more likely to hold.

As seen in (18) and (19), a large Λ tends to make the equal disribution outcome both unstable and undesirable. Nevertheless, there may be some cases in which the equal distribution outcome is desirable and yet unstable, and concentration will occur (Figure 1C). If this is the case, regional policies may need to be designed to restore the stability of balanced growth. On the other hand, the equal distribution outcome may be stable and yet undesirable (Figure 1B). In such a case, appropriate regional policies should pick one region as a showcase and encourage people to agglomerate, in order to take full advantage of economies of scale. To explore such possibility further, we will now turn to more specific examples.

Example 1. Suppose that the low-tier utility function U is a Cobb-Douglas: $\Theta(\bullet) \equiv 1$ and $e(p_E, p_W) = (p_E p_W)^{1/2}$. Then, $\Psi(L^E/L^W) = L^W/L^E$ and, from (11) and (12), the standard-of-living index becomes

$$V^E = \begin{cases} K' \Omega^{\frac{\mu}{2}} (L^E)^{\frac{1-\mu}{\sigma-1}} & \text{if} \quad 0 \le L^E \le \frac{1}{1+\Omega} , \\ K'(1-L^E)^{\frac{\mu}{2}} (L^E)^{\frac{1-\mu}{\sigma-1} - \frac{\mu}{2}} & \text{if} \quad \frac{1}{1+\Omega} < L^E < \frac{\Omega}{1+\Omega} , \\ K' \Omega^{-\frac{\mu}{2}} (L^E)^{\frac{1-\mu}{\sigma-1}} & \text{if} \quad \frac{\Omega}{1+\Omega} \le L^E \le 1 . \end{cases}$$

In what follows, we ignore a non-generic case, $\Lambda = 1$. If $\Lambda < 1$, then $L^E = 0$, 1/2, and 1 are all stable. (In this case, there are two other states, in which the standard-of-living in the two regions are equalized, but they can be shown to be unstable.) If

$$\Lambda > 1 , \tag{18-1}$$

only $L^E = 0$ and $L^E = 1$ are stable. On the other hand, concentration would be undesirable if

$$\Lambda < \frac{\log \Omega}{\log 4} . \tag{19-1}$$

These conditions are illustrated in Figure 1, which shows that, depending upon the parameters, the economy may be in any of the following four different regimes. In Regime A, illustrated in Figure 1A, $L^E = 1/2$ is stable and most desirable among the three stable outcomes. In Regime B (Figure 1B), it is stable but less desirable than the concentration outcome. In Regime C (Figure 1C), it is most desirable and yet unstable. In this case, individuals, in their pursuit of higher standard-of-living, would reallocate and end up agglomerating in one region, which makes them worse off compared to the equal distribution outcome. This is the case of self-defeating concentration. Finally, in Regime D (Figure 1D), concentration will occur and it is good to occur.

This result is immediately applicable to the analysis of one aspect of economic development. Imagine that, initially, the population is equally divided between East and West. Furthermore, Λ is so small that this is stable and desirable: the economy is in Regime A. Let us now suppose that Λ begins to go up gradually: as the economy develops, economies of scale become more important. Service sectors account for a larger share of national income, and numerous innovations lead to services of highly specialized character. As Λ becomes larger than one, the equal distribution becomes unstable: a slight perturbation would suffice to push the economy out of this outcome, and the economy would experience a regional agglomeration. Note that a small change in Λ across the horizontal line $\Lambda = 1$ creates a catastrophic change in the regional patterns of the economy, as in Krugman (1991a). The efficiency implications of concentration, however, depends on the value of Ω. If $\Omega < 4$, then the problem is the lack of concentration when $\Lambda < 1$; thus, further increase in Λ could eliminate this problem. On the other hand, when $\Omega > 4$, concentration turns out to be self-defeating; only after a later stage of development, when Λ becomes sufficiently large, such concentration will be desirable. This thought experiment thus suggests that, when regional differences are large, some policies designed to counter tendencies of uneven regional growth may be called for during the transition phase of economic development.

Cobb-Douglas preferences have a strong property in that both E and W are essential, U(0,x)

$= U(x,0) = 0$: that is, if one of the two goods is not consumed at all, an increase in consumption of the other good would not raise the level of the utility. It implies that individuals prefer strongly consuming at least a little bit of both E and W. One may wonder whether this property is responsible for the self-defeating concentration result for a high Ω. To answer this question, we now turn to our second example.

Example 2. Suppose now that the low-tier utility function U is a CES: $\Theta(\bullet) \equiv \theta$ and $e(p_E, p_W) = (p_E^{1-\theta} + p_W^{1-\theta})^{1/(1-\theta)}$. Let us recall the basic property of a CES. When $\theta < 1$, any indifferent curve is bounded away from the axes; that is, E and W are essential goods; when $\theta > 1$, indifferent curves intersect with the axes, meaning that E and W are non-essential goods. Again, we ignore a non-generic case, $\Lambda\theta = 1$. The equal distribution outcome, $L^E = 1/2$, is unstable and only the concentration outcomes, $L^E = 0$ and $L^E = 1$, become stable if

$$\Lambda^{-1} < \theta , \tag{18-2}$$

and yet the equal distribution outcome is the most desirable one if

$$\Lambda^{-1} > \log 2 \Big/ \log\left[\frac{1+\Omega^{1-\theta}}{2}\right]^{\frac{1}{1-\theta}} = \log 2 \Big/ \int_1^{\Omega} \frac{dv}{v^{\theta}+v} \equiv \phi(\Omega,\theta) . \tag{19-2}$$

Figure 2a illustrates these conditions for a given Ω. The stability condition is given by the 45-degree line, below which only $L^E = 0$ and $L^E = 1$ are stable outcomes. The other curve, $\Lambda^{-1} = \phi(\Omega,\theta)$, is monotone increasing in θ and crosses the 45-degree line only once; it is asymptotic to $\Lambda^{-1} = \theta - 1$, as θ tends to infinity. Concentration is undesirable above this curve. An increase in Ω shifts this curve down, and hence makes self-defeating concentration more likely.

Three points about Figure 2a deserve special emphasis. First, $\theta > 1$ does not rule out the possibility of self-defeating concentration: the essentiality of E and W is not at all necessary for self-defeating concentration. Second, when Ω is sufficiently high, the essentiality of E and W becomes almost sufficient for concentration to be self-defeating. To see this, note that

$$\lim_{\Omega\to\infty}\phi(\Omega,\theta) = Max\{\ \theta - 1\ ,\ 0\ \}\ .$$

Hence, Regime D requires $\theta > 1$ as Ω tends to infinity. This is to say that, if E can be produced only in East and W only in West, the essentiality of E and W implies that concentration, when it occurs, has to be self-defeating. Third, Regime B disappears as Ω tends to infinity: that is, with a sufficiently large degree of regional differences, the equal division outcome is stable only when it is desirable.

As in the previous example, we can think of a thought experiment, in which the economy is initially in Regime A and Λ goes up gradually. As the economy moves down in Figure 2a, the incentive to concentrate becomes larger. As the economy crosses the 45-degree line, there is a catastrophic change in its regional patterns. Whether the economy passes through Regime C, the case of self-defeating concentration, depends on θ and Ω. The condition is given by $\phi(\Omega,\theta) < \theta$, or

$$\log\Omega\ >\ \frac{\log(\,2^{1/\theta} - 1\,)}{1 - \theta} \tag{20}$$

The right hand side expression is decreasing in θ, as plotted in Figure 2b. Thus, as economies of scale become more important, the economy would go through the stage of self-defeating concentration when Ω is sufficiently high. With a larger degree of regional differences, concentration could be self-defeating. Our conclusion earlier obtained in the Cobb-Douglas case thus carries over to more general CES cases.

What may seem counterintuitive about condition (20), or Figure 2b, is that in order for self-defeating concentration to happen, regional differences of East and West should be larger when the two tradeable goods are less substitutable. This is entirely due to the peculiarity of CES preferences. What the condition in (19-2) said is that, for a given Λ, regional differences need to be large when the two tradeable goods are more substitutable in order for concentration to be less desirable than the equal distribution outcome. In fact, if $\theta \geq \Lambda^{-1} + 1$, then self-defeating concentration never occurs. The reason why (20) shows the seemingly counterintuitive result is that, when we change θ, we change both the desirability of concentration and the stability of the equal distribution outcome at the same time. As shown in Figure 2a, $\phi(\Omega,\theta)$ has a slope less than 45-degree at the intersection. That is, with a smaller θ, a larger increase in Λ would be necessary for restoring (18-2) than for restoring (19-2). As a result, we need a large Ω in order for concentration that does occur to be undesirable.

That this property is peculiar to the CES specification can easily be seen if we rewrite the condition for self-defeating concentration for a general low-tier utility function in the following way:

$$\frac{1}{\Theta(1)} < \Lambda < \frac{1}{\log 2} \int_1^{\Omega} \frac{dv}{\exp\left[\int_1^v \Theta(u)\, du/u\right] + v} . \tag{21}$$

The first inequality reproduces equation (18), the condition for instability of $L^E = 1/2$, while the second reproduces (19), the condition for undesirable concentration. That the second inequality is equivalent to (19) can be verified by simply differentiating the right hand side of (19) twice, and using the Euler's Theorem. Note that the stability of the equal distribution outcome depends solely on $\Theta(1)$, while its desirability depends on the entire functional form of Θ. A CES function imposes a very strong restriction in that each side of the inequalities cannot be controlled independently. More generally, if one considers a Θ function which declines as the relative price moves away from unity, then self-defeating concentration becomes more likely. In particular, if E and W are both essential goods, that is,

$$\lim_{u \to \infty} \Theta(u) = \lim_{u \to \infty} \Theta(1/u) < 1 ,$$

then the right hand side of (21) is an unbounded function of Ω, so that the concentration outcome becomes undesirable for a sufficiently large degree of regional differences. For a general Θ function, this could be consistent with a large $\Theta(1)$.

One cannot dismiss such a Θ function as contrived or farfetched. Imagine that E is fish and

W is beef. Consumers may not care much about the ratio of fish and beef consumption as long as the ratio is between, say, one half and two. On the other hand, they may prefer strongly to consume at least a little bit of both fish and beef. Such preferences can be represented by indifference curves which are flat around the 45-degree line and bounded away from the axes. These preferences have a Θ function with the property mentioned above. That is, relative demand responds elastically around $p_E/p_W = 1$, and yet both goods are essential. If this is the case, self-defeating concentration becomes inevitable.

4. Modelling Dynamics Explicitly.

The analysis in the previous section relies on the stability. We ignore any population distribution at which $V^E - V^W$ is zero but locally increasing in L^E. We argue that such a distribution is unstable. We have already discussed one justification: with a locally increasing standard-of-living index function, an even arbitrary small coalition of individuals would gain from migration. This is only but one justification, however. Alternatively, one could rule out such a distribution by asserting that it is an artifact of a continuity of the set of individuals. This assumption, while technically convenient, is not a realistic feature of the model. If the integer constraint on the number of individuals is taken seriously, the interior solution of $V^E = V^W$ can generally be achieved only approximately. The positive slope of the index function, however, implies that, at an approximate solution, all individuals have an incentive to move into one region. Hence, equilibria of an economy with a large, but finite number of individuals do not exist anywhere near the interior distribution at which standard of living is locally increasing in population. In other words, such an interior equilibrium of the limit economy could not be a limit point of a sequence of equilibria of large, but finite economies. One could also rule out such a distribution based on its perverse comparative static properties; the population in East must decline in order to restore the equilibrium when some changes in parameters make East more attractive. This also suggests that it is unstable under occasional perturbations in a process of simple migration dynamics. For example, suppose, in the spirit of the evolutionary game literature, that individuals naively gravitate towards the region with the higher standard-of-living. Then the economy always converges to an interior equilibrium at which the index function is negatively sloped, or to one of the two end points.

A complete treatment of this issue, however, would need an explicit analysis of a dynamic process in which individuals make rational migration decisions. Suppose that individuals in this economy cannot all move at once; that there is some kind of adjustment cost that limits the rate at which the population can shift. Thus an individual who chooses to locate in one region or the other is stuck with the choice, at least for a while. Then, she would be concerned not only with the current levels of standard of living in the two regions, but also with the future levels. But standard of living in each region at any point in time depends on the population distribution; this means that each individual's reallocation decision depends on her expectations about the future decisions of others. In what follows, we will present a model of labor migration dynamics with this feature, based on Matsuyama (1991, 1992a, b), and demonstrate that an interior population distribution at which the index function has a positive slope is indeed unstable in such migration processes.

Let us imagine that the opportunity for an individual to reallocate arrives randomly; it follows the Poisson process with λ being the mean arrival rate. Furthermore, it is assumed that the process is independent across individuals and there is no aggregate uncertainty. Let L_t be the East population and $V_t^E - V_t^W \equiv F(L_t)$ be the difference between the standard-of-living in East and West as of time t. When the opportunity arrives, an individual chooses the region that offers a higher expected discounted utility over the expected duration of her commitment. Thus, individuals who are free to choose at time t would commit to East if $Q_t > 0$ and to West if $Q_t < 0$ and are indifferent if $Q_t = 0$, where

$$Q_t = (\lambda+\delta)\int_t^{\infty} F(L_s)\, e^{(\lambda+\delta)(t-s)} ds \ , \tag{22}$$

with $\delta > 0$ being the discount rate. Therefore, $\{L_t\}_{t=0}^{\infty}$ is an equilibrium population dynamics for a given initial distribution, L_0, if its right hand derivative satisfies

$$\frac{d^+L_t}{dt} \in \begin{cases} \{\lambda(1-L_t)\} & if\ Q_t > 0, \\ [-\lambda L_t\ ,\ \lambda(1-L_t)] & if\ Q_t = 0, \\ \{-\lambda L_t\} & if\ Q_t < 0, \end{cases} \tag{23}$$

for all $t \in [0,\infty)$. Equation (23) states that all individuals in West (resp. East), if given the opportunity, move to East (resp. West), when $Q_t > 0$ (resp. $Q_t < 0$).

To solve this equilibrium dynamics, let us differentiate (22) to yield

$$\dot{Q}_t = (\lambda+\delta)\,[Q_t - F(L_t)] \ . \tag{24}$$

Figures 3a through 3d show that the phase portraits defined by equations (23) and (24). Note that Q increases above Q = F(L) and declines below it, while L increases above Q = 0 and declines below it. In all cases, there are at least three steady states: (L,Q) = (0, F(0)), (L,Q) = (1/2, 0), and (L,Q) = (1, F(1)), indicated by black dots in the figures. (We do not provide the proof for the statements that follows; the proof is rather intricate, but can be done along the lines given in Matsuyama 1991, 1992b). Some equilibrium paths for some L_0 are depicted by heavily barbed curves. When the index function is negatively sloped at L = 1/2, the cases corresponding to Regimes A and B, there are generically three qualitatively different types of equilibrium dynamics. Figures 3a through 3c illustrate each of them. In all these cases, L = 1/2 is locally stable in that, if L_0 is sufficiently close to 1/2, there is an equilibrium path along which L_t converges to 1/2. In Figures 3a and 3b, it is the only equilibrium path. Figure 3d illustrates dynamics when the index function is positively sloped at L = 1/2, the case corresponding to Regimes C and D. In this case, unless L_0 =

1/2, L_t converges either to 0 or to 1 along any equilibrium path. Hence, in Regime C or D, the entire population tends to concentrate in one region. And once concentration happens, then it cannot be broken.

Incidentally, Figure 3d shows that, for some initial conditions, there are multiple equilibrium paths, some of which converge to 0, while others converge to 1. This shows that the initial distribution of population does not necessarily help to predict where concentration occurs, once the expectations of individuals are explicitly incorporated in their migration decisions.[6] For example, suppose that East is initially more populous of the two and offers a higher standard-of-living. If everybody was convinced that West will be the destination of migration and started moving into West, then standard-of-living in West would eventually become higher and the belief that West is the land of opportunity would turn out to be a self-fulfilling prophecy. Similar indeterminacy arises for the cases in which the equal distribution outcome is locally stable, as shown in Figures 3a through 3c.

One may find the indeterminacy of equilibrium dynamics disturbing, but for the purpose of this paper, it does not pose any problem. Essential for our argument in the previous section is that, in Regimes C and D, the equal ditribution outcome is unstable and only the concentration outcomes are stable. Our prediction is that concentration does occur; we do not need to say where.

5. Concluding Remarks.

In this paper, we have presented a model of a two-region economy with two tradeable goods and differentiated nontradeable services. The benefits of increasing variety in nontradeable services tend to work for regional concentration, while regional differences in productivity of the tradeable goods sectors tend to work against concentration. Among others, we have demonstrated that undesirable concentration occurs in some cases; with freedom to choose where to live, individuals in

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[6]This possibility is also noted by Krugman (1991c, d) in similar adjustment dynamics.

this economy concentrate into one region in their pursuit of better life, and end up becoming worse off. The possibility of such self-defeating concentration depends on the share of nontradeable services in expenditure, economies of scale in nontradeable service sectors, regional differences in labor productivity in tradeable goods sectors, and substitutability of the tradeable goods in consumption (and their essentiality). We have identified the conditions for self-defeating concentration in terms of these parameters.

A natural next step is to explore the role of national governments in their regional policies. One can think of a long list of policies that may be relevant in this context. For example, interregional income transfers and subsidies for those moving to poor regions should be useful for restoring the stability of balanced regional growth. Regional allocation of public investment in infrastructure is also of great importance.[7] One cautionary remark seems appropriate: self-defeating concentration does not necessarily imply that the government should redirect public investment in favor of underdeveloped regions. Such an investment policy may prevent self-defeating concentration, but the government could also reduce the cost of concentration by increasing investment in favor of developed regions. Going back to the example of the island economy, suppose that the whole population concentrates in West and that this is an undesirable situation. On one hand, the government may be able to induce people to go to East by providing a minimum level of services in East. On the other hand, the government may also be able to use its resources to build harbors in West, improving the productivity of its fishing industry. To identify an appropriate regional investment policy thus requires a careful study of costs and benefits analysis. It is hoped that the model presented in this paper or some variations of it would prove useful for this purpose.

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[7]For the literature on regional investment policies, see Takahashi (1993) and the work cited there.

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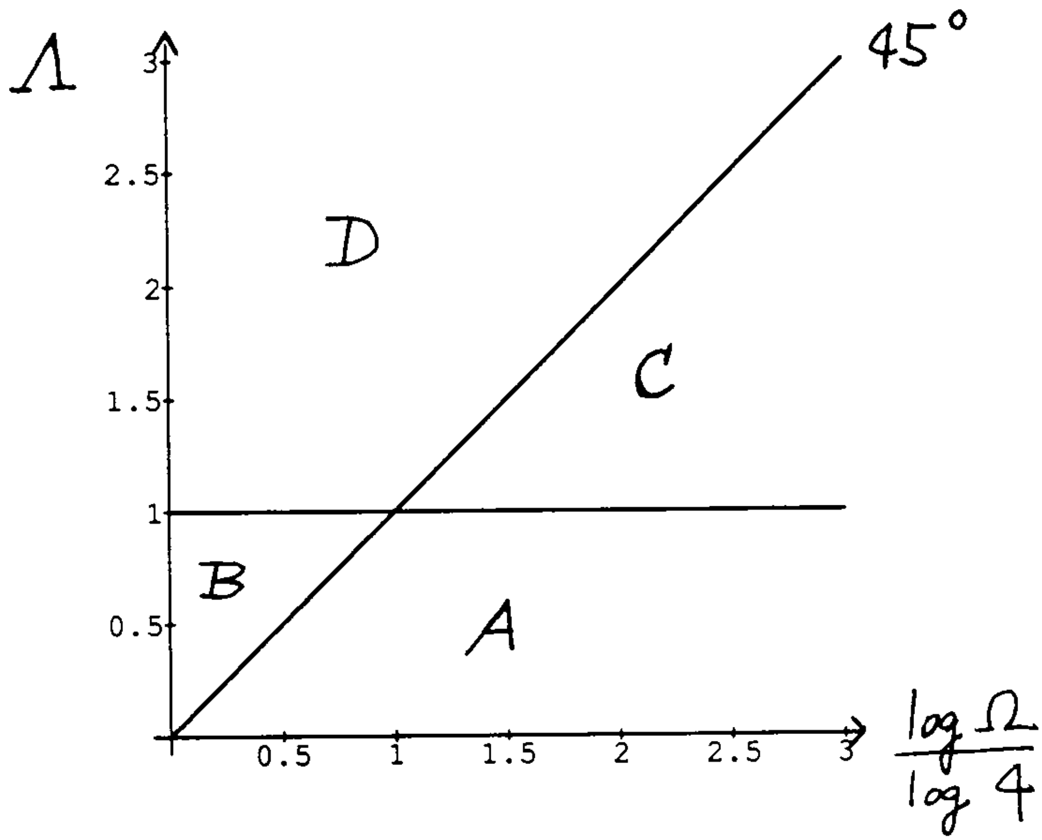

Figure 1 : Cobb-Douglas Preferences

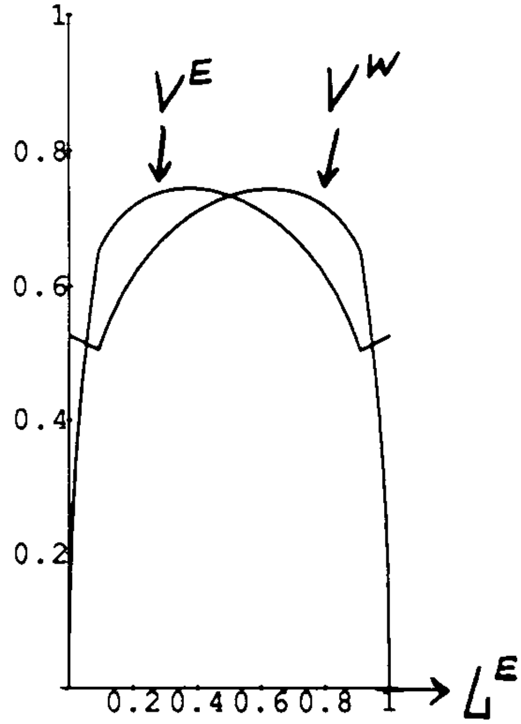

Figure 1A: ($\Omega = 10$, $\mu = \frac{5}{9}$, $\sigma = 2$)

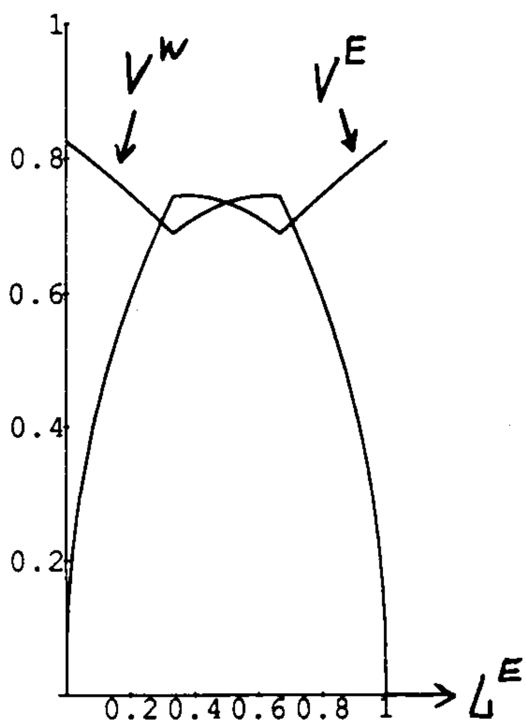

Figure 1B: ($\Omega = 2$, $\mu = \frac{5}{9}$, $\sigma = 2$)

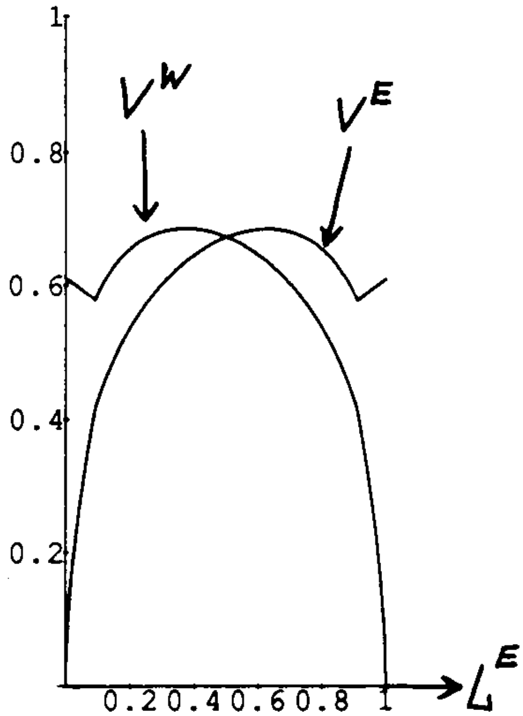

Figure 1C:
($\Omega=10, \mu=\frac{3}{7}, \sigma=2$)

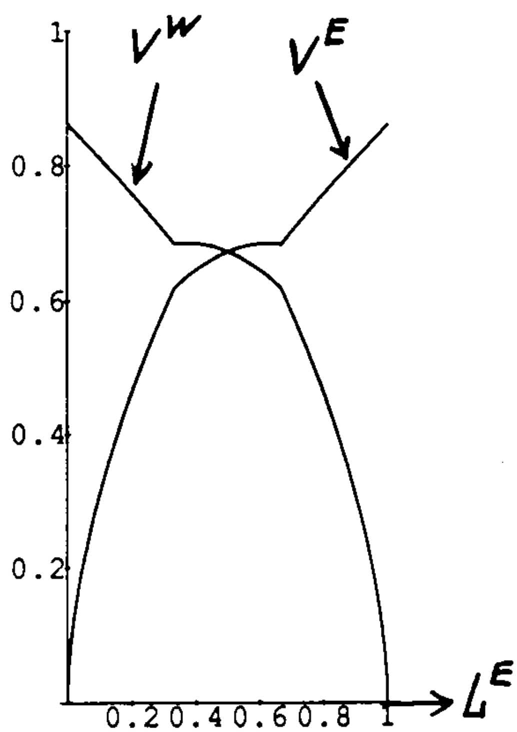

Figure 1D:
($\Omega=2, \mu=\frac{3}{7}, \sigma=2$)

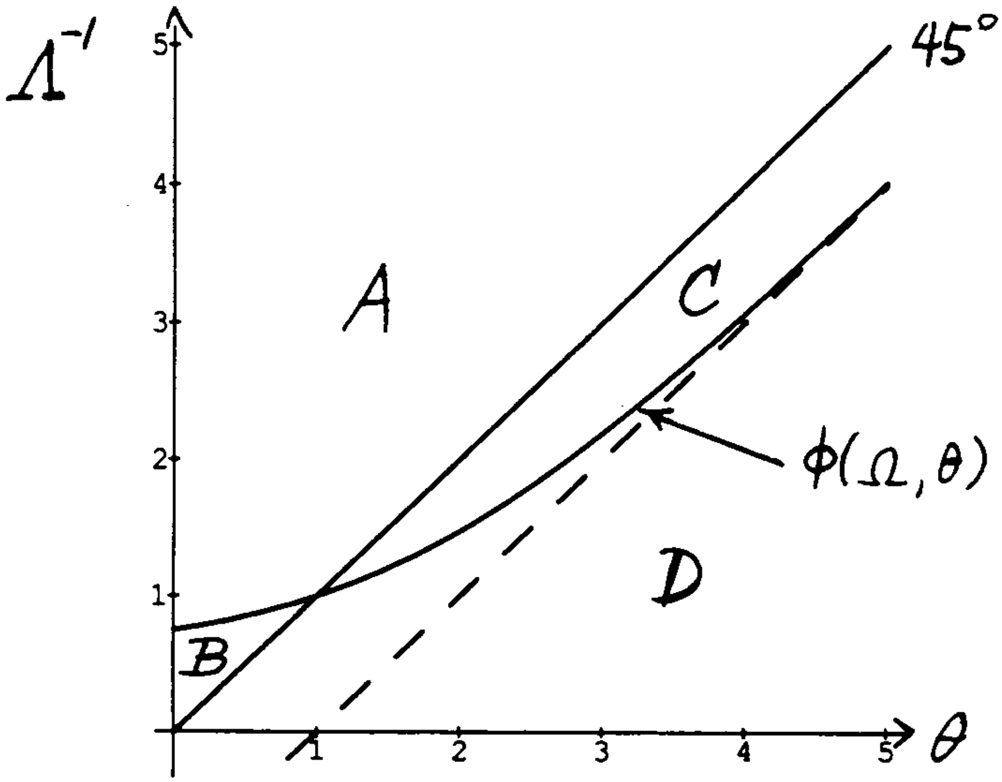

Figure 2a: CES Preferences ($\Omega = 4$)

Figure 2b: CES Preferences

Fig. 3a

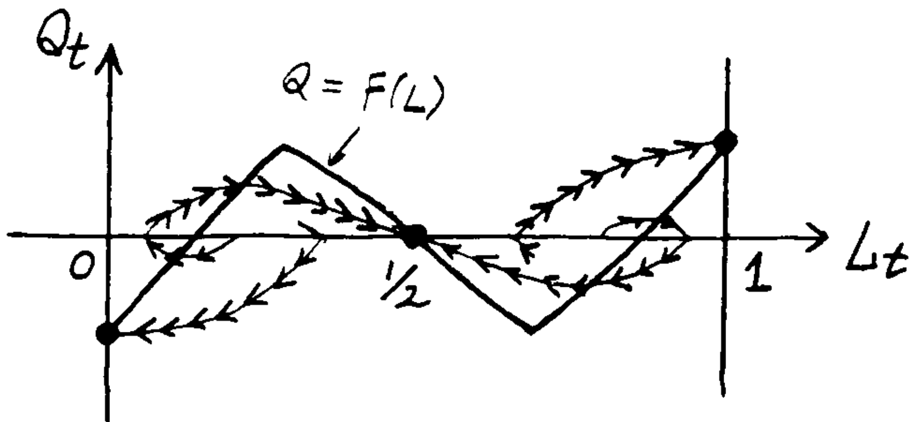

Fig. 3b

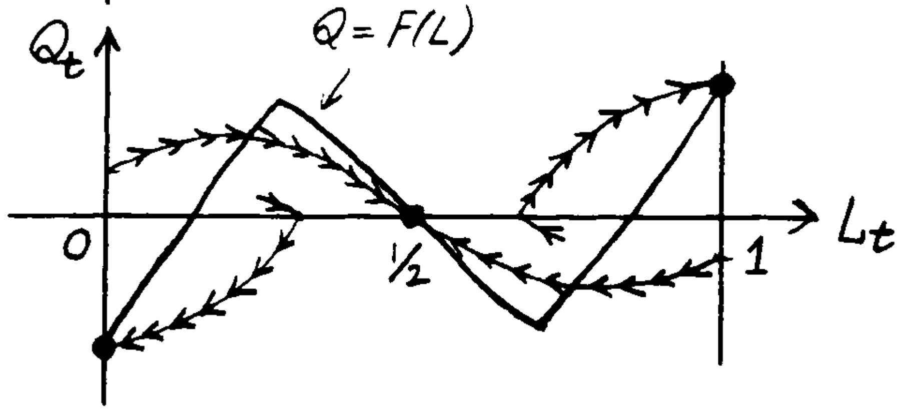

Fig. 3c

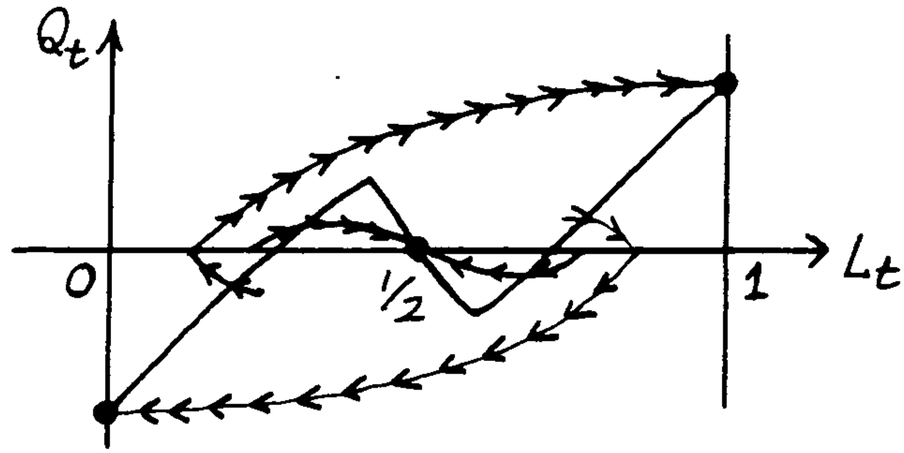

Fig. 3d