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"Design Innovation and Fashion Cycles"

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Abstract

A model of fashion cycles is developed in which fashion is used as a signaling device in a "dating-game". We assume that there is a designer (monopolist) who can create new designs at a positive fixed cost and zero marginal cost. Designs are durable commodities. We show the existence of equilibria of the following form: Every T periods a new design is innovated. Over time the price of the design falls and it spreads to more and more agents. Once sufficiently many agents own the design it is profitable to create a new design and a new fashion cycle begins.

We also examine the case when there is competition among many potential designers and show that there are equilibria in which fashion changes less frequently and the price of fashion remains bounded above marginal cost.

1 Introduction

Appearance is an important component of most durable consumption goods. Large amounts of resources are devoted to the development of designs for clothing, cars, furniture, or electronic equipment. These resources are not primarily used to make those goods more functional, rather their goal is to let the product appear *fashionable*. By fashion we generally mean the opaque process which identifies certain designs, products or social behavior as "in" for a limited period and which replaces them with infallible regularity by new designs, new products and new forms of social behavior.

If the consumption of a fashionable item is removed from its specific social context then changes in fashion do not entail any improvement in product quality. In his essay on fashion Georg Simmel writes:

> *Fashion is merely a product of social demands (...). This is clearly proved by the fact that very frequently not the slightest reason can be found for the creations of fashion from the standpoint of an objective, aesthetic or other expediency. While in general our wearing apparel is really adapted to our needs, there is not a trace of expediency in the method by which fashion dictates. (...) Judging from the ugly and repugnant things that are sometimes in vogue, it would seem as though fashion were desirous of exhibiting its power by getting us to adopt the most atrocious things for its sake alone". (Simmel (1904) p. 544).*

Simmel goes on to argue that the lack of practical use is part of the definition of "fashion". Thus fashion and fashion cycles can only be understood if consumption is considered as a social activity.

> *[Fashion] satisfies the need of differentiation because fashions differ for different classes - the fashions of the upper stratum of society are never identical with those of the lower; in fact, they are abandoned by the former as soon as the latter prepares to appropriate them. (Simmel (1904) p. 543).*

Examples of fashion cycles can be found throughout history. Braudel (1981) notes that fashion resulted to a large extent from the desire of the privileged to distinguish themselves.

whatever the cost, from the masses that followed them. According to a Sicilian who passed through Paris in 1714, *"Nothing makes noble persons despise the gilded costume so much as to see it on the bodies of the lowest men in the world"* (Marana (1883)). So the upper class had to invent new "gilded costumes", or new distinctive signs, whatever they might be, every time complaining that *"things have changed indeed, and the new clothes being worn by the bourgeois, both men and women, cannot be distinguished from those of persons of quality"* (Marquis de Paulmy (1774)). As a consequence, observers reported, *"tailors have more trouble inventing than sewing"* (Marana (1883)).

The purpose of fashion is to facilitate differentiation of "types" in the process of social interaction. The demand for new designs is derived from the desire of agents to interact with the "right" people. At the same time, fashion is accompanied by a process of continuous innovation, in which new designs are developed at sometimes large cost only to be replaced by others. With the arrival of every new design, previous fashions become obsolete and suddenly appear as what they were to begin with: ridiculous and useless.

We propose a simple model that captures the described features of fashion. We consider a monopolist (the designer) who can create new designs for a product such as a dress. There is a fixed cost of redesigning the product. Buyers like dresses not for their own sake, but because they allow them to signal their own quality. Customers want to signal their own quality to other customers because they are involved in a matching game in which each person would like to match up with a high quality person rather than a low quality person (for example, a mating game).

More precisely, we assume that there is a population of buyers each of whom can be one of two types, say "high" or "low". There is a positive complementarity associated with mate quality, so that a high type's utility loss of meeting a low type rather than a high type is greater than the corresponding loss of a low type. When a consumer wears the new dress other consumers will observe this and draw inferences about her type. Thus, if the price of the dress is high enough, it will allow the high types to at least partially separate from the low types. Those who wear the new dress in equilibrium are members of the "in" group, which contains a relatively high fraction of high types. Those who do not wear the dress have a lower proportion of high types. An "in" person will insist on dating another "in" person. The dress therefore acts as a screening device of potential partners.

The product sold is a *durable good* of which the consumer can use exactly one unit at a time[1]. The production cost is zero, so the producer's only cost is designing the product. As with standard durable goods monopolists, the designer lowers the price of the dress over time since he cannot commit to maintaining a high price. The dress is therefore sold to more and more low types and consequently the compositions of the "in" and the "out" groups change over time. Eventually the "in" group will be so large that it becomes worth while for the designer to design a new dress, which can be sold at the high price again. By innovating the designer introduces a new signaling device and hence destroys the value of the previous design.

The model predicts deterministic fashion cycles of fixed length. For large fixed costs, fashion cycles are long. To recover the fixed cost the designer has to sell the fashion at a high price which in turn requires that the design stays fashionable for a long period of time, and hence sufficient time must pass between innovations. If the fixed costs are small then the cycle is short. In the limit, for zero fixed costs, the designer will create a new fashion every period.

There are two possible cases: the "egalitarian" case, in which fashion spreads over the whole population before a new innovation occurs, and the "elitist" case, in which the latest design is sold only to the high types. In the latter case, once all the high types have acquired a design, the designer sells it at a zero price to the low types and at the same time introduces a new design. Thus low types will acquire only designs that are "out of fashion", that is, designs which are no longer used by high types as signaling devices. An elitist fashion cycle will be the equilibrium outcome if the benefit of meeting other high types is much larger for high types than for low types and if the period length between possible price changes is not too small. We can interpret the first parameter as a measure of the inequality, e.g., in the human capital endowment, between high and low types. The relevant length of "one period" depends on the institutional arrangement of the market under consideration. If the

[1]While most clothing products are not perfectly durable, fashionable clothes are usually replaced long before they are worn out. Clothes, as are cars, are indivisible goods of which at most one unit can be used at a time.

trading period is very small, then fashion spreads very quickly and in most periods the "in" group contains a large portion of the low types.

We extend the analysis also to the case where there is competition between designers. First, it is observed that even if potential competitors are free to enter the design market, *one* possible outcome is that one designer is designated as a fashion czar and behaves like a monopolist. If all consumers belief that only the fashion czar is capable of creating "fashion" then this will be the equilibrium outcome. We observe fashion czars in the form of trademarks. As was pointed out by Becker and Murphy (1993) a trademark is one method of "artificially" creating a monopoly over the production of a good.

Second, we show how true competition may actually help to keep prices above marginal cost and allow fashion cycles to be more stable, i.e., competition reduces the speed at which new innovations are introduced. If there are two or more competing designs sold then observing a lower price indicates to the high types that the corresponding design will "go out of fashion" (i.e., will not be used by high types anymore). Hence high types will purchase an alternative design. The fact that the high types have abandoned a design is devastating for its market value. The design has lost its value for all consumer, since the only reason for purchasing a design is the increased probability of meeting high types. Therefore attracting low types by lowering the price may be impossible.

Note that designers compete along an unusual dimension: The designer whose clients are more likely to be high types will also be more attractive to future buyers. This effect, which is not present with a design monopoly, will lead to prices above marginal cost and higher equilibrium prices than in the monopoly case.

We do not allow imitation of successful designs. Imitation would give designers an additional incentive to create periodically a new fashion. Clearly imitation is an important force behind the creation of new designs. However, through the creation of brand names, designers can at least partially insulate themselves from the competition with potential imitators. In this paper we consider the case where the designer has well defined property rights over his innovations. It turns out that even in this case fashion cycles will occur.

1.1 Relation to the Literature

In his analysis of static demand curves, Leibenstein (1950) distinguishes the "bandwagon" and the "snob" effect. The bandwagon effect describes the idea that demand for a commodity may increase if others are consuming it while the snob effect refers to the extent to which demand for a commodity is decreased because others are consuming it. Similar consumption externalities have been studied by Becker (1974), Frank (1985), Jones (1984), and Schelling (1978).

The demand for design commodities in the present model will display *both* the snob and the bandwagon effects even though there is no direct consumption externality. If more high types purchase the design, it will be more valuable to all consumers while if more low types purchase the design, it will be less valuable to all consumers. Since high types purchase the design commodity first, for low demands there will be a bandwagon effect, while for large demands there will be a snob effect.

Becker (1991) and Becker and Murphy (1993) derive bandwagon and snob effects by assuming that consumers care about who else consumes a particular good. Bagwell and Bernheim (1993) consider a model in which consumers signal their wealth by purchasing conspicuous commodities. In Bagwell and Bernheim's model consumers can signal both with the quantity and with the type of good they consume. If the single crossing property is satisfied then consumers signal their wealth by consuming large amounts rather than by choosing expensive brands. In contrast to Bagwell and Bernheim we assume that only one unit of a conspicuous good can be consumed at a time. This assumption is satisfied for many conspicuous consumption goods, like clothing, or cars. Our approach shares with Becker and Murphy and with Bagwell and Bernheim that a fashion is introduced by a price setting producer.

Bikhchandani, Hirshleifer and Welch (1992) and Banerjee (1992), (1993) show how in a model of sequential choice rational consumers can be lead to imitate the choice of consumers who move first. In this interpretation it is a particular buyer (or a group of buyers) who dictate a new fashion or social norm and changes in fashion are explained by exogenous shocks.

Matsuyama (1992) and Karni and Schmeidler (1990) examines the dynamics of a random matching game between conformists and non-conformists. The authors show that there are equilibria with cyclical demand variations which can be interpreted as fashion cycles. In a related paper, Conlisk (1980) considers the interaction between optimizers and imitators in a changing environment.

A central question that did not receive attention in the literature is why producers spend large amounts of resources on periodic changes in their design. The current paper addresses this question. Further, instead of assuming a consumption externality, we demonstrate how this externality can arise endogenously. Finally, since we consider a dynamic model of price setting firms, the model predicts price cycles for fashion commodities which can be compared to actual price movements of fashion commodities.

The present paper is also related to the literature on durable goods monopolies (see, for example, Coase (1972), Gul, Sonnenschein and Wilson (1986)). The price cycles predicted by the model are similar to the price cycles analyzed in Sobel (1984), Conlisk, Gerstner and Sobel (1984), and Sobel (1991). The reason for a sale in their model is the periodic desire of the monopolist to sell to the large mass of low valuing buyers that have accumulated on the market. In the present model sales occur because design commodities go out of fashion and buyers anticipate this.

1.2 Examples

Most major fashion-houses fit into the described pattern of design innovation and price cycles. The fashion house *Armani* has three different "lines" of fashion products: *Armani Via Borgo Nuovo*, *Armani*, and *Emporio Armani* [2]. The three lines differ in prices and in designs but not in the type of clothing they offer. New designs are introduced first in the top line (Armani Via Borgo Nuovo) at a very high price, and later passed on to the lower priced lines. Currently, e.g., the new jacket design will only be offered by *Armani Via Borgo Nuovo*, while *Emporio Armani* still offers the jackets that were fashionable in previous years. *Armani* is therefore an illustration of fashion cycles very similar to the ones predicted by the model. Similar patterns can be found for many other fashion houses.

An alternative interpretation of the described design innovations is improvements in

[2] This line is called *Mani* in the US.

product quality, for which there is little value from the point of view of consumption. For example, one might think of some technical innovations in consumer electronics or cars as "useless" technical innovations. Nevertheless, if a class of goods is used as a signaling device in the process of social interaction these seemingly "useless" innovations may be very valuable to its consumers. Thus the current model predicts "overinvestment" in product quality, in the sense that the cost of the quality improvements may exceed the gain in "consumption value" if consumption is removed from its social context.

2 The Model

We consider a society of many consumers, each of whom is either a high type (type h) or a low type (type l). Let $q \in [0,1]$ denote a generic consumer. If $q \leq \alpha$ then q is a high type and if $q > \alpha$ then q is a low type. Depending on the interpretation, the type of an individual may refer to her education, entertainment skills, or human capital.

The purpose of a consumer in this model is to "date" another consumer. We assume that there is a matching mechanism, to be specified below, that matches consumers into pairs[3]. The utility of a match to a consumer depends on the type of the partner she is assigned to and on her own type: $u(i,j)$ denotes the utility of a type i consumer matched with a type j consumer. Both types prefer to be matched with high types, i.e. $u(i,h) > u(i,l)$ for $i = h, l$. Moreover, high types value a match with high types more than low types. This idea is expressed in the following assumption.

$$\text{(Complementarity)} \qquad u(h,h) - u(h,l) > u(l,h) - u(l,l) \tag{1}$$

Let $v^h \equiv u(h,h) - u(h,l)$ and $v^l \equiv u(h,l) - u(l,l)$. Hence (Complementarity) is equivalent to $v^h > v^l$. [4].

[3]Alternatively we could assume that consumers are matched into groups of size N, $N \geq 2$. Groups of size 2 simplify the analysis, but the same results could be obtained for arbitrary N.

[4]An example of this preferences is a situation where the type refers to a human capital endowment. After a match is made, consumers engage in some form of production which involves complementarities. I.e., let x_h and x_l denote the high and low types human capital endowment and let $f(x^i, x^j)$ denote the payoff to i when she is matched with j. The assumption is that the two inputs are complements. (See also Becker (1981) for a related discussion.) As Kremer (1993) points out, (Complementarity) is also satisfied if the type

Consumers are endowed with money which they can spend on a "design commodity". Suppose a consumer type i is matched with a consumer type j and spends m units of money on a design. Her total utility is $u(i,j)-m$. Designs do not directly enter the utility function. This assumption captures the idea that design innovations do not provide any direct improvement in the quality of the product.

There is one type of firm, a designer, who can create a designs $n \in \{1,2,...\} = Z$. To create a new design, a fixed cost of $c > 0$ must be paid. Once design n has been innovated, the designer can produce indivisible units of it at 0 marginal cost. Consumers can purchase arbitrarily many units of a design but can only use one unit of it. The idea is that the design is attached to a commodity like a dress or a car of which at most one unit can be used at a time. All consumers observe which design an individual is using and therefore designs can be used as a signaling device in the matching process.

2.1 Matching

Consumers who use the same design will be randomly matched into pairs. A consumer who does not use any design will be described as using design "0". Consequently, the "design" $n = 0$ is available to every consumer at no cost.

This definition of the matching process is incomplete since it does not specify a match for a consumer who is the only one to use a given design. We assume that such a consumers is matched with consumers who uses no design. In addition, we assume that there is always a small (measure zero) group of consumers of type l who use no design.

Let $\mu_i(n), i = l, h$ denote the fraction of consumers of type i using the design n.

(i) If $\mu_l(n) + \mu_h(n) > 0$ then the probability of meeting a high type for a consumer who uses n is given by $\mu_h(n)/(\mu_h(n) + \mu_l(n))$.

(ii) If $\mu_l(0) + \mu_h(0) = 0$ then the probability of meeting a high type for a consumer who uses no design ($n = 0$) is zero.

(iii) If $\mu_l(n) + \mu_h(n) = 0$ then the probability of meeting a high type for a consumer who uses design n is equal to her probability of meeting a high type when she uses no

of a consumer refers to her probability of being infected by a sexually transmitted disease.

design ($n = 0$).

Condition (ii) also defines the outside option of a consumer who does not purchase any design while all other consumers purchase some design. This specification of the matching technology ensures that no matter which design a consumer uses, she will not stay without a match. The analysis below remains valid even if parts (ii) and (iii) of the above matching technology are replaced by alternative conditions. The conditions chosen simplify the notation and the analysis below.

2.2 The Demand For Fashion

Suppose a new design n is offered and all consumers have the same endowment of old designs. Suppose further that individuals only care about their current utility, i.e. they do not take into account future uses of the design. What is the demand for the new design?

(Complementarity) implies that if q is willing to purchase the design then all consumers $q' < q$ are willing to purchase the design. If all consumers in $[0, q)$ use the design, then the maximum q is willing to pay is given by q's utility gain from meeting a high type times the increase in the probability of meeting a high type by using the new design. (Note that since all consumers have the same endowments of old designs, every old design used in equilibrium will lead to the same payoff.) Hence we can define the inverse demand for the new design at q as the maximum price that consumer q is willing to pay for the design if all consumers $q' \in [0, q]$ are using the design.

First suppose that $0 \leq q \leq \alpha$. Thus q is a high type and if she purchases the design she is matched with a high type with probability 1. If she does not purchase the design then she finds herself in a pool of consumers with a $(\alpha - q)/(1 - q)$ fraction of high types. Hence the maximum consumer q is willing to pay for the design is $v^h \cdot \left(1 - \frac{\alpha - q}{1 - q}\right) = v^h \frac{1-\alpha}{1-q}$.

Suppose that $\alpha < q \leq 1$. Then consumer q is a low type and purchasing the design gives her an α / q chance of meeting a high type. She will be matched with a low type with probability one if she does not purchase the design. Therefore the maximum consumer q is willing to pay is $v^l \frac{\alpha}{q}$.

The two cases are summarized by the function f.

$$f(q) = \begin{cases} \frac{1-\alpha}{1-q} v^h, & \text{if } 0 \leq q \leq \alpha; \\ \frac{\alpha}{q} v^l, & \text{if } 1 \geq q > \alpha. \end{cases}$$

The function $f(q)$ can be interpreted as the one period inverse demand function for a new design. $f(q)$ gives the maximum amount consumer q is willing to pay for the benefits of the design in the current period, given that consumers $[0, q]$ are purchasing the design and consumers $(q, 1]$ are not purchasing the design.

Insert Figure 1 here

Note that for $q \leq \alpha$, the demand function has a positive slope. This is the case since the more high type consumers purchase a new design the smaller is the chance of meeting a high type without the new fashion. By purchasing the fashion, high types impose a negative externality on consumers that do not buy the new fashion and this externality is responsible for the positive slope in the first part of the demand function. This effect can be interpreted as a "bandwagon effect" (Leibenstein, 1950).

At $q = \alpha$ the inverse demand function reaches a maximum. At this point all high types purchase and use the fashion whereas all low types do not. Thus the consequence of not purchasing the design is to be matched with a low type with probability 1.

For $q > \alpha$, the demand function has the usual negative slope since the more low types purchase the fashion, the less advantageous it is to wear the new fashion. In this case every additional purchase imposes a negative externality on the group buying the new fashion since the probability of meeting a high type conditional on displaying the design decreases. Hence in this range of the demand function we observe a "Snob effect" (Leibenstein (1950))[5].

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[5] In the simple two type case analyzed here, the bandwagon and the snob effect are conveniently separated: for $q < \alpha$ there is no snob effect whereas for $q > \alpha$ there is no bandwagon effect. In the general case with a continuum of different types both effects are present simultaneously since an increase of the "in" group will always imply a decrease in the average quality of both the "in" and the "out" group. Suppose the utility of consumer q from meeting q' is given by a symmetric function $u(q, q')$. We can construct the inverse demand function $f(q)$ also for this general case. The slope of f in the general case depends on the functions $u(q, \cdot)$.

Even if all consumers purchase the fashion, the utility from buying the fashion is strictly positive, i.e. $f(1) > 0$.

3 Fashion Cycles

To develop a model of fashion cycles we embed the static analysis of the previous section in a dynamic game. For this section we assume that there is one designer who can create at most one design in any period. Since in the present framework all designs are equivalent and there are no direct preferences for any specific design we can assume without loss of generality that designs are created "in order", i.e. design n is created only if design $n-1$ has previously been innovated.

Every period is structured in the following way.

- First the designer decides whether to innovate and at what prices to offer his designs.
- Then, consumers decide which designs to buy or sell. We assume that the owners of old designs can sell them if they wish to. Thus, the designer acts as a market maker who meets the excess demand in every period.
- Finally, consumers decide which design to use and pairs are formed according to the matching technology described below. At the end of every period the pairs separate.

Implicit in the definition of the game is that histories of individuals are unobservable and hence the matching technology cannot condition on past designs used by a consumer.

We denote by n_t the number of designs that have been created up to period t, i.e., in period t designs $n \leq n_t$ have been innovated and the designs $n > n_t$ have not. Even though in period t at most t designs have been innovated, we define p_t^n for all n and set $p_t^n = \infty$ for $n > n_t$.

By y_t^n we denote a consumer's purchase (or sale) of design n in period t and by x_t^n we denote a consumers endowment of design n at the end of period t. Thus $x_t^n = x_{t-1}^n + y_t^n$.

If $u(q,\cdot)$ is *concave* for all q, then $f(q)$ will be increasing (the bandwagon effect dominates the snob effect), whereas if $u(q,\cdot)$ is *convex* for all q, then $f(q)$ will be decreasing. Thus, if $u(q,q')$ is linear for all q then $f(q)$ is a constant.

We also define $x_t = (x_t^1, x_t^2, ...)$ as the vector of endowments and $y_t = (y_t^1, y_t^2, ...)$ as the vector of purchases in period t.

Every period, each consumer will choose the design that gives her the highest chance of meeting a high type. Thus let $\mu_t(x_t)$ denote the probability of meeting a high type when the consumer owns x_t and she chooses the optimal design. Clearly, μ_t is not exogenous but the result of equilibrium strategies.

The payoff of a consumer of type $i = l, h$ in period t is then

$$r^i(x_t, y_t, p_t, \mu_t) = (1 - \delta)[\mu_t(x_t)u(i, h) + (1 - \mu_t(x_t))u(i, l)] - p_t y_t \quad (2)$$

We normalize the utility of each match by $(1 - \delta)$ where $(1 - \delta)$ should be thought of as measuring the length of one period. The overall payoff of the consumer is then given by the discounted sum of the period payoffs:

$$\sum_{t=1}^{\infty} \delta^{t-1} r^i(x_t, y_t, p_t, \mu_t) \quad (3)$$

The payoff of the designer the discounted sum of revenues from his designs minus the incurred fixed costs. If λ_t^n denotes the (excess) demand for design n in period t and $\lambda_t = (\lambda_t^1, \lambda_t^2, ...)$ then the payoff of the designer is

$$\sum_{t=1}^{\infty} \delta^{t-1}[p_t\lambda_t - c \cdot (n_t - n_{t-1})] \quad (4)$$

If a consumer is the only person to purchase a design then she will be matched with a random consumer from the pool of individuals who use no design. The design does not improve the quality of the consumer's match in this case and therefore has no value. Thus a design is only valuable to consumers if a coordination problem is solved. We assume that the designer can coordinate demand for his latest design. Part of the innovation cost c should be interpreted as expenses for marketing and advertising to achieves the coordination of consumers to the largest demand. We also assume that whenever the designer creates a new design he cannot simultaneously advertise old designs and hence the coordination of the demand for the old designs breaks down. Consequently, we restrict our attention to equilibria in which designs other than the latest innovation are sold at a zero price.

We consider subgame perfect equilibria that satisfy:

- The consumer purchase decision for the latest demand depends only on the current price and on the consumer's endowment in the current period[6] Therefore consumer demand for design n_t can be characterized by an acceptance function $P(\cdot)$ such that consumer q will purchase exactly one unit if and only if $p \leq P(q)$.

- For design n_t the realized demand in any period will be the maximal demand consistent with equilibrium and for any design $n < n_t$ the equilibrium price is zero.

We will call equilibria that satisfy these two properties *Weak Markov Coordination Equilibria*[7] *(WMC equilibria)*.

Let $M(\delta)$ be the payoff to the designer in a WMC equilibrium if the designer is committed to exactly one innovation and the discount factor is δ. In this case we can interpret the game as a standard durable goods monopoly with the demand function $f(q)$[8].

Theorem 1 *Under the maintained assumptions, there exists a WMC equilibrium. Moreover, in every WMC equilibrium the designer innovates an infinite number of times if $c < M(\delta)$.*

The Theorem says that there is a subgame perfect equilibrium to the dynamic game that satisfies the stationarity and coordination assumptions described above. Moreover, if costs of innovation are smaller than $M(\delta)$ then WMC equilibria capture a crucial feature of fashion cycles: design innovation never stops.

3.1 Properties of Fashion Cycles

In a WMC equilibrium the maximal demand will be realized for every price of the latest design. Therefore $P(\cdot)$ is a decreasing function[9].

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[6]This assumption corresponds to the familiar notion of Weak Markov equilibrium in the literature on bargaining with asymmetric information and the durable goods monopoly. (See Gul, Sonnenschein and Wilson (1986)).

[7]For a formal definition of WMC equilibria see the Appendix.

[8]Since we restrict to WMC equilibria we can apply Theorem 1 in Gul, Sonnenschein and Wilson (1986) to show existence and uniqueness of $M(\delta)$, even though $f(\cdot)$ is not monotone.

[9]The assumption of (Complementarity) implies that high types have a strict incentive to purchase the design if a low type consumer purchases the design. In addition, we can rearrange consumers of each type

The following equation defines the net present value of profits of the monopolist, $R(q)$, when all consumers $q' \leq q$ have purchased the current design n_t. Consumer demand is stationary and all previously innovated designs have an equilibrium prize of zero. Therefore payoffs in the subgame following any innovation are identical to the payoffs after the first innovation[10] and q summarizes the payoff relevant history.

$$R(q) = \max_{y \in [q,1]; \pi \in [0,1]} \{(1-\pi)[P(y)(y-q) + \delta R(y)] + \pi \cdot [R(0) - c]\} \tag{5}$$

π denotes the probability of innovation. The designer will innovate ($\pi = 1$) whenever the payoff at the beginning of the game ($R(0) - c$) exceeds the payoff from selling the design for one more period. When the designer innovates he makes the old design obsolete by selling it at a zero price. If the designer does not innovate then he continues to sell the previously created design and chooses an optimal volume of sales, $(y - q)$, for the current period.

The assumption that old designs are sold at a zero price greatly simplifies the analysis but clearly is not essential for the intuition of planned obsolescence in our model. If the designer could coordinate demand for old designs then innovation might not lead to full obsolescence of old designs. The qualitative conclusion, however, would remain unchanged: since the new design is always sold to high types, innovation would decrease the value of old designs[11].

Let $t(q)$ and $\pi(q)$ be the solution to the optimization problem in (5). Consider a point q where $t(q)$ and $\pi(q)$ are single valued[12]. Let $q' = t(q)$, then equation 6 has to hold:

$$P(q) \quad \geq \quad (1-\delta)f(q) + \delta(1-\pi(q))P(q')$$

so that first consumers (with respect ot their name) purchase first. We also assume that deviations by sets of consumers of measure zero do not affect the equilibrium, and hence we can assume (without loss of generality) that $P(\cdot)$ is a left continuous, decreasing function.

[10] Since all designs $n < n_t$ have a price of zero, consumers are indifferent between any design $n < n_t$ used in equilibrium by some positive fraction of consumers. Thus we can assume without loss of generality that all consumers who do not own n_t use design $n_t - 1$.

[11] If the new design is sold to both high and low types then the old design must have a zero price. This follows from the fact that in this case no high type is using the old design (by complementarity).

[12] Since $t(\cdot)$ is monotone, it is single valued except possibly at a countable set of q. Similarly $\pi(\cdot)$ will be unique except for possibly one point q at which the designer may be indifferent between innovating and not innovating.

$$P(q) = (1-\delta)f(q) + \delta(1-\pi(q))P(q') \tag{6}$$
$$\text{if } P(q) \text{ is strictly decreasing at } q$$

If $P(q)$ is strictly decreasing, i.e. q is the marginal consumer at $p = P(q)$, then Equation 6 says that the consumer pays the value of the use of the design in the current period plus its discounted expected resale value. If $P(q)$ is constant at q, i.e., q is not the marginal consumer at $p = P(q)$, then a strict inequality may hold since at the price $P(q)$ some consumers $q' > q$ will purchase and therefore the current utility of owning the design may be larger than $(1-\delta)f(q)$. We have to allow for this possibility since $f(q)$ is upward sloping for $q \leq \alpha$.

When $\pi(q), t(q)$ is multiple valued the monopolist should play a possibly mixed strategy such that if p_{-1} was the price charged in the previous period then the expected price and the innovation probability π satisfy

$$\begin{aligned} p_{-1} &\geq (1-\delta)f(q) + \delta(1-\pi)p \\ p_{-1} &\leq (1-\delta)f(q') + \delta(1-\pi)p,\ q' > q. \end{aligned} \tag{7}$$

Such a mixed strategy justifies the decision of q to purchase in the previous period and for all $q' > q$ not to purchase.

For every design there is a sequence of prices $(p_1, ..., p_T)$, where in the first period after innovation the monopolist charges the price p_1 and sells to the first q_1 consumers, in the second period after innovation the design is sold at a price p_2 to consumers in $(q_1, q_2]$ and so on. After the design has been sold for T periods a new innovation occurs. We call a fashion cycle "elitist", if the low type never purchase the currently fashionable design. If low types purchase the currently fashionable design in some periods, we call the fashion cycle "egalitarian".

Proposition 1 *Suppose $c < M(\delta)$. Consider any new design along a WMC equilibrium path.*

(i) The price of the design is strictly decreasing over time until a new innovation occurs. In the period prior to a new innovation either all high types and no low types have purchased the design (the elitist case), or all consumers have purchased the design (the egalitarian case).

(ii) A lower bound for the number of periods between consecutive innovations is given by $\log(1 - c/\alpha f(\alpha))/\log(\delta)$*. Furthermore, (for a fixed discount factor), if the fixed cost of innovation is sufficiently small, then a new design will be created every period.*

(iii) If $v^h - v^l$ *is sufficiently large, (for a fixed discount factor and a fixed* $\alpha > 0$*), then the fashion cycle will be elitist.*

(iv) The designer will never randomize between innovating and not innovating along the equilibrium path. Furthermore, except possibly in the initial period after innovation, there will be no randomization between prices along the equilibrium path.

Item (i) of Proposition 1 implies that whenever the fashion cylce is longer than 2 periods, high types will sometimes be matched with low types, and therefore inefficient matches will be made in equilibrium.

The second part of the proposition is concerned with the length of fashion cycles. If innovation costs are large, the price of the design must be high to make an innovation worth while and hence the design must stay fashionable for many periods. With small innovation costs the temptation of innovation is too large to sustain sales over more than one period[13]. In this case the model is equivalent to the repeated selling of a perishable good and the equilibrium outcome is simply the one shot monopoly outcome: the good is sold to all the high types at a price of $f(\alpha)(1 - \delta)$.

The third item of Proposition 1 suggests that if the distribution of wealth or human capital is very unequal (and hence the difference between v^h and v^l is very large) then we should expect the fashions of the upper stratum of society to be different from those of the lower. This relation between "inequality" and the pattern of fashion cycles may explain the fact that, e.g. in the clothing industry, fashion cycles tended to be elitist prior to the 20-th century while they appear to be egalitarian today.

[13] If part of the innovation cost is the cost of coordinating consumers by means of advertising, then it is misleading to interpret c as a technological parameter of the fashionable good. In this interpretation the "innovation cost" cannot be reduced to zero by choosing a commodity for which design changes pose no technological problem.

3.2 Two Examples of Fashion Cycles

In this section we present two simple numerical examples of fashion cycles.

Example 1. An elitist fashion cycle. In this example, the designer innovates every third period. The parameters for this example are: $\delta = .9; c = 5.71; \alpha = 1/2; v^h = 60, v^l = 10$. The price sequence is given by:

$$p_{3t+1} = 14.58, p_{3t+2} = 10.54, p_{3t+3} = 6$$

A new design is created in periods $3t+1, t = 0, 1, \ldots$. The measure of consumers buying the design is given by the following sequence:

$$q_{3t+1} = .298, q_{3t+2} = .4135, q_{3t+3} = .5$$

Hence the design is sold to 3/5 of the high types in the initial period after innovation and then spreads over the next two periods until after 3 periods all of the high types have purchased the design. Then the designer innovates and at the same time makes the old design freely available to all consumers and the fashion cycle starts again with a new innovation.

Figure 3 indicates the function $P(q)$ for this example.

(Insert Figure 3 here)

In the first and second period after innovation the design does not spread among all the high types. This is surprising since for $q \in [0, 1/2]$, $(1-\delta)f(q)$ is increasing in q. Hence by selling to more high types the designer could increase the value of the fashion in the current period. The reason he decides not to do so is that there is an opposing effect: selling to more consumers will imply that the design stays fashionable for fewer periods. Hence, in order to increase the sales volume, the designer has to decrease the price not because the buyers value the good less but rather because large sales imply a bigger incentive to innovate and hence destroy the value of the design.

In this example the low types never purchase the currently fashionable design. It is only when a design has gone "out of fashion" when the low types will purchase (at a zero price) the design.

Example 2. An egalitarian fashion cycle. In this example the fashionable designs also spreads among the low types. The parameters of the example are: $\delta = .9$; $c = 2.44$; $\alpha = 1/2$; $r^h = 40$, $r^l = 10$. Again there will be a 3 period fashion cycle:

$$p_{3t+1} = 7.6,\ p_{3t+2} = 4.9,\ p_{3t+1} = 1$$

A new design will be created every 3 periods and the fractions of consumers buying the current designs is given by the sequence:

$$q_{3t+1} = .3577,\ q_{3t+2} = 1/2,\ q_{3t+3} = 1$$

After 2 periods the design is sold to all the high types and in the third period of the fashion cycle the design is sold to the low types. After that a new design is created and a new cycle starts.

3.3 Universal Fashions

Some examples of fashionable products have the feature that after a short period of time almost everybody in the relevant group of consumers owns the fashionable item[14]. In our model this implies that fashion does not significantly alter the social interaction, i.e., the matches that are made when consumers use the designs are almost identical to the matches that would be made if there were no fashion at all. In the following we show that this phenomenon is reproduced by the model if we let the period size be very short[15].

Proposition 2 *Suppose $f(1) > c > 0$. For every $\epsilon > 0$, if the period length is sufficiently small (i.e., δ is sufficiently close to one) then*

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[14] Examples of this phenomenon are CDs of popular musicians like Michael Jackson among teenagers, the latest model of *Swatch* watches in among Italian youths, *Barbie* dolls, and other "cheap" fashions.

[15] As the period length becomes small also the utility per period shrinks. We can think of every period as consisting of a large number of short independent matches for every consumer. If the period size is small then the number of interactions that take place in one period is also small. Thus rather than the duration of each match going to zero we can think of the number of interactions as going to zero.

(i) the payoff of the designer is smaller than ϵ;

(ii) the fraction of consumers who own the currently fashionable design is larger than $1-\epsilon$ in all but an ϵ fraction of periods.

If periods are very short then the initial phase of a fashion cycle is similar to the predictions of the Coase conjecture for durable goods monopolies. Most consumers buy within a short period after innovation and most consumers pay essentially the same price for the new design. (This follows from the fact that the price of a design is equal to the value of its current use for the marginal consumer plus its discounted resale value). After this initial phase the sales volume of the design will be very small but positive for many periods. In particular, it will be just sufficient to keep the designer from creating a new design. Since the payoff of the designer is very small also the sales volume can be very small without giving the designer an incentive to innovate. The price for the design commodity decreases over time until it almost reaches zero. At this point the designer introduces a new design and a new fashion cycle starts. Why do consumers purchase fashion in this case? Anybody who does not use the fashion is matched with a low type with probability one. Hence even though fashion does not permit the separation of types, the fashionable item becomes a "must" since without it a consumer is identified as a low type.

Figure 4 describes a typical price cycle when the period length is very small. Note that τ indicates calendar time, rather than the number of periods. Innovations are assumed to occur at times τ_1 and τ_2.

(Insert Fig. 4 here)

The welfare implications of a short period length are very different from the standard durable goods monopoly. The logic of the dynamic design monopoly essentially precludes successful separation of types. In the next section we show how competition may succeed in restoring the ability of fashion to separate types.

4 Competition

In this section we discuss outcomes that arise if there are many potential designers. Suppose $j \in \{1, 2, ...\}$ denotes the designer and Z^j describe the collection of designs that j can innovate. We assume designers cannot imitate each others products, i.e., the sets Z^j are disjoint. The definition of the game is exactly like above with the exception that now many designers may sell their innovations simultaneously.

4.1 A Fashion Czar

Even in a situation of potential competition the analysis of the previous section, i.e. monopoly, may be the equilibrium outcome. Suppose some random process designates one particular designer as a "fashion-czar" and all consumers believe that only this designated designer is capable of creating "fashion". Such beliefs will be self-enforcing and no consumer will have an incentive to purchase designs from any other designer. Hence even though there are other designers capable of creating designs, only the fashion-czar can create fashion.

Suppose designer $\hat{j}$ is designated as a fashion czar. We assume that only designer $\hat{j}$ can coordinate the demand for fashion. In this case profitable entry is not possible irrespective of the price and innovation policy of $\hat{j}$. Therefore designer $\hat{j}$ is a monopolist and the game reduces to the one analyzed in the previous section. Hence even though competition is possible, the fact that fashion always involves the coordination of many consumers may prevent competition from actually taking place.

4.2 More than one successful designer

Here we describe equilibria in which true competition between designers occurs. In particular, we construct equilibria in which competition implies a *lower bound* on the price for designs. This lower bound will allow the high types to separate themselves from the low types. Competition will imply an increase in variety of the designs marketed. Hence two or more designers will simultaneously market designs that attract similar types of consumers. This increase in variety is without additional benefit to the consumers and hence from the point of view of efficiency, wasteful.

The following proposition describes a collection of subgame perfect equilibria in which only the high types purchase the latest design and innovation is accompanied by a sale of the previous design at a zero price. The interval between successive innovations is indeterminate. In particular, there is an equilibrium in which only one innovation occurs by every designer in the market.

Recall that $(1-\delta)v^l$ is the one-period benefit of a design to a low type if all high types and no other low types are purchasing the design.

Proposition 3 *Suppose $c < \alpha v^l/2$. Let (T, N) satisfy $(1-\delta^{T+1})v^l/N \geq c$, $N \geq 2$. The following price and innovation sequence constitute equilibrium play of a subgame perfect equilibrium.*

(i) N designers create a new design every T periods.

(ii) The price of any design t periods after its creation is given by $v^l(1-\delta^{T+1-t})$ if $t \leq T$; if $t > T$ then the price of the design is zero.

(iii) In the period of an innovation, every high type consumer purchases one of the new designs. Low types never purchase designs at a strictly positive price.

In this equilibrium, the designers innovate and sell to the high types in the period of innovation. Then, for $T-1$ periods no sales occur until in period T a new design is introduced and the old design is sold at a zero price to the low types. New designs are sold at a price of $v^l(1-\delta^{T+1})$ to all high types. Note that in this equilibrium high types always meet high types and low types are always matched with low types. Hence from the point of view of matching, an efficient allocation is achieved.

How can the designers in this equilibrium resist the temptation to sell to the low types after having sold a new design to the high types? Suppose τ periods prior to a new innovation designer j tries to attract low types. Suppose he lowers the price to $0 < p < v^l(1-\delta^\tau)$. At this price some low types have an incentive to purchase the design, if some high types keep using this design. In this case the fraction of low types purchasing the design will be such that every low type consumer is indifferent between purchasing and not purchasing.

But this implies that *all* the high types who use the design have an incentive to sell it and purchase an alternative design at a price $r^l(1 - \delta^t)$.[16] This in turn has the consequence that the design is without value for the low types since it can no longer guarantee a positive probability of meeting a high type.

After a reduction in the price below the equilibrium price high types respond by selling the design and purchasing an alternative design and low types do not purchase the design. Thus lowering the price will not generate any positive demand for the design and designers are unable to attract low types at a non-zero price.

The behavior of consumers after an attempt of a designer to attract "low" types is reminiscent of the behavior of clients of the Italian designer *Fiorucci*. *Fiorucci* started in the 70's as a designer with a young upper middle class clientele. Around 1980 the designer tried to attract a broader group of customers by selling his fashion in department stores and lowering the price. The consequences were disastrous. Essentially *Fiorucci* was abandoned by all customers and after a brief struggle went bankrupt. In the context of the present paper the abrupt change in pricing policy was interpreted by the "high" types that *Fiorucci* was going out of fashion and hence the high types switched to different designers. But this also implied that the design was of no value to the "low" types who *Fiorucci* tried to attract.

5 Conclusions

This paper provides a model of fashion industries based on the idea that fashion is used as a signaling device in social interactions. The model reproduces several stylized facts associated with fashion industries:

- A design is most desirable when it is new. Over time the price of any design declines.
- When a new fashion arrives, the old design becomes obsolete and sells at a very low price relative to its introductory price.
- Design innovations occur with deterministic regularity. The clothing industry, for example, "innovates" every year. Such design changes cannot be explained by the

[16] This follows from the fact that $r^h > r^l$ and that at the equilibrium price for any other design every low type is indifferent between purchasing and not purchasing.

(necessarily stochastic) arrival of new ideas which improve previous products. In our model design "innovations" are arbitrary changes to the look of a commodity. The new design does not improve the old in any dimension and therefore, innovations can and will occur with precise regularity.

In the paper we examine two different market conditions for the designer, monopoly and perfect competition. We show that competition can have surprising effects when it applies to markets of fashion goods. Competition may at the same time keep prices high and reduce the frequency with which new products are introduced on the market. This effect comes about since a decisive factor in the competition between designers is the average "quality" of their customers. A designer whose clients are mostly low types will be less attractive to prospective buyers than a designer whose clients are mostly high types. We show that if a designer tries to expand his clientele by catering to low types, the average quality of his clients may collapse since high types switch to competing designs and leave him with a design that nobody wants. Thus attracting low types by lowering the price may not be profitable for a designer facing competition.

A monopoly designer on the other hand, cannot be deterred from passing the design on to the low types once high types have purchased it. Since consumers anticipate this, prices may be lower than in the competitive case. If the designer cannot commit to minimum period length within which he does not change prices or innovate then the design will spread very quickly throughout the whole population, prices will be low, and fashion will not be a useful signaling device. In this case, consumers would be better off by banning the use of all fashion.

This may explain the frequent attempts to regulate apparel. Sumptuary legislation, for example, which existed throughout Europe during the Middle Ages and early modern times, regulated the apparel that members of the lower classes were allowed to wear. Both in England and France velvets and silks were forbidden for certain classes and limitations of expenditure for clothing according to rank, income or both were in place (Vincent (1934)). Sumptuary laws were explicitly passed *"to restrain extravagance, which was considered not only displeasing to God but economically ruinous to individuals"*(Vincent (1934)).

In the context of our model, sumptuary legislation can be interpreted as a way of avoiding wasteful fashion cycles. If the lawmaker can easily decide which social groups should interact, then sumptuary legislation will be efficient in the sense that the maximal gains from social interaction will be realized without waste of resources on design innovations[17]. Thus in societies with a well defined class structure one would expect sumptuary laws while in a society for which it is impossible for the lawmaker to identify the efficient matches, one would not expect these laws.[18]

The analysis of this paper focuses on the simple case, where consumers can only be one of two types. Moreover, the analyzed equilibrium has the designer market only one design at a time. In the example of *Armani* the designer markets at least three designs simultaneously and thereby provides more precise signaling devices than the designers in our model. Marketing several designs simultaneously will be important in a setting with a continuum of different types trying to interact. Such a framework would allow us to analyze the interaction between the distribution of types and the resulting fashion cycles.

A Appendix

A.1 Monopoly

A.1.1 Strategies and Equilibrium

To simplify the definition of histories and strategies we assume that all agents can observe each others actions. However, we will insist on strategies being anonymous, i.e. the deviations of a measure zero set of agents do not affect equilibrium outcomes. Note that the interaction between consumers is entirely determined by the matching technology which determines matches using the currently displayed designs of consumers. Therefore, information about individual consumer histories is irrelevant for all agents and we can interpret

[17]See also Becker and Murphy (1993) for a similar interpretation of sumptuary legislation.

[18]"In Massachusetts the enactment of sumptuary laws extended from about 1634 to 1676, and at that time in spite of repeated efforts at enforcement the courts were already beset with widespread disobedience. The laws did not embody such detailed regulation of dress as did those in Europe, but they expressed a similar desire to maintain distinctions between an upper and a lower class". (Vincent 1934)

the game as one in which only the designer's action and total sales can be observed.

A history in period t is a sequence of prices, a sequence of innovations, a sequence of purchases by consumers, and a sequence of display decisions by consumers. Let H^t denote the set of histories in period t. A pure strategy σ_t^d for the designer in period t is a map from histories to prices and innovations, $\sigma^d = (\sigma_t^d)_{t=1}^{\infty}$. The consumer's action in every period consists of a purchase/sales decision $y_t^n \in \{-1, 0, 1\}$ for all n and a display decision $z_t \in \{0, 1, ...\}$. A strategy in period t for consumer q, $\sigma_t(q)$, is a map from histories (including the actions taken by the designer in period t) to purchases and displays. To incorporate the feasibility restrictions into the payoff function we will assume that whenever the consumer chooses to sell more than x_t^n units of a design the payoff will correspond to the sale of x_t units, hence $x_t^n(q) = \max\{x_{t-1}^n(q) + y_{t-1}^n(q), 0\}$. Similarly, whenever a consumer chooses a design that she does not own then this is equivalent to choosing no design. As a function of q, $\sigma_t(q)$ is assumed to be measurable with respect to the Borel σ-algebra on $[0, 1]$. $\sigma(\cdot) = (\sigma_t(\cdot))_{t=1}^{\infty}$.

Note that $(\sigma(\cdot), \sigma^d)$ induces a sequence $(\mu_t(x))$ and hence we can define the payoff for consumer q as $V^q(\sigma^d, \sigma(\cdot), \sigma(q))$. Similarly $V^d(\sigma^d, \sigma(\cdot))$ is the payoff of the designer.

A *subgame perfect equilibrium* is a strategy pair $(\sigma^d, \sigma(q), q \in [0, 1])$ such that for all t and for any history h^t, $V^d(\sigma^d|_{h_t}, \sigma(\cdot)|_{h_t}) \geq V^d(\sigma'^d|_{h_t}, \sigma(\cdot)|_{h_t})$ for all σ'^d. Furthermore, $V^q(\sigma^d{}_{|h_t}, \sigma(\cdot)|_{h_t}, \sigma(q)|_{h_t}) \geq V^q(\sigma^d|_{h_t}, \sigma(\cdot)|_{h_t}, \sigma'(q)|_{h_t})$ for all $\sigma'(q)$. In order to ensure existence of an equilibrium we have to allow the monopolist to mix at any stage of the game. It should be clear to the reader how to extend the above definitions when mixed strategies are allowed.

In the following we show the existence of a subgame perfect equilibrium with the following properties:

(i) Deviations of sets of consumers of measure zero do not affect equilibrium play.

(ii) Demand for the design n_t can be characterized by an acceptance function $P(\cdot)$. $P(\cdot)$ is a non-increasing left continuous function that satisfies Equations 6 and 7.

(iii) Along the equilibrium path, for any $n < n_t$, $p_t^n = 0$.

Since old designs have a zero price the designer's behavior along the equilibrium path is a map from histories to prices for the latest design and innovations, i.e. $\sigma_t^d : H^{t-1} \to \{0, 1\} \times \mathbf{R}_+$. Since Equations 6 and 7 are satisfied, we can assume $y_t \in \{0, 1\}$ along the equilibrium path. Moreover, since (by the coordination assumption) design n_t is always guarantees a better match than any design that was previously innovated we can eliminate the consumer's "display decision" and simply assume that any consumer who owns the currently fashionable design will use it. Any old design used in equilibrium will give consumers the same payoff. Hence we can assume that all consumers who do not own the latest innovation use design $n_t - 1$.

A.1.2 Proofs:

Theorem 1 *There exists a subgame perfect equilibrium satisfying requirements (i)-(iii) above (WMC equilibrium). Moreover, if $c < M(\delta)$, then innovation occurs infinitely often in any WMC equilibrium.*

First we show that the system of equations given by (5), (6)and (7) has a solution.

Lemma 1 *There exist $R(\cdot|V)$ and $P(\cdot|V)$ of such that*

$$R(q|V) = \max_{y \in [q,1]; \pi \in [0,1]} \{(1 - \pi)[P(y|V)(y - q) + \delta R(y|V)] + \pi \cdot V\}$$

and $P(\cdot|V)$ satisfies 6 resp. 7.

Proof: Let q be the largest q such that $\max\{(1 - q)f(1), (\alpha - q)f(\alpha\} = (1 - \delta)V$. Then for $q \in (q, 1]$ we define $P_q'(q|V) = (1 - \delta)f(q)$ and $R_q(\cdot|V) = V$. Note that P_q' satisfies equations 6 and 7 on $(q, 1]$. Moreover, given V, for every $q > q$ it is optimal to innovate, i.e. to get V instead of $(y-q)P_q(y|V)+\delta V$. By a direct application of the argument of the proof of Lemma A.3 in Ausubel and Deneckere (1989) (which in turn builds on Fudenberg, Levine and Tirole (1985, Lemma 3) P_q', R_q can be extended to $[0, 1]$. Note that the $P'(\cdot)$ obtained by this procedure need not be monotonic. However, since the definition of R implies that the designer will never choose a q such that $P'(q)$ is increasing we can take $P(q|V)$ to be the smallest non-increasing, left continuous function such that $P(q|V) \geq P'(q|V)$. Therefore the existence of the pair $P(\cdot|V), R(\cdot|V)$ is established. $\square$

For a family of real-valued functions (J_n), $J_n : X \to Y$ we make the following definitions: Let $G(J_n) =$ Graph of J_n and

$J_n(x) = \text{ conv}\{y : y = \lim_{i\to\infty} J_n(x_i)$, for some convergent sequence $(x_i)\}$.

$B_\epsilon(J_n) = \{(x', y') \in X \times Y : \|(x', y') - (x, y)\| < \epsilon$ for some $(x, y) \in G(J_n)\}$

Then we define:

$$\rho(J_n, J) = \inf\{\epsilon > 0 : G(J) \subset B_\epsilon(J_n) \text{ and } G(J_n) \subset B_\epsilon(J)\}$$

Lemma 2 *Suppose $q < 1$, $R, R^n, P, P^n, V > 0, V^n > 0$ are given. Suppose further that for all $\delta > 0$, there is an n such that $\|R^n_q(\cdot|V^n) - R_q(\cdot|V)\|_\infty < \eta$ and $\rho(P^n_q, P_q) < \eta$, then for all $\epsilon > 0$ there is an N such that for $n \geq N$, $|R^n(0|V^n) - R(0|V)| < \epsilon$.*

Proof: In the following we suppress V as an argument of P and R. Let $q' \geq q - (1 - \delta)(R^n_q - 2\eta)/f(\alpha)$. Then for large n we have

$$R^n(q) = \max\{(1 - \pi^n)(y^n - q)P^n(y^n) + \delta R^n(y^n) + \pi^n V^n\}$$

$$R(q) = \max\{(1 - \pi)(y - q)P(y) + \delta R(y) + \pi V\}$$

Note that $V^n \to V$ since $\rho(P^n_q, P_q) \to 0$. This implies that there is a q^n, q such that for $q > \sup\{q, q^n\}$, innovation is optimal and for $q < \inf\{q, q^n\}$, innovation is not optimal. Since $q^n \to q$ we can find $(q_j)_{j=1}^J \in [q', q]$ such that $\pi(q_j) = \pi^n(q_j)$, for all j, $q_j - q_{j+1} < \eta/f(\alpha)$ and $t^n(q_j), t(q_j)$ are single valued for all $n \geq n'$. For small η we can apply a generalization of the Theorem of the Maximum due to Ausubel and Deneckere (1993) to obtain $|t(q_j) - t^n(q_j)| < \gamma$ if η is small. Further note that since $R^n > V^n > 0, R > V > 0$ we know that $|q_j - t(q_j)|$ and $|q_j - t^n(q_j)|$ are bounded away from zero. Therefore $\|R^n_{q'} - R_{q'}\|_\infty < \eta$ implies that $|P^n(t^n(q_j)) - P(t(q_j))| \leq \eta \cdot k$ for some $k < \infty$ and all j. Since P^n, P are decreasing functions this implies that for all $\hat{q}$ with $q > \hat{q} \geq q' + \eta/f(\alpha)$ we have $\rho(P^n_{\hat{q}}, P_{\hat{q}}) < \eta \cdot k'$ for some $k' < \infty$. Furthermore $\|R^n_{\hat{q}} - R_{\hat{q}}\| < \eta$. Since $q - \hat{q} > \beta > 0$ independent of q, the Lemma follows by finite induction.□

Lemma 3 *$R(0|V)$ is continuous in V, for all $V > 0$.*

Proof: Let $V^n \to V > 0$. Further let q^n, q be such that $(1 - \delta)f(1)(1 - q^n) = V^n$, $(1 - \delta)f(1)(1 - q) = V$. Then $q^n \to q$. Then for $q = q - \epsilon$ we have for $n \geq N$ $\|R^n_q(\cdot) - R_q(\cdot)\|_\infty < \delta$ and $\rho(P^n_q, P_q) = 0$. Now we can apply Lemma 2 and the Lemma follows.□

Lemma 4 *Suppose $c < M(\delta)$. Then if $V = \alpha f(\alpha)/(1-\delta)$ then $R(0|V) - c \leq V$. Furthermore, if $V_k \to 0$ then for large k, $R(0|V_k) - c > V_k$.*

Proof: The first part of the lemma follows from the fact that $(1-\delta)f(\alpha)\cdot\alpha$ is the highest one period payoff achievable. Hence for $(1-\delta)V = (1-\delta)\alpha f(\alpha)$, the monopolist will innovate every period and $R(0|V) - c = \alpha f(\alpha)/(1-\delta) - c \leq V$.

For the second part fix any $1 > q > \alpha$. Then we claim that $P^k(q) \equiv P(q|V_k) \geq f(1)$ for small V_k. Suppose to the contrary that $P(q|V_k) \leq f(1) - \epsilon$ for all V_k.

Let $q_0 = 1$, $P_0 = f(1)(1-\delta)$. Let q_1 be defined by $(1-q_1)P_0 = (1-\delta)V^0$. Let P_1 be given by $P_1 = (1-\delta)f(q_1) + \delta P_0$. $V^0 = V_k$ and $V^1 = (q_2 - q_1)P_1 + \delta V_k$. Let q_2 satisfy the following equation:

$$(q_1 - q_2)P_1 + \delta V_k = (1-q_1)P_0 + \delta V_k \tag{8}$$

First note that for $q < q_2$ the left hand side of (8) is strictly bigger than the right hand side. Thus for $q \leq q_2$, $P(q|V_k) \geq P_1$. This follows from the fact that $(q' - q)f(q)$ is increasing for $q > q' > \alpha$ and therefore the right hand side is an upper bound on the profit of the designer if he sells to more than q_1 households in the following period. Furthermore, $q_1 \to 1$ as $V_k \to 0$. Now suppose that, $q_{t-2}, q_{t-1}, q_t, P_{t-1}, V_{t-1}$ are determined and $P(q) \geq P_{t-1}$ if $q \leq q_t$, furthermore suppose $q_t \to 1$ as $k \to \infty$. Let P_t, V_t, q_{t+1} be given by:

$$P_t = (1-\delta)f(q_t) + \delta P_{t-1}; V^t = (q_t - q_{t-1})P_t + \delta V^{t-1} \tag{9}$$

$$(q_{t+1} - q_t)P_t + \delta V^t = (q_{t+1} - q_{t-1})P_{t-1} + \delta V^{t-1} \tag{10}$$

we have $P(q) \geq P_t$ for $q \leq q_{t+1}$. Again this is the case since for $q < q_{t+1}$ the lhs of Equation 10 is strictly bigger than the rhs and the rhs is the upper bound to the payoffs of the monopolist if he decides to sell to more than q_t households. Moreover $P_t > (1-\delta)f(1)$, and since $V^{t-1} \to 0$, $V^{t-2} \to 0$ we have $q_{t+1} \to 1$ as $k \to 1$ and $V^t \to 0$. Hence for any $q < 1$, $P(q|V_k)$ cannot stay bounded away from $f(1)$ for infinitely many k. Hence for any arbitrarily small q, ϵ, $P(q'|V_k) \geq f(1) - \epsilon$ for large k and $q' \leq q$. (In the following we will write P^k, R^k, t^k instead of $P(\cdot|V_k), R(\cdot|V_k), t(\cdot|V_k)$.

Now consider the solution to the one innovation durable goods monopoly problem that leads to a payoff of $M(\delta)$ and denote with $\hat{R}(\cdot)$, $\hat{P}(\cdot)$, and $\hat{t}(\cdot)$ the corresponding value function, acceptance function, and policy function. Let $\tilde{q}$ be such that for all $q' \geq \tilde{q}$

$$f(q)(q - q') + \delta f(1)(1 - q) < f(1)(1 - q'), \forall q \in [q', 1] \tag{11}$$

Note that since f' is bounded there exists such a $\tilde{q}$. Further note that $\hat{t}(\tilde{q}) = 1$.

Now we claim that for every $\gamma > 0$ there is a K such that for $k > K$, $t^k(\tilde{q}) \geq 1 - \gamma$. To see this chose ϵ, η so that $(f(1) - \epsilon)(1 - \tilde{q} - \eta) > f(q)(q - \tilde{q}) + \delta f(1)(1 - q)$ for all $q \leq 1 - \gamma$. Since the lhs of 11 reaches a maximum at $q = 1$ such ϵ, η exist. Note that for large k, $P^k(1 - \eta) > f(1) - \epsilon$, and $P^k(t^k(1 - \eta)) > f(1) - \epsilon$. The rhs of 11 is an upper bound to the profit of the designer if he sells to fewer than $1 - \gamma$ consumers, while the lhs is the payoff of selling to $1 - \eta$ consumers for large k, and thus it follows that $t^k(\tilde{q}) > 1 - \gamma$ for sufficiently large k. Since $t^k(\cdot), \hat{t}(\cdot)$ are monotonic we can conclude that for $\tilde{q} \leq q \leq 1 - \gamma$

$$(1 - \delta)f(q) + \delta f(1 - \gamma) \geq P^k(q) \geq (1 - \delta)f(q) + \delta(f(1) - \epsilon)$$

and

$$\hat{P}(q) = (1 - \delta)f(q) + \delta f(1)$$

Hence for large k, $\rho(P^k_{\tilde{q}}, \hat{P}_{\tilde{q}}) < \eta$ and $\|R^k_{\tilde{q}} - \hat{R}_{\tilde{q}}\|_\infty < \eta$ and Lemma 2 can be applied to yield that for large k, $R^k(0|V_k) \geq \hat{R}(0) - \epsilon > c$. Since $V_k \to 0$ the Lemma follows. □

Proof of the Theorem 1: For $c \geq M(\delta)$ let the equilibrium strategy prescribe that no innovation occurs. If the designer deviates and innovates, no further innovation will occur and the designer receives the payoff $M(\delta)$ by following the equilibrium strategy corresponding to a standard durable goods monopoly with demand function $f(q)$ and the assumption that the monopolist can coordinate demand. Clearly the designer has no incentive to innovate.

If $c < M(\delta)$ then by Lemma 3 and Lemma 4 there exists a V^* such that $R(0|V^*) - c = V^*$. Thus 5 and 6, resp. 7 are satisfied for $R(\cdot) = R(\cdot|V^*)$, $P(\cdot) = P(\cdot|V^*)$. Moreover $V^* > 0$ by Lemma 4. Thus $P(\cdot|V)$, $R(\cdot)$ support a WMC equilibrium if all designs $n < n_t$ are sold at a price of 0.

It remains to show that in equilibrium designs $n < n_t$ can be assigned a price of zero. For all designs $n < n_t$ assume the following strategies: If $p^n_t = 0$ for all $n < n_t$ then all consumers who did not purchase design n_t purchase and use design $n_t - 1$. If $p^n_t > 0$ for some n the all consumers who own design n sell it to the designer and use design 0. All consumers who did not purchase the design n_t use design 0. Thus it is optimal for the designer to set $p_t^n = 0$ for all $n < n_t$. Consumer behavior is clearly optimal.

Since $V^* > 0$ a new design must be created at least every T periods for some $T < \infty$, since otherwise the discounted profits for the monopolist would be smaller than V^* for some t. Thus the second part of the Theorem follows. $\square$

Proof of Proposition 1:

*(i)*Since payoffs in every period are positive there have to be positive sales in every period. Further, if $p_t = p_{t+1}$ then optimality of the designer's strategy is violated, hence prices have to be strictly decreasing along the equilibrium path. For the second part note that since innovation occurs in the following period we have $P(q_T) = (1-\delta)f(q_T)$. Since $(q-q_{T-1})f(q)$ attains a maximum at α (for small q_{T-1}) or at 1 (for large q_{T-1}) the proposition follows.

(ii) By Equations 6 and 7, $(1-\delta^T)\alpha f(\alpha)$ is the maximal revenue from a design. This implies that $(1-\delta^T)\alpha f(\alpha) \geq c$, where T is the number of periods between two successive innovations, since otherwise it is otimal not to innovate. For $c = 0$, we have that $R(0) \geq \alpha f(\alpha)$. Let q be such that for $q > q$ the designer creates a new design and for $q \leq q$ the designer continues to sell the old design. Then we must have that $\max_q (q-q)(1-\delta)f(q) \geq (1-\delta)R(0)$. This implies that $q = 0$. Clearly, for c close to zero q is close to zero. But for small q and for $y \leq q$, $P(y) \cdot y + \delta R(y) \leq R(y)$ since the $R(y)$ is bounded below by $\alpha f(\alpha) - c/(1-\delta)$. Hence it is optimal for the designer to sell to all high types instead of restricting the sale to $[0, q]$.

(iii) Note that $R(0) - c \geq \alpha \cdot v^h(1-\delta) - c/(1-\delta) = \alpha \cdot f(\alpha) - c/(1-\delta)$ is a lower bound to the payoff of the designer. Since v^l is an upper bound of the per period gains to selling to low types we can choose a v^h sufficiently large, so that $(1-\delta)R(0) - c > v^l$ which implies that it can never be optimal to sell to low types.

(iv) First suppose that $0 < \pi(q) < 1$ along the equilibrium path. Then by choosing any $q' < q$ the designer will no longer be indifferent between innovation and not innovation, i.e. $\pi(q') = 0$ and therefore $P(q') > P(q) + c$ for all $q' < q$. Thus q can not be optimal. The fact that there will be no randomization among prices except in the initial period is a property of the equilibrium path of a durable goods monopoly (see Ausubel and Deneckere, (1989)).

□

Proof of Proposition 2: Since $\epsilon > 0$ we have for some $\gamma > 0$ and for $T(\delta)$ the number of periods between successive innovations $1 - \delta^{T(\delta)} \geq \gamma > 0$.

Claim 1. Let V^d denote the payoff to the designer when starting the game. As $\delta \to 0$, $V^d \to 0$. To see this note that $(1-\delta)V^d \leq \max\{(1-q_1)(1-\delta)f(1), (\alpha - q_1)(1-\delta)f(\alpha)\}$. Further optimality requires that along the equilibrium path

$$P_t(q_t - q_{t-1}) + \delta V_t^d \geq P_{t-1}(q_{t-1} - q_{t+1}) + \delta V_{t-1}^d \tag{12}$$

where P_t and V_t^d denotes the price, resp. the expected payoff, t-periods prior to a new innovation. But this implies that

$$(P_t - P_{t-1})(q_t - q_{t+1}) \geq (1-\delta)V_t^d \tag{13}$$

or

$$q_t - q_{t-1} \geq \frac{(1-\delta)V_t^d}{(1-\delta)f(q_t) - (1-\delta)P_{t-1}} \tag{14}$$

Note that $V^d \leq V_1^d$ and V_t^d is increasing in t, and therefore if V^d stays bounded away from zero as $\delta \to 1$ then there is a bounded number of periods in which the inovation is sold. This contradicts the observation that $1 - \delta^{T(\delta)} \geq \gamma > 0$. Hence $V^d \to 0$ and hence $(q_1 \to 1)$.

Claim 2. For all $\epsilon > 0$ there is a $\bar{\delta}$ such that for $\delta > \bar{\delta}$ and for $t > \tau(\delta)$, $q_t > 1 - \epsilon$ where $\tau(\delta)$ satisfies $\delta^{\tau(\delta)} = 1 - \epsilon$.

Suppose the contrary. Then there is a sequence $\delta_k \to 1$ such that if t_k is the integer part of $\tau(\delta_k) + 1$ then $q_{t_k} \leq 1 - \epsilon$ for all k. First we show that this implies that for all k, $R^k(q_k) \geq \epsilon'$, for some $\epsilon' > 0$. Note that along the equilibrium path $P_t \geq f(1)(1 - \delta^{\tau}) - \epsilon$, where τ are the remaining periods for which the design is purchased. Let $T(\delta)$ denote the number of periods for which the design is sold. Then for $R^k(q_k) \to 0$ we must have that

$$\sum_{t=\tau(\delta)}^{T(\delta)/2} q_t^k \to 0 \text{ and } \sum_{t=T(\delta)/2}^{T(\delta)} q_t^k \geq \epsilon \tag{15}$$

Hence along the equilibrium path all of the remaining sales from period $\tau(\delta)$ on must be crowded in the second half of the remaining "life-time" of the design. Now observe that for $q_{t+1}, q_t, q_{t-1} \neq \alpha$ Equations 12 and 14 must hold with equality. If strict inequality were to hold then by increasing q_{t+1} the designer could increase his profit. This is the case since

$\frac{\partial}{\partial q}[(q-q')f(q)] > 0$ for $q > q' > 0, q \neq \alpha$. But if inequality 14 holds with equality then $q_t - q_{t+1} \geq k(q_{t'} - q_{t'+1})$ for all $t' < t$ and for some constant $k > 0$. But this contradicts 15 if ϵ/γ is sufficiently small. Hence $R^k(q_t^k) > \epsilon' > 0$.

To obtain a contradiction to $q_t^k \leq 1 - \epsilon$ for all k suppose that after innovation the designer charges the sequence of prices $\{\hat{p}_k\}$, with $p_k = f(\alpha)(m-k)/m, k = 0, 1, \ldots, m-1$ with $1/mf(\alpha) \leq \epsilon/4 \cdot \epsilon'$. Since $\{\hat{p}_t\}$ comes within $\epsilon/4 \cdot \epsilon'$ of the available surplus, selling after τ with $\delta^\tau > 1 - \epsilon$ to more than ϵ consumers cannot be optimal since more than ϵ' of the available surplus remains. Thus Claim 2 follows. □

A.2 Competition

Histories and strategies are like in the monopoly case with the exception that now in every period all designers choose innovations and prices simultaneously. It should be clear to the reader how to extend the above definitions of strategies and of subgame perfect equilibrium to this case. Let n_t^i denote the latest design innovated by designer i, i.e., all designs $n^i \in Z^i$ such that $n^i \leq n_t^i$ have been innovated in period t.

Proof of Proposition 3: The following strategies constitute a subgame perfect equilibrium of the game:

- N designers create a design every T periods, i.e. in periods $kT + 1, k = 0, 1, 2, \ldots$. For $t = kT + t', t < T$ $p_t^{n_t^i} = v^l(1 - \delta^{T+1-t'})$ and $p_t^{n^i} = 0$, $n^i < n_t^i$

- Every T periods high types purchase a new design. Every designer sells to a α/N fraction of high types. Purchases occur only in periods $1, T+1, 2T+1$, etc.

- If designer i deviates in period $t = kT + t', t' < T$ to a price $p_t^{n_t^i} < v^l(1 - \delta^{T+1-t'})$, then all high types who own n_t^i purchase design n_t^j offered by some $j \neq i$ and sell the design n_t^i.

- If designer i creates a new design in any period $t \neq kT + 1$ for some $k = 0, 1, \ldots$ then no purchases occur until period $t + \tau$, where τ is the smallest number such that $t + \tau = kT + 1$ for some k.

- The strategy of all designers is constant independent of the subgame they are in. In particular, after a price deviation of one designer, *all* designers return to the strategy played along the equilibrium path.

It is easily checked that the strategy outlined above constitutes a subgame perfect equilibrium. □

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