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"Unobserved Delegation"

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UNOBSERVED DELEGATION

by

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and
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ABSTRACT

The paper describes situations where commitment via delegation is beneficial, even when the delegation is unobservable and the players have the option to play the game themselves. The potential for such benefits depends on the type of delegation, incentive versus instructive, the possibility of repetition, and the probability of observability.

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Unobserved Delegation.

Introduction.

Delegating the authority to play a game on behalf of the original players is a common practice in strategic situations. The potential benefits of using delegates as a commitment device has already been emphasized by Schelling (1956,1960) and has since been extensively discussed in the literature.[1] While the advantage players may have from commitment through delegation is rather intuitive what is surprising is the magnitude of such benefits. The possibility of delegates to condition their strategy on the observed contracts of other delegates actually leads to a full folk theorem (see Fershtman, Kalai and Judd (1991)).

In many situations, however, the delegation contract, or even its existence, is not observable. Players may hire agents to play the game on their behalf and yet the contract they sign may not be public information. In a recent paper Katz (1991) analyzed the strategic delegation problem with unobservable contracts. The intuitive problem pointed out by Katz, is that if the delegation contract is unobservable, players would endow their own preferences to their delegates, making the delegation non effective.

The main concern of this paper is to examine situations under which the possibility to commit through unobserved delegation does affect the outcome of the game. Our research points out that the answer to the above question critically depends on: (i) The type of the delegation, (ii) whether the delegation is completely unobservable or there is a small probability that it will be observed, (iii) the game being one shot or a repeated and (iv) on the equilibrium concept

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[1] See for example Brander and Lewis (1986), Brander and Spencer (1985), Bolton and Scharfstein (1990), Fershtman and Judd (1987), Gal-Or (1992), Green (1990), Katz (1991), Sklivas (1987) and Vickers (1985).

used in the analysis.

(i) We distinguish two types of delegation. The first is **incentive delegation** where a delegate is provided with an incentive scheme and then chooses a strategy that maximizes his own payoff. The second type is **instructive delegation** where a delegate receives a program or a set of instructions on how to act and then he just carries out the program. An example of incentive delegation is a manager of a firm who is provided with an incentive scheme that links his compensation to the performance of the firm. An example of instructive delegation is a sales clerk in a store who is provided with prices at which to sell the goods.[2] The two types of delegations, although closely related, are shown to perform differently as a commitment device. The paper demonstrates that when unobserved, incentive delegation is more effective than instructive delegation. We find that the set of perfect equilibria of a one shot game drastically changes when the players are allowed to use unobservable incentive delegation. With instructive delegation, following the intuition suggested by Katz, the set of perfect equilibria is unaffected.

(ii) While most of the paper considers delegation which is unobservable with certainty, an important class of games are ones in which the delegation is likely to be unobserved. We consider a delegation game in which there is a small probability that the delegation contract will be observed. The paper demonstrates that such a probability, no matter how small, affects dramatically the possibility of commitment through delegation[3].

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[2] Within the sales area both types of delegation exist. This is the distinction between a sales person in a car dealership or in a stereo store who has the power to make a deal and whose compensation depends on sales, and a sales clerk in a store who works for a fixed hourly wage and just carries out the instructions that he is given.

[3] This seems to be in opposition to a recent result by Bagwell (1993). In his model a player is informed of the action of his opponent but with a small probability of error. This small probability turns out to make the information useless and the outcome of the game is the same as the outcome of

(iii) As suggested above, in the one shot game we study a player cannot benefit from unobserved instructive delegation. This is no longer the case, however, when the game is repeated, a situation common in real life. The commitment through unobserved instructive delegation can be beneficial to a player even if the game repeats itself only twice. We study first a twice repeated game and a delegate version of the same game. In the delegate version one of the players commits initially to unobserved **stationary** instructions of how he wishes the two stage game to be played on his behalf. The other player is not informed of the choice made. We show that the set of perfect equilibria of the delegate version of the game is very different from the original game. In our game the delegating player's payoff is significantly improved. Next we study a more sophisticated version of the delegate game. In this version, one of the players can initially choose to either leave stationary instructions of how to play the game on his behalf or he can play the two stage game himself, i.e. self representation. In either case his opponent is not informed whether he is facing unknown instructions or the actual player. It turns out, that in this game too the player with ability to delegate will improve significantly his (perfect) equilibrium payoffs when compared to the original game without delegation.

Commitment through delegation may also be limited by the possibility of renegotiating the delegation contract (see for example Dewatripont (1988), and Green (1990)). The paper concludes with a short discussion of the stationarity and renegotiation aspects.

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the game without information. We have a small probability that the player be informed of the action an opponent, however when he is informed, the information is accurate, and he knows that it is accurate. In our model the small probability of accurate information does affect the outcome of the game drastically. Bonanno (1992) studies a deterrence game with a similar phenomenon.

1. The Setting.

Consider the following two person game denoted by G_1. There is one seller and one buyer. The seller owns one unit of a good and the buyer wishes to buy exactly one unit. The value of the good for the seller is 0 and for the buyer is 100. We assume that it is a one shot game and it is the seller who offers a price p which the buyer either accepts or rejects. If he accepts then there is a trade, the seller's payoff is p, and the buyer's is 100-p. If the offer is rejected then there is no trade and both players' payoff is zero.

We further assume that the smallest distinguishable unit of money is 1 (for example 1 cent) and we restrict the set of permissible prices to be p=1,2,...,100.

Let G_2 be the 2 time repetition of G_1. The buyer wishes to buy a new unit of the good in each period, and the seller has exactly one unit to offer in each period. Formally, at every period the seller offers a price and the buyer responds by accepting or rejecting. We denote the seller's price offer at period t by $p_t \in \{1,...,100\}$ while the buyer's (or his agent) response at period t is denoted by $R_t \in \{y,n\}$, t=1,2. The strategies in the repeated game are such that at every period t, the seller's offer p_t and the buyer's response R_t are functions of the history of the game until period t. The utility of the two players in the game G_2 is the sum of their per period payoffs.

All the subgame perfect equilibria of the game G_1 (or G_2) have the seller offers the prices 100 or 99 and the buyer accepts. That is, the seller, through his ability to commit first receive essentially all the surplus. This property simplifies the discussion on the importance of delegation. Since any surplus received by the buyer in the games with buyer-delegation will indicate beneficial delegation.

Consider now the game G_1 but when the buyer can use a delegate that will purchase on his behalf and further assume that the delegation contract is observable to the seller before he determines the asking price. In such a game it is obvious that any price can be supported as a Nash equilibrium. But there is, however, a unique subgame perfect equilibrium. Since the agent's contract is observable, the seller can find out what is the highest price that he can offer and still have the good purchased. At equilibrium, the seller will not ask for a price that he knows for sure will be rejected. Thus the only subgame perfect equilibrium is for the buyer to commit to the price 1. The commitment can be achieved by either signing a contract with the agent that yields positive compensation only at a price 1, or giving the agents instructions to accept only the price 1. The equilibrium payoffs of such a game is that the seller receives 1 and the buyer with his agent receives 99.

The above conclusion relies very much on the observability of the delegation contract. The rest of the paper studies the same question, but when the delegation contract or even its existence may be not observable.

2. Unobserved Delegation in the One Shot Game.

We consider two variations of the delegation game associated with the above seller-buyer game:

The Incentive-Delegation Game: This game, denoted by D_1, is a 3-player game with the seller and the buyer as above but with a third player who we refer to as the agent or the delegate of the buyer. At the first stage of the game the buyer provides the agent with a contract that determines the agent's compensation as a function of the outcome of the game. The existence of the delegation contract is known to the seller, but the specifics are unobservable to him. In the second stage the agent plays the game G_1 on behalf of the buyer. The final payoffs of this game are as before but with the following correction: the agent receives the payoff according to the contract he has, and this compensation is subtracted from the buyer's payoff. We will be dealing with perfect equilibrium and mixed strategies, and for this reason it is mathematically convenient to restrict the set of possible delegate contracts to be finite. We do so by restricting the set of all possible compensation levels to be finite. Formally, we assume that there is a finite set F and that a <u>contract</u> is a function $f:\{1,...,100\} \rightarrow F$. We will further assume that F contains 0 (no compensation) and some positive values, with the smallest one being δ.

We assume that buying the good requires some effort by the agent. The disutility of such an effort is e. So when the delegate buys the good at a price p, under the contract f his net payoff is f(p)-e. The buyer's net payoff is 100-p-f(p) and the seller's payoff is p. If the delegate does not buy the good, everybody's payoff is zero. We assume that $e<\delta$ and thus buying the good at a price p with compensation $f(p)\leq 0$ is not profitable for the delegate (i,e. $f(p)-e<0$) while buying at a price p with $f(p)\geq\delta$ is profitable. The role of this assumption is to serve as a natural tie breaking rule to avoid dealing with agent indifference of buying at zero compensation.

<u>The Instructive-Delegation Game</u>: We define the associated Instructive-Delegation game, M_1, as a 2-player game with the seller and the buyer as above but with a preliminary step in which the buyer gives instructions that a machine - like delegate follows in playing the game G_1 on behalf of its principle. The instructions specify an acceptance set, denoted by A, such that for all $p \in A$ the response is R_1=yes and for every $p \notin A$ the response is R_1=no. Since there are 100 possible prices, there are 2^{100} different sets of instructions. The seller, who is aware that the

game was delegated, does not observe the chosen A.

We will say that the delegation game is characterized by strategic delegation if its equilibrium outcomes are different from the game without delegation.

Proposition 1: (i) (Incentive-Delegation with Unobservable Contracts): The (trembling hand) perfect equilibrium of the incentive-delegation game with unobservable contracts, D_1, is characterized by strategic delegation. Specifically, while sequential rationality in the game G_1 implies that the only equilibria is for the seller to ask for 100 or 99 and for the buyer accept, in the game D_1 with unobservable contract the price P=1 can be supported as a perfect equilibrium payoffs.

(ii) (Instructive-Delegation with unobservable commitment): At all perfect equilibria of the instructive-delegation game with unobservable instructions, M_1, the delegate's instructions are to accept any price not greater than 99 and the seller asks for the price 99 or 100. Thus there is no strategic delegation and the buyer cannot benefit from commitment via delegation.

Proof: (i) We will show that the price 1 can be the outcome of a (trembling hand) perfect equilibrium. Consider the following strategies: For the buyer to submit a contract f( ) such that $f(1)=\delta$ and $f(x)=0$ for any $x>1$. For the seller to ask p=1 and for the delegate to say yes for any compensation function g( ) and a price p with $g(p)\geq\delta$ and to say no otherwise. It is easy to see that the above are Nash equilibrium strategies. To show perfection of the above equilibrium we need to find arbitrarily close interior strategies (assigning positive probability to every feasible action at every information set) to which the above pure strategies are best response. Let ε be a positive small number and consider the following trembles. To simplify notation we do not normalize the probabilities of mixed strategies to add up to one.

The buyer assigns probability 1 to the equilibrium contract f(·) and probability ε to any other contract. The seller demands the price 1 with probability 1 and any other price, p=2,...,100 with probability ε^3. The delegate , at the information set described by the contract f( ) and the price 1 follows the equilibrium strategy of saying yes, with probability 1 and deviates from it, says no, with probability ε^2. At every other information set (g,p) he follows the equilibrium strategy with probability 1 and deviates with probability ε.

It is easy to see that the delegate's equilibrium strategy is best response to the strategies (with the trembles) at every information set. Similarly, for sufficiently small ε, submitting the price p=1 is the seller's (unique) best response.

For the buyer it is clear that any best response g( ) must have $g(1)\geq\delta$, otherwise he is loosing on the equilibrium path. If $g\neq f$ and yet $g(1)\geq\delta$ then if no trembles occur then f and g yield the same payoffs. Comparing f and g when trembles occur we restrict ourselves to the most likely deviations of magnitudes ε and ε^2. Under the contract g the delegate will reject the price offer 1 with probability ε while with f he would only reject it with probability ε^2. □

(ii) The game M_1 is a 2-player game with each player choosing an action only once. Thus at a perfect equilibrium no player uses (weakly) dominated strategies. Consider any strategy pair such that the buyer's instructions are to reject some prices less than or equal 99. This strategy is dominated by the strategy of providing the instructions to accept all prices. Thus such instructions cannot be a part of a perfect equilibrium. The only equilibrium that does not use dominated strategies is the one in which the buyer give the instructions to accepts all prices less or equal 99 and the seller asks for the price 99, or 100, depending on whether 100 is or not acceptable. □

Proposition 1(ii) is equivalent to Katz's (1991) analysis. In order to capture the above intuition Katz considered a different equilibrium called a "rational agent Nash equilibrium". Under this notion the agent always acts rationaly and never trembles. Thus it resembles our definition of instructive-delegation game in which agents are not strategic players but are treated as robots that carry on instructions[4].

In proposition 1 we used trembling hand perfect equilibrium. Recall that such an equilibrium is always a sequential equilibrium. However under unrestricted sequential equilibrium, instructive delegation, even in the one shot game, would become beneficial. For example, in the game M_1 of proposition 1(ii), the buyer could give the acceptance set {1}, the seller holding the belief that {1} is the acceptance set chosen by the buyer, asks for $p_1=1$. This is obviously a sequential equilibrium yielding most of the surplus to the buyer.

As proposition 1 demonstrates, even in a one shot game, incentive delegation may already affect the outcome of the game. Therefore, the rest of this paper is devoted to studying beneficial instructive delegation.

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[4] Note that while Katz changed the equilibrium concept we change the game and stay with the same equilibrium concept. We note however that the concept of rational agent Nash equilibrium requires more than just sequential rationality on the agent's behalf. Sequential rationality is satisfied in our incentive delegation game and yet, as Proposition 1(i) indicates, the (perfect) equilibrium is characterized by strategic delegation.

3. Instructive Delegation with Unlikely Observation.

For every $\rho>0$ we consider a delegation game M_ρ of our seller and buyer game. As in M_1, in the first stage of M_ρ the seller chooses an acceptance set, $A\subseteq\{1,.....,100\}$, which describes the prices at which he wishes the good to be bought on his behalf. However, in the second stage of M_ρ, nature chooses, with probability ρ, to reveal the chosen acceptance set A to the seller, and with probability $1-\rho$ not to reveal it. In the third stage the seller has to choose a requested price, p, when he did not receive information about A, and a price function, p(A), for any acceptance set A revealed to him by nature. The payoffs (to the buyer and the seller respectively) for any choice of strategies A and $(p,p(\cdot))$ are determined according to the expected values in the natural way. With probability $1-\rho$ it is: $(100-p,p)$ if $p\in A$ but (0,0) if $p\notin A$. And with probability ρ it is : $(100-p(A),p(A))$ if $p(A)\in A$ but (0,0) if $p(A)\notin A$.

In other words the above game models the possibility that the instructions chosen by the buyer are revealed to the seller with probability $\rho>0$. It is surprising that no matter how small ρ is, there is a perfect equilibrium of M_ρ, which yields the total surplus to the buyer. Note that with $\rho=0$ we are back in the case of Proposition 1, where the entire surplus is gained by the seller.

To verify the above claim consider the following strategies. The buyer's acceptance set is $\{1\}$. The seller's choices are $(1, B(\cdot))$ with B defined by $B(A) = \max\{x: x \in A\}$ for every $A\neq\phi$ and $B(\phi) = 100$. Clearly at such an equilibrium the selling price is 1 and the resulting payoff to the buyer and seller respectively are (99,1).

Proposition 2 (Beneficial instructive delegation under arbitrarily small probability of observability): The pair of strategies {1}, (1,B(·)), yielding the surplus to the buyer, is a perfect equilibrium of M_ρ for every $\rho > 0$.

Proof: It is easy to see that the above strategies are a Nash equilibrium. Now consider any trembles of the buyer and the seller. Say the buyer submits the acceptance set {1} with probability 1 and every other acceptance set with probability ε. The seller, when he does not learn the acceptance set , demands the price p=1 with probability 1 and every other price with probability ε. Similarly, whenever he is told the identity of the acceptance set A he ask for B(A) with probability 1 and all other prices with probability ε.

It is clear that if ε is sufficiently small then {1} is the buyer's best response instruction. With probability 1-ρ the seller will not discover the identity of the acceptance set and will demand p=1 with the highest probability. So having $1 \in A$ is optimal. But with probability $\rho > 0$ he will discover the identity of the acceptance set A and any $A \neq \{1\}$ will yield the buyer lower profits at the seller's equilibrium strategies. Also the seller's equilibrium strategies are best response. When he does not know the acceptance set demanding the price p=1 is a better response than any other price. When he knows A and $A \neq \phi$, demanding the maximum acceptable price is the only optimal response. When nothing will be accepted, $A = \phi$, demanding p=100 is as optimal as any other price. □

4. Delegation in Repeated Game.

In this section we study beneficial repeated instructive delegation. We consider a two periods game to emphasis that a small number of repetitions is already sufficient to obtain such benefits. Note also that since the game G_2 is of finite length, at any subgame perfect equilibrium the seller offers in every period the price of 99 or 100, and the buyer accepts this price. So as in the one shot game, without delegation, essentially all the surplus is obtained by the seller.

We first study a two period version of the instructive delegation game, M_1, studied before. The new game, M_2, is described as follows. In the first stage the buyer chooses a time independent acceptance set, $A \subseteq \{1,2,...,100\}$, consisting of the prices he wishes the item purchased on his behalf. Following this choice, but without being informed of the identity of A, the seller demands a price p_1 for the good. p_1 is then accepted if and only if $p_1 \in A$. Observing the outcome of the first offer, the seller makes a second period offer p_2. Again p_2 is accepted if and only if $p_2 \in A$. In each period the payoffs of an accepted offer to the buyer and the seller respectively are $(100-p_i, p_i)$ i=1,2, while an unaccepted offer results in (0,0) payoffs. Both players wish to maximize the sum of the periods payoffs.

<u>Proposition 3 (Repeated Unobservable Instructive-Delegation)</u>: In the game M_2 the perfect equilibrium is characterized by strategic delegation. Specifically, for any price x, $1 \leq x < 100$, there is a perfect equilibrium in which the good is sold at both periods at the price x.

<u>Proof</u>: The proof can be easily obtained by modifying the proof of proposition 4 that follows.□

The difference between the one period delegation game and the two period game is that in the two period game the agent's first period action may inform the seller of the type of instructions the agent has. More specifically, first period trembles by the seller, can only affect the one period outcome of the one shot game. While in the two period game, first period trembles, through information revelation about the acceptance set, may have significant effect on the second period outcome. For example, in the one shot game, the buyer should include 99 in his acceptance set. It can only increase his payoff in the case that the seller "trembles" and demands the price 99. But in the two period game, if 99 is rejected in the first period, the seller will learn that it is not in the acceptance set, and will have to make a better deal for the buyer in the second period. While if 99 was accepted in the first period, the seller will demand and receive 99 in the second period. It is worth noting however, that on the equilibrium play path, the seller will not learn which prices are excluded from the acceptance set. It is merely the possibility to make such a check, which occurs only off the equilibrium path, that facilitates the strategic delegation.

An important feature of the game M_2, discussed in proposition 3, is the restriction to a stationary acceptance set. One may think that this should not alter the outcome of the game since the behavior of the buyer in the equilibrium of the original game, G_2, was to accept any price at every period, i.e. a stationary $A=\{1,2,...,100\}$. Yet the discussion in the previous paragraph points that it is the knowledge of the seller that the acceptance set has to be stationary in the game M_2, which facilitates the threat of learning and thus the beneficial delegation. This is no longer the case in the next game we study, which allows for stationary delegation but also for unrestricted self representation.

The delegation game with a possible self representation, SM_2, will be similar to M_2, but with the modification that in the beginning, instead of choosing an acceptance set A, the buyer can choose also to play the game himself. Formally, in the first stage the buyer can choose any acceptance set $A \subseteq \{1,....,100\}$ or a strategy of self representation, SR. Following this choice, but without being informed of it, the seller demands a price p_1. If the buyer's choice was to use instructive delegation with the acceptance set A, then p_1 is accepted if and only if $p_1 \in A$. If the buyer chose the strategy of self representation, SR, then he can accept or reject the price p_1. The seller, observing only the outcome of his first offer and not knowing if the response was one of a delegate or the original buyer, now proposes a price p_2. Again, if the original buyer's choice was a given acceptance set A then the offer is accepted if and only if $p_2 \in A$. And if his original choice was SR, then he can either accept or reject the offer. The overall payoffs of the game are the same as in M_2.

Proposition 4 (Instructive-Delegation with the Possibility of Self Representation): In the game SM_2, perfect equilibria allow strategic delegation. Specifically, for any price x, $1 \le x < 100$, there is a perfect equilibrium with the good being sold in both periods at the price x.

Proof: Consider the following strategies:

(i) In the first period the buyer chooses the acceptance set $T_x \equiv \{1,...,x\}$.

(ii) If the buyer is representing himself then his strategy in the first period is $R_1(p_1)$=yes if and only if $p_1 \in T_x$. At the second period the buyer's strategy is to accept any price.

(iii) The seller offers the price $p_1=x$ at the first period and at the second period $p_2(p_1,R_1)=100$ if $p_1>x$ and R_1=yes or if $p_1=x$ and R_1=no. For every other (p_1,R_1) $p_2=x$.

It is easy to see that the above strategies are a Nash equilibrium supporting the price x at both periods. To show perfection we construct trembles to which the above strategies are best response at every information set. As before we do not normalize the probabilities of mixed strategies to add up to one.

The buyer: with probability 1 he chooses the acceptance set T_x, with probability ε he chooses the acceptance set $\{1,..,x\text{-}1,x+1,..,100\}$ and with probability ε he chooses the acceptance set $\{x\}$. All other possible acceptance sets are chosen with probability of ε^2. Self representation, SR, is chosen with probability ε^3.

When the buyer represents himself his first period trembles are: if $p_1>x$ to respond yes with probability ε and if $p_1\leq x$ to respond no with probability ε. In the second period the trembles are to respond no with probability ε for any p_2.

For the seller in period 1: $p_1=x$ with probability 1, and all the other prices with probability ε. For the second period seller: if $p_1>x$ and R_1=yes, or $p_1=x$ and R_1=no, then $p_2=100$ with probability 1 and all the other values with probability ε^3. At all other information sets the seller offers $p_2=x$ with probability 1 and all the other values with probability ε^3.

We will now show that the best response property holds for all players in every information set, if ε is small enough. When comparing two individual strategies and letting ε be arbitrarily small the comparison can be done lexicographic by first examining how the strategies compare against the main strategies of the opponents. If there is a tie we can compare their performance against strategies assigned the probability ε and continue lexicographically in this fashion.

<u>The buyer</u>: Suppose the acceptance set A is a best response to the strategies with the trembles specified above. Clearly $x\in A$, for otherwise it will be an inferior response to the equilibrium strategies. Suppose that for some $y<x$, $y\notin A$. With probability ε the seller will choose $p_1=y$, it will be rejected and in the second period the good will be sold at a price $p_2=x$. If $y\in A$,

then in the same event that p_1=y the good will be sold at a price y in the first period and still at a price x in the second period, resulting in a higher profits for the seller. Thus $\{1,2,..,x\} \subseteq A$.

Now suppose that for some y>x, $y \in A$. With probability ε the seller will demand p_1=y, the offer will be accepted, in the next period the seller will demand p_2=100 so the total buyer's payoff is 100-y+0. If $y \notin A$, then under the same event, y will be rejected in the first period but an offer of p_2=x will be accepted in the second period, yielding a total buyer payoff of 100-x which is greater than 100-y. Thus $A=T_x$.

It remains to be shown that T_x is a better response than SR. On the play path, without trembles, both T_x and SR yield the same total payoff to the buyer. The next important trembles are of order ε. These can come about with only the seller trembling at the first period, asking for $p_1 \neq x$, or with only the self representing buyer, responding no to p_1=x. In the first case, only the first period seller trembles, SR and T_x yield the same payoffs. While in the second case, the self representing agent receives a zero payoff and T_x yields him 2(100-x)>0.

<u>The first period seller</u>: any price below x will be accepted with probability 1 and will be followed by p_2=x which will also be accepted. Thus the first period seller is better off offering x than any price below x. Any price above x will be rejected with probability 1 and will be followed by p_2=x at the second period and thus p_1=x is an optimal action.

<u>The second period seller</u>: first consider the seller at information set with p_1>x and R_1=yes or the information set S_1=50 and R_1=no. The equilibrium strategy in such information sets is to offer the price 100. Observing acceptance of a price above 50 (or the rejection of the price 50) implies that the buyer trembled and gave the wrong instructions or he chose to represent himself and then trembled. Given the assumed perturbation, and computing posterior probabilities based

on the above observations, the dominant tremble to be considered is that the acceptance set $A=\{1,2,..,x-1,x+1,..,100\}$ was chosen. Thus in such a case indeed the optimal strategy is to offer the price $p_2=100$. Consider now the seller at the information sets with $p_1>x$ and R_1=no or $p_1\leq x$ and R_1=yes. In such information sets with probability 1 there was no trembles and thus offering $p_2=x$ is the only optimal action. Now consider the information sets $p_1<x$ and R_1=no. Observing a rejection of $p_1<x$ implies that the buyer trembled and provided different instructions or that the buyer represents himself and trembled at the first period. Given the above perturbation the most likely tremble is that the buyer chose the acceptance set $\{x\}$. In such a case offering $p_2=x$ is the only optimal action.

Self representing buyer: clearly at the first period saying yes to any $p_1\leq x$ is the only optimal action. If $p_1>x$ than saying no is the only optimal action since accepting such a price will be followed by $p_2=100$. For the second period, accepting any price less than or equal to 100 is clearly a buyer's optimal choice. □

Concluding Remarks.

As we demonstrated, beneficial unobservable incentive delegation can be obtained even in a one shot game. This result should continue to hold in games allowing a small probability of detection, and in repeated games, without restricting the permissible contract e.g., stationary contracts. Beneficial unobservable instructive delegation, on the other hand, does not exist in the one shot game, and this paper demonstrated situations in which it does exists.

If there is arbitrary small, yet positive, probability of the instructions being observed, then beneficial instructive delegation is possible. However, if there is zero probability of observability, even if the game is repeated, beneficial instructive delegation required the use of some stationarity assumptions. The mildest version of such an assumption was in the game SM_2, where a player could play the game himself, without any restriction, but had the option to use stationary instructions. It is the possibility of learning, due to repetition, and the potential value of learning, due to stationarity, which together facilitate beneficial delegation. In the model we studied, without the above stationarity assumption, beneficial instructive delegation does not exist.

Stationary instructions are observed in real life. In the salesperson example, previously discussed, the salesperson's instructions are usually time invariant. One possible explanation for stationary instructions is the complexity of nonstationary acceptance sets. In our model, if there are n periods, there are substantially more than 2^{100n} possible nonstationary instructions.

Commitment through delegation may be limited by the possibility to renegotiate the delegation contract. If the possibility of renegotiation between a player and his delegate exists and is common knowledge, then delegation seems to loose its value as a commitment devise. Delegation reversed the order of the moves in the game while renegotiation reverts it back to the original order.

The situation is more complex when renegotiation is not possible with certainty, or at least is not common knowledge. Such uncertainties may be due to technical limitations of the environment and the technology of communication. But they may also result from strategic choices made by the players who may find it beneficial to commit not to renegotiate. The existence of such commitment devices and their observability seems an important subject for further research.

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