Discussion Paper No. 1036

COOPERATION BY INDIRECT REVELATION THROUGH STRATEGIC BEHAVIOR*

by

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Revised March 1993

* I wish to thank Itzhak Gilboa and Morton Kamien for helpful comments, as well as Leo Hurwicz and Roger Myerson, whose courses in the J.L. Kellogg School of Management inspired this study.

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ABSTRACT

The paper deals with a one-shot prisoners' dilemma when the players have an option to go to court but cannot verify their testimonies. To solve the problem a second stage is added to a game. At the first stage the players are involved in the prisoners' dilemma and at the second stage they play another game in which their actions are verifiable. In such a setup the information about the actions chosen at the prisoners' dilemma stage can be revealed through strategic behavior of the players during the second stage. A mechanism for such revelation in the extended game is described. It provides an existence of a unique sequential equilibrium, which may be obtained by an iterative elimination of dominated strategies and has a number of desirable properties.

Prisoners' dilemma, a non-cooperative game whose nickname is attributed to A.W. Tucker, is probably the most notorious example in the game theory. One can find its discussion in practically any book which touches upon the game theoretic concepts. An extensive description of the game may be found, for example, in such monographs as Luce and Raiffa (1957), Rapoport and Chammah (1970), Kreps (1990), Fudenberg and Tirole (1991), Myerson (1991).

The popularity of the prisoners' dilemma can be explained by the fact that it represents a classical example of a contradiction between individual and collective interests, moreover such pattern of interaction may serve as a quite realistic description of many economic and social conflicts. A "rational" choice of an individual player, if he acts unilaterally, results in a loss of efficiency compared the outcome of an optimal choice for this player under possibility of enforceable cooperation between the players. Because of that, cooperation is always desirable from the game theoretic point of view. In economics, instead, cooperation between agents may lead to inefficiency to prevent which many countries introduce antitrust laws. Nevertheless, even such legislation does not rule out certain types of agreements. A specific example of that kind will be considered in the next section.

Since the first presentation of the prisoners' dilemma there have been many attempts to "solve" it, i.e. to find a way for cooperation. Most of these endeavors are summarized by Tsebelis (1990). Different efforts can be split into four groups.

The first group of arguments relies on belief that there is no such thing as the prisoners' dilemma in real life, and such a model arises as an inappropriate representation of the conflict. For example, Stinchcombe (1980) asserts that people do not try to maximize their goals, and therefore do not follow rationality assumptions adopted by game theory. As a result prisoners' dilemmas are being solved without people even realizing that they were faced with such problems. Similar views are shared by many proponents of evolutionary biology, where it is frequently assumed that in the prisoners' dilemma the players are concerned about issues other than just self-interest (see Maynard Smith, 1982). As a result the payoff matrix has to be modified and cooperation becomes attainable through individually rational behavior. Another branch within the first group can be represented by Howard's (1970) metagames theory. Howard allows for certain mental experiments which bring cooperation about. His theory is criticized as irrational on the grounds that the mental experiments carried out by a player do not have any causal effect on the rival, and when this mental illusion is set aside the players still make their choices independently, thus there is still no reason for them to cooperate. The theories of this group either refute prisoners' dilemma per se or offer a solution which is not satisfactory.

Another approach suggests introduction of communication with monitoring and binding contracts. The players may sign a binding contract which leads to such a modification of the conflict that cooperation is a Nash equilibrium in the modified game (see, for example, Myerson, 1991). Just communication or signing a contract is not enough – it must be binding. The latter can be ensured by existence of an authority which can monitor players' actions and impose obligatory sanctions when the deviations from the terms of the agreement occur. If the fines (punishments) to deviators are sufficiently large, cooperation is trivially enforceable.

The third way to solve the dilemma is to admit that the game is played repeatedly. Luce and Raiffa (1957) reported that in the conducted experiments on human behavior in repeated prisoners' dilemma, people usually cooperate in some periods. Later it was proven that such behavior is "irrational" in a sense that in any finite repetition of the game the only equilibrium is for both players never to cooperate. However, if the game is played infinitely many times or information is incomplete (i.e. there is a small probability that the game will stop after each round) there are conditions under which always to cooperate for both players is an equilibrium.

Finally, Brams (1975) introduces asymmetry in informational structure of the game, making one of the players a principal and another a follower. The principal can commit himself to a certain strategy, while the follower adapts to the principal's choice. If the players can almost always correctly predict rivals' choices, then there is a place for cooperation. This solution, though trivial, does not seem satisfactory.

In this paper we use an approach that does not precisely fit any of the groups sketched above. First of all, we assume that the dilemma really exists and do not require repetitions. Then we allow for a contract and existence of an authority which can levy compulsory fines (or grant rewards) depending on the obedience by the contract. However, the actions chosen by the players can not be monitored by this authority, which makes the enforcement task non-trivial. Instead of direct modification of the payoff matrix of the game, we assume that the players are involved in some other activity besides their interaction in the prisoners' dilemma. So we try to get additional information, necessary for assurance of cooperation, from monitoring the choices of the players outside the dilemma.

More specifically, we deal with a one-shot prisoners' dilemma in which the players have an option to go to court but cannot verify their testimonies. The question is what kind of a mechanism can be designed to enforce cooperation in such a game. In search for an answer the original game is extended into a two-stage game, where at the first stage the players are involved in the prisoners' dilemma and at the second stage they play another game in which their actions are verifiable. The payoffs at both stages are publicly known, and each player knows before choosing a strategy for the second stage, what his opponent's choice was at the first stage. In such a setup the information about the actions chosen at the prisoners' dilemma stage can be revealed through strategic behavior of the players during the second stage. A mechanism for such revelation in the extended game is described. It provides an existence of a unique sequential equilibrium, which may be obtained by an iterative elimination of dominated strategies and has a number of desirable properties.

The paper does not suggest a way to resolve the dilemma once and for all. Our point is, rather, that in certain economic situations, in which participants are involved in prisoners' dilemma and cooperation is both legitimate and socially desirable, it might be achieved by traditional social institutions even without perfect monitoring, incomplete information or infinite repetition. Instead, we require that the players be able to think "one period" ahead and that there was some relationship between the prisoners' dilemma played today and the game that will be played tomorrow. A remarkable fact is that even a very slight connection between the stages provides extensive opportunities for cooperation if the court and the players behave strategically.

Both verbal and formal description of the model under consideration are presented in section 2. Section 3 contains discussion of an example as well as some general observations. Intuitive explanation of the solution approach is in section 4. The main results in general form are given in section 5.

2. Setup of the Problem

Let us consider an economic situation as follows. There are two firms. Firm one produces a high-tech product and comes up with an idea, which is a breakthrough in a conventional technology. To implement this idea a sophisticated component is required, which firm one can not develop itself. Moreover, it does not know precisely what the parameters of the required component should be. So firm one turns for assistance to firm two, which specializes in production of equipment similar to what firm one seeks. However, for the second firm to develop exactly what is needed, its access to the firm's one prospective technology is necessary. On the other hand, firm two has to spend substantial resources on research and development to fulfil the order of the firm one. As a result, the two sides sign a contract, which, in particular, stipulates that for a specific period after the component is developed by firm two, firm one will buy it exclusively from firm two and not order the component (whose specifications are already known by that time) from firm two's competitors. Firm two, in return, is obligated not to promote the new component to firm one's competitors, since such action would be practically equivalent to revelation of the firm one's innovation.

Each firm has two possible strategies: either to stick with the agreement or to breach it. If only one firm abides by the contract, then a deviator gets additional profits by expanding market where it buys (were it firm two) or sells (were it firm one), while its opponent incurs losses. In case when both sides deviate, they both lose compared to the profit levels they would get if no violations occurred. Hence, the interaction between the firms has a nature of the prisoners' dilemma.

In case of a contract violation a victim can go to court. But the problem is, that, though the court knows what profit each firm earns in any of the four outcomes, firms' actions are not verifiable in court, i.e. the court does not know which outcome occurred indeed. However, after the firms get their payoffs from interaction, they may take loans from a bank, which brings a positive profit of α per dollar. The size of the loan is determined by each firm, with the only restriction imposed by the bank that the loan has to be backed-up in full by the firm's assets. The court can verify how much each firm has borrowed.

The question is, whether this information suffices to develop a feasible mechanism (i.e. a system of penalties and compensations), depending only on verifiable information, such that a unique equilibrium behavior for the firms is to cooperate and not to go to court. By feasibility of a mechanism we mean the following two properties. Firstly, a penalty to a firm in any given state does not exceed the assets of the firm in that state. Secondly, in any state the net penalty paid by the two firms together is non-negative, i.e. there is no need in outside subsidies to implement a court order.

To provide an answer to this question we need to formulate a model of the problem, with which we are going to proceed.

We assume two players: I and II,- who are risk neutral and have preferences represented by utility functions depending only on each player's own assets. Without loss of generality, one may identify utilities with the assets values. The players are involved in a two-stage game.

At the first stage they are engaged in a prisoners' dilemma, described by the following matrix:

	C	D
C	A1, A2	B1, B2
D	C1, C2	D1, D2

where rows correspond to the strategies of player I (C stands for cooperation and D for deviation) and columns represent the respective choices for player II. For this matrix to have the structure of the prisoners' dilemma we need that C1 > A1 > D1 > B1 and B2 > A2 > D2 > C2.

Assume that the firms have endowments big enough for their assets to remain positive after the first stage even if some payoffs in the matrix above are negative, i.e. we exclude a possibility of bankruptcy after the first stage. Mathematically that means that the utility levels, with which the players enter the second stage are given by the matrix

	C	D
C	a1, a2	b1, b2
D	c1, c2	d1, d2

(1)

in which all numbers are positive.

At the end of stage one the players know the outcome of this stage and the choices made by both of them. Moreover, they receive payoffs of their interaction.

At the second stage each player decides how much money to borrow from the bank (the possible size of the loan is any amount between 0 and assets of the firm, given by an appropriate number in (1); each borrowed dollar brings $\alpha > 0$ dollars of profit by the end of the second stage) and whether to go to court or not.

At each stage the players make their choices simultaneously and are not aware of the rival's decision.

If any player comes to court (accusing the opponent in breaching the agreement), the court knows matrix (1) and the choices of the players at the second stage of the game, i.e. whether one or both firms complained; what assets could each firm have after the first stage of the game in any possible outcome; and the amounts borrowed by each firm. Based on this information the court orders each player either to pay a fine or to get a retribution from the rival. Each set of information determines a case denoted by i. If case i occurs the court rules out that player I has to pay a fine of f_i and player II has to pay g_i. The players receive compensations when the corresponding numbers are negative.

The system of punishments and rewards used by the court is publicly known before the game starts, while the value of α becomes revealed only at the second stage of the game.

Summarizing the above, we have two players. We will use arabic numbers referring to the players in subscripts. The set of strategies for player k (k = 1,2) is

$$\Sigma_k = \{(p_k, x_k, s_k) \in P \times \mathbf{R}^{P \times P} \times S^{P \times P} |\ 0 \le x_k \le \pi_k\},$$

where P = {C, D} is the first stage choice; S = {G, S} is the last decision a player makes, namely, to go to court (G) or to stay out (S); and $\pi_k(p_1,p_2)$ is a payoff of the first stage given by matrix (1). Let player k choose a strategy $(p_k,\ x_k,\ s_k)$. Then his total payoff is $\pi_k(p_1,p_2) + \alpha x_k - q_{ik}$, where q_{ik} is a penalty levied by the court, depending on payoff matrix, and information about players' choices available to court: $q_{ik}\colon i \to \mathbf{R}$, where $i \in I \equiv \pi \times x_1 \times x_2 \times S \times S$; $\pi\colon P \times P \to \mathbf{R}_+^2$, $\pi(p_1,p_2) = (\pi_1(p_1,p_2),\ \pi_2(p_1,p_2))$, $p_1,p_2 \in P$; $q_{ik} = f_i$ if $k = 1$ and $q_{ik} = g_i$ if $k = 2$. (We index q, f and g by i rather than made i an argument for notational convenience.)[1]

[1]Since each player's decisions are made sequentially the appropriate solution concept would be a subgame perfect or sequential equilibrium. We choose the latter one because it is stronger (See, for example, Myerson (1991), pp. 184–185). That is why from now on whenever we say "equilibrium" we will mean a sequential equilibrium and "equilibrium strategy" will stand for an equilibrium

The objective is to develop a mechanism (to construct f_i and g_i for all $i \in I$) such that the equilibrium strategy for the player k would be (C, •, S). It might also be desirable to have $\pi_k(C,C)$ in the second place in the equilibrium strategy above; to ensure uniqueness of the equilibrium; and to have for any $\alpha \in \mathbf{R}$ and any $i \in I$, $q_{ik} \leq \pi_k(p_1,p_2) + \alpha x_k$ and $f_i + g_i \geq 0$.

A mechanism satisfying these and some additional properties will be described in section 5, but let us first consider an example.

3. An Example of Mechanism Design

We would like to start with two simple observations. They have a general nature, but are part of the preliminary assumptions of the model, hence we include and discuss them before turning to a specific example.

Observation 1. If for k = 1,2 ranges of x_k do not depend on (p_1,p_2), then there is no sequential equilibrium in which players cooperate during the first stage[2].

The proof is an immediate consequence of the following consideration. If the stages of the game are completely independent one from another (which is the case if in the setup of section 2 ranges of x_k do not depend on (p_1,p_2)), then the only possible sequential equilibria for the two-stage game are those that consist of choices which would be equilibria in the stage games were they played

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strategy in a sequential equilibrium.

[2]It is possible to construct a mechanism which allows for cooperation without the borrowing stage (which is the same as the assumption of x_k's independence from (p_1,p_2).) Such mechanism, however, can be implemented only in Nash equilibrium which lacks subgame perfection and, hence, is not sequential.

separately. Since the unique equilibrium in the classical prisoners' dilemma is deviation for both players, any sequential equilibrium of the two-stage game with completely independent stages, should include choices of D at the first stage.

Observation 2. If (p_k, x_k, s_k) is k's equilibrium strategy in a sequential equilibrium, which satisfies feasibility, then $x_k = \pi_k(p_1, p_2)$, for some $p_1, p_2 \in P$.

By feasibility, for any $\alpha \in \mathbf{R}$ and any $i \in I$, $q_{ik} \leq \pi_k(p_1, p_2) + \alpha x_k$. That allows us to write $q_{ik} = a_{ik} + b_{ik}(\alpha x_k)$, where a_{ik} is the part covered by the first stage payoff, and $b_{ik}(\alpha x_k)$ is to be paid from the profit generated by a loan. To meet feasibility requirement above, we need $b_{ik} \leq 1$ for all i and k, since α can be arbitrary large. Next, notice that in each case of fixed π, x_1 and x_2 there is a possibility that both players stay out of court (i.e. $b_{ik}=0$, $k = 1, 2$). Taking into account that $\alpha > 0$, this suffices to create an incentive for each player to borrow as much as possible. At the same time x_k conveys some information about firm k's assets, which, in turn, may be used to determine punishment for the firm. This information is, however, discrete and can signal about one of the four states. In equilibrium player k chooses a signal which is the best for himself under a given system of punishments and selects the highest value of x_k compatible with that signal. This is exactly what observation 2 says.

Since observation 2 is so "obvious for the players", from now on we will ignore a possibility for a player k to announce $x_k \neq \pi_k(p_1, p_2)$, for some $p_1, p_2 \in P$[3].

The above implies that for a given π (matrix (1)) the court has to deal

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[3]This will add significantly to simplicity and clarity of exposition without any loss of generality. Of course, if we drop this assumption, a mechanism has to be adjusted accordingly. All results will still,however, hold.

with not more than 48 cases (in the other 16 situations no one goes to court and, hence, for all these i's $f_i = g_i = 0$.) Restrictions applied by the bank together with the structure of π reduce this number further down to 33.

To work out a desired mechanism we can simplify the representation of the game by splitting the second stage into 4 subgames, corresponding to the 4 outcomes of stage one. We can consider each subgame separately, since the players are by that time fully aware of what had happened at the first stage, while the court order depends on the first stage outcome only through the second-stage decisions of the players.

Let the matrix (1) be as below:

	C	D
C	10, 10	4, 12
D	12, 4	5, 5

.

We will denote the subgames of the second stage as follows: the subgame corresponding to the outcome (C,C) of the first stage will be called II.A; to outcome (C,D) – II.B; to outcome (D,C) – II.C; to outcome (D,D) – II.D. Extensive form representations for these subgames are shown in figures 1 – 4.

Let us explain the figures. Each node of the tree is denoted by two numbers: a latin number indicating who moves at this node, and an arabic number showing the number of the node within the tree. Information sets are shown by ovals. Branches corresponding to equilibrium moves are drawn in bold. There are two numbers at each end node of the tree: the upper one designates a payoff to player I, and the bottom one stands for a payoff to player II, if the game ends at this node. The payoffs will be explained later, after the mechanism is introduced.

It is convenient to represent the mechanism in the form of a table, which prescribes punishments to the players for each possible case.

Table I

x_1	x_2	s_1	s_2	f_i, g_i	x_1	x_2	s_1	s_2	f_i, g_i
4	4	G	G	$4+3\alpha, 2\alpha$	5	5	S	G	$5+5\alpha, -2.5\alpha$
		G	S	$-2\alpha, 4\alpha$	5	10	G	G	$2.5\alpha, 4+10\alpha$
		S	G	$4+4\alpha, -2\alpha$			G	S	$-3.5\alpha, 4+5\alpha$
4	5	G	G	$4+3\alpha, 2.5\alpha$			S	G	$2.5\alpha, 5\alpha$
		G	S	$-2.5\alpha, 5\alpha$	10	4	G	G	$10+10\alpha, 2\alpha$
		S	G	$4+4\alpha, -2\alpha$			G	S	$5, 4\alpha$
4	10	G	G	$2\alpha, 10\alpha$			S	G	$4+10\alpha, -4-10\alpha$
		G	S	$-3\alpha, 5\alpha$	10	5	G	G	$10\alpha, 2.5\alpha$
		S	G	$4\alpha, 5\alpha$			G	S	$5\alpha, 5\alpha$
4	12	G	G	$-12-12\alpha, 12+12\alpha$			S	G	$5\alpha, -2.5\alpha$
		G	S	$-3\alpha, 6\alpha$	10	10	G	G	$10\alpha, 10\alpha$
		S	G	$4\alpha, 6\alpha$			G	S	$5\alpha, 5\alpha$
5	4	G	G	$4+5\alpha, 4+3\alpha$			S	G	$5\alpha, 5\alpha$
		G	S	$-2\alpha, 4+4\alpha$	12	4	G	G	$12+12\alpha, 2\alpha$
		S	G	$5+5\alpha, -2.5\alpha$			G	S	$6\alpha, 4\alpha$
5	5	G	G	$4+3\alpha, 4+3\alpha$			S	G	$4+10\alpha, -4-10\alpha$
		G	S	$2.5\alpha, 5+5\alpha$					

The mechanism above determines the payoffs as in the figures 1 – 4 and as a result the equilibrium strategy for the second stage of the game (Since this equilibrium is obtained by backward induction, it can be extended to a sequential equilibrium.) Notice that, whatever happens at the first stage, the optimal behavior for each player is to truthfully announce his payoff received from the prisoners' dilemma and go to court only if according to his announcement he was cheated against. This property makes the players' behavior at the second stage internally consistent. Another nice characteristic of the proposed mechanism is that this is a pure strategy equilibrium, i.e. beliefs of the players in any information set are unambiguous, which eliminates the problem of their concordance.

Needless to say, a feasible mechanism with such properties is not unique. To see whether it helps to achieve cooperation in prisoners' dilemma let us address the first stage of the game, the extensive form of which is represented in figure 5. Since for player I C dominates D, in any sequential equilibrium player II believes with probability 1 that he is at the node II.2. Thus, for (C,C) to be played in a sequential equilibrium, one needs α to be at least .5. If this condition is satisfied we have a unique, feasible, pure strategy sequential equilibrium for the two-stage game, in which the players cooperate in the prisoners' dilemma phase.

Having finished with this example, we still have several questions to answer. Is the mechanism described an optimal one, i.e. can we change it to relax the constraint on α? (It will follow from the corollary to the proposition 4 of section 5 that we can ensure cooperation with the payoff matrix as in the example for any $\alpha > 0$.) When will a similar mechanism work for general case? When can cooperation be obtained as part of a mixed strategy sequential equilibrium? These issues, as well as the problem of construction of an optimal mechanism of such type, will be dealt with in section 5. But before addressing these problems, let us discuss the basic intuition underliyng the idea of the proposed mechanism.

4. A Bit of Intuition, Or Where Strategic Considerations Come From

As was mentioned above, one can always achieve cooperation in the example considered in section 3 (though not through the system of punishments described there.) Why and how does it work? What is the intuition underlying the numbers in Table I?

To shed some light on these and similar questions let us consider the case when both players cooperate in the first period (the logic for the other cases is similar.) Why should the players truthfully choose the loan amounts and not go to court? Suppose that player I decided to frame his opponent, so he borrowed only 4, pretending that he had been cheated against and went to court. Player II obviously borrowed 10 and stayed home. The court sees the signals and realizes that one of two things had happened: either player II cheated but pretends that he cooperated, or player I wants to mimic a victim while in fact he is not (which is the true case). Since the court cannot distinguish between these two scenarios it has to work out a system ruling out both of them. This objective can be accomplished by the following decision. First of all when the loans are 4 and 10 the player who gets 10 receives exactly the same punishment as he would have received if borrowed 12 (more precisely, the difference between the two fines should be such that it is still better for player II to borrow 12 rather than 10). That makes it unprofitable for player II to pretend. On the other hand, the player who borrowed 4 gets nothing (or any other amount, possibly negative, which makes him better off had he announced 10). This makes cheating for player I unprofitable. So we are done. Notice that this mechanism does not presume full revelation of the true state, but ensures a desired outcome through strategic behavior. Thinking in a similar fashion we can find the conditions that rule out false signaling in other cases as well.

The next step is to combine all these interrelated restrictions together. It might be done in many ways, since each case admits certain flexibility in punishments. Having in mind that we would like to ensure cooperation for the possibly widest range of parameters, one can easily construct a mechanism which is optimal in that sense. Being fully aware of what and how to do we are ready to proceed with the last section.

5. Mechanism Design for the General Case

Let us turn back to the first stage payoff matrix of the general form, given by (1). Even in that case one can always construct a mechanism, which brings about sequential equilibria for the second stage subgames satisfying desirable properties.

Proposition 3. For any payoff matrix (1) there is a mechanism with the following properties:

a) it implements sequential equilibria for the second stage subgames;

b) equilibrium for each subgame is a unique pure strategy equilibrium;

c) sequential equilibria, compatible with this mechanism, are such that, whatever had happened at the first stage, the players want to reveal their true payoffs, and go to court if and only if according to what they reveal they were cheated against;

d) the mechanism is feasible.

Proof. The game trees, describing desirable mechanism, are presented at figures 6 - 9. The cases are numbered according to the following table:

Table II

x_1	x_2	s_1	s_2	i	x_1	x_2	s_1	s_2	i
b1	c2	G	G	1	d1	d2	S	G	18
		G	S	2	d1	a2	G	G	19
		S	G	3			G	S	20
b1	d2	G	G	4			S	G	21
		G	S	5	a1	c2	G	G	22
		S	G	6			G	S	23
b1	a2	G	G	7			S	G	24
		G	S	8	a1	d2	G	G	25
		S	G	9			G	S	26
b1	b2	G	G	10			S	G	27
		G	S	11	a1	a2	G	G	28
		S	G	12			G	S	29
d1	c2	G	G	13			S	G	30
		G	S	14	c1	c2	G	G	31
		S	G	15			G	S	32
d1	d2	G	G	16			S	G	33
		G	S	17					

If the trees as in figures 6 – 9 can be obtained through a choice of f_i and g_i, then an equilibrium for each subgame is achieved by iterative elimination of dominated strategies, and hence is a unique, pure strategy sequential equilibrium. Part c) follows directly from the form of the equilibria, so the only two things left to be shown are existence of appropriate f_i's and g_i's and feasibility.

It is clear that this problem is equivalent to solvability of a system of linear inequalities: if for a given non-terminal node n m_n is the number of choices available at this node, then we need m_n-1 inequalities to ensure the right choice at n (not all of such inequalities for all n's are independent), and for every end-node we need three inequalities to guarantee feasibility, namely $f_i + g_i \geq 0$ and $q_{ik} \leq \pi_k(p_1,p_2) + \alpha x_k$, where q_i equals to f_i or g_i. This system happens to be solvable and one of the solutions is presented in table III below (where t is a number between 0 and 1, close to 1).

Table III

i	f_i	g_i	i	f_i	g_i	i	f_i	g_i
1	$b1+\alpha tb1$	$c2+\alpha tc2$	12	$\alpha b1$	$\alpha b2/2$	23	$a1/2$	$\alpha c2$
2	$-\alpha c2/2$	$c2+\alpha c2$	13	$b1+\alpha d1$	$c2+\alpha c2/2$	24	$b1+\alpha a1$	$\max\{-b1-\alpha a1, \alpha(c2-a2)/2\}$
3	$b1+\alpha b1$	$-\alpha b1/2$	14	$-\alpha c2/2$	$c2+\alpha c2$	25	$\alpha a1$	$\alpha d2/2$
4	$b1+\alpha tb1$	$c2+\alpha td2$	15	$d1+\alpha d1$	$-\alpha d1/2$	26	$\alpha a1/2$	$\alpha d2$
5	$-\alpha d2/2$	$c2+\alpha d2$	16	$b1+\alpha td1$	$c2+\alpha td2$	27	$\alpha a1/2$	$\max\{-\alpha a1/2, \alpha(d2-a2)/2\}$
6	$b1+\alpha b1$	$-\alpha b1/2$	17	$-\alpha d2/2$	$d2+\alpha d2$	28	$\alpha a1$	$\alpha a2$
7	$\alpha b1/2$	$c2+\alpha a2$	18	$d1+\alpha d1$	$-\alpha d1/2$	29	$\alpha a1/2$	$\alpha a2/2$
8	$\max\{-\alpha a2/2, \alpha(b1-a1)/2\}$	$c2+\alpha ta2$	19	$\alpha d1/2$	$c2+\alpha a2$	30	$\alpha a1/2$	$\alpha a2/2$
9	$\alpha b1$	$\alpha a2/2$	20	$\max\{-c2-\alpha ta2, \alpha(d1-b1)/2\}$	$c2+\alpha ta2$	31	$c1+\alpha c1$	$\alpha c2/2$
10	$-b2-\alpha b2$	$b2+\alpha b2$	21	$d1/2$	$\alpha a2/2$	32	$\alpha c1/2$	$\alpha c2$
11	$-c2-\alpha tb2$	$c2+\alpha tb2$	22	$a1+\alpha a1$	$\alpha c2/2$	33	$b1+\alpha tc1$	$-b1-\alpha tc1$

A direct check shows that the mechanism given by Table III satisfies the desired properties. ■

Proposition 4. In a unique, pure strategy sequential equilibrium that has the properties specified in proposition 3, both players cooperate in the prisoners' dilemma stage if the following conditions are satisfied:

$$\alpha > (b2 - c2 - a2)/a2,$$

$$\alpha > (c1 - b1 - a1)/a1,$$

and either $\alpha > (d2 - 2c2 - b1)/(c1 + c2)$ or $\alpha > (d1 - 2b1 - c2)/(b1 + b2)$.

Proof. If one takes into account the analysis of proposition 3 and looks at figure 10, it is clear that the conditions above are equivalent to the first stage choice of (C, C) by iterative elimination of dominated strategies. ■

Corollary. Cooperation at the prisoners' dilemma stage is attainable for any $\alpha > 0$, if the following relationships hold:

$$c2 + a2 \geq b2,$$

$$b1 + a1 \geq c1,$$

$$\text{and either } 2b1 + c2 \geq d1 \text{ or } 2c2 + b1 \geq d2.$$

Proof follows immediately from proposition 4 and the fact that under the corollary conditions the right hand sides of the inequalities in proposition 4 become negative.

Since the conditions specified by the corollary hold for our example from section 3, we could conclude that it was always possible to achieve cooperation having such prisoners' dilemma matrix. This also means that the mechanism proposed for that example in section 3 was not optimal.

Proposition 5. The mechanism presented in Table III is optimal in a sense that one can not relax the restrictions on α, given in proposition 4, and still have a unique pure strategy sequential equilibrium; which is feasible and in which the players cooperate in the prisoners' dilemma stage; take loans up to full amount, revealing their true first stage payoffs; and go to court if and only if according to what they reveal they were cheated against.

Proof. It follows from figure 10, that to relax restrictions on α, given matrix (1), one should simultaneously minimize

$$-f_{33},\ -f_{16},\ -g_{11},\ -g_{16},\ f_{11},\ g_{33} \tag{2}$$

subject to restrictions on choices in the second stage subgames and feasibility. It is easy to see that under such constraints, the minimal values for the parameters in (2) are attained in the mechanism presented in Table III. ■

Proposition 6. There is no mechanism which allows for cooperation and satisfies the properties stipulated in proposition 3 if

$$\text{either } \alpha \le \min\{(c1 - b1 - a1)/a1,\ (d1 - 2b1 - c2)/(b1 + b2)\},$$

$$\text{or } \alpha \le \min\{(b2 - c2 - a2)/a2,\ (d2 - 2c2 - b1)/(c1 + c2)\}.$$

Proof. As could be seen from figure 10, the conditions above imply that for at least one player defection dominates cooperation. ■

Proposition 7. If conditions of neither proposition 4 nor proposition 6 hold then there is a unique sequential equilibrium, where the players randomize between cooperation and defection at the first stage and for the second stage play according to prescription of proposition 3.

The proof is obvious.

Corollary. There is always (for all $\alpha > 0$) a possibility for cooperation if

$$b1 + a1 \ge c1 \text{ or } 2b1 + c2 \ge d1$$

$$\text{and } c2 + a2 \ge b2 \text{ or } 2c2 + b2 \ge d2.$$

Proof follows from the fact that under these conditions, only negative α could satisfy inequalities of proposition 6. That means that either proposition 5 or proposition 7 hold, which implies possibility for cooperation.

Let us conclude with a remark about the assumption of risk neutrality, made mostly for simplicity in section 2.

Remark. The results of propositions 4 - 7 (together with their corollaries) hold for any players who have utility functions strictly increasing in wealth, regardless of their risk attitude. This is true, since these results are based on dominance arguments only. Also, if the players are risk neutral then α may be viewed not only as a sure net rate of return, but also as expected profit per each borrowed dollar.

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Stage II.A

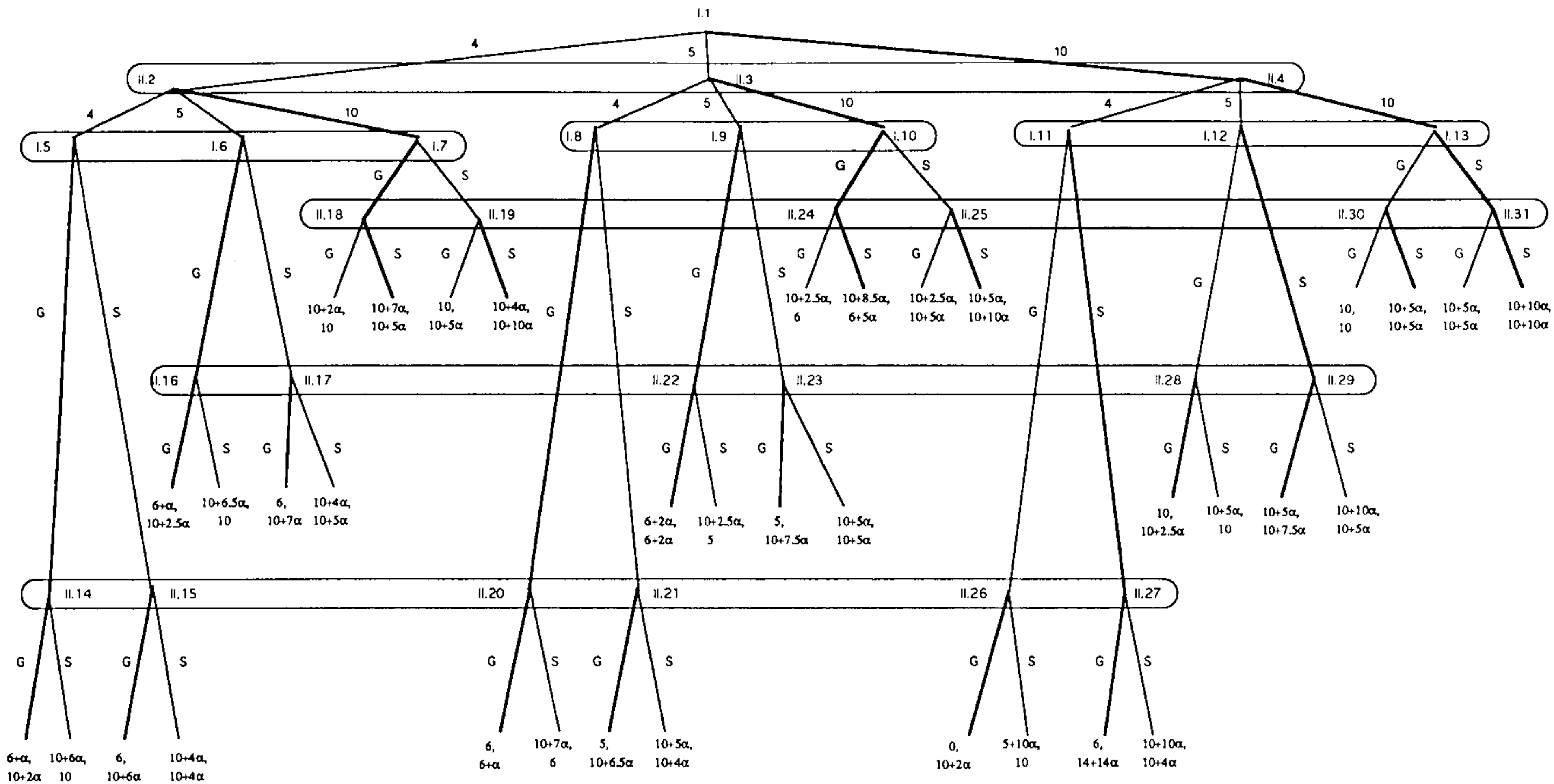


Figure 1.

Stage II.B

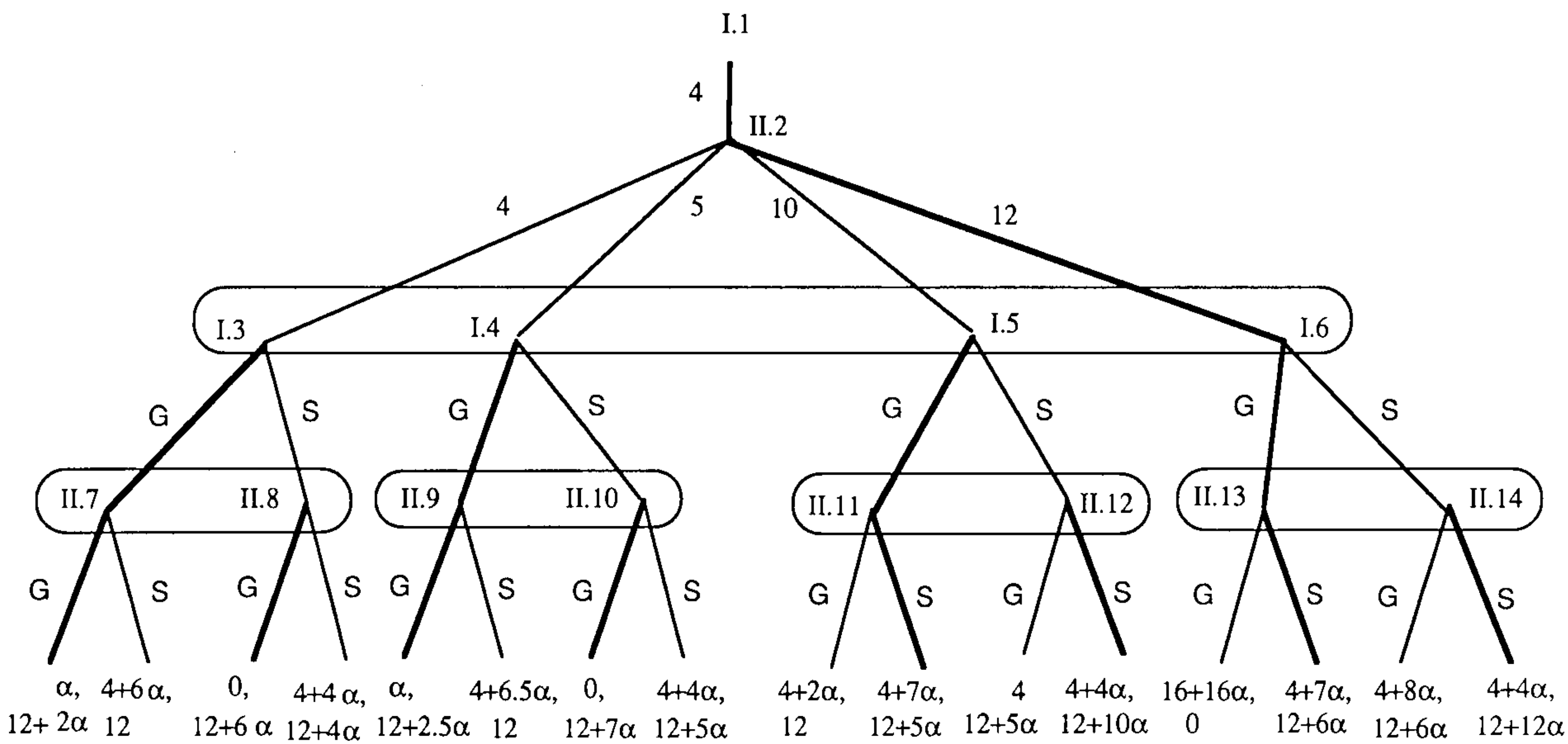


Figure 2.

Stage I

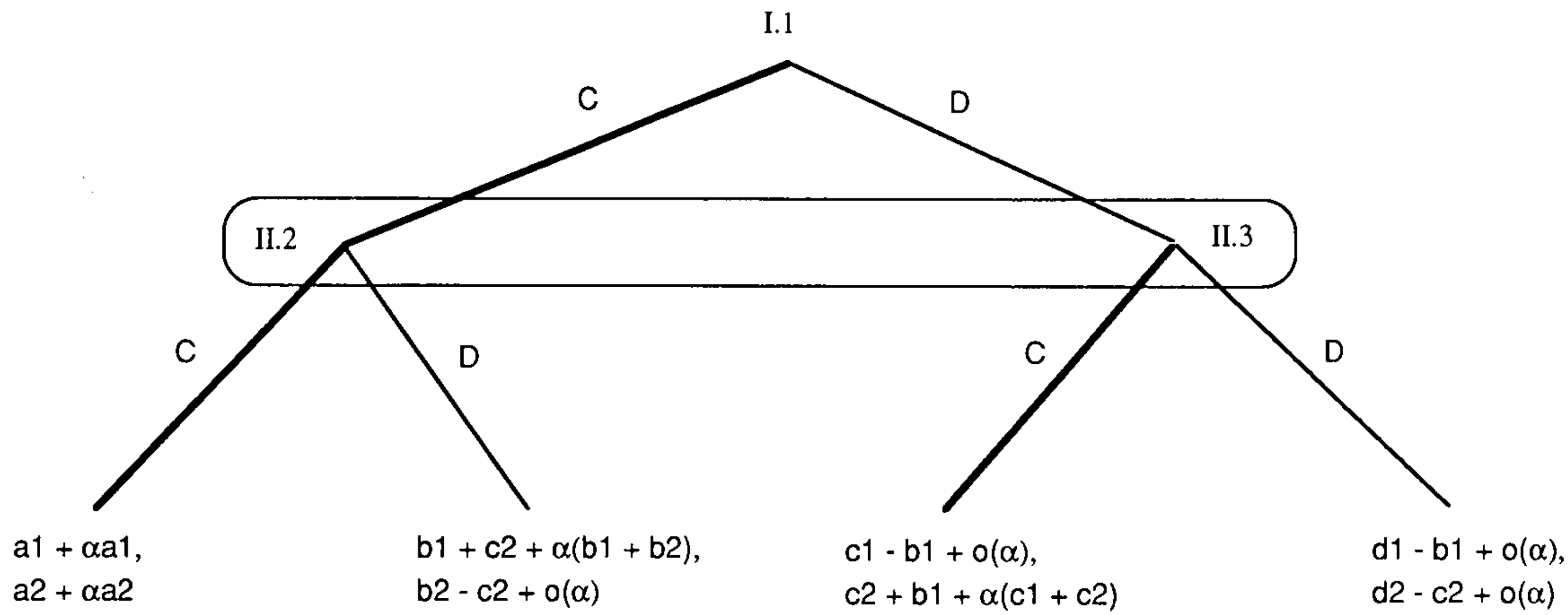
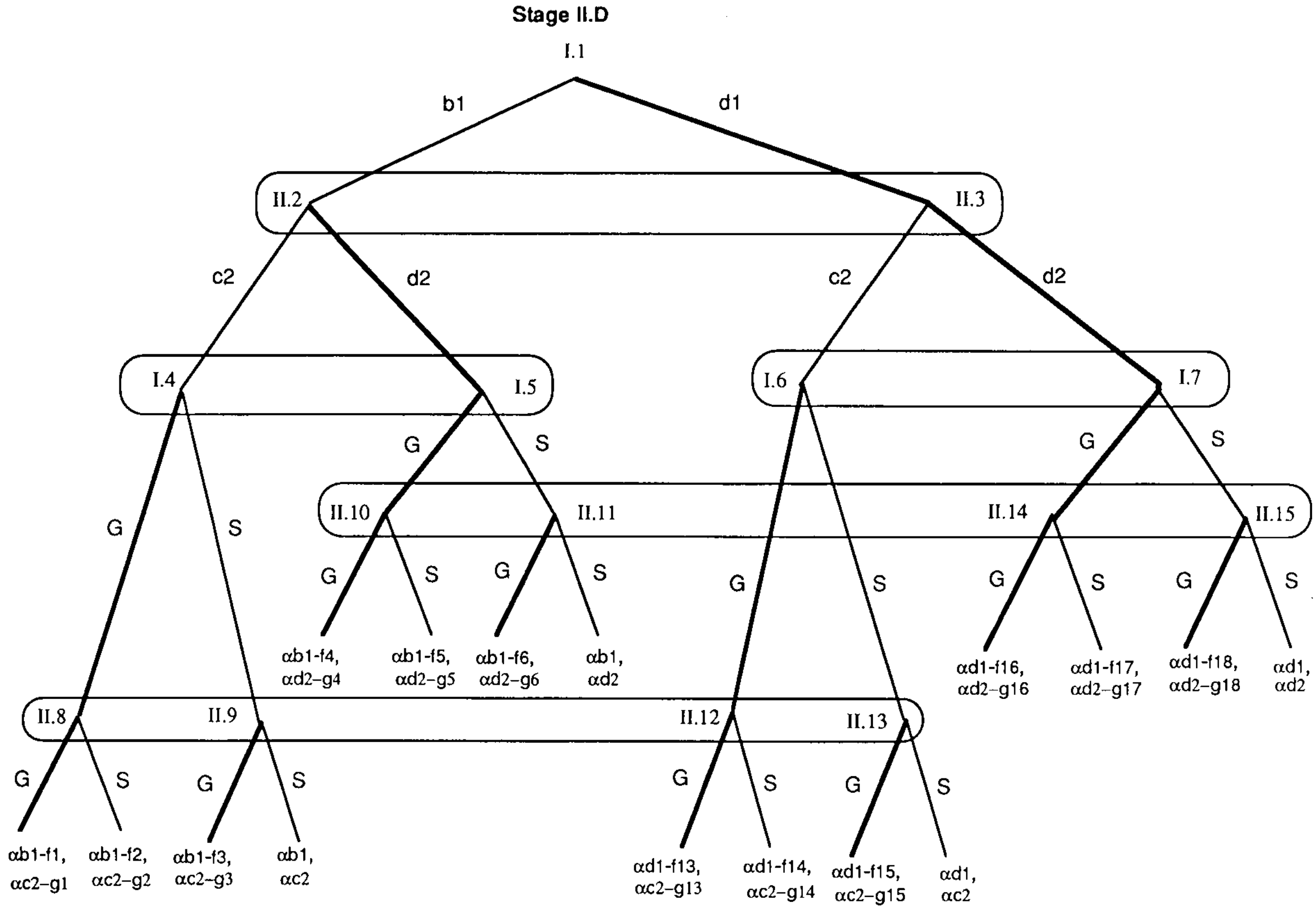


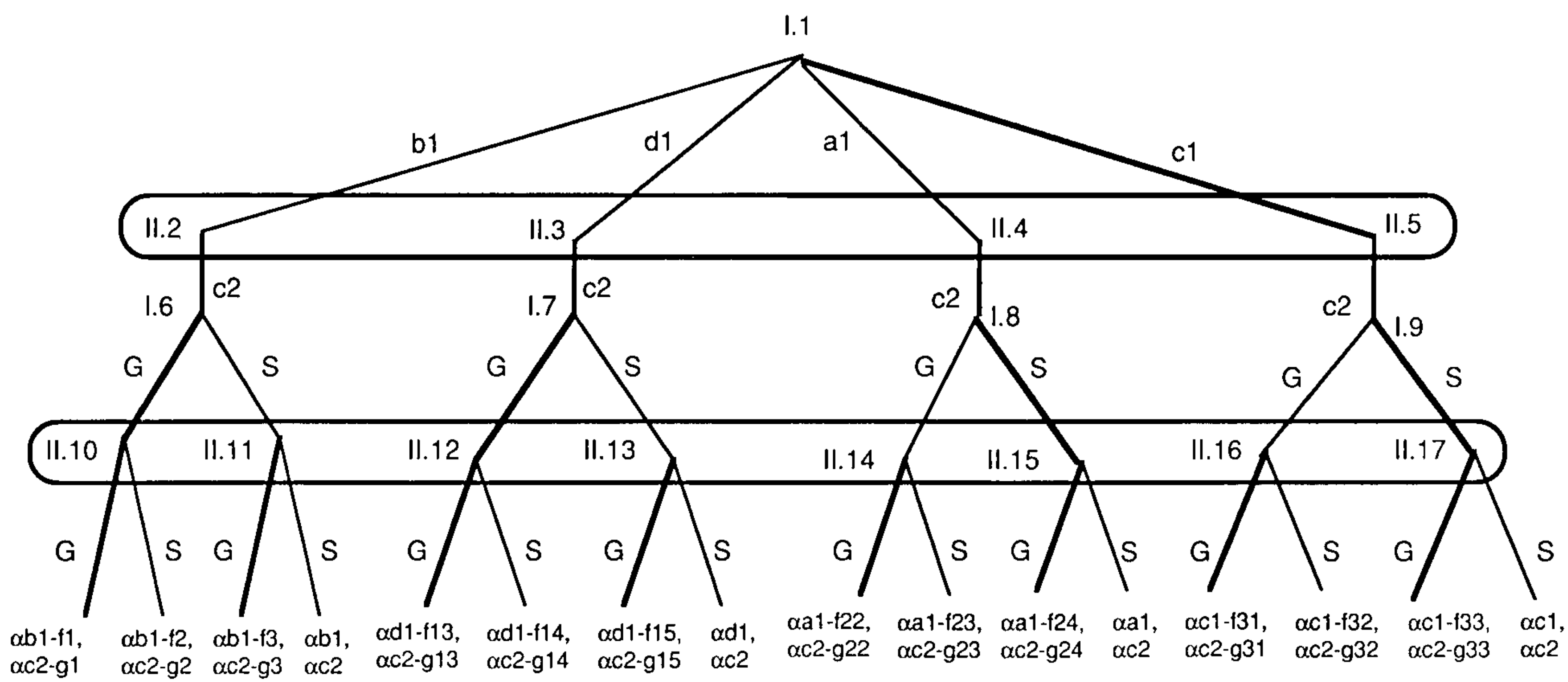
Figure 10.



To get total payoffs, the payoffs of this subgame should be increased by d1 for player 1 and by d2 for player 2.

Figure 9.

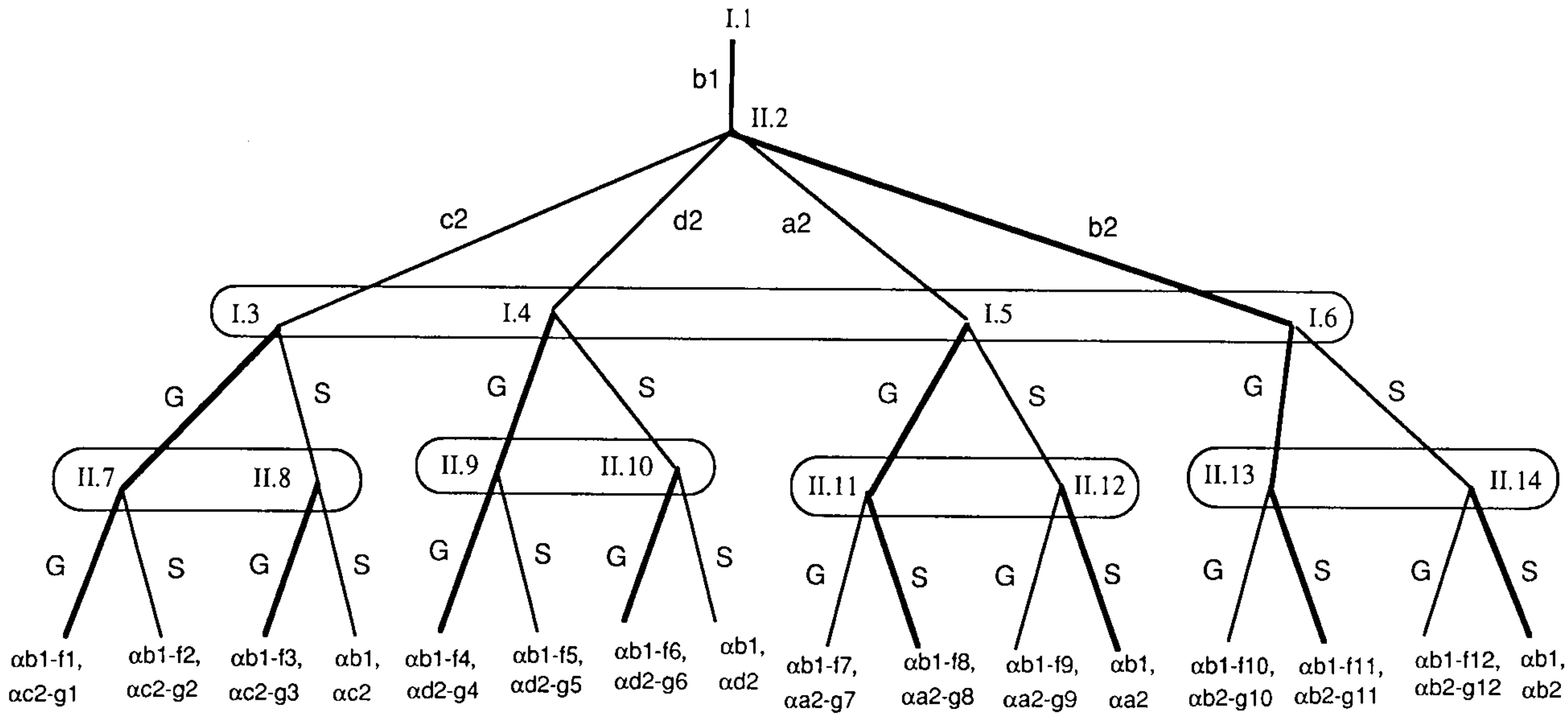
Stage II.C



To get total payoff for the players in this subgame one should increase payoffs of player 1 by c1 and payoffs of player 2 by c2

Figure 8.

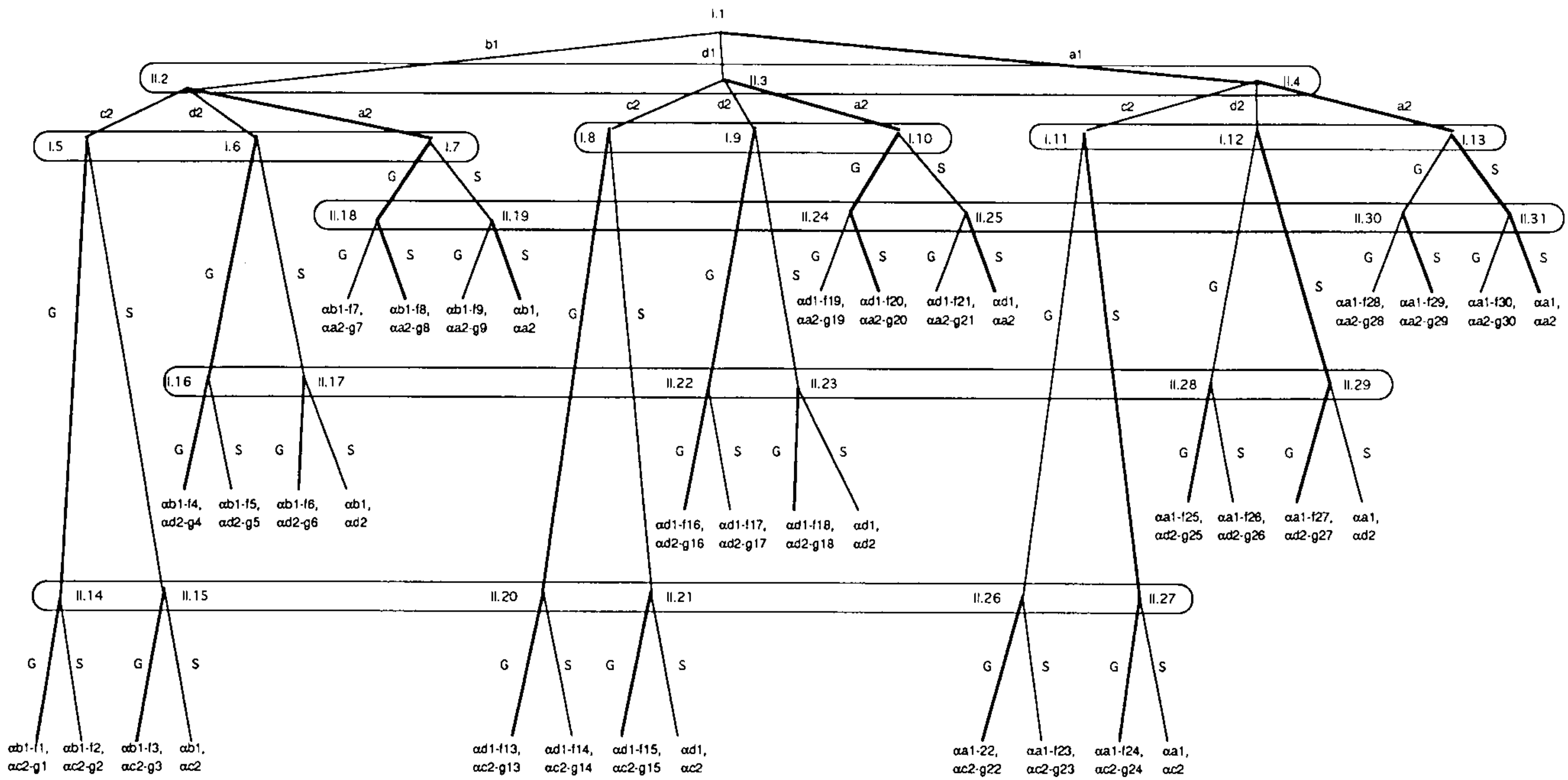
Stage II.B



To get the total payoffs for this subgame one should increase payoff of player I by b1 and payoffs of player II by b2.

Figure 7.

Stage II.A



To get the total payoffs for the players in this subgame one should increase payoffs of player I by a1 and payoffs of player II by a2.

Figure 6.

Stage I

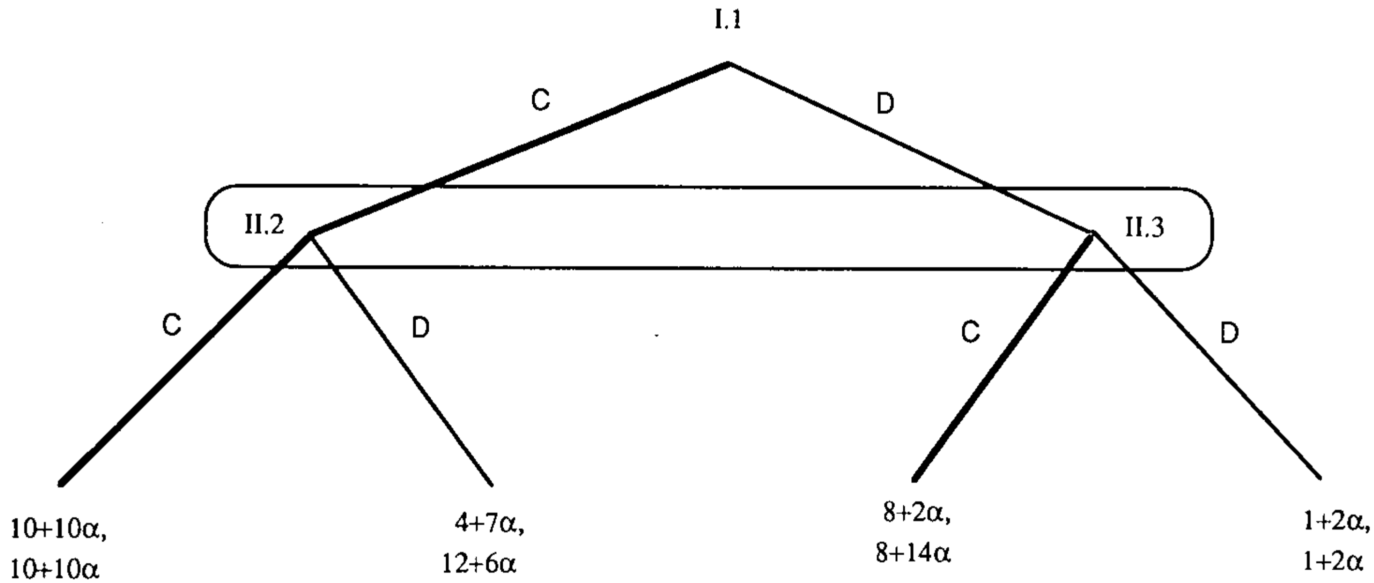


Figure 5.

Stage II.D

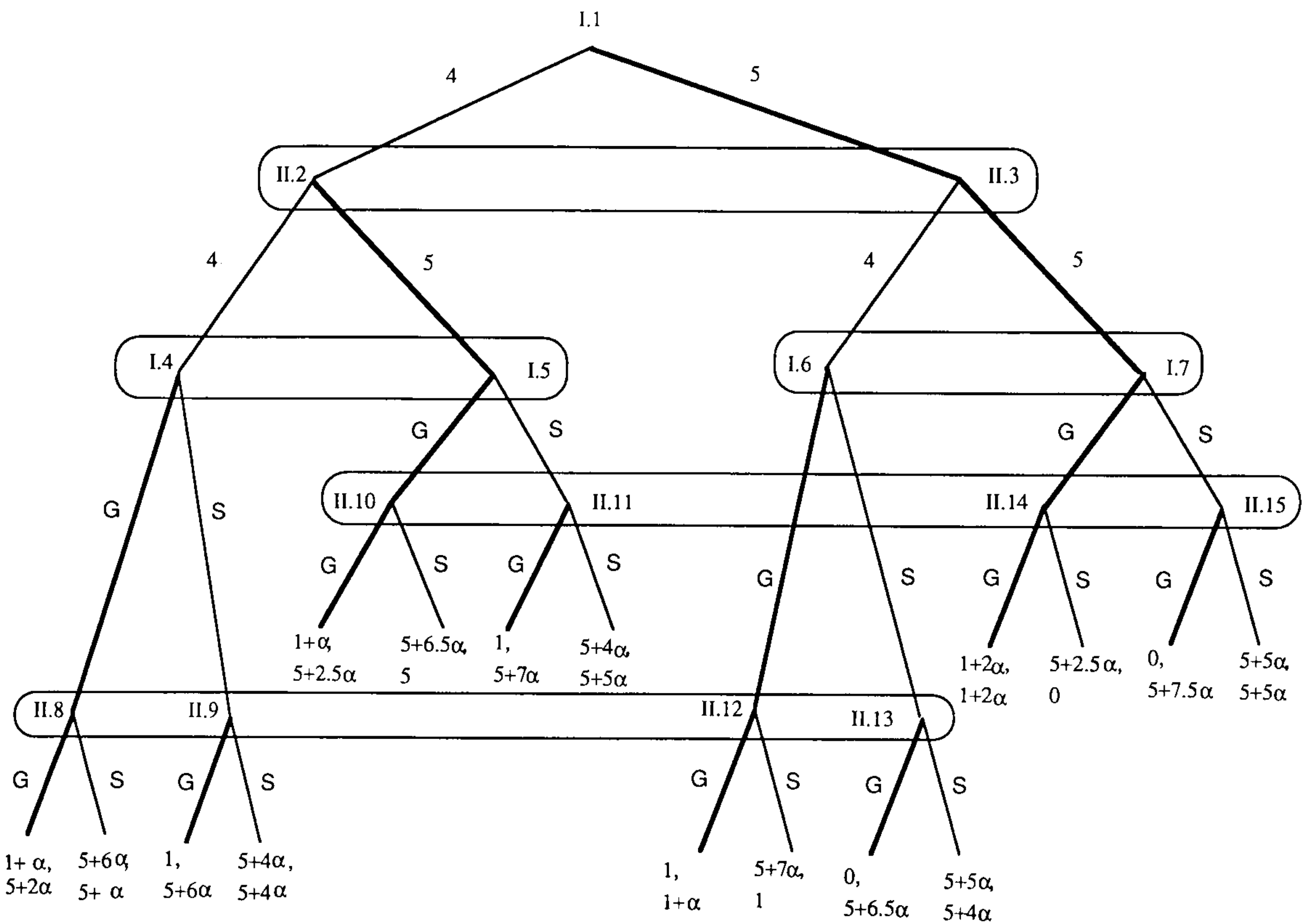


Figure 4.

Stage II.C

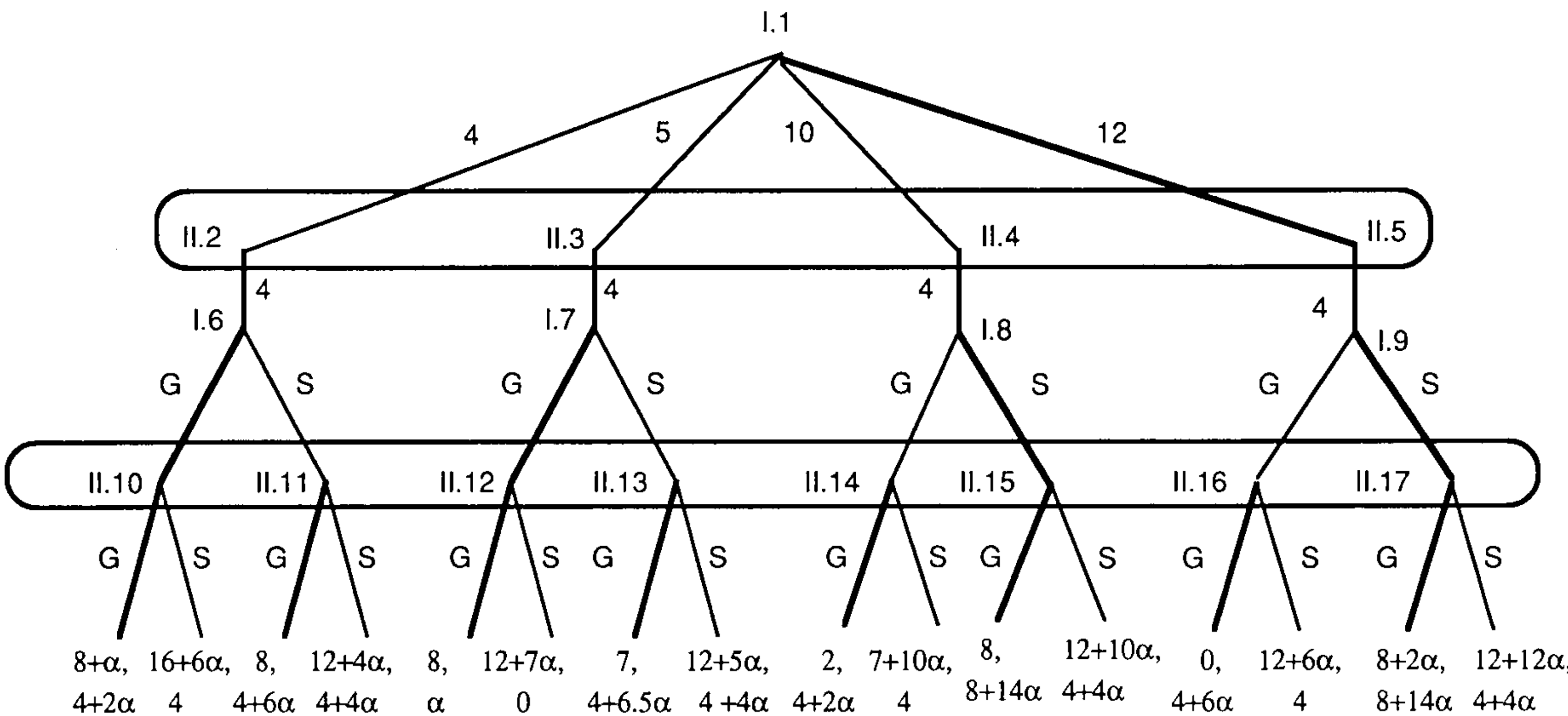


Figure 3.