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CAMPAIGN SPENDING WITH IMPRESSIONABLE VOTERS

by

Rebecca B. Morton* and Roger B. Myerson**

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*Department of Political Science, Texas A&M University, College Station, Texas 77843-4348.

**Department of Managerial Economics and Decision Sciences, Kellogg Graduate School of Management, Northwestern University, Evanston, Illinois 60208.

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Abstract. We consider a model of two-candidate elections with a one-dimensional policy space. Spending on campaign advertisements can directly influence voters' preferences, and contributors give the money for campaign spending in exchange for promised services if the candidate wins. We find that the winner of the election depends crucially on the contributors' beliefs about who is likely to win, and the contribution market tends towards nonsymmetric equilibria in which one of the two candidates has no chance of winning. If the voters are only weakly influenced by advertising or if permissible campaign spending is small, then the candidates choose policies close to the median voter's ideal point, but the contributors still determine the winner. Uncertainty about the Condorcet-winning point (or its nonexistence) can change these results and generate equilibria in which both candidates have substantial probabilities of winning.

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*Department of Political Science, Texas A&M University, College Station, Texas 77843-4348.

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1. Introduction

As candidates compete for votes in an election campaign, they also compete for contributions to finance their campaigns. To develop a theory of elections, we need to understand the effects of both of these levels of competition, and the interactions between the two levels.

It is difficult to develop an intuitively reasonable and logically complete model of elections that integrates fund-raising competition with vote-getting competition. Models with both types of competition generally have difficulty explaining how campaign expenditures affect voting decisions. Typically, campaign expenditures are assumed to play an informational role, either related to the policy position of the candidates or non-policy qualities (like honesty) of the candidates. For example, in Austen-Smith (1987) and Hinich and Munger (1989) campaign expenditures reduce the variance in voter expectations of a candidate's policy position while voters' expectations of the candidate's position are correct and unchanged by expenditures.[1] Yet the variance-reduction technology is not completely satisfactory since it is unclear why a voter whose expectation of the candidate's position is always correct would have uncertainty about that position. In contrast, Cameron and Enelow (1992) assume that the information conveyed is non-policy.[2]

Another approach suggested by Austen-Smith (1990) and following signaling theory in economics, is to view campaign spending (purchasing advertisements in the media) by a candidate as a signal to the voters about his (or her) qualities. In Coates (1989), for instance, the candidate with the greatest expenditures is assumed to have received the most contributions from interest groups that care about the candidate's policy position. Voters can therefore assume that the candidate's position is closest to that of the contributing interest groups since all groups have identical policy preferences in his model. However, if there are contributing interest groups on both sides of the issue, the size of expenditures is no longer a "clear" signal of policy position.

Moreover, if campaign contributions are "service-induced," that is, given in return for private services, as in Baron (1989a,b, 1991), Hinich and Munger (1989), and Snyder (1991), then the signalling approach begs the question, should the primary message from campaign spending be that the candidate who spends the most has also sold the most promises to special-interest contributors? It is hard to imagine that voters would find candidates that provide such favors attractive. If so, then campaign spending should be a negative signal that repels voters, and all candidates should spend nothing in equilibrium.

A different approach is followed in this paper. We allow that voters may be directly affected by campaign spending, at least to some small extent. That is, instead of interpreting campaign spending as a signal of some other utility-relevant variable, we suppose that a voter's utility may depend directly on the advertisements that he has observed. Advertisements can give voters a sense of contact or familiarity with the candidates. We assume here that, other things being equal, a voter would prefer the election of a candidate with whom the voter has a greater sense of such contact or familiarity. Thus, a voter's utility for the outcome of an election may depend directly on the amount that the winning candidate has spent for campaign advertising, as well as on the winning candidate's substantive policy positions. Alternatively, we could justify the influence of campaign expenditures directly upon voter utility as serving an informational role such as discussed above.

In rational-choice modeling, "rationality" is strictly interpreted as meaning that each agent acts consistently so as to maximize the expected value of some utility function. In this strict technical sense, there is nothing "irrational" about an assumption that voters' utility functions can be directly influenced by advertising. Nevertheless, we agree that there is something unappealing about such an assumption. We may be attributing a kind of vulgarity or bad taste to the voters, and it would not be surprising if bad outcomes follow from assuming that agents have such bad taste. To answer such concerns, the utility dependence on campaign spending is expressed here only in a term that is multiplied by a parameter δ, which we explicitly take towards zero in our analysis. Our results show that, even if the effect of advertising on voters' utilities is arbitrarily small, the competition for campaign financing may be crucial to determining the winner of the election.

It should be noted that our placing campaign spending directly into the voter's utility functions is the assumption implicit in the standard service-induced models of Baron (1989a,b, 1991), Baron and Mo (forthcoming) and Snyder (1991). In these models campaign expenditures are generated by providing private services to interest groups and the utility of voters is directly a function of the plurality of campaign expenditures received. However, the interaction between campaign spending and policy position selection is ignored in these models or candidates are constrained *a priori* in policy positions. The implicit assumption is that candidates choose policy positions without anticipating the effect that such positions may have on the campaign contributions generated, or that the policy positions of the candidates are exogenously determined. Essentially, only the subgame after policy positions are taken is analyzed. In this paper we consider the entire game; we find that the campaign contribution subgame has important implications for the policy position taking of the candidates.

2. The basic model

We consider an election with two candidates, whom we number 1 and 2. We suppose that there is a one-dimensional set of government policy options which are denoted by real numbers. During the campaign, each candidate i first chooses a number x_i, which is the candidate's policy position. We assume that the two candidates choose their policy positions simultaneously and independently. Then, after competitively raising campaign funds, each candidate i spends some amount s_i to buy campaign advertising, where s_i is a nonnegative number.

Each voter h has an ideal point θ_h that denotes the government policy option that he (or she) would most prefer. If candidate i wins the election, then the utility payoff to a voter h with ideal point θ_h will be

$$\delta s_i - (x_i - \theta_h)^2,$$

where δ is a given parameter (common to all voters) which is assumed to be greater than zero. So voter h likes the outcome better when the winning candidate has advertised more, and when the winning candidate advocates a policy position that is closer to the voter's ideal point θ_h. Notice that each voter's utility depends on the spending and the policy position of the winning candidate only.

We describe the large population of voters by a measure space, and we assume that voters' ideal points are distributed over some interval. In this section, we assume that this distribution has positive density over some interval, and its median is M. That is, M denotes the median voter's ideal point, and this number M is assumed here to be common knowledge among all candidates and campaign contributors. (In Section 4, we will consider the case where candidates and campaign contributors may be uncertain about the location of the median voter's ideal point.)

After the campaign, when the policy positions (x_1, x_2) and the levels of campaign spending (s_1, s_2) are known by all voters, the election is held, and each voter votes for a candidate. The winner of the election will be the candidate who gets the most votes (i.e., who gets votes from the larger measure of voters). So each voter should vote for his preferred candidate, because doing otherwise would be a dominated strategy for the voter.

Given the positions (x_1, x_2) and the campaign spending levels (s_1, s_2), a voter with ideal point θ_h would vote for candidate 1 only if

$$\delta s_1 - (x_1 - \theta_h)^2 \geq \delta s_2 - (x_2 - \theta_h)^2$$

If $x_1 \leq x_2$, then this inequality is algebraically equivalent to

$$\theta_h \leq \frac{(x_1 + x_1)}{2} + \frac{\delta(s_1 - s_2)}{2(x_2 - x_1)}.$$

Thus, the number

$$\Theta(x_1, x_2, s_1, s_2) = \frac{(x_1 + x_2)}{2} + \frac{\delta(s_1 - s_2)}{2(x_2 - x_1)}$$

divides the set of voters' ideal points into two sets. The voters whose ideal points are less than $\Theta(x_1, x_2, s_1, s_2)$ will vote for the candidate who has chosen the lower policy position x_i, while the voters whose ideal points are greater than $\Theta(x_1, x_2, s_1, s_2)$ will vote for the candidate who has chosen the higher policy position. The voters with ideal point $\Theta(x_1, x_2, s_1, s_2)$ are indifferent between the two candidates, and would be willing to randomly vote for either candidate. (We assume here that there are no costs of voting.) The majority block is the side that includes the median point M. Thus, candidate 1 will win if

$$\delta s_1 - (x_1 - M)^2 > \delta s_2 - (x_2 - M)^2.$$

Candidate 2 will win if

$$\delta s_1 - (x_1 - M)^2 < \delta s_2 - (x_2 - M)^2.$$

On the other hand, if

$$\delta s_1 - (x_1 - M)^2 = \delta s_2 - (x_2 - M)^2$$

then we should expect a close race in which either candidate may win.

In this section, we assume that each candidate's campaign spending is exactly equal to the amount of money that he raises from contributors. That is, campaign contributions are not diverted to the private use of the candidate, and the candidate does not use any of his own funds. Furthermore, we assume that contributors give money to a candidate purely in return for the promise of private favors or preferential government service if the candidate wins. The level and value of preferential government service is assumed to be unrelated to the policy position of the candidate in the unidimensional issue space.[3] The type of campaign contributions postulated are generally called "investor contributions" or "service-induced" contributions. We do not explicitly model campaign contributions that are ideologically motivated; that is given to affect the probability that a preferred candidate wins. Instead we assume that contributors believe that their contribution is not large or significant enough to affect the outcome of the election.

We assume that candidates are constrained to meet pre-election commitments; Baron (1989b) argues that actual campaign expenditures might be loans paid off by contributors once favors are delivered. Following Snyder (1990), we assume that the winning candidate will be able to offer a stock of political services that are worth a total present-discounted value W to their recipients.[4] To raise funds for the campaign, each candidate sells his promises of these services in a competitive market, where contributors buy them as investment goods.[5] So let p_i denote the perceived probability that candidate i will win the election, as assessed at the point in time when the candidates are selling their promised services to raise campaign funds. Then each candidate i will raise funds equal to Wp_i, and so the campaign spending by candidate i will be

$$s_i = Wp_i.$$

Of course, $p_2 = 1 - p_1$ in this two-candidate race, so we may express each candidate's campaign spending in terms of candidate 1's probability of winning:

$$s_1 = Wp_1, \quad s_2 = W(1 - p_1). \tag{1}$$

The candidates raise campaign funds after choosing their policy positions, so each candidate's perceived probability of winning may depend on the policy positions chosen by both candidates. Thus, we may write p_1 as a function of the two policy positions $p_1(x_1, x_2)$, such that

$$0 \le p_1(x_1, x_2) \le 1, \tag{2}$$

for each pair of policy positions (x_1, x_2).

Let us now add the assumption that contributors have rational expectations about the outcome of the election when they buy their service promises from the candidates. Then, for any pair of policy positions (x_1, x_2), candidate 1's win probability $p_1(x_1, x_2)$ must satisfy the following conditions:

$$\text{if} \quad \delta W p_1(x_1, x_2) - (x_1 - M)^2 > \delta W(1 - p_1(x_1, x_2)) - (x_2 - M)^2$$

$$\text{then} \quad p_1(x_1, x_2) = 1; \tag{3}$$

and

$$\text{if} \quad \delta W p_1(x_1, x_2) - (x_1 - M)^2 < \delta W(1 - p_1(x_1, x_2)) - (x_2 - M)^2$$

$$\text{then} \quad p_1(x_1, x_2) = 0. \tag{4}$$

On the other hand, if

$$\delta W p_1(x_1, x_2) - (x_1 - M)^2 = \delta W(1 - p_1(x_1, x_2)) - (x_2 - M)^2$$

then the race is expected to be close, and the contributor's perceived probability of candidate 1 winning $p_1(x_1, x_2)$ could be any number between 0 and 1.

We assume that each candidate chooses his position so as to maximize his probability of winning. That is, candidate 1 wants to maximize p_1, and candidate 2 wants to minimize p_1 (that is, to maximize $p_2 = 1 - p_1$). So in a (pure-strategy) Nash equilibrium, the candidates must choose positions $\bar{x}_1$ and $\bar{x}_2$ at the beginning of the campaign such that, for any other positions x_1 and x_2,

$$p_1(\bar{x}_1, \bar{x}_2) \ge p_1(x_1, \bar{x}_2) \text{ and } p_1(\bar{x}_1, \bar{x}_2) \le p_1(\bar{x}_1, x_2). \tag{5}$$

That is, neither candidate can unilaterally increase his probability of winning by changing his position. Thus, an equilibrium of the election is a specification of positions $\bar{x}_1$ and $\bar{x}_2$ for the two candidates, and a specification of a win-probability function $p_1(\cdot,\cdot)$ such that the conditions (2)-(5) are satisfied for all possible positions x_1 and x_2.

Given any pair of positions (x_1, x_2), we now characterize the probabilities $p_1(x_1, x_2)$ that can satisfy conditions (2)-(4) in the subgame after the candidates have chosen their policy positions. Conditions (2)-(4) have three kinds of solutions that we must consider: candidate 1 may be expected to win for sure; or candidate 2 may be expected to win for sure; or both candidates may have positive probabilities of winning.

We can have a solution in which $p_1(x_1, x_2) = 1$ if and only if

$$\delta W - (x_1 - M)^2 \geq - (x_2 - M)^2,$$

because otherwise condition (4) would force $p_1 = 0$. Similarly, we can have a solution in which $p_1(x_1, x_2) = 0$ if and only if

$$-(x_1 - M)^2 \leq \delta W - (x_2 - M)^2,$$

because otherwise condition (3) would force $p_1 = 1$. Finally, we can have a solution in which $p_1(x_1, x_2)$ is between 0 and 1 only if

$$\delta W p_1(x_1, x_2) - (x_1 - M)^2 = \delta W(1 - p_1(x_1, x_2)) - (x_2 - M)^2,$$

so that the hypotheses of both conditions (3) and (4) are not satisfied. This equation is equivalent to

$$\delta W(2p_1(x_1, x_2) - 1) = (x_1 - M)^2 - (x_2 - M)^2.$$

So there is a solution in which $0 < p_1(x_1, x_2) < 1$ (making $2p_1(x_1, x_2) - 1$ between -1 and 1) if and only if

$$-\delta W < (x_1 - M)^2 - (x_2 - M)^2 < \delta W.$$

Thus, there are three cases to consider, in a subgame after the candidates have chosen positions x_1 and x_2. If

$$\delta W < (x_1 - M)^2 - (x_2 - M)^2 \tag{6}$$

then the only possibility is that candidate 2 is expected to win for sure (that is, $p_1(x_1, x_2) = 0$). If

$$-\delta W \leq (x_1 - M)^2 - (x_2 - M)^2 \leq \delta W \tag{7}$$

then there are three possible equilibrium probabilities in the subgame: candidate 1 may be expected to win for sure ($p_1(x_1, x_2) = 1$), or candidate 2 may be expected to win for sure ($p_1(x_1, x_2) = 0$), or we may have an intermediate solution in which

$$p_1(x_1, x_2) = \frac{1}{2} + \frac{[(x_1 - M)^2 - (x_2 - M)^2]}{2\delta W}. \tag{8}$$

(At the boundaries of the interval in this case, this intermediate solution becomes redundant with one of the other two.) Finally, if

$$(x_1 - M)^2 - (x_2 - M)^2 < -\delta W \tag{9}$$

then the only possibility is that candidate 1 is expected to win for sure ($p_1(x_1, x_2) = 1$). To summarize, we have proven the following theorem.

Theorem 1. In any equilibrium, for any pair of candidates' positions (x_1, x_2), candidate 1's conditional probability of winning given these positions must be either $p_1(x_1, x_2) = 1$ or $p_1(x_1, x_2) = 0$ or

$$p_1(x_1, x_2) = \frac{1}{2} + \frac{[(x_1 - M)^2 - (x_2 - M)^2]}{2\delta W}.$$

Any one of these three equations can hold if the quantity

$$(x_1 - M)^2 - (x_2 - M)^2$$

is between $-\delta W$ and δW. If this quantity is greater than δW, however, then $p_1(x_1, x_2)$ must equal 0. If this quantity is less than $-\delta W$ then $p_1(x_1, x_2)$ must equal 1.

The multiplicity of solutions in the case where condition (7) applies give us multiple equilibria in the first stage where the candidates choose their policy positions. The set of all possible equilibrium outcomes $(\bar{x}_1, \bar{x}_2, p_1(\bar{x}_1, \bar{x}_2))$ is easy to characterize, however. In fact, we can now prove the following theorem.

Theorem 2. There exists an equilibrium in which a candidate chooses the position $\bar{x}_i$ and then has a positive probability of winning the election if and only if

$$M - (\delta W)^{\frac{1}{2}} \leq \bar{x}_i \leq M + (\delta W)^{\frac{1}{2}}.$$

In an equilibrium where both candidates choose positions $(\bar{x}_1, \bar{x}_2)$ that satisfy these inequalities, candidate 1's probability of winning $p_1(\bar{x}_1, \bar{x}_2)$ may be 1 or 0 or

$$\frac{1}{2} + \frac{[(\bar{x}_1 - M)^2 - (\bar{x}_2 - M)^2]}{2\delta W}.$$

Proof. Notice first that a candidate i who wins with positive probability in equilibrium cannot be expected to choose a position $\bar{x}_i$ that is outside the interval

$$[M - (\delta W)^{\frac{1}{2}}, M + (\delta W)^{\frac{1}{2}}],$$

because otherwise the other candidate could win for sure by choosing a position at M. (For example, if $(x_1 - M)^2 > \delta W$ and $x_2 = M$, then condition (6) would be satisfied and so candidate 2 would win.)

Conversely, for any pair $(\bar{x}_1, \bar{x}_2)$ such that

$$M - (\delta W)^{\frac{1}{2}} \leq \bar{x}_1 \leq M + (\delta W)^{\frac{1}{2}}$$

and

$$M - (\delta W)^{\frac{1}{2}} \leq \bar{x}_2 \leq M + (\delta W)^{\frac{1}{2}},$$

we can construct a function $p_1(\cdot,\cdot)$ such that $(\bar{x}_1, \bar{x}_2, p_1(\cdot,\cdot))$ together form an equilibrium satisfying (2) - (5). If $(\bar{x}_1 - M)^2 \leq \delta W$ then no value of x_2 can make the quantity $(\bar{x}_1 - M)^2 - (x_2 - M)^2$ greater than δW, so there exists an equilibrium in which

$$p_1(\bar{x}_1, x_2) = 1, \quad \forall x_2 \neq \bar{x}_2.$$

Similarly, if $(\bar{x}_2 - M)^2 \leq \delta W$ then no value of x_1 can make the quantity $(\bar{x}_1 - M)^2 - (x_2 - M)^2$ less than $-\delta W$, so we can have

$$p_1(x_1, \bar{x}_2) = 0, \quad \forall x_1 \neq \bar{x}_1.$$

We can then let $p_1(\bar{x}_1, \bar{x}_2)$ be 1 or 0 or the intermediate solution in equation (8), and the equilibrium condition (5) will be satisfied. To complete the construction, the function $p_1(x_1, x_2)$ can be defined at all other points (where both $x_1 \neq \bar{x}_1$ and $x_2 \neq \bar{x}_2$) in any way allowed by our subgame rationality conditions (3) and (4). Q.E.D.

The essential idea behind this result is as follows: When both candidates choose policy positions that are within $(\delta W)^{\frac{1}{2}}$ of the median point M, then either candidate could win if he got all the campaign contributions. Thus, the contributors are essentially in a coordination game. Each contributor wants to contribute to the candidate to whom all the other contributors are giving, because no one wants to pay for promises from a loser, and a candidate will lose if no one contributes to his campaign. Such coordination games have multiple equilibria, and the candidates' positions can influence the selection of the focal equilibrium that the contributors realize, in the subgame after the candidate's positions are announced. In the above construction, if one candidate i has deviated from his expected position $\bar{x}_i$ but the other candidate has not, then the contributors focus on the equilibrium in which candidate i is perceived as a sure loser and therefore cannot raise any campaign funds.

The probability function $p_1(\cdot,\cdot)$ is discontinuous in the above construction, but this property is unavoidable, because there is no way to construct a function $p_1(\cdot,\cdot)$ that satisfies conditions (3) and (4) and is continuous everywhere. To prove this impossibility, consider first $p_1(M, x_2)$, as a function of x_2 alone. When $(x_2 - M)^2 > \delta W$ then we must have $p_1(M, x_2) = 1$. When $(x_2 - M)^2 \leq \delta W$ then we can have $p_1(M, x_2)$ equal to either 1 or 0 or $1/2 - (x_2 - M)^2/(2\delta W)$, but the latter two possibilities never approach the value of 1 that is needed outside the interval. Thus, the only continuous function that takes the permissible values is the constant function $p_1(M, x_2) = 1$, for all x_2. A similar argument can be made, however, to show that the only way to make $p_1(x_1, M)$ continuous for all x_1 is to have $p_1(x_1, M) = 0$, for all x_1. Thus, we get a contradiction, because $p_1(M, M)$ cannot be both 0 and 1.

In the case where condition (7) applies, the intermediate solution defined by equation (8) may seem theoretically appealing, because it treats the two candidates more symmetrically. We now show, however, that this intermediate solution cannot be applied in an open neighborhood of an equilibrium outcome.

Theorem 3. There does not exist an equilibrium $(\bar{x}_1, \bar{x}_2, p_1(\cdot,\cdot))$ such that $p_1(x_1, x_2)$ satisfies the intermediate-solution equation (8) for all (x_1, x_2) in an open ball around $(\bar{x}_1, \bar{x}_2)$.

Proof. If the theorem were false, then there would exist an interval such that $\bar{x}_1$ is in the interior of this interval and such that

$$p_1(x_1, \bar{x}_2) = \frac{1}{2} + \frac{[(x_1 - M)^2 - (\bar{x}_2 - M)^2]}{2\delta W}$$

for all x_1 in this interval. But in any region where this equation is satisfied, $p_1(x_1, \bar{x}_2)$ is a strictly convex function of x_1. Candidate 1 wants to choose x_1 so as to maximize p_1, and the maximum of a strictly convex function on an interval cannot be in the interior of the interval. So choosing the position $\bar{x}_1$ could not be optimal for candidate 1 against $\bar{x}_2$. Q.E.D.

In essence, the intermediate solution creates a kind of instability because it makes each candidate want to move his policy position farther away from the median voter's ideal point. In the intermediate solution, the candidates' relative abilities to attract campaign contributions are supposed to counterbalance any differences in their policy appeal to the median voter. Thus, moving a candidate's policy position away from the median voter's ideal point M requires more counterbalancing campaign contributions, which in turn is possible only if the candidate's probability of winning is increased. Because each candidate's objective is to maximize his probability of winning, the intermediate solution implies that each candidate can benefit from unilaterally moving farther away from M. Ultimately such moves must take us to (or beyond) the boundary of the region where the equation (8) is applied. Such analysis may also explain the separation of candidates that Baron and Mo (1991) found in a more general model of electoral competition, because Baron and Mo systematically focused on equilibria that correspond to these intermediate solutions.

Theorem 1 tells us that, when the candidates policy positions are not too far from each other (in the sense of condition (7)), the likely winner may be determined not by the preferences of voters but rather by the collective beliefs of the campaign contributors. In this sense, the most important part of a candidate's strategy may be to develop a strong image in the eyes of potential contributors. Theorem 2 tells us that this incentive to manipulate contributors' perceptions can drive the candidates into picking any pair of positions within some interval around the median voter's ideal point. However, Theorem 1 allowed the possibility that, rather than swinging decisively to one candidate or the other, the contributors might have relatively balanced perceptions of the two candidates. It might have seemed that the possibility of such intermediate perceptions might have allowed for equilibria in which the candidates could be less concerned about the contributors' beliefs. But Theorem 3 tells us that, in any equilibrium outcome, either the contributors are decisively swinging to one of the two candidates, or else there must be some arbitrarily small perturbations of the candidates' positions that would cause the contributors to decisively swing to one candidate or the other. In this sense, Theorem 3 shows that manipulating the contributors' perceptions must be of primary concern to the candidates, in any equilibrium.

The strong conclusions of Theorem 3 may be modified somewhat if we admit the possibility that the contributors' beliefs may also depend on some random variable, which we may call the <u>momentum factor</u>, that is publicly observed only after the candidates choose their policy positions. Rational expectations still implies that, after the contributors get all their information about the policy positions and the momentum factor, they must believe either that p_1 is 1, or that p_1 is 0, or that p_1 is the intermediate value described in Theorem 1. However, the possibility that these beliefs depend on the momentum factor implies that the candidates cannot necessarily predict p_1 when they choose their policy positions. Thus, there can be a pure-strategy equilibrium in which the candidates are expected to choose positions $(\bar{x}_1, \bar{x}_2)$ and in which the intermediate solution would be applied with positive probability at any pair of positions near $(\bar{x}_1, \bar{x}_2)$. To support such an equilibrium, however, the probability of the $p_1 =$ 0 beliefs must be positive and increasing whenever candidate 1 unilaterally deviates from $\bar{x}_1$, and the probability of the $p_1 = 1$ beliefs must also be positive and increasing whenever candidate 2 unilaterally deviates from $\bar{x}_2$. That is, there must be at least a significant possibility that deviations from his predicted campaign position may turn the momentum factor decisively against a candidate in the eyes of his contributors.

The parameter δ represents the degree to which voters are impressionable or manipulable by campaign advertisements. As δ approaches 0, our model approaches one in which campaign spending has only an infinitesimal impact on voters' preferences. However, our results about the decisiveness of the campaign contributors hold for any positive value of δ. The only change that occurs as δ gets smaller (when W, the value of services to contributors, is held fixed) is that the interval of policy positions that candidates may choose in equilibrium becomes narrower. That is, if the voters are less impressionable by campaign advertisements, then the candidates must choose policy positions that are closer to the median voter's ideal point. After the candidates have both taken such close positions, however, the contributors remain the ultimate determinants of who wins the election.

The effects of decreasing W, the value of offered services, are the same as the effects of decreasing δ, because W appears in our analysis only multiplied by δ. So decreasing the value of services that the winner can offer to contributors forces the candidates to take positions in a narrower interval around the median voter's ideal point. After the candidates have both located in this narrower interval, however, the contributors' perceptions remain the crucial decisive factor in determining who wins the election.

Our model can offer a simple explanation of incumbency advantages. We only need to add the assumption that the contributors always act according to a focal equilibrium in which the incumbent's probability of winning is 1 whenever he picks a policy position that is close enough to the median voter's ideal point (i.e., between $M - (\delta W)^{\frac{1}{2}}$ and $M + (\delta W)^{\frac{1}{2}}$. In our model, the contributors' beliefs that the incumbent will win can become a self-fulfilling prophecy.

3. Limits on campaign spending

The above model can be easily extended to consider the effect of legal upper bounds on campaign spending. Notice first that campaign spending affects voters' decisions only through the difference between the utility effects of the candidates' campaign advertising, which is δs_1

$- \delta s_2$. In the basic model above, condition (1) implied that this utility difference could be related to the perceived probability p_1 through the simple formula

$$\delta s_1 - \delta s_2 = \delta W(2p_1 - 1).$$

Now, to introduce a legal maximum on campaign spending, let B denote the maximum amount that each candidate is allowed to spend on campaign advertising, where B is between 0 and W. Then condition (1) in Section 2 must be rewritten as follows:

$$s_1 = \text{minimum}\{Wp_1, B\}, \quad s_2 = \text{minimum}\{W(1 - p_1), B\}.$$

Thus, the relationship between the utility difference $\delta s_1 - \delta s_2$ and candidate 1's perceived probability of winning now becomes

$$\delta s_1 - \delta s_2 = \text{minimum}\{\delta Wp_1, \delta B\} - \text{minimum}\{\delta W(1 - p_1), \delta B\}.$$

As p_1 increases from 0 to 1, this utility difference increases from $-\delta B$ to δB. To achieve a utility difference from advertising that equals

$$(\bar{x}_1 - M)^2 - (\bar{x}_2 - M)^2,$$

which is necessary for an intermediate solution in which both candidates have a positive probability of winning, p_1 must now satisfy the equation

$$(\bar{x}_1 - M)^2 - (\bar{x}_2 - M)^2$$

$$= \text{minimum}\{\delta Wp_1(x_1, x_2), \delta B\} - \text{minimum}\{\delta W(1 - p_1(x_1, x_2)), \delta B\}.$$

Thus, the following generalization of Theorem 2 can be proven by a straightforward extension of the arguments in Section 2.

Theorem 4. Suppose that the upper bound B on campaign spending satisfies $0 < B < W$. Then there exists an equilibrium in which a candidate chooses the position x_i and then has a positive probability of winning the election if and only if

$$M - (\delta B)^{\frac{1}{2}} \leq x_i \leq M + (\delta B)^{\frac{1}{2}}.$$

In any equilibrium, after the candidates choose positions (x_1, x_2) that both satisfy these inequalities, one of the following three conditions must hold:

either $p_1(x_1, x_2) = 1$, or $p_1(x_1, x_2) = 0$, or $p_1(x_1, x_2)$ satisfies the equation

$$(x_1 - M)^2 - (x_2 - M)^2 =$$

$$\text{minimum}\{\delta W p_1(x_1, x_2),\ \delta B\} - \text{minimum}\{\delta W(1 - p_1(x_1, x_2)),\ \delta B\}.$$

So the effect of a bound on campaign spending is very similar to the effect of a decrease in W, in that both force the candidates to take positions closer to the median voters' ideal point, but both still permit multiple equilibria in which contributor's beliefs can decisively determine the winner of the election. Only the intermediate ($0 < p_1 < 1$) solution is somewhat changed. However, the same argument used to prove Theorem 3 can also be applied here to show that the intermediate solutions are unstable, in that they reward a candidate for moving farther from the median voter's ideal point.

4. Uncertainty about the median point

One extreme feature of our basic model is that, once we move away from the unstable intermediate solutions, we find that the campaign contributors are sure which candidate will win, and the other candidate gets no contributions at all. This extreme outcome is moderated when we consider a slightly more complicated model in which there is uncertainty about the median voter's ideal point.[6]

So let us now consider a model that differs from the basic model in Section 2 only in that the candidates and campaign contributors have some uncertainty about the location of the median voter's ideal point. Let us suppose that, when the candidates choose their positions and when the contributors make their campaign contributions, the median voter's ideal point is considered to be a random variable with a normal distribution that has mean μ and variance σ^2, for some given parameters μ and σ. (The model discussed in Section 2 corresponds to the special case where $\mu = M$ and $\sigma = 0$.) Let $\Phi_{\mu\sigma}$ denote the cumulative distribution function for this normal distribution:

$$\Phi_{\mu\sigma}(x) = \int_{-\infty}^{\frac{(x-\mu)}{\sigma}} (2\pi)^{-\frac{1}{2}} e^{-\frac{y^2}{2}} dy.$$

(Here $e = 2.71828\ldots$ and $\pi = 3.14159\ldots$.)

Once the candidates' policy positions (x_1, x_2) and levels of campaign spending (s_1, s_2) are specified, the voting preferences of a voter with any ideal point θ_h can be determined as in Section 2. In particular, a voter with ideal point θ would be indifferent between voting for candidate 1 and voting for candidate 2 if and only if

$$\delta s_1 - (x_1 - \theta)^2 = \delta s_2 - (x_2 - \theta)^2. \qquad (10)$$

The number θ that solves this equation (10) may be called the <u>indifference point</u>. If $x_1 < x_2$ then all voters with ideal points that are less than this indifference point will prefer to vote candidate 1, and so candidate 1 will get a majority if the median voter's ideal point is less than the indifference point. Conversely, if $x_1 > x_2$ then candidate 1 will get a majority (from all voters with ideal points greater than the indifference point) if the median voter's ideal point is greater than the indifference point. Thus, when θ is the indifference point that satisfies equation (10), candidate 1's probability of winning must be

$$p_1 = \Phi_{\mu\sigma}(\theta) \qquad \text{if } x_1 < x_2, \tag{11a}$$

$$p_1 = 1 - \Phi_{\mu\sigma}(\theta) \qquad \text{if } x_1 > x_2. \tag{11b}$$

As in Section 2, equilibrium in the market for campaign contributions (where each candidate raises funds for campaign spending, by selling promises of favors worth W if he wins the election) requires the additional conditions

$$s_1 = Wp_1 \text{ and } s_2 = Wp_2 = W(1 - p_1).$$

Thus, the indifference-point equation (10) is

$$\delta\, Wp_1 - (x_1 - \theta)^2 = \delta\, W(1 - p_1) - (x_2 - \theta)^2.$$

Solving this equation for p_1 in terms of θ gives us

$$p_1 = 0.5 + \left[\theta - \frac{(x_1 + x_2)}{2}\right]\frac{(x_2 - x_1)}{\delta W}. \tag{12}$$

This equation tells us what the contributors to the candidates' campaign funds must believe about the probability of candidate 1 winning, if the difference in the two candidates' campaign spending is to be such that a voter with ideal point θ would be indifferent between the two candidates.

So in any rational-expectations equilibrium, given the candidates' positions x_1 and x_2, the win probability p_1 and the indifference point θ must together satisfy conditions (11ab) and (12). For now, let us consider the case where $x_1 < x_2$, so (11a) and (12) are the conditions for a rational expectations equilibrium.

<u>Theorem 5.</u> Suppose that we are given candidate positions x_1 and x_2 such that $x_1 < x_2$. Then the conditions (11a) and (12) for a rational-expectations equilibrium have at least one solution and have at most three solutions. If there are three solutions, then there is at least one solution such that $\theta > \mu$ and $p_1 > 0.5$, and there is at least one other solution such that $\theta < \mu$ and $p_1 < 0.5$. Furthermore, if x_1 and x_2 are close enough so that

$$\frac{|(\mu + 2\sigma - x_1)^2 - (\mu + 2\sigma - x_2)^2|}{2\delta W} < 0.4772 \quad \text{and}$$

$$\frac{|(\mu - 2\sigma - x_1)^2 - (\mu - 2\sigma - x_2)^2|}{2\delta W} < 0.4772,$$

then there must exist three solutions, including one solution in which p_1 is less than 0.0228, another solution in which p_1 is greater than 0.9772, and a third in which $0.0228 < p_1 < 0.9772$.

Proof. We get a solution with $p_1 = \Phi_{\mu\sigma}(\theta)$ for each θ such that

$$\Phi_{\mu\sigma}(\theta) = 0.5 + \left[\theta - \frac{(x_1 + x_2)}{2}\right]\frac{(x_2 - x_1)}{\delta W}. \tag{13}$$

The right-hand side of this equation is an increasing linear function of θ, with a range of values that covers the whole set of real numbers, but the left-hand side is an increasing continuously differentiable function of θ that is bounded between 0 and 1. Thus, these two functions of θ must be equal for at least one value of θ.

Between any two solutions, there must exist a value of θ where the slope of $\Phi_{\mu\sigma}(\cdot)$ is equal to $\frac{(x_2 - x_1)}{\delta w}$ (which is the slope of the linear function of θ on the right-hand side of (13) above). There cannot be more than two such values of θ, because the slope of $\Phi_{\mu\sigma}(\theta)$ increases monotonically from 0 to $\frac{1}{\sigma(2\pi)^{\frac{1}{2}}}$ as θ increases from $-\infty$ up to μ, and the slope of $\Phi_{\mu\sigma}(\theta)$ decreases monotonically back down to 0 as θ increases from μ to $+\infty$. Thus, there cannot be more than three solutions.

Notice that the right-hand side of (13) can be rewritten

$$0.5 + \left[\theta - \frac{(x_1 + x_2)}{2}\right]\frac{(x_2 - x_1)}{\delta W}$$

$$= 0.5 + \frac{\left[(\theta - x_1)^2 - (\theta - x_2)^2\right]}{2\delta W}.$$

The first inequality in Theorem 4 implies that, when $\theta = \mu + 2\sigma$, the right-hand side of (13) is less than $0.9772 = \Phi_{\mu\sigma}(\mu + 2\sigma)$. But as θ goes to $+\infty$, the left-hand side of (13) goes to 1 while the right-hand side goes to $+\infty$, so there must exist a solution in which $\theta > \mu + 2\sigma$ and $p_1 > 0.9772$. The second inequality implies that, when $\theta = \mu - 2\sigma$, the right-hand side of (13) is greater than $\Phi_{\mu\sigma}(\mu - 2\sigma) = 0.0228$. But as θ goes to $-\infty$, the right-hand side of (13) goes to 0 while the left-hand side goes to $-\infty$, so there must exist another solution in which $\theta < \mu - 2\sigma$ and $p_1 < 0.0228$. Finally, because the difference between the two sides of (13) changes sign between as θ goes from $\mu - 2\sigma$ to $\mu + 2\sigma$, there must exist a third solution in which θ is between $\mu - 2\sigma$ and $\mu + 2\sigma$. Q.E.D.

Thus, when the two candidates choose positions that are close together, we find three very different rational-expectations equilibria, just as in Section 2. If the difference in the candidate's policy positions is small, then small differences in campaign spending can influence many voters, and a belief among contributors that some candidate is much more likely to win can become a self-fulfilling prophecy. Because of the uncertainty about the median voter's ideal point, however, each candidate must always have some positive probability of winning, even in the equilibrium where the campaign contributions are heavily favoring his opponent.

As in Section 2, the multiplicity of equilibria after policy positions are chosen gives us a multiplicity of equilibria in the first stage, when the candidates independently choose their policy positions. To verify whether there is a pure-strategy equilibrium in which any given positions $(\bar{x}_1, \bar{x}_2)$ are chosen by candidates 1 and 2 respectively as follows, it suffices to suppose that, if either candidate i deviated unilaterally from his predicted position $\bar{x}_i$ then the contributors and voters will behave thereafter according to the subgame equilibrium that is worst for candidate *i*. If the intermediate equilibrium at $(\bar{x}_1, \bar{x}_2)$ gives each candidate a probability of winning that is higher than the best that he can get from his worst equilibrium after any deviation, then $(\bar{x}_1, \bar{x}_2)$ can be a pure-strategy outcome at the first stage. (For example, if σ is small enough that $\frac{\sigma}{(\delta W)^{\frac{1}{2}}} < 0.0375$, then any pair of positions between $\mu - .9(\delta W)^{\frac{1}{2}}$ and $\mu + .9(\delta W)^{\frac{1}{2}}$ could be such pure-strategy equilibrium outcomes, essentially because the inequalities in Theorem 4 can be satisfied for any x_1 and x_2 in this interval.) We do not attempt here a general characterization of all subgame perfect equilibria for this model with uncertainty. Instead, we consider now one example which may be viewed as the equilibrium that is the most discriminatory against candidate 1, so that we can evaluate the maximal impact that arbitrarily biased contributors' perceptions can have on the election.

So let us consider here an equilibrium in which, after the candidates' policy positions have been chosen, the contributors will focus on the solution that has the lowest value of p_1, as long as candidate 2's policy position is at the expected median voter's ideal point μ. If x_2 is different from μ, however, the contributor's behavior would shift to the solution in which p_1 is maximal. Thus, at the first stage of this scenario, candidate 2 should locate at $x_2 = \mu$. If candidate 1 chose a position x_1 that was also very close to μ, then the right-hand side of equation (12) would have slope close to 0 (as a function of θ), and so the lowest-p_1 solution to (11ab) and (12) would have θ very far from μ, and thus p_1 would be very close to 0. So candidate 1 must prefer to move some positive distance away from μ.

To find the optimal position for candidate 1 in this scenario, we differentiate equations (11a) and (12) with respect to x_1, keeping $x_2 = \mu$, and we set $\frac{\partial p_1}{\partial x_1} = 0$. (Notice that both θ and p_1 depend on x_1 in these equations.) Differentiating (11a) at the optimum gives us

$$0 = \frac{\partial p_1}{\partial x_1} = \Phi'_{\mu\sigma}(\theta)\frac{\partial \theta}{\partial x_1},$$

and so

$$0 = \frac{\partial \theta}{\partial x_1},$$

because $\Phi'_{\mu\sigma}(\theta)$ is always positive. Then differentiating (12) at the optimum gives us

$$0 = \frac{\partial p_1}{\partial x_1} = -\frac{\left[\theta - \frac{(x_1 + x_2)}{2}\right]}{\delta W} - \frac{(x_2 - x_1)}{2\delta W},$$

which simplifies to

$$\theta = x_1.$$

Thus, when $x_2 = \mu$ and the lowest-p_1 solution is always anticipated in fund-raising stage after the policy positions have been chosen, the optimal position x_1 for candidate 1 must satisfy the equations

$$p_1 = \Phi_{\mu\sigma}(x_1) = 0.5 - \frac{(x_1 - \mu)^2}{2\delta W}.$$

This equation has one solution in the range $x_1 < \mu$, which maximizes p_1 for candidate 1 in this scenario. (A second solution at $x_1 = \mu$ corresponds to a local minimum for p_1.)

To tabulate these solutions, it is helpful to transform our variables. Let us define

$$\alpha = \frac{\sigma}{(\delta W)^{\frac{1}{2}}}, \text{ and } z_1 = \frac{(\mu - x_1)}{\sigma}.$$

Then z_1 and p_1 depend on the parameter α according to the equations

$$p_1 = \Phi_{01}(-z_1) = \frac{(1 - (\alpha z_1)^2)}{2}, \tag{14}$$

where $\Phi_{01}(\cdot)$ denotes the cumulative distribution for the standard normal distribution that has mean 0 and standard deviation 1. Solutions to (14) are shown in Table 1 for selected values of the parameter α. Notice that this crucial parameter α is the ratio of σ, which measures uncertainty about the median voter's ideal point, and $(\delta W)^{\frac{1}{2}}$, which measures the possible distance of equilibrium outcomes from the median voter's ideal point when this point is known.

[INSERT TABLE 1 ABOUT HERE]

When α is substantially larger than 1, then the contributors' uncertainty about the median voter's ideal point is large, and so they assess a substantial probability that the median voter's ideal point may be far from x_2, in the same direction as x_1 is from x_2. In this event, the median voter would care strongly about even small policy changes towards his ideal point, and so he might not be swayed from voting for candidate 1 even if candidate 2 got almost all the money for campaigning. Thus, when α is large, candidate 1 can guarantee himself a substantial probability of winning in any rational expectations equilibrium, by choosing a policy position different from x_2 but close to μ.

On the other hand, if α is close to 0, then the contributors' uncertainty about the median voter's ideal point is small. In this case, as in Section 2, contributors' perceptual biases against either candidate can sustain a rational expectations equilibrium in which this candidate has a very small chance of winning. The rightmost column in Table 1 shows that, as α becomes small, the distance between μ and the optimal position for candidate 1 in this worst-case equilibrium approaches the bound $(\delta W)^{\frac{1}{2}}$ that we found in Section 2.

Given any value of α, if candidate 2's position were known to be anything other than μ, then candidate 1 could still guarantee himself a worst-case probability of winning that is not less than the number in the p_1 column of Table 1, by locating z_1 standard deviations from μ on the side away from x_2. Thus, the equilibria that are summarized in Table 1 are the worst for candidate 1 among all the pure-strategy rational-expectations equilibria of this model.

5. Conclusions

For elections involving two candidates and a one-dimensional policy space, theorists since Hotelling [1929] have observed that the candidates have strong incentives to both pick policy positions that are very close to the ideal point of the median voter. This policy convergence result in the Hotelling model should lead us to expect that other non-policy variables may assume crucial roles in determining which of these two policy-similar candidates will actually win the election. In this paper, we have found that such closeness of policy positions may cause campaign spending on image advertising, and the contributions that fund this campaign spending, to become the decisive factors that determine who wins the election. This result can hold even if such campaign spending has only a weak effect on voters' preferences.

We have also shown that, when contributions from service-motivated contributors are so decisive, the market for campaign contributions may tend towards extreme equilibria in which contributors overwhelmingly favor one candidate over the other. Which candidate will be so favored is then a question of selection among multiple equilibria, which must be understood in terms of Schelling's [1960] focal-point effect. That is, the winner of the election may be determined primarily by some environmental factor (such as incumbency, or lead in an early poll) that can lead the contributors to focus on a self-fulfilling prophecy that one particular candidate will win. Thus, a candidates' most important political activity may be his manipulation of contributors' beliefs by such focal factors, at the beginning of the campaign.

These conclusions depend on the assumption that there exists a known policy position (the median voter's ideal point) that could beat any other position if there were no campaign advertising, and which thus strongly attracts the candidates to choose similar policy positions. In Section 4, we found that uncertainty about the median voter's ideal point can reduce the importance of contributors' perceptions, and can thus also reduce the importance of focal factors like incumbency which affect the contributors' perceptions. With such uncertainty, for any policy position that one candidate might take, the other candidate can find another position such that, with substantial probability, a majority of voters may prefer the latter position strongly enough to vote for it even when the first candidate greatly outspends the second. Similar effects may be expected in multidimensional policy spaces where a Condorcet-winning policy position does not exist. Analysis of the multidimensional case would require consideration of randomized-strategy equilibria, however, which would greatly complicate our analysis.

Our results do have important positive and normative implications about the role of campaign expenditures upon electoral outcomes. First and foremost, our results imply that in elections in which campaign contributions are primarily given for private favors (investor contributions) and the electoral results are strongly influenced by such contributions, then the candidates involved have chosen convergent positions, close to that desired by the median voter. Thus, the existence of campaigns apparently driven by campaign spending may actually indicate electoral efficiency. Secondly, our results suggest that incumbency advantages may simply reflect interest groups' use of incumbency as a focal point of coordination and that incumbents may have significant electoral advantages even when challengers offer not much of a difference in policy position from the incumbent.

$\alpha = \dfrac{\sigma}{(\delta W)^{\frac{1}{2}}}$	$z_1 = \dfrac{(\mu - x_1)}{\sigma}$	p_1	$\dfrac{(\mu - x_1)}{(\delta W)^{\frac{1}{2}}}$
0.3	3.33	.0004	.999
0.5	1.95	.026	.98
1	0.73	.232	.73
2	0.20	.421	.40
4	0.05	.480	.20
8	0.012	.495	.10

TABLE 1. Functions of candidate 1's optimal position x_1 and candidate 1's probability of winning p_1, when candidate 2's position is μ, the lowest-p_1 equilibrium is anticipated in the contributions market, and the median voter's ideal point is a normally distributed random variable with mean μ and standard deviation σ.

ENDNOTES

1. Husted, Kenny, and Morton (1992) find evidence that campaign spending does provide information on candidate's policy positions. In particular, they consider how campaign spending affects the errors voters make in predicting Senatorial positions. They estimate predicted positions by party and state for Senators up for reelection. If the Senator's position is close to the predicted, the Senator's campaign spending significantly reduces errors in voter perception of the Senator's position while the Senator's campaign spending increases voter errors when the Senator is far from the predicted position. Challenger spending has the opposite effect.

2. See Morton and Cameron (1992) for a more extensive review of the empirical and theoretical literature on campaign expenditures.

3. That is, to the extent that favors are rivalrous and affect other contributing interest groups the assumption is that the effects are widespread and not significant enough to induce a response from a particular group or voters. One may argue that voters' utility may be decreasing in the size of the sum of such favors as noted above in our discussion of the signalling approach; such an assumption would simply lower the size of delta in our model.

4. In Baron (1989a,b) candidates are unconstrained a priori in the amount of services they can offer but instead receive disutility from providing services and thus the value from winning declines as the amount of services provided increases.

5. The implied assumption is that the competition in providing "favors" across many elections is such that the demand curve for each candidate for "favors" is perfectly elastic. However, as Morton and Cameron (1992) note, if candidates have monopoly power over the provision of favors once elected then the competition within an electoral contest for contributions might lead to zero campaign contributions in equilibrium unless there is a constraint on the supply of potential services. We assume that the supply is constrained by W.

6. Such uncertainty may result if we assume as in Ledyard (1984) that voters may rationally abstain and that candidates are uncertain about the costs of voting.

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