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On the R and D Process and Decisions

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## ABSTRACT

The R and D process is a non-repetitive stochastic information gathering, processing and utilizing system with learning during which cost is incurred with the expectation of enjoying future benefits. It involves collection of information about some relevant aspects of the world through inquiry, processing it through a discovery phase, translating the processed information into a useful form through a design phase and finally producing new products or processes from the design, each of these activities being stochastic. Selection of the inquiring and production processes and control of the discovery and design phases constitute the decisions that have to be made in the conduct of R and D. The general framework presented here is based on the concepts in the economics of information, management of processing systems and the stochastic control theory and is intended to facilitate a greater understanding of the total R and D process and fruitful solutions to the important decision problems in the broader context.

## On the R and D Process and Decisions

S. D. Deshmukh and S. D. Chikte

### 1. INTRODUCTION

In recent years economists, engineers and management scientists have been interested in the problems related to conduct of basic and applied Research and Development (R and D). Economists have studied the relation between an industry's market structure and the manner in which it invents, accepts and utilizes new technology. Incentive for the socially desirable creative activity depends on the degree of competitiveness in the industry and appropriability of inventions (see Arrow [3], Kamien and Schwartz [15]). For a firm once induced to innovate optimal timing of innovation, which determines the speed with which the creative activity takes place, also depends on the actions of the rival firms seeking a similar discovery (see Barzel [8], Kamien and Schwartz [16], Scherer [25]). Upon creation, the rate at which the innovations are demanded, accepted and utilized in the industry is determined by the follower's learning process. This diffusion process depends on the nature and costs of the communication channels as well as on the expected profitability of adopting the new techniques (see Arrow [4], Mansfield [20], ch. 4). These factors along with the current state of technology and the availability of resources (scientists and capital equipment) form the economic environment within which the R and D manager has to operate. Within this environment the R and D process may be considered as an activity of producing and applying knowledge (see Arrow [3], Brandenburg and Stedry [9], Olivera [24] and Rosen and Souder [25]).

A natural first problem facing the R and D manager is to choose the kind of knowledge his process should produce, taking into account the probability, time and the cost of successfully producing it and its expected value to the environment. This is the problem of selection of R and D projects to be worked on and is usually attacked by management scientists using the capital budgeting methods (see Baker and Pound [7], Freeman and Gear [12], Weingartner [27] and references therein).

Upon determining the optimal set of proposed projects, the next problem is the selection of an approach to (i.e. a method of completing) each project. Here the parallel strategy of carrying on more than one approach to a project simultaneously and sequentially eliminating all but one of them has been stressed. Such a strategy provides better information for selection, maintains options and hedges against the uncertainty about the project's outcome, (see Abernathy and Rosenbloom [1], Marschak, Glennan and Summers [21], ch. 5 and Nelson [23]).

Once an approach has been selected for each project the problem of planning the allocation of resources over the entire time horizon for an individual project has been attacked using the optimal control theory, (see Kamien and Schwartz [16] and Lucas [19]).

As the projects proceed, learning takes place, uncertainty about its outcome is reduced and the resources have to be reallocated on the basis of the current information till the projects are complete. For this purpose a stochastic control policy has to be formulated (see Gittins [13] and Hess [14]). Also, depending on the environmental factors such as competitors' actions, it may be better to market the knowledge produced before

the completion of the project. This determines the optimal development stopping policy (see Deshmukh and Chikte [9]).

The solution of this sequence of problems forms the set of interrelated decisions which the R and D manager has to make, the choice of each decision affecting feasibility and effectiveness of others. The objective of this paper is to provide a unified conceptual framework within which the R and D process and the managerial decisions in face of various uncertainties involved may be analyzed. The framework is based on the concepts in the theory of management of processing systems (see Deshmukh, Murphy and Tuite [10]), economics of information systems (see Marschak [22]) and stochastic control theory (see, for example, Aoki [2], Åström [6] and Kushner [18]).

## 2. THE R AND D PROCESS

The objective of research is to produce information so as to reduce uncertainty about the true state of the environment by making and processing an observation. Development involves translation of the research findings into concrete specifications of actions (e.g. blueprints) and then actual production yielding new products or processes. For example, making physical, chemical and optimal observations yields some understanding of the atomic structure of Uranium. Based on this information constructing theories and physical models and testing them by controlled experiments is a discovery phase. Designing an atomic reactor for producing useful energy is a design activity followed by its actual fabrication phase resulting in the technological advance. Similarly, understanding an existing economic structure, by collecting relevant data is the inquiring activity, mathematically

modeling it in competitive framework, say, and testing the validity of the model by econometric methods is a discovery activity, while economic policy making based on this model is a design activity and its implementation is analogous to the production activity. Thus the entire process of a technological advance consists of inquiring or data collection to yield relevant information, processing it by the discovery activity, translating the processed information into designs by the design activity and finally producing concrete products and processes according to the designs. The inquiry and discovery phases together may be called the research activity, while the design and production phases constitute the development activity. Though in reality the above distinction is somewhat fuzzy, such a separation seems natural and convenient for our purposes. The total process requires the use of capital (e.g. experimental equipment) and labor (e.g. skilled scientists) resources, considerable uncertainty involved in its operation being its significant feature.

Let  $\mathcal{A}$  be the set of all possible states of the environment  $S$  to be interpreted broadly to include all phenomena investigated and possibly controlled as well as those affecting the R and D process itself including the present technology. Let  $\mathcal{B}$  be the  $\sigma$  field of subsets of  $\mathcal{A}$  of interest. In general, denoting by  $S_t$  the state at time  $t$ ,  $\{S_t : t \in (-\infty, \infty)\}$  is a stochastic process; usually the aspects of the environment (i.e. components of  $S_t$ ) being investigated do not change appreciably over the time span of the R and D process. Let  $Q^*$  be the probability measure on  $(\mathcal{A}, \mathcal{B})$  representing the current knowledge about the environment.

Let  $\mathcal{A}$  be the set of all actions that could be produced in face of the environment, where an action  $A \in \mathcal{A}$  is interpreted as a new technology (i.e. new products or processes). Let  $\mathcal{A}$  be a  $\sigma$  field of subsets of  $\mathcal{A}$  of interest. Information about  $S$  is sought through the research process, information for producing a response  $A$  to  $S$  is sought through the development process, while  $S$  itself affects and could in turn be affected by the entire research and development process. For example, meteorological research itself could be affected by bad weather, while at the same time it could lead to the development of methods for modifying weather patterns such as artificial rain making. Similarly, in the armament race the other nations' actions significantly affect and are also affected by our own R and D efforts in the field of strategic arms.

Information about true  $S$  is the input to as well as the output of the discovery process. Though information as an economic commodity has peculiar properties (see Arrow [3]), for our purposes it can be characterized by its attributes of form and quantity of the information and time and place at which the information is available. The form of a piece of information represents that part of the state of the environment it seeks to describe and details of the description. Typically, the form of information specifies a partial description of true  $S$ . The form is specified by a measurable partition  $X$  of  $\mathcal{A}$  according to the component (s) of the environment and their details that information refers to; thus each set  $x$  in a partition  $X$  is a cylinder over a complete description of specific environmental components of interest. For example, if the information refers to the tensile strength of a metal, a typical set  $x$  in the partition  $X$  would specify those  $S$ 's

which describe the tensile strength of the metal to be  $500 \pm 10$  psi plus all other possible irrelevant data about the environment. More components of  $S$  of interest and/or more detailed description of the components would mean a finer partition of  $\mathcal{A}$  and correspond to a more complete information about true  $S$ . Let  $\mathcal{X}$  be the set of all partitions  $X$ .

For a given form  $X$ , the quantity of information about that form can be identified with a probability distribution  $Q(\cdot)$  over  $\mathcal{F}$ , the smallest  $\sigma$  field containing all sets in that partition  $X$ . This specifies the amount of uncertainty about the relevant aspect of the true state of the environment and is determined by the current state of knowledge regarding that aspect of the environment and possible imperfections (noise) in the inquiring system. For example, if the inquiry is noiseless, then on sets in  $X$  we have  $Q = Q^*$  which is then extended to the probability  $Q$  defined on  $\mathcal{F}$ . For example, the tensile strength of the above metal is  $500 \pm 10$  psi with probability  $Q(x)$ , thus defining a probability distribution of the tensile strength. Let  $\mathcal{Q}$  be the set of all such probability measures defined on measurable subsets of  $X$ 's. Thus the form and quantity attributes refer to the completeness and noiselessness of the information.

Finally, time and place at which the information is available also affects its value. The earlier a piece of information is available the greater is its value, due to the discount rate. This time utility can be enhanced by increasing the speed with which research takes place by stepping up the rate of expenditure of resources. Let the time  $T$  at which the information is available be an element of the set  $\tau = (-\infty, \infty)$ . Similarly, the information available at a place where it can be developed into information



leading to production of an economically useful product or a process is valuable. Such a place utility can be increased through a diffusion process utilizing superior communication channels. Let the place  $P$  at which the information is available be an element of the set  $\theta$ .

Thus a piece of information is an element  $I = (X, Q, T, P)$  of the product space  $\mathcal{I} = \mathcal{X} \times \mathcal{Q} \times \mathcal{T} \times \theta$  with its product  $\sigma$  field  $\mathcal{X}$ . The details of interest in  $S$  are summarized in the first component of  $I$ , inherent uncertainty about the true  $S$  with respect to those details is represented by the second component of  $I$ , while the third and the fourth component of  $I$  show the time and place where this information is available. These four will be called the information attributes.

Information  $I$  about the state of the environment  $S$  is collected through an inquiring (data collection) activity  $\epsilon \in \mathcal{E}$ , which includes partitioning  $\mathcal{I}$ , making observations, computing posterior  $Q$  and transmitting it for use in the  $R$  and  $D$  process at a desired time and place. Due to the noise involved in the inquiry process  $\epsilon$  is a transition probability from  $\mathcal{I}$  to  $\mathcal{I}$ , i.e. for fixed  $S \in \mathcal{I}$ ,  $\epsilon(S, \cdot)$  is a probability measure on  $\mathcal{X}$  and for any  $B \in \mathcal{X}$   $\epsilon(\cdot, B)$  is  $\mathcal{B}$ -measurable.

The discovery process is defined to be a transition probability  $\eta$  from  $\mathcal{I}$  to  $\mathcal{I}$ , so that, for any input information  $I \in \mathcal{I}$ , the processed information is determined according to the probability distribution  $\eta(I, \cdot)$  over  $\mathcal{X}$ . The transition probability  $\eta$  depends on the characteristics of the scientific equipment and researchers utilized. These capital and labor resources are tapped from the current technology and manpower available as a part of the environment. The inherent uncertainty regarding the form, quantity, time and

place of the output information, for given input information and resources, is captured in  $\eta$ . Let  $\mathcal{N}$  denote the set of all feasible discovery processes attainable with the available resources. The pair  $(\epsilon, \eta)$  may be called the research process denoted by  $\mu$ . (In practice, it is sometimes difficult to draw a line between inquiry and discovery.) Given  $S \in \mathcal{S}$  and  $E \in \mathcal{E}$ , we have  $\mu(S, E) = \int \epsilon(S, dI) \eta(I, E)$ .

The capital equipment and scientists used in this discovery process have certain capability for processing different forms of information (e.g. a physicist with a nuclear reactor can handle atomic energy problems but is unable to discover properties of an organic compound). The output form may be different from the input form due to possible unexpected discoveries, e.g. nylon was accidentally discovered during the research study of condensation of polymers. The resources used also have certain capacity for processing different quantities of information by reducing its uncertainty (e.g. an inexperienced chemist without a computer can not obtain very precise knowledge about a drug), the exact amount of reduction being unknown. The speed with which they can process the information affects the time at which the output information is available. Finally, their transmission and communication abilities determine the location of the information diffusion. At any time  $t$  let  $K_t$  and  $L_t$  denote the capability, capacity, speed and communication ability of capital and labor resources. These will be called the information processing characteristics of the resources. Thus, the information processing characteristics of the resources and the way in which they are utilized determine the manner in which the attributes of the input information are transformed into those of the output information. Let  $I' = (X', Q', T', P')$  denote the output of the discovery process.

During this random transformation process the capital equipment depreciates due to wear and tear, while the labor skills appreciate due to learning, in terms of changed capability, capacity, speed and transmission ability. Given the input information  $I_{t-1}$  the amount of learning that takes place during a particular stage in the process depends upon the information output  $I_t$  of that stage and enhances the capability, capacity, speed and communication ability of human resources to process  $I_t$  at the next stage. The better the performance of the given stage in terms of producing information with better form, quantity, time and place attributes the more is the learning during that stage. The transition probability with respect to all the four components of  $I_t$  depends on the cumulative learning through  $t$ . During the course of a typical R and D project learning is erratic because at each stage the input-output relationship is stochastic. (If this relationship were deterministic, then this concept of learning would coincide with the usual one in a repetitive production process, where it is a function of the number of repetitions of the activity or, equivalently, the amount of time spent repeating the activity (see Arrow [5])). While learning improves the information processing characteristics of labor resources, those of the capital resources deteriorate due to depreciation during processing. Thus  $\eta$  evolves and is controlled through time, stochastically transforming not only the input information but also itself, exhibiting a higher level of adaptation. This peculiarity of the research process seems difficult for mathematical modeling. Nevertheless, a conceptual model will be outlined in the next section.

Once the information with modified form, quantity, time and place attributes has been produced, it serves as the input to the subsequent design process, which takes this information about the environment and translates it into information about new products and processes.

This information emerging out of the design process also has the form, quantity, time and place attributes. The form of this information is the type and the details of the proposed technological change. The form may be described by a measurable partition  $Y$  of  $\mathcal{A}$  according to the type and details of new products and processes that information refers to, i.e. each set  $y$  in  $Y$  is a cylinder over the complete description of specific products and processes to be produced. For example, a set  $y$  in  $Y$  corresponds to a complete physical and chemical description of an electronic component, disregarding other products and processes. Let  $\gamma$  be the set of all measurable partitions of  $\mathcal{A}$ . For a given form  $Y \in \gamma$ , the quantity of information about that form can be summarized in a probability distribution  $R(\cdot)$  over  $\mathcal{L}$ , the  $\sigma$  field generated by the sets in  $Y$ . This specifies the amount of uncertainty about the concrete new product or process and depends upon the quality of the design in terms of the amount of the technological advance possible, as measured by the expected value of the benefit function to be described in the next section. For example,  $R$  concentrated on a "highly desirable" new technology for producing solar energy would mean a good design of the advance with respect to efforts for finding new energy sources. Let  $\mathcal{R}$  be the set of all such probability measures. Finally, the time  $T''$  and place  $P''$  at which this information is available with respect

to the production process also affect its ultimate value, where  $T'' \in \tau = (-\infty, \infty)$  and  $P'' \in \theta$  as before. Thus, the output information regarding  $\mathcal{A}$  resulting from the development process is an element  $J = (Y, R, T'', P'')$  of the product space  $\mathcal{J} = \mathcal{Y} \times \mathcal{R} \times \tau \times \theta$  with its product  $\sigma$  field  $\mathcal{L}$ .

The design process is defined to be a transition probability  $\gamma$  from  $\mathcal{J}$  to  $\mathcal{J}$ . Thus, given the information  $I' \in \mathcal{J}$  about the environment obtained by the research process, the design process yields information about new products and processes according to the probability distribution  $\gamma(I', \cdot)$ , symbolizing the uncertainty in the design process. Let  $\Gamma$  be the set of all such design processes attainable with the available resources. As in the case of discovery the transformation  $\gamma$  evolves stochastically through time and depends upon the capability, capacity, speed and the communication ability of resources (capital equipment and engineers) and the manner in which they are utilized.

Let  $\alpha$  denote the physical transformation of the information  $J$  into a concrete new technology. Due to the uncertainty in the production process,  $\alpha$  is a transition probability from  $\mathcal{J}$  to  $\mathcal{A}$ . Let  $\mathcal{X}$  be the set of the available production processes. The combined design and production process  $(\gamma, \alpha)$  may be called the development process  $\nu$ . (Again, in reality, the distinction between  $\gamma$  and  $\alpha$  is blurred.) Thus,  $\nu$  is a transition probability from  $\mathcal{J}$  to  $\mathcal{A}$  defined by  $\nu(I', B) = \int \gamma(I', dJ) \alpha(J, B)$  where  $I' \in \mathcal{J}$  and  $B \in \mathcal{A}$ .

Thus the research process  $\mu = (\epsilon, \eta)$  stochastically produces useful information  $I' \in \mathcal{J}$  by inquiry  $\epsilon$  and discovery  $\eta$ . The development process  $\nu = (\gamma, \alpha)$  stochastically makes use of this information to produce technological advance  $A \in \mathcal{A}$  by design  $\gamma$  and production  $\alpha$  (see Fig. 1).

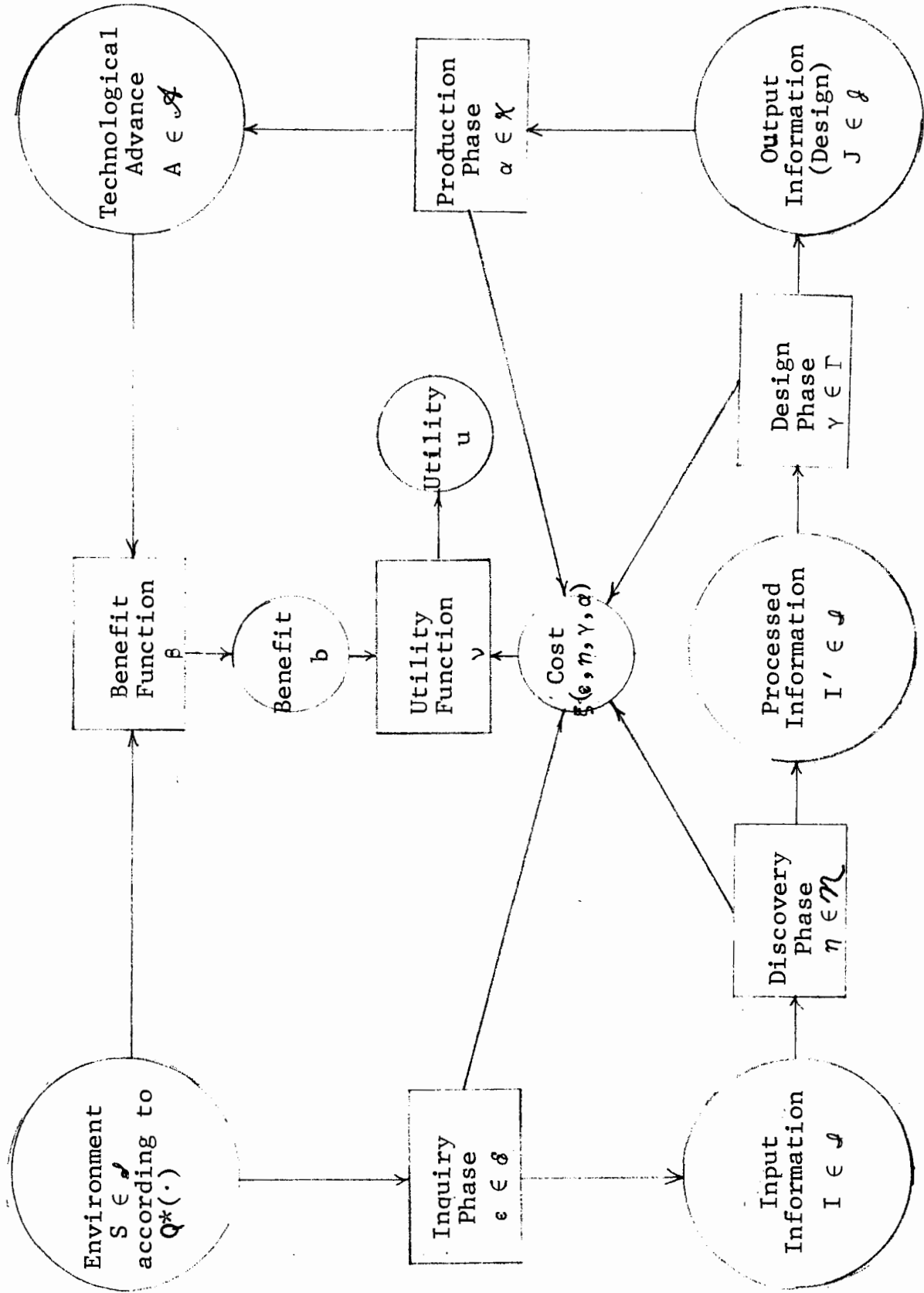


Fig. 1. The Process of R and D

It is usually reasonable to assume that the inquiry and production activities,  $\epsilon$  and  $\alpha$ , consume negligible time in comparison with the discovery and design activities  $\eta$  and  $\gamma$ . The next section considers the decisions affecting  $\mu$  and  $\nu$  with respect to a given payoff function.

### 3. THE R AND D DECISIONS

The benefit from a technical advance depends on the present state of knowledge and technology and the true unknown aspects of nature. For example, the value of developing a rotary auto engine today is much lower than it would have been before Ford's invention of his first model. Similarly, if the engine design is based on inaccurate estimate of the true properties of the metal it will be less valuable. The economic value of basic research by itself is difficult to estimate; therefore it is assumed that the output of R and D is of economic value only if it culminates in a concrete new product or a process. The value may include, for example, the reduction in the production cost due to the discovery of a more efficient process or increase in profit as a result of an improved product. The value of a technological advance is thus represented in terms of a measurable benefit function  $\beta: (\mathcal{A}, \mathcal{B}) \times (\mathcal{C}, \mathcal{D}) \rightarrow E^1$ ,  $E'$  being the real line with its Borel  $\sigma$  field. This is a natural generalization of a binary valued benefit function resulting in "success" or "failure" of a "project" often considered in the literature.

The total expected cost of a technological advance to be denoted by  $\xi(\epsilon, \eta, \gamma, \alpha) = \xi_1(\epsilon) + \xi_2(\eta) + \xi_3(\gamma) + \xi_4(\alpha)$  consists of the cost  $\xi_1(\epsilon)$  of carrying out the data collection and inquiring

activity  $\epsilon$ , the cost  $\xi_2(\eta)$  of the discovery activity  $\eta$ , the design activity cost  $\xi_3(\gamma)$  and the production and implementation activity cost  $\xi_4(\alpha)$ . For example, fine observations about several aspects of the environment requires sophisticated and expensive measuring instrumentation. Improved and quick results of the R and D process require a commitment of larger budgets and superior scientific personnel. Increasing capability, capacity, speed or communication ability of resources increases the cost of the R and D process, while yielding better attributes of the output information. Similarly, translating detailed specifications of a new technology into several new products and processes necessitates the use of precise and costly production methods.

The preferences over the above sets of benefits and costs are represented in terms of the utility function  $\kappa$  yielding utility  $\kappa(\beta(S,A), \xi(\epsilon, \eta, \gamma, \alpha))$ . If the R and D process takes place according to  $(\epsilon, \eta, \gamma, \alpha)$ , then the expected payoff is given by  $u(\epsilon, \eta, \gamma, \alpha) =$

$$\int \int \int \int \kappa(\beta(S_{T''}, A_{T''}), \xi(\epsilon, \eta, \gamma, \alpha)) \mu(S_T, dI') \nu(I' dA_{T''}) \\ P^*(S_T, dS_{T''}) Q^*(dS_T)$$

where  $I = (X, Q, T, P)$ ,  $I' = (X', Q', T', P')$  and  $P^*$  is the transition probability distribution governing the stochastic process  $\{S_t\}$ .

If the expected payoff is assumed to be increasing in the expected benefit and decreasing in the expected cost, it can be shown (see Marschak [22]) that the utility function is decomposable into



$$\begin{aligned} \kappa(\beta(S,A), \xi(\epsilon, \eta, \gamma, \alpha)) &= \beta(S,A) - \bar{c}(\xi_1(\epsilon) + \xi_2(\eta) \\ &\quad + \xi_3(\gamma) \\ &\quad + \xi_4(\alpha)) \end{aligned}$$

so that the expected payoff can be expressed as

$$\omega(\epsilon, \eta, \gamma, \alpha) = B - \bar{c}K$$

where B is the expected benefit, K is the expected cost and  $\bar{c}$  is a constant.

The objective is to maximize this expected payoff by sequentially making decisions that affect the transition probabilities  $\epsilon$ ,  $\eta$ ,  $\gamma$  and  $\alpha$ , where at each stage the cost is incurred and at the end of the entire process the benefit is collected. (This feature is shared by models of economic growth, investment in R and D being made at the cost of foregoing present consumption in anticipation of a superior technological advance.)

The decisions regarding the inquiring system  $\epsilon$  and the production process  $\alpha$  involve only their selection from the sets  $\mathcal{E}$  and  $\mathcal{X}$  of the available ones.

On the other hand, the discovery process  $\eta$  and the design process  $\gamma$  evolve through time as a result of controlling  $(K_t, L_t)$ , the capability, capacity, speed and communication ability of the resources and changes in these due to learning and depreciation taking place during the process

Consider the discovery process that is being observed and controlled at discrete points in time, for simplicity. At any time t the information processing characteristics of the resources

$(K_t, L_t)$  and the attributes of the information being processed  $I_t$  constitute the state of the system  $(K_t, L_t, I_t)$ . Based on this a decision  $d_t \in D$  has to be made at time  $t$  using a decision rule  $\delta_t$ , so that  $d_t = \delta_t(K_t, L_t, I_t)$ . The decision is regarding a change in capability (e.g. hiring a consulting specialist), capacity (e.g. renting a larger computer or laying off scientists), speed (e.g. increasing laboratory work hours) and communication ability (e.g. movement of scientific personnel for delivering lectures, attending conferences and seminars, encouraging research publications etc.). In particular, the decision  $d_t$  indicates whether to continue the discovery process or to stop it by reducing the speed of resources to zero and feed the output information into the subsequent design process. The decision by itself does not determine the new information processing characteristics  $(K_{t+1}, L_{t+1})$  due to learning and depreciation effects. As indicated in the previous section learning and depreciation taking place during the stage depends on  $I_t$  and  $I_{t+1}$ , which together with the decision  $d_t$  yield new capital and labor characteristics. Symbolically, the system dynamics may be written as

$$(K_{t+1}, L_{t+1}) = \phi_t(K_t, L_t, I_t, d_t, I_{t+1})$$

$$I_{t+1} = \psi_t(K_t, L_t, I_t)$$

where  $\phi_t$  and  $\psi_t$  denote stochastic transformations. These stochastic transformations compounded together over time yield what we have called  $\eta$ , the discovery process that stochastically transform  $I$  into  $I'$ . The cost incurred during this stage of the process is denoted by  $\xi_{2t}(K_t, L_t, I_t, d_t)$  which includes the cost of making a change in

$(K_t, L_t)$  by selecting the decision  $d_t$  as well as the cost of processing the information  $I_t$  using  $(K_t, L_t)$ . The total expected cost incurred during the discovery process using policy  $\Delta = \{\delta_t : t \geq T\}$  is

$$\xi_2(\eta) = E_{\Delta} \left[ \sum_{t=T}^{T'} \xi_{2t}(K_t, L_t, I_t, d_t) \mid K_T, L_T, I_T = I \right]$$

The dynamics, costs and the related decisions of the design process  $\gamma$  may be described similarly.

Thus the decisions during the course of an R and D process involve selection of the inquiring system  $\epsilon$ , control of the discovery and the design processes  $\eta$  and  $\gamma$  over time and finally the selection of the production process  $\alpha$ . Due to uncertainties characteristic of an R and D process and the sequential and adaptive nature of decisions involved, the stochastic control theory and the dynamic programming formulation provide a natural framework for making these decisions optimally. For each  $J \in \mathcal{J}$  let  $V_4(J)$  be the maximum payoff obtained beyond the design stage by selecting the best production technology  $\alpha_J^*$ , i.e.

$$V_4(J) = \text{Max}_{\alpha \in \mathcal{X}} \left[ - \xi_4(\alpha) + \iiint \alpha(J, dA) \right. \\ \left. \begin{array}{l} \mathcal{B}(S_{T''}, A) \\ P^*(S_T, dS_{T''}) \quad Q^*(dS_T) \end{array} \right].$$

At any time  $t$  during the design phase let  $V_{3t}(K_t, L_t, J_t)$  be the optimal payoff from  $t$  onwards, given the state of the system  $(K_t, L_t, J_t)$ , satisfying the recursive relation

$$V_{3t}(K_t, L_t, J_t) = \text{Max}_{d_t \in D} \left\{ - \xi_{3t}(K_t, L_t, J_t, d_t) + \right. \\ \left. E \left[ V_{3(t+1)}(K_{t+1}, L_{t+1}, J_{t+1}) \mid K_t, L_t, J_t, d_t \right] \right\}$$

where the conditional expectation is computed using the stochastic relations  $(K_{t+1}, L_{t+1}) = \phi'_t(K_t, L_t, J_t, J_{t+1}, d_t)$

and  $J_{t+1} = \psi'_t(K_t, L_t, J_t)$ .

and  $V_{3T''}(K_{T''}, L_{T''}, J_{T''}) \stackrel{\Delta}{=} V_4(J_{T''})$

where  $T''$  is the optimal stopping time of the design process. Similarly, the optimal value functions  $V_{2t}$  and  $V_1$  associated respectively with the discovery and inquiry phases can be defined. The recursive relations for the optimal decisions during the inquiry and the discovery phases then follow a similar pattern and will not be elaborated here. Specific structural properties of the optimal decision rules could be derived from the above formulation by assuming specific properties of the system dynamics and the cost and benefit functions, and will be taken up elsewhere.

We may relate the decisions usually considered in the literature to those in the above formulation as follows and summarize them in Figure 2. Selection of the inquiring system  $\epsilon$ , the first decision in our model may be called the project selection problem, for it determines the kind of information (i.e. types of projects) that will be processed. However, due to the sequential nature of the process, this decision cannot be made optimally until we know that the succeeding ones will be made optimally. Given  $I = (X, Q, T, P)$  the next problem of determining the initial characteristics of the resources  $(K_T, L_T)$  may be called the approach selection problem, for it determines the manner in which the information will be processed. Given the initial state  $(K_T, L_T)$

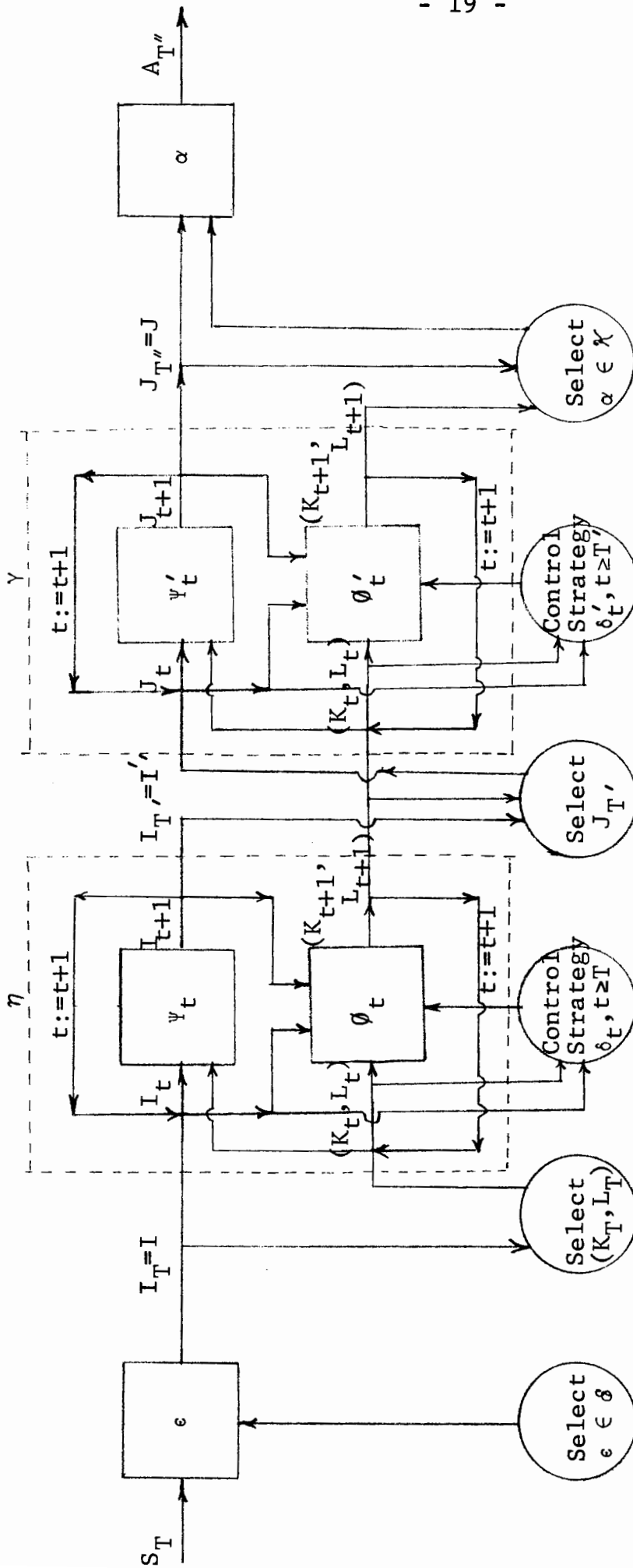


Fig. 2. The Dynamics of R and D Decisions

the problem of optimally controlling the discovery and design processes through time is the project scheduling and control problem, since it determines how resources are allocated dynamically during the course of the project, taking into account the current environment, progress, uncertainties, learning and depreciation. Finally, given the output design  $J$ , selection of  $\alpha$  indicates the choice of production methods to translate the design into a concrete advance. All of these being interrelated decisions optimal choice of one must involve taking others into consideration. Also, due to the stochastic nature of the discovery and design phases, only dynamic resource allocation seems natural. Thus, for example, planning the total resource allocation and the budget for an R and D project at its outset can be near optimal only if the uncertainties involved are so minor as to justify assuming them away or if administrative and political costs are considered. Otherwise, the only kind of a plan that can claim optimality is a stochastic control strategy.

#### 4. REMARKS

We have considered research and development as a non-repetitive stochastic information gathering, processing and utilizing activity with learning and depreciation during which cost is incurred with the expectation of future benefits. These distinguishing features of the R and D process have been implicitly recognized in the literature in scattered places and some specific details have been modeled and studied, depending on the biases of the authors' disciplines. We have attempted to provide a common interdisciplinary framework

for capturing the most essential features of the R and D process and analyzing the problems arising out of its conduct. The model has been based on concepts developed by management scientists, economists and electrical engineers. It is hoped that the conceptual framework and the generality of the model presented here will facilitate a greater understanding of the total R and D process and fruitful solutions to the important decision problems in the broader context.

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