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SMALL DEVIATIONS FROM MAXIMIZING BEHAVIOR

IN A

SIMPLE DYNAMIC MODEL

by

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ABSTRACT

The basic intuition that motivates this paper is that the presence of non-maximizing agents creates incentives for maximizing agents to take advantage of them, and when "frictions" are sufficiently small, these incentives might translate seemingly small deviations from maximizing behavior into non-negligible effects. This paper explores this intuition by looking at a simple dynamic model, which in reduced form can be described by the elementary demand-supply paradigm. The dynamic model allows to capture explicitly the special efforts that the maximizing agents devote to gain at the expense of the non-maximizing ones. In a model with inflexible entry process, it is shown that, when market frictions are relatively insignificant, small deviations from maximizing behavior have substantial impact on market outcomes. In a model with flexible entry process, the price effect of deviations from rationality is dampened by adjustments in entry. Yet these deviations result in first order efficiency loss, in contrast to the second order loss that one would expect from looking at standard static models.

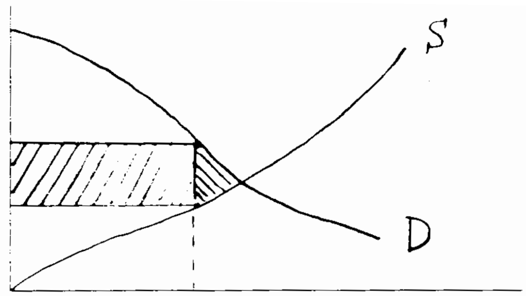
Small Deviations from Maximizing Behavior in a Simple Dynamic Model

1. Introduction

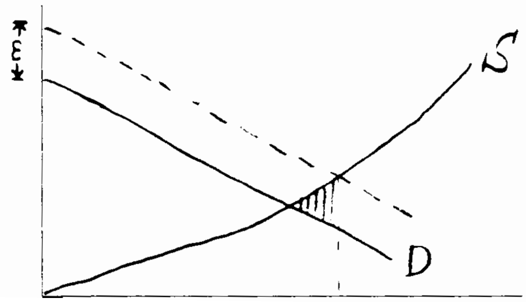
Even adherents of the fundamentalist approach to economic theory would probably agree that the assumption of maximizing behavior is just a simplification and that the predictions of a sensible model should be robust to small deviations from maximizing behavior. This paper focuses on a particular aspect of the possible effects of such deviations. The basic intuition that motivates this paper is that the presence of non-maximizers creates incentives for maximizers to take advantage of them, and when "frictions" are sufficiently small, these incentives might translate seemingly small deviations from maximizing behavior into non-negligible effects.

This paper explores that intuition in the context of a simple dynamic model in which trade takes place over time through a process of pairwise meetings of sellers and buyers. This model has three attractive features for the discussion at hand. First, by looking explicitly at the process of price formation, this model allows to capture naturally the notion of a mistake that agents can make in deciding on prices. Second, the dynamic nature of this model allows to capture explicitly the efforts that maximizing agents make to take advantage of non-maximizing ones. Third, this model has the standard textbook diagram of demand and supply curves as its reduced form -- this diagram captures the participation decisions of agents in the steady state equilibrium of the model--and this will allow to present the results in an elementary way.

Looking at the reduced form, without explaining yet how it corresponds to the underlying dynamic model, one result is depicted by the following diagram.



If all agents are perfect maximizers, the equilibrium price and volume of trade will correspond to the usual intersection of the curves. If buyers are not perfect maximizers in the sense that a fraction f of them might make ε-errors in "bargaining" over the price, then the equilibrium volume of trade will correspond to Q in the diagram. There will be two types of efficiency loss: one on the order of ε^2 due to suboptimal volume of trade (captured by the shaded triangle); the other is on the order of $f\varepsilon$ per-capita (captured by the shaded rectangle). This observation contrasts with a familiar intuition which is based on the static model and according to which deviations from maximizing behavior have only second order effect on welfare. To see this contrasting intuition, consider the standard textbook diagram of demand and supply curves, and suppose, for example, that the buyers in this market behave as if their reservation values were actually higher by an ε than their true values. In the following diagram actual behavior is described by the dashed D' curve, while maximizing behavior is described by the solid D curve. The welfare loss is captured by the shaded triangle which is on the order of ε^2



In this sense and in contrast to the result reported above, here the welfare loss is only of second order in ε.

Going back to the result of this paper, the additional deadweight loss means that the potential to make errors is fully translated into efficiency loss. This loss is caused by the extra efforts of sellers to take advantage of the buyers' mistakes, and it is reminiscent of the idea due to Posner(1975) that competition for monopoly position translates the monopoly profit into efficiency loss.

Another result concerns a special case in which the reduced form is captured by demand and supply curves which are inelastic at prices between some common sellers' reservation value and some common buyers' reservation value. This corresponds to a scenario in which the gains from trade for all agents exceed the value of any outside alternatives. In this case, non-maximizing behavior of the type described above does not create any efficiency loss, but it leads to an extreme distribution of the surplus whereby the price is driven to the buyers' reservation value and sellers appropriate the entire surplus.

The question of whether and under what circumstances small deviations from rationality can have substantial impact on the outcomes of economic interactions has been considered by Akerlof and Yellen [1985a,b], Colinsk [1980], Haltiwanger and Waldman [1985], Russel and Thaler [1985]. What this paper adds to that literature is a dynamic model which is geared towards capturing the intuition that some amount of non-maximizing behavior in a relatively frictionless market triggers responses from the maximizing agents who try to take advantage of it and in the process affect the outcome in new ways.

The model used in this paper is a relatively straightforward modification of the pairwise search and bargaining model which was probably first analyzed by Diamond and Maskin(1979) and Mortensen(1978). The version used here is more directly related to Rubinstein and Wolinsky(1985) and even more so to the model and insights of Gale(1987).

2. The Basic Model

2.A. The benchmark case of perfect maximizers

The market is envisioned as an ocean of agents who meet pairwise to transact. There are two populations: sellers who seek to sell a unit of some indivisible good, and buyers who seek to buy a unit of that good. The market operates over a sequence of dates labelled by $t=0,1,2...$ At each date sellers and buyers are matched randomly pairwise. Matched agents decide through the following simple bargaining game whether and at what price to trade. One of the matched agents is selected randomly, with probability ½, to propose a price; the other agent responds to that price offer with acceptance or rejection. Upon acceptance, the agents trade at the agreed price and leave the market; upon rejection, they separate and return to the pools of the unmatched. The intrinsic value of a unit is 0 for sellers and 1 for buyers, so that an exchange of a unit for the price p yields utilities p and 1-p to the seller and the buyer respectively. Agents discount future benefits using a discount factor δ. It is assumed that there is no aggregate randomness in the sense that frequencies of realizations of random variables over the population will be identified with the distributions of these variables (e.g., exactly ½ of all matched sellers get to propose in each date, etc.) .

Let S(t) and B(t) denote the numbers of sellers and buyers present in the market at date t. The evolution of these numbers is determined by the initial conditions, the rates of new arrivals, and the rates of departure, which depend in turn on the meeting technology and the behavior of all agents. Let S(0) and B(0) denote the initial stocks of agents, and let D=S(0)-B(0).

Initially, we assume that the entry process is exogenous: there are constant and equal flows of M new sellers and M new buyers arriving each period. This together with the fact that sellers and buyers depart from the market in equal numbers implies that the difference in their stocks, S(t)-B(t), will remain constant and equal to the historical difference D, for all t. This assumption will be relaxed in Section 3 below where entry decisions will be made endogenous and responsive to market conditions.

The meeting technology is captured by the function K[S(t),B(t)] describing the total number of meetings in a period, as a function of the stocks of agents present. To avoid discussion of unimportant details, we shall restrict attention to the specific technology,

$$K[S(t),B(t)] = \min\{S(t),B(t)\},$$

whereby in each period the maximum possible number of pairs are being matched and only the excess agents on the long side of the market are left unmatched. It will be obvious, however, that this restriction does not play a qualitative role and the points made below are valid for a large class of well behaved technologies.

The behavior of a seller (buyer) at some date t is characterized by a price he will offer, if he is matched and it is his turn to propose, and the minimal (maximal) price he will accept, if it is his turn to respond at this date. Thus, in the aggregate, agents' behavior at date t is summarized by the four distributions of price offers and acceptances of sellers and buyers respectively. These distributions determine in the obvious way the evolution of S(t+1) and B(t+1) from S(t) and B(t). The state of the market at date t consists of these four distributions and the numbers S(t) and B(t). The market is in a steady state if the state is constant over time. An agent's strategy describes what prices he will offer and what prices he will accept at each date, as a function of the state. A market equilibrium is a steady state situation such that each agent's strategy is optimal, given the state of the market which is in turn the outcome of all agents' behavior.

Consider any market equilibrium. Since, by definition, this equilibrium is a steady state, we can omit the time variable t from the equilibrium magnitudes. Let a_S and a_B denote the probabilities that sellers and buyers respectively have of meeting an agent of the opposite type at any period in that equilibrium. Note that

$$(1) \qquad a_S = K(S,B)/S; \qquad a_B = K(S,B)/B.$$

Let V_S and V_B denote the maximized expected utilities of unmatched sellers and buyers respectively at the beginning of any period in that equilibrium. Since the equilibrium strategies are required to be optimal after any possible history, a buyer would accept any price below $p=1-\delta V_B$ and a seller would accept any price above $p=\delta V_S$. Therefore, in this equilibrium, at any date, the prices $p=1-\delta V_B$ and $p=\delta V_S$ will be proposed by proposing sellers and buyers respectively and would be accepted by their counterparts. Thus,

$$(2) \qquad V_S = a_S[(1 - \delta V_B)\tfrac{1}{2} + \delta V_S\tfrac{1}{2}] + (1 - a_S)\delta V_S$$

$$(3) \qquad V_B = a_B[(1 - \delta V_S)\tfrac{1}{2} + \delta V_B\tfrac{1}{2}) + (1 - a_B)\delta V_B$$

Since at equilibrium every meeting is concluded with a transaction, the number of departures is equal to the number of meetings and the steady state condition of equality between the outflow and the inflow reduces to

(4) $K(S,B) = M$.

Equations (1)-(4) together with the initial condition $S-B=D$ characterize the market equilibrium when all agents are maximizers. We shall be interested in situations in which the trading process does not involve large <u>frictions</u>. In this model small frictions are captured by a discount factor near 1. Solving this system for the limiting case as δ approaches 1, and noting that $\lim V_S$ is actually the price (denote it by p) at which all transactions are carried out in the limit, we get

(5) $$p = \lim V_S = 1-\lim V_B = B/(2B + D)$$

The particular specification of the meeting technology, $K(S,B)=\min\{S,B\}$, together with (4) imply $M=\min\{S,B\}=\min\{B+D,B\}$. Substituting into (5) we have

(6) $$p = \lim V_S = 1 - \lim V_B = \begin{cases} M/(2M + D) & \text{if } D>0 \\ (M-D)/(2M-D) & \text{if } D<0 \end{cases}$$

<u>2.B. Non-maximizing behavior</u>

Suppose that, occasionally, a buyer might miscalculate his response and be willing to accept price offers which are higher than what is optimal for him to accept. Specifically, in each meeting in which it is the buyer's turn to respond, there is probability f that the buyer will underestimate his continuation value, V_B, and act as if it were $(1-\varepsilon)V_B$. In other words, he will agree to accept prices up to $1-\delta V_B(1-\varepsilon)$ rather than just up to $1-\delta V_B$ as correct optimization would dictate. Otherwise, all agents behave optimally in all other occasions. The occurrence of an error will be assumed independent of all other random events in this model. This form of "irrationality" is of course ad-hoc. One may think of many other variations, such as errors made both in accepting and proposing, and errors made only by certain buyers and not others, etc. However, the qualitative points made below will be valid for many such variations and we adopt one such variation for performing concrete computations. We shall return to discuss this assumption at a later stage after its exact role will become clear.

The notion of equilibrium remains the same except that, in decision nodes in which he is in the non-optimizing state, a buyer's strategy is required to be only ε-optimal. In other nodes the buyer's behavior is fully optimal and even takes into account the possibility that in the future he might err and make a suboptimal decision. Now, in any possible market equilibrium, buyers always offer δV_S, always agree to any price up to $1-\delta V_B$, and a fraction f also agree to any price below $1-\delta_B(1-\varepsilon)$, while sellers always agree to any price above δV_S in their turn to respond. Different possible market equilibria may differ only with respect to sellers' behavior in their turn to propose. The only two relevant alternative equilibrium strategies for the seller are to propose the price $p=1-\delta V_B$, or to propose $p=1-\delta V_B(1-\varepsilon)$ in the hope that the buyer is in the non-maximizing state. If, at the market equilibrium, the former strategy is optimal for the sellers, then the market equilibrium is characterized by (1)-(4) as above. However, to account for the cases in which the latter strategy or both strategies are optimal at the market equilibrium, conditions (2)-(4) are replaced by the following.

$$
(2')\quad \begin{cases} U_S = a_S[(1-\delta V_B)\tfrac{1}{2} + \delta V_S \tfrac{1}{2}] + (1-a_S)\delta V_S \\ W_S = a_S[f(1-\delta V_B(1-\varepsilon))\tfrac{1}{2} + \delta V_S \tfrac{1}{2}] + [1-a_S + a_S(1-f)\tfrac{1}{2}]\delta V_S \\ V_S = \mathrm{Max}\{U_S, W_S\} \end{cases}
$$

$$
(3')\quad V_B = a_B[(1-\delta V_S)\tfrac{1}{2} + \{(1-g)\delta V_B + gf\delta V_B(1-\varepsilon)\}\tfrac{1}{2}] + [1-a_B + a_B(1-(1-g)-gf)\tfrac{1}{2}]\delta V_B
$$

$$
(4')\quad K(S,B)[1+1-g+gf]\tfrac{1}{2} = M
$$

where $g \in [0,1]$ satisfies: if $U_S > W_S$, then $g=0$; if $U_S < W_S$, then $g=1$.

U_S and W_S are the respective expected utilities of a seller who always demands the price $1-\delta V_B$ and of one who always demands $1-\delta V_B(1-\varepsilon)$. The variable g captures the fraction of the seller population who, at the equilibrium, demand the price $p=1-\delta V_B(1-\varepsilon)$. The remaining fraction, 1-g, demand of course $1-\delta V_B$. Note that, if g=0, then $V_S=U_S$ and this system reduces to (2)-(4). If g=1, then $V_S=W_S$. In this case the difference from (2) is that here the sellers attempt to take advantage of the non-maximizing buyers by insisting on $p=1-\delta V_B(1-\varepsilon)$. The first expression on the RHS of the (2') equation which defines W_S accounts for the possibility that a particular seller will reach an agreement in the current period. This will happen if the seller meets a buyer and either it is the seller's turn to propose and the buyer is non-maximizer, or it is the buyer's turn to propose. The former event occurs with probability $a_S f \tfrac{1}{2}$ and in it the unit will be traded for the price $p=1-\delta V_B(1-\varepsilon)$; the latter event occurs with probability $a_S \tfrac{1}{2}$ in which case the unit is traded for the price $p=\delta V_S$. The second expression accounts for the possibility of this seller will not reach an agreement in the current period. This event occurs with the complementary probability $[1 - a_S + a_S(1 - f)\tfrac{1}{2}]$ and in it the seller gets the discounted continuation value of δV_S. If $g \in (0,1)$, the equilibrium is mixed: both strategies are optimal for sellers, and the equilibrium value of g is determined so as to exactly maintain the balance $V_S=U_S=W_S$.

Condition (3') is explained analogously by accounting for the possibility that some or all sellers may "ambush" the non-maximizing buyers. Note that the continuation value V_B which determines the buyer's optimal acceptance behavior accounts for the possibility that this buyer might err in the future.

Finally, condition (4') differs from (4) due to the fact that, in their turn, some or all sellers (fraction g) make proposals which are acceptable only to non-maximizers. This means that only some of the meetings are concluded with a transaction and hence slows the rate at which agents depart from the market. Specifically, the LHS of (4') states that the fraction of all meetings that are concluded with a transaction consists of the ½ of all meetings in which the buyers propose, the (1-g)½ in which the sellers demand the price $p=1-\delta V_B$, and the gf½ of all meetings in which sellers demand the price $p=1-\delta V_B(1-\varepsilon)$ from buyers who happen to be in the non-maximizing state.

The following proposition collects the relevant features of the market equilibrium in this environment.

<u>Proposition 1</u>:

(i) If δ is sufficiently small, the equilibrium is characterized by (1)-(4).

(ii) If δ is sufficiently close to 1, then in any market equilibrium V_S is near 1 and V_B near 0. (I.e., $\forall v>0 \exists \delta(v)$ s.t. $\forall \delta \geq \delta(v)$, $V_B<v$ and $V_S>1-v$ in any equilibrium.)

<u>Proof</u>: First, observe from (4'), (1) and the meeting technology that in all equilibria a_S and a_B are bounded away from 0. To verify part (i), note that, if δ is sufficiently small, then since a_S is bounded away from 0, we have

$U_S > W_S$. Therefore, g=0 and the market equilibrium is characterized by the system (1)-(4). To verify part (ii), let v>0 and suppose that there is a sequence of δ approaching 1 and a corresponding sequence of market equilibria in which $V_B > v$. Observe from (2') that, when V_B is bounded away from 0 and δ sufficiently close to 1, then $W_S > U_S$. Therefore, in these equilibria g=1. Solving (1),(2'),(3') for V_S and V_B, under the assumption g=1, we get

$$(7) \qquad V_S = \frac{a_S f[2(1-\delta) + \delta a_B \varepsilon (1+f)]}{2(1-\delta)[2(1-\delta)+\delta a_B(1+f\varepsilon)] + \delta a_S f[2(1-\delta)+\delta a_B \varepsilon (1+f)]}$$

$$(8) \qquad V_B = \frac{2(1-\delta) a_B}{2(1-\delta)[2(1-\delta)+\delta a_B(1+f\varepsilon)] + \delta a_S f[2(1-\delta)+\delta a_B \varepsilon (1+f)]}$$

Since a_S and a_B are bounded away from 0, it follows that, as δ approaches 1, $\lim V_B = 1 - \lim V_S = 0$. This contradicts our supposition, and hence implies that, when δ is sufficiently close to 1, the equilibrium values of V_S and V_B are arbitrarily close to 1 and 0 respectively. QED

In this model market frictions are captured by the discount factor δ: the smaller it is, the larger is the cost of search and hence the more significant is the friction. Thus, proposition 1 says that, when the market frictions are sufficiently significant, the presence of non-maximizers does not affect the market outcome. However, when these frictions are relatively insignificant, the presence of non-maximizers has a substantial effect: essentially, the entire surplus is appropriated by the sellers.

When there are non-optimizing buyers, sellers have an opportunity to try to "catch" such a buyer and take advantage of his mistake. The seller's gain from such behavior is determined by the magnitude of the buyer's error, εV_B;

the cost associated with this behavior is the delay, whose expected duration depends on the frequency of non-optimizing behavior, f, and the disutility it causes depends on δ. Now, for any given level of $V_B>0$, sufficiently high δ's will render the cost of delay small in comparison to the gain εV_B and hence will make it profitable for sellers to wait for an "irrational" buyer. But such behavior on the part of all sellers will depress the payoff to the maximizing buyers as well, and so V_B will drop. The system will be equilibrated once the gain εV_B dropped enough to strike a new balance with the cost of delay.

The upshot is that a small extent of non-maximizing behavior as captured by small values of ε and f might have a great impact in a market with small frictions, because maximizing agents will attempt to take advantage of it and in the process alter significantly the market outcome. Put differently, what are seemingly small deviations from maximizing behavior are actually not small when the frictions are sufficiently insignificant.

Finally, consider the efficiency of the market outcome. Since, by assumption, the rates of participation in this market are exogenously fixed, the total surplus is determined by the extent to which the potential gains from trade are exploited. Obviously, for $\delta<1$, there is surplus loss in the magnitude of $1-V_S-V_B>0$ per transaction which captures the resources spent on search. But $\lim V_S=1-\lim V_B$ means that the potential gains from trade are fully exhausted in the limit. Thus, while the deviations from maximizing behavior have a pronounced effect on the price, they do not involve any welfare loss.

3. Endogenous Entry

So far entry into the market has been modeled as an exogenous process: M new entrants flow into each side of the market regardless of the terms of trade. Endogenous entry decisions might be expected to affect the results in two ways. First, the extreme distribution of surplus arising above might be offset through adjustments in entry. Second, the possibility of suboptimal rates of participation introduces another source of potential distortions.

Suppose that in each period new sellers and buyers arrive at the gates of this market and decide, once and for all, whether to enter or to pursue an alternative opportunity. Agents differ with respect to their utility from the alternative. Let $M_i(V)$, i=S,B, denote the numbers of sellers and buyers respectively, who value their alternative opportunity at V or less. Assume that the functions M_i are constant over time, continuous, strictly increasing and that $M_i(0)=0$. The functions M_i replace the parameters M and D as primitives of the model, and M and D will be now determined in the system. The assumption that M_i is an increasing function captures the idea that better terms of trade for a type i agent (i.e., higher V_i) draw a larger entry flow of agents of this type.

Consider first the case in which all agents are perfect maximizers. The equilibrium is characterized by conditions (1),(2)-(4) as before, but now there is an additional condition on the equilibration of the entry rates,

$$(9) \qquad M = M_B(V_B) = M_S(V_S).$$

Analogously, when buyers are not perfect maximizers in the manner considered throughout, the market equilibrium is characterized by (1), (2')-(4') and (9). The fact that V_B appears as the argument of M_B means that the entry of buyers is assumed to respond to the correct value of participation, which takes into account the fact that they are liable to make errors later on. This seems like the right rational-expectations assumption for this case. But it will also be obvious that an alternative assumption whereby buyers' entry decisions ignore the possibility of error would lead to the same qualitative results.

For later reference, it is useful to note that the assumptions $M_i(0)=0$ and M_i strictly increasing imply that there is a unique $v>0$ such that $M_S(v)=M_B(1-v)$. Furthermore, if $V_S>v$ and $V_B<1-v$, then $M_S(V_S)>M_B(V_B)$; if $V_S<v$ and $V_B>1-v$, then $M_S(V_S)<M_B(V_B)$.

Proposition 2:

If δ is sufficiently close to 1, the equilibrium behavior of all sellers is to always propose $p=1-\delta V_B(1-\varepsilon)$. I.e., the market equilibrium is characterized by the system (1), (2'), (3') and (9), where $g=1$ and $V_S=W_S$.

Proof: Consider a sequence of δ approaching 1 and a corresponding sequence of equilibria.

Claim: Over this sequence a_S approaches 0.

Proof: Suppose to the contrary that there is a subsequence of the equilibria over which $a_S>A>0$. Observe from (2') that over this subsequence $\lim V_S \geq 1-(1-\varepsilon)\lim V_B$. This is because, with a_S bounded away from 0, a seller can guarantee that much in the limit by sticking to the strategy of always demanding $p=1-\delta V_B(1-\varepsilon)$ and refusing any other offer. Also, it is always true that $\lim V_B \leq 1-\lim V_S$. Combining these two inequalities we get $\lim V_B \leq 1-\lim V_S \leq (1-\varepsilon)\lim V_B$ which implies $\lim V_B=0$ and $\lim V_S=1$. But this means that, far enough in the sequence, $M_S(V_S)>M_B(V_B)$ so that (9) is violated, contradicting the supposition that a_S is bounded away from 0. ■

Suppose that over the said sequence of equilibria $g<1$. In this case $V_S=U_S$ and solving the system (2'),(3') we get

$$V_S = \frac{a_S[2(1-\delta) + \delta a_B g f \varepsilon]}{2(1-\delta)[2(1-\delta)+\delta a_B(1+gf\varepsilon)] + \delta a_S[2(1-\delta)+\delta a_B g f\varepsilon]}$$

$$V_B = \frac{2(1-\delta)a_B}{2(1-\delta)[2(1-\delta)+\delta a_B(1+gf\varepsilon)] + \delta a_S[2(1-\delta)+\delta a_B g f\varepsilon]}$$

The above claim together with the matching technology, $K(S,B)=\min\{s,B\}$, and (1) imply that, when δ is near 1, $a_B=1$. Substituting this into the above expressions, we get that, over sequences of equilibria such that $\lim V_S$ and $\lim V_B$ exist, they are

$$\lim V_S = \frac{\lim[g a_S/2(1-\delta)]f\varepsilon}{1 + \lim g f\varepsilon + \lim[g a_S/2(1-\delta)]f\varepsilon}$$

$$\lim V_B = \frac{1}{1 + \lim g f\varepsilon + \lim[g a_S/2(1-\delta)]f\varepsilon}$$

Notice that over a sequence such that $\lim V_S$ exists, $\lim[g a_S/2(1-\delta)]$ exists and $0<\lim[g a_S/2(1-\delta)]<\infty$, since $\lim[g a_S/2(1-\delta)]=0$ or ∞ implies $\lim V_S=0$ or 1, violating (9).

Next observe that $g<1$ means that demanding the price $p=1-\delta V_B$ is at least as good a strategy for the seller as the alternative of demanding the price $p=1-\delta V_B(1-\varepsilon)$. That is,

$$1-\delta V_B \geq f[1-\delta V_B(1-\varepsilon)] + (1-f)\delta V_S$$

Rearranging, we get

$$1-\delta V_B-\delta V_S \geq \delta V_B f\varepsilon/(1-f).$$

Taking limits and substituting the above derived values of $\lim V_S$ and $\lim V_B$,

$$\lim g f\varepsilon \geq f\varepsilon/(1-f)$$

But since $\lim g \leq 1$ and $1/(1-f)>1$, this is a contradiction. This means that the supposition, that there is a sequence of equilibria over which $g<1$, is false. Therefore, for sufficiently large δ, in all equilibria $g=1$, which proves the proposition. QED

Thus, when δ is near 1, sellers' equilibrium behavior is to demand in their turn the price $p=1-\delta V_B(1-\varepsilon)$ in the hope that the buyer they are facing is in the non-maximizing state. Note that $\lim[a_S/2(1-\delta)]<\infty$ means that, for δ near 1, the sellers' meeting probability, a_S, is small. The reason is as follows. The above described behavior of sellers exerts an upwards pressure on the price. If δ is near 1 and a_S is not too small, this pressure will drive the price near 1. But this is incompatible with equilibrium, since then the flow of entering sellers will exceed the flow of entering buyers. The system will be equilibrated through the emergence of a large excess stock of sellers, S-B, which exerts a counter pressure on the price by reducing a_S and hence weakening the sellers' position.

As a corollary from Proposition 2, we have

Proposition 3:

$$\lim V_S = x^* f\varepsilon(1+f)/[1+f\varepsilon+x^* f\varepsilon(1+f)]; \quad \lim V_B = 1/[1+f\varepsilon+x^* f\varepsilon(1+f)],$$

where x^* is the solution to $M_S(xf\varepsilon(1+f)/[1+f\varepsilon+xf\varepsilon(1+f)]) = M_B(1/[1+f\varepsilon+xf\varepsilon(1+f)])$

Proof: It follows from proposition 2 that, for δ near 1, the equilibrium values of V_S and V_B are given by (7) and (8). Taking limits we have

$$\lim V_S = \frac{\lim[a_S/2(1-\delta)]f\varepsilon(1+f)}{1 + f\varepsilon + \lim[a_S/2(1-\delta)]f\varepsilon(1+f)}$$

$$\lim V_B = \frac{1}{1 + f\varepsilon + \lim[a_S/2(1-\delta)]f\varepsilon(1+f)}$$

Recall that over a sequence such that $\lim V_S$ exists, $\lim[ga_S/2(1-\delta)]$ exists and $0<\lim[ga_S/2(1-\delta)]<\infty$. Now, letting $x=\lim[a_S/2(1-\delta)]$ and x^* be the value of x for which (9) holds in the limit, i.e., $M_S(\lim V_S)=M_B(\lim V_B)$, we get the expressions in the statement of the proposition. QED

Notice that $\lim V_S + \lim V_B < 1$. This implies immediately the following observation.

<u>Corollary</u>: Even in the limit, as δ approaches 1, there is a non-vanishing efficiency loss at the rate of

$$1 - (\lim V_S + \lim V_B) = f\varepsilon/[1+f\varepsilon+x^*f\varepsilon(1+f)] > 0.$$

The source of this inefficiency is the waiting costs of sellers. The above noted fact that $\lim[ga_S/2(1-\delta)]<\infty$ means that $\lim a_S=0$. The low a_S is the consequence of a large excess of the number of sellers over that of the buyers and it translates to longer expected delays between consecutive matchings of a seller. That is, at equilibrium, the balance in entry is achieved when the extra gain that sellers make at the expense of buyers' mistakes is being whittled away by inefficiently longer delays.

4. Discussion

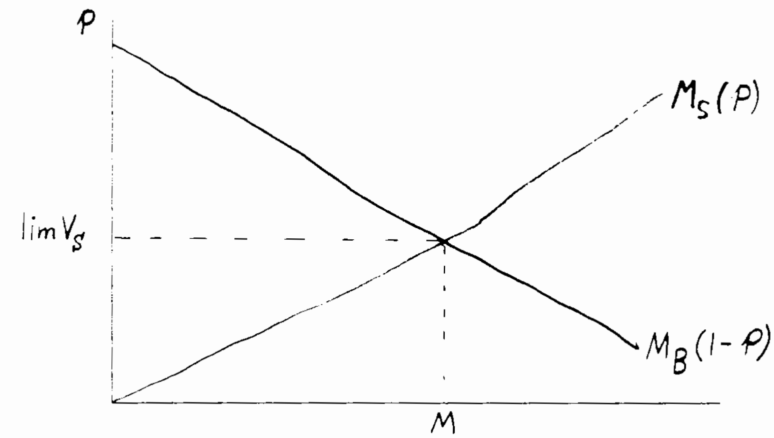
<u>Identifying the model with the demand-supply paradigm</u>

Let us first explain the sense in which the dynamic model of this paper corresponds to the familiar demand-supply apparatus (this correspondence has been pointed out by the above cited Gale(1987)). Consider the benchmark case in which all agents are perfect maximizers and assume the above endogenous entry process. In this case the limiting market equilibrium, as δ approaches

1, is characterized by the price $p = \lim V_S = 1 - \lim V_B$ as given by (6) and the entry flow M which is given by the limit version of (9), $M = M_S(\lim V_S) = M_B(\lim V_B)$. The latter can be written as

$$(10) \qquad M = M_S(p) = M_B(1-p)$$

As we have already mentioned, the properties of M_S and M_B imply that there is a unique p and hence a unique M which satisfy (10). Given these values of p and M, equation (6) only determines D, which is also endogenous when entry is. The following figure describes (10) in a form that corresponds to the familiar textbook supply-demand picture.

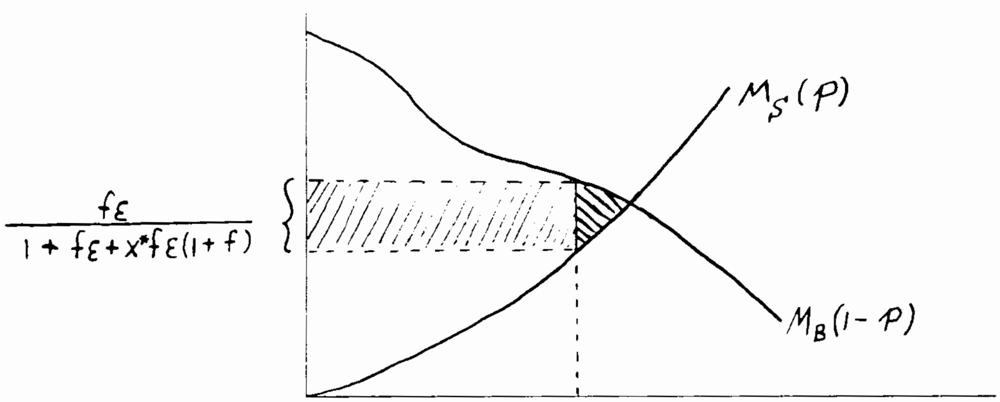


Like in the traditional supply and demand curves, the axis are inverted: the argument p -- the price -- is plotted on the vertical axis, while the corresponding flows of agents -- the quantities -- are drawn on the horizontal axis. The supply and demand curves, $M_S(p)$ and $M_B(1-p)$, describe the periodic flows of entering sellers and buyers as functions of price. The equilibrium of the model corresponds to the equilibrium in this "flow market" as depicted by the intersection of the curves.

Consider now the case in which buyers are not perfect maximizers, as described above. As argued by proposition 3, in this case $\lim V_S < 1 - \lim V_B$, the appropriate version of (10) is

$$M = M_S(p_S) = M_B(1-p_B), \tag{10'}$$

where $p_S = \lim V_S$ is the seller's price and $p_B = 1 - \lim V_B$ is the buyer's price, in a sense that will become clear. Notice that the transactions are not all performed at one price: at a fraction $1/(1+f)$ of the transactions the price is p_S, while at the remaining fraction the price is $p_B/(1-\epsilon)$. Thus, p_S and p_B should be interpreted as the expected full prices received by the sellers and incurred by the buyers respectively. For example, p_S accounts for the expected transaction price that the seller receives minus the expected search cost incurred in waiting. The following figure depicts the flow market equilibrium in this case.



The two types of inefficiency

The last figure captures two types of inefficiency caused by the non-maximizing behavior of buyers. The shaded triangle captures the potential gains from trade that are being lost due to too little entry, which in turn reflects the wedge between the seller's and buyer's prices. The shaded rectangle reflects the losses due to sellers' waiting time wasted in the system: the difference between the expected price paid by buyers, p_B, and the expected utility received by sellers is the seller's search or waiting cost. The explanation for this waste is clear: rent seeking behavior of sellers results in a great excess supply of sellers who seek to exploit the "irrationality" of buyers. This excess supply imposes waiting costs on the sellers, and the system is equilibrated only once these waiting costs completely exhaust the gains they make at the expense of buyers' mistakes. This point is reminiscent of the point made by Posner(1975) and others that monopoly profit actually represents waste, because, in competition for the monopoly position, rent seekers would expend resources until all this profit is exhausted.

The first type of efficiency loss (the triangle) is on the order of ε^2; the second type of loss (the rectangle) is a linear function of $f\varepsilon$, which by the corollary to proposition 3 is $(p_B - p_S)M = Mf\varepsilon/[1+f\varepsilon+x^*f\varepsilon(1+f)]$. Thus, if we think of small deviations from maximizing behavior, i.e., small ε and f, then the second type of loss is significantly more important than the first.

The "smallness" of the deviations from maximizing behavior

The title of this paper speaks of "small" deviations. This is in the sense that the parameters ε and f can be small in absolute value. However, it is important to note that when δ is near 1 these parameters are not small relative to the frictions. That is, when we take limits with respect to δ, the parameters ε and f are kept constant. It is this fact which makes it worthwhile for maximizing agents to incur the cost of the frictions in order to capitalize on the ε magnitude errors of the non-maximizing. Thus, the observations of this paper pertain to a situation in which the deviations from maximizing behavior, while possibly small in absolute value, are sufficiently significant relative to the frictions of the trading process.

5. Further Discussion of Some Simplifying Assumptions

Many of the specific assumptions made above were helpful in obtaining a concrete example which can be solved explicitly, but are not essential for the qualitative points. The important element is that some of the buyers are sometimes liable to err and trade at higher prices than they would if they were maximizing, and that when the frictions are relatively insignificant, sellers would "ambush" those buyers. This basic element does not depend on whether all buyers err with some probability, as we assume, or only some fraction make errors while other do not; whether buyers err only in their turn to respond or also when they propose; whether they only err in being too soft or also in the other direction; and what the precise functional form of the error is.

Only certain buyers are inconsistent maximizers

Consider, for example, the case in which only certain buyers are inconsistent maximizers, while the rest maximize consistently. Suppose that the inconsistent maximizers constitute a fraction t of each entering batch of M buyers, and that they always make the ε errors described above. Here the fraction f of non-maximizing buyers in the steady state population is determined endogenously and is in general different from t, since the different types of buyer stay in the market for different durations. Recall that g denotes the fraction of sellers who demand the price $1-\delta V_B(1-\varepsilon)$. Since the non-maximizers agree to this price, they reach an agreement in their first match. The maximizers, who reject this price, take on the average $2/(2-g)$ matches to transact, since at any given match they transact only with probability 1-g/2. This probability consists of the probability 1/2 that they propose and of the probability (1-g)/2 that they are offered the price $1-\delta V_B$. Therefore, the fraction f is given by

$$(11) \qquad f = t/[t + (1-t)2/(2-g)] = t(2-g)/[t(2-g) + 2(1-t)].$$

Now, system (2')-(4') is adapted to this case by replacing (3') with

$$V_B = a_B[(1-\delta V_S)\tfrac{1}{2} + (1-g)\delta V_B\tfrac{1}{2}] + [1-a_B + a_B g\tfrac{1}{2}]\delta V_B$$

and substituting f from (11) into (4'). The equations (2') remain unmodified. Since f is bounded away from 0, the qualitative results reported above continue to hold for this case as well.

Two-sided errors

The assumption that only buyers err plays a more important role in that it is responsible for the asymmetries inherent in the results, i.e., that with exogenous entry the sellers appropriate the entire surplus and that with endogenous entry there is an excess of sellers that creates the delays. However, this asymmetry only serves to sharpen the observation concerning the aggregate effects of the non-maximizing behavior. If we assumed instead that both sellers and buyers are liable to make such mistakes, then the forces identified above will work on both sides of the market and the outcome will be determined by their relative strength. To see this point more concretely, assume that sellers err in a similar manner. Let f_i, ε_i and g_i, $i=S,B$, denote the counterparts of f, ε and g which are now indexed by S and B since they appear on both sides. The counterpart of (2') will now be

$$(2'') \quad \begin{cases} U_S = a_S[1-\delta V_B+g_B f_S \delta V_S(1-\varepsilon_S)+(1-g_B)\delta V_S]\tfrac{1}{2} + [1-a_S+a_S(1-(1-g_B)-g_B f_S)\tfrac{1}{2}]\delta V_S \\ W_S = a_S[f_B(1-\delta V_B(1-\varepsilon_B))+g_B f_S \delta V_S(1-\varepsilon_S)+(1-g_B)\delta V_S]\tfrac{1}{2} + \\ \qquad\qquad [1-a_S+a_S(1-f_B+1-(1-g_B)-g_B f_S)\tfrac{1}{2}]\delta V_S \\ V_S = \mathrm{Max}\{U_S,W_S\} \end{cases}$$

Since the structure is now symmetric, the counterpart of (3′) will be completely analogous (just exchange B's for S's everywhere). This new system can be solved in the same way as above. For the case of the exogenous entry process discussed in Section 2, there is a limit (as δ approaches 1) market equilibrium such that $g_S=g_B=1$ and

(12) $\lim V_S = f_B\varepsilon_B/(f_S\varepsilon_S + f_B\varepsilon_B); \quad \lim V_B = f_S\varepsilon_S/(f_S\varepsilon_S + f_B\varepsilon_B).$

To verify this note that, if there is such an equilibrium, the relevant equations will be the W_S and W_B parts of (2′′) and (3′′) respectively, with $g_S=g_B=1$. That is,

$$V_S = a_S[f_B(1-\delta V_B(1-\varepsilon_B))+f_S\delta V_S(1-\varepsilon_S)\delta V_S]½ + [1-a_S+a_S(2-f_B-f_S)½]\delta V_S$$

$$V_B = a_B[f_S(1-\delta V_S(1-\varepsilon_S))+f_B\delta V_B(1-\varepsilon_B)\delta V_B]½ + [1-a_B+a_B(2-f_S-f_B)½]\delta V_B$$

Solving these equations and taking the limit as δ approaches 1, we get the expressions in (12). Thus, in the limit, at a seller's turn to propose, the expected value of following the $g_S=1$ strategy is $f_B(1-\lim V_B(1-\varepsilon_B))+(1-f_B)\lim V_S$, while the value of the alternative is $1-\lim V_B$. Since $\lim V_S=1-\lim V_B$, we have $f_B(1-\lim V_B(1-\varepsilon_B))+(1-f_B)\lim V_S > 1-\lim V_B$. Thus, the $g_S=1$ strategy is indeed optimal for the seller, in the limit, and analogously the $g_B=1$ is optimal for the buyer. Since the inequality is strict, this is also true for $\delta<1$ but sufficiently close to 1, implying that this behavior is an equilibrium for such values of δ.

Notice from (12) that the surplus is allocated here according to the relative frequencies and magnitudes of the errors. If, for example, $\varepsilon_S=\varepsilon_B$ and $f_B>f_S$, then the share of sellers is larger. This is of course consistent with the result of proposition 1 which refers to the extreme case where only buyers are liable to err. It is interesting to contrast the expressions here with those in (6). Like in Proposition 1, here too, in the limit when the market is nearly frictionless, the equilibrium "prices" depend only on data concerning the errors and so are independent of the parameters M and D which fully determine these magnitudes when there are no errors. In particular, this means that, if the extent of the non-maximizing behavior is the same on both sides of the market, i.e. $\varepsilon_S=\varepsilon_B$ and $f_B=f_S$, then the mistakes do not necessarily "cancel out" in the sense that here $\lim V_S=\lim V_B=\frac{1}{2}$, which is different from the magnitude in (6) unless D happens to be 0.

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SMALL DEVIATIONS FROM MAXIMIZING BEHAVIOR IN A SIMPLE DYNAMIC MODEL

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