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PUBLIC EDUCATION AND CAPITAL ACCUMULATION

by

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Abstract

I study an overlapping generations model where physical and human capitals are used in production and can be accumulated by witholding resources from current consumption. Human capital is accumulated through a schooling system which can be financed either by private expenditures or by taxes on current income or by a combination of both. In a political equilibrium with majority voting, the median voter may approve of public school financing as an instrument to solve a "free rider problem". It improves the skills of next period's workers which in turn increases the expected return on capital, something which could not be achieved by means of private school financing. Public schools, moreover, turns out to be an instrument for intergenerational income distribution so that they may be preferred to private schools for this motive as well. The model is shown to display a poverty trap (poor societies vote to invest too little in education) as well as persistent growth. I am able to fully characterize the global dynamics of the model, which delivers a number of interesting and potentially testable hypotheses on the relation between income growth, capital accumulation and the development of public education. I also endogenize the dynamic behavior of school attendance rates, as well as the choice between public and private financing of schools in the presence of parental altruism.

A particular attention is paid to the different performances of a publicly provided school system vis-a-vis a publicly financed school system. It is shown that, under very general conditions, the latter tends to create a better environment for the accumulation of human capital as it fosters support for public education through the voting mechanism. All throughout the paper I concentrate on specific functional forms that allow for a closed form solution of the equilibrium dynamics, but all the important results can be shown to apply for a general class of utility and produciton functions.

1. Introduction

The model I submit is predicated on the idea that publicly financed school systems were introduced to provide a solution to an important free-rider problem affecting the accumulation of physical and human capital.

In a society in which physical and human capital are complementary factors of production, the owners of next period's capital stock have a vested interest in the level of human capital of the future *average* worker. In the absence of a credit market where the young generation could borrow against future earnings, the equilibrium level of private financing of education will be less than required by productive efficiency. Even if parental altruism were present, still the amount of private expenditure going to the school system would be less than appropriate as the private agents would have no incentive to consider the increase in the return on capital that an extra dollar spent on education entails.

While these observations apply to all forms of education, in this paper I prefer to interpret the word "school" as referring to primary and secondary education only. Available evidence (see e.g. Psacharopoulos (1985, 1989) and references therein) suggests this is where the social rate of return is at its highest level, where public financing is most concentrated, and where attendance levels near 100% in developed countries. It is also roughly consistent with the allocation of political rights I assume later on.

I make no claim of originality for this fundamental idea: scholars in the economics of education and in the human capital literature have long stressed the fact that the *main* purpose of a public school system is to provide society with an educated and skillful workforce (see e.g Becker (1975), Butts (1978) Friedman (1962), Stiglitz (1974), to name only a few).

My own contribution amounts to proposing a dynamic general equilibrium model which formalizes this old idea, and to investigate its theoretical implications. The ultimate purpose of the exercise is to develop a theory of the dynamic interactions between education and economic growth, which could be used to explain our observations of the real world. While this is not yet attempted here, I have nevertheless chosen to forego theoretical generality and to adopt specific functional forms that allow for explicit solutions and quantitatively testable predictions.

The scope of the present study is also limited by the fact that I try to provide a *positive* (as opposed to normative) model of schooling: hence I do not examine the important deadweight losses in welfare arising from the taxation of income, a topic which is better left for a separate investigation.

The basic structure of the model is that of an OLG world where individuals live for three periods: young, middle age and old. When young they are endowed with a unit of time which they can use either to go to school (thereby accumulating some productive skills) or to have leisure. During their middle age they are also endowed with a unit of labor time which they inelastically supply to the productive sector. The labor income so obtained is divided between current consumption, saving (in the form of physical capital) and education of the children. When old they just consume the return on the accumulated stock of physical capital (πs) and then die. Production of the homogeneous good is obtained by means of a standard neoclassical production function $Y = F(K, H)$ which uses physical capital (K) and effective labor (H) (one unit of labor time multiplied the level of human capital) as inputs. The growth rate of human capital depends on the amount of time and physical resources devoted to it: when the latter are zero human capital shrinks, whereas it may grow if enough inputs are applied to the schooling process. In particular I assume that a positive growth rate of aggregate human capital can be obtained by investing a finite amount of resources. Perpetual growth is therefore possible.

I find it reasonable to assume that young people attending school cannot borrow from the older generations. Credit markets to finance private investments in human capital do exist, especially in the United States, but they seem to concentrate on the latest stages of the learning process. They are almost non-existent in Europe and in the rest of the world, where a large portion of higher education is publicly financed (but not necessarily publicly provided, see James (1992)). Furthermore I am mostly concerned with primary and secondary education, for which these markets are almost never available (according to West (1991, p. 168) Japan is an important exception; see also Ehrlich and Lui (1991) for a very different theoretical interpretation of this matter).

While the current owners of capital have little or no interest in its future productivity, the middle age individuals have a *collective* interest in fostering society's stock of human capital. This can be achieved by instituting a publicly financed school system. Such an explanation of public schooling does not depend on the assumption that the educational process generates positive external effects. Quite obviously this would further increase the social value of investing in education but, per se, the latter is already justified by the presence of factor complementarities in production and by the absence of perfect capital markets.

The crucial point is that the pecuniary externality can be internalized by means of some institutional mechanism. Nevertheless, assuming the existence of a central state with the power to collect taxes does not fully solve the problem. This creates a distributional conflict between generations, as they hold opposite interests over the kind and amount of taxes that should be levied. I proceed by assuming that such conflicts are solved by means of majority voting: the agents that are alive and have electoral rights in each period are asked to vote on an income tax level and on the allocation of its revenues.

This leads to a number of strategic considerations which I circumvent by assuming a weak form of myopia in the voters' decision making process. More precisely I assume that the middle age individuals, while perfectly able to recognize the impact of taxation on the current consumption-saving decisions, fail to recognize its indirect effect on the *future* tax rates. In other words they take next period tax rate as given and select their best response to it.

I justify this assumption on the ground of tractability (see Section 2 for a more complete discussion), together with the firm convinction that the qualitative results would not change if one were able to characterize the equilibria arising when the full burden of strategic rationality is imposed upon the voting agents.

The median voter turns out to be the representative agent from the middle age generation who is in fact the only one facing a meaningful tradeoff when selecting a fiscal policy. Given the choice of particular functional forms for the production and utility functions, the global dynamics of the system can be computed explicitely by solving the middle age political optimization problem and feeding it back into the competitive equilibrium relations.

As I am also interested in the studying the sinergies between private and public financing, I extend the basic model to allow for parents that care about their children and, in particular, about the level of education they achieve. I formalize this by simply introducing the human capital of the young into the utility function of their parents.

Exception made for the last section, I always assume the existence of a representative agent in each generation. Hence, I do not examine the intragenerational income distribution problems associated with voting on public education, which are instead the object of a number of recent studies, e.g. Eckstein and Zilcha (1991), Gloom and Ravikumar (1992), Perotti (1990), Saint-Paul and Verdier (1991).

A number of interesting issues in the theory of economic development can be addressed with the model I just described.

The process of economic growth is intrinsically intergenerational as it involves the transferring of an ever growing stock of resources from the old to the young generations. In the representative agent infinite horizon model this problem is solved at once by assuming a bequest motive that effectively transforms individuals into infinitely lived dinasties. If one believes this solution is too simple, then one faces the problem of explaining persistent growth from within an OLG framework where reproducible resources are not bequeathed. It is well known (Boldrin (1992), Jones- Manuelli (1992)) that, contrary to the infinite horizon model, persistent growth will not obtain if the technology is modeled as a one sector with constant returns to scale. Other channels for the transferring of income/wealth from the old to the young generations have therefore been proposed. This paper models (private and public) provision of education as one such intergenerational wealth transfer.

A consequence of this hypothesis is that while the presence of parental altruism may lead one to believe that these transfers are fully voluntary, the political equilibrium I study suggests that a portion of them is most likely not. I show in fact that, even when privately financed schools are a viable option, a majority of voters may find it rational to maintain funding of education by means of income taxes as the latter (forcedly) transfer income from the old to the middle and young generations. Public schooling, therefore, is not just an instrument for intragenerational income distribution, but also one of intergenerational income redistribution. And the latter may well be the key element guaranteeing a persistent increase in per-capita income.

The last statement is based on the idea that growth is due to the sinergies between human and physical capitals and to the absence of effective bounds on the accumulated stock of the first. This implies, though, that not only continuous accumulation but also the lack of it should be explainable from within our framework. A number of authors have developed models in which growth is due to the accumulation of human capital (Azariadis-Drazen (1990), Caballe-Santos (1991), Chamley (1992), Lucas (1988) and Uzawa (1965)). Among them, only Azariadis and Drazen build a model which explains both the presence and the absence of growth. It is based on the idea that the existence of a positive externality in the production of human capital may create a threshold mechanism such that countries below the threshold are unable to grow while those above will, (see Boldrin (1991) for some reservations about the viability of such an approach). The problem has a very simple solution in the context I study, where the existence of a poverty trap is independent from the presence of any kind of externality. It is related instead to the lack of initial income which renders the economic agents unwilling to invest in the future generations (either privately or by means of an income tax) and/or to its inadequate distribution which determines a wicked anti-growth alliance between the destitute part of the society and the old owners of the physical stock of capital.

The two reasons I adopt to justify the provision of education (altruism and increased productivity) are perfectly compatible in the sense that in equilibrium we should typically observe both private and public expenditure contributing to the accumulation of human capital. The model can be extended to deliver just that. I use it in section three to show that the intergenerational transfer implicit in public financing may result in a "crowding out" of private expenditure under appropriate circumstances. I show also that, while at high income levels a relatively inefficient public system will be replaced by private schools, the former may come into existence at very low income levels and help the economy to jump-start the growth process. This would not occur if private financing were the only option.

In a static model of school expenditure Peltzman (1973) has argued that the public *provision* of education may result in an equilibrium with *less* total expenditure in schooling due to the fact that using the services provided by public institutions prevents families from appropriately supplementing them with private contributions. This is related to the important debate about the effects that a fiscally supported system of educational vouchers would have on the accumulation of human capital (see e.g. Friedman (1962)).

In section four I introduce heterogenous agents into the basic model and modify the technology to incorporate Peltzman's observation. While this may have the umpleasant consequence of creating a multiplicity of political equilibria (as pointed out already in Stiglitz (1974) I try to avoid them by making a number of simplifying (but, I believe, empirically justifiable) assumptions. It turns out that, quite generally, not only the demand for education would go down but that also the total amount of public financing would diminish, as the strategic considerations implicit in the political process generate a much smaller equilibrium tax rate. This happens because the impossibility of supplementing public funds with private ones lead the wealthier portion of the middle age generation to escape from the public system altogether. When this "run to the suburbs" occurs, the only portion of public financing this group is willing to support is the one affecting the future return on the aggregate capital stock. By shifting downward the whole distribution of preferred tax rate this must yield an equilibrium with less public financing of education. I believe this observation may be of some relevance for the current political debate about the impact that the introduction of vouchers would have on the quality of education received by the poorest group of our society.

The plan of the paper is the following. The next Section introduces the basic model, studies its equilibria and characterizes their long-run evolution. Section 3 adds parental altruism and stresses the intergenerational redistributive aspect of the public school system as a further motivation for voters' support. The long-run relation between per-capita income and educational spending is emphasized. In Section 4 I look further into this issue along the lines of the previous paragraph. Section 5 concludes.

2. Voting on Public Financing of Education.

2.1 The Basic Model.

I will study an economy composed of overlapping generations of identical agents living for three periods. Each generation is composed of a continuum of atomless individuals, and population growth is such that each generation is $1+n$ times larger than the previous one. At the beginning of period $t = 0$ two generations are alive: the old people (of total size $1/(1+n)$) and the middle age ones (of size 1). A new generation of young individuals (of size $(1+n)$) is then born. The old agents already own the initial stock of capital K_0, while the middle age are endowed with human capital H_0. These are the two factors of production. The young individuals can only spend their time at school (if such an institution exists) acquiring human capital. When middle-age they will work and carry out consumption-saving decisions. When old they will just consume the return on their accumulated stock of physical capital.

For the time being I assume that the amount of time available to young people is inelastically dedicated to education and that no utility is obtained from leisure activities. I also assume that individuals have identical skills and identical initial human capital and that parents are completely selfish and do not care for the education of their children. These and other restrictions will be removed in the subsequent sections.

The technological capabilities of society are described by two production functions, one for the homogeneous consumption-investment good and the other for the accumulation of human capital. The first has the standard Cobb-Douglas form:

$$Y_t = K_t^{\alpha}(H_t)^{1-\alpha},$$

while I write the second as:

$$\lambda_t = h_{t+1}/h_t = \frac{(\epsilon + z_t)^{\gamma}}{(1+n)},$$

Capital letters are being used to denote aggregate variables and lower case letters denote per-capita values. So $y_t = Y_t/(1+n)^t$ is income per-capita which is equal to $k_t^{\alpha} h_t^{1-\alpha}$. Most of the ensuing analysis will be carried out in per-capita units. The parameters ϵ and γ are both positive and less than one and z_t denotes the quantity of per-capita physical resources devoted to education.

As I mentioned in the Introduction, borrowing against future income from human capital is impossible in this economy: nobody is willing to lend to the young people to attend school. For the purpose of explicitely calculating the equilibria I assume the utility function to be separable and logarithmic. Most results would be preserved by using separable and homothetic utility fucntions.

The life-cycle optimization problem for an agent born in period $t-1$ is then

$$U_{t-1} = \max\{\log c_t + \delta \log c_{t+1}\} \tag{2.1}$$

$$\begin{aligned} \text{subject to} : c_t + s_t &= (1-\tau_t)\omega_t \\ c_{t+1} &= (1-\tau_{t+1})\pi_{t+1}s_t = \tilde{\pi}_{t+1}s_t \end{aligned}$$

where τ_t is the tax rate in place during period t, individual labor income is $\omega_t = w_t \cdot h_t$, w_t is the wage rate and π_t is the period t net return on capital. The consumer's behavior is summarized by the following policies:

$$\begin{aligned} c_t &= \frac{1-\tau_t}{1+\delta}\omega_t \\ s_t &= \frac{\delta(1-\tau_t)}{1+\delta}\omega_t \\ c_{t+1} &= \frac{\delta(1-\tau_t)}{1+\delta}\omega_t \cdot \tilde{\pi}_{t+1} \end{aligned}$$

Competitive equilibrium in the inputs and output markets, together with the fact that the total expenditure on schooling $((1+n)^t z_t)$ has to be financed by taxes on current income $(\tau_t \cdot Y_t)$ yield:

$$\begin{aligned} \omega_t &= (1-\alpha)k_t^\alpha h_t^{1-\alpha} \\ \pi_t &= \alpha k_t^{\alpha-1} h_t^{1-\alpha} \\ k_{t+1} &= \frac{\delta(1-\tau_t)}{(1+\delta)(1+n)}(1-\alpha)k_t^\alpha h_t^{1-\alpha} \\ h_{t+1} &= \frac{h_t}{(1+n)}\left(\epsilon + \tau_t k_t^\alpha h_t^{1-\alpha}\right)^\gamma \end{aligned}$$

2.2 The Political Equilibrium.

To close the model and move on to the study of its dynamic implications, we need to determine the level of taxation τ_t. This will be done by means of voting on the tax rate: at the beginning of each period t all the citizens who are entitled to, cast their vote on the government's fiscal policy. The selected tax rate then gets implemented, consumption-saving decisions are made and the process repeats itself again and again in all the subsequent periods.

A mildly realistic interpretation of the model recommends treating the young generation as composed of individuals which have not yet entered college. In all the countries I am familiar with, voting rights are acquired between the age of eighteen and twentyone: consequently I will assume that young people do not participate in the political decision making process. The assumption is of no harm, as the same conclusions would be reached even if I allowed the young agents to exercise some political power. As for the middle age and the old individuals I will assume they all have equal voting rights.

I assume that a simple majority is required to reach a decision: the tax rate collecting the majority of votes will be implemented. This still leaves open the question of how a rational agent should decide to cast its vote in an environment such as the one I have described. This requires making assumptions about the set of available actions, their impact on the aggregate state variables and the notion of equilibrium adopted by the representative voter.

To address these issues one needs to appeal to game theoretical arguments. In what I call the "strategic voter" model, it is assumed that agents realize that their vote, by affecting the tax rate, will influence the future state of the economic system. This, in turn, will affect future agents decisions about fiscal policies: today's taxes impact on current investment decisions, which in turn will modify future's fiscal policies. Rational voting on the part of utility maximizing agents, therefore involves taking into account not only the impact that current tax rates have on future state variables but also the indirect effect this has on future tax rates.

Obviously this argument is of practical relevance only for the median voters' decision process, as all the other voters will understand that in equilibrium their votes do not matter. In the model being examined, the median voter is the representative member of the middle age generation so it is his choice of an optimal strategy that needs to be formalized. Set

$$u_t(\tau_t, \tau_{t+1}) = \log\big((1-\tau_t)\omega_t - s_t(\tau_t, \tau_{t+1})\big) + \delta \log\big((1-\tau_{t+1})\pi_{t+1}(\tau_t, \tau_{t+1})s_t(\tau_t, \tau_{t+1})\big)$$

Then the equilibrium tax rates sequence will be derived as the perfect Nash equilibrium of a game involving an infinite number of players: the middle age generations alive in the periods $t = 0, 1, \ldots$. The following "infinitely nested" set of optimization problems formalizes the median voter decision in period t:

$$\max_{\tau_t} \; u_t(\tau_t, \tau_{t+1}) \tag{2.2}$$

$$\text{subject to : } \tau_{t+1} = \arg \max_{\tau_{t+1}} \; u_{t+1}(\tau_{t+1}, \tau_{t+2})$$

$$\text{subject to : } \tau_{t+2} = \arg \max_{\tau_{t+2}} \; u_{t+2}(\tau_{t+2}, \tau_{t+3})$$

$$\text{subject to : } \tau_{t+3} = \ldots \text{ etc. } \ldots$$

An equilibrium is then a sequence of functions $\tau_t^*(\cdot)$ such that $\tau_t = \tau_t^*(\cdot)$ solves (2.2) given $\{\tau_0^*(\cdot), \tau_1^*(\cdot), \ldots, \tau_{t-1}^*(\cdot), \tau_{t+1}^*(\cdot), \ldots\}$. The political equilibrium so obtained is then a perfect Nash equilibrium along which each middle age generation choses the tax rate τ_t optimally by fully discounting the effect it will have upon τ_{t+1} and so on.

In spite of its good theoretical attributes, I will *not* make use of the strategic voter assumption to close the model. The motivation is, essentially, one of mathematical tractability: the hypothesis that each generation chooses its optimal strategy $\tau_t^*(\cdot)$ by taking also into account the effects it will have on the value of $\tau_{t+1}^*(\cdot)$ makes it impossible (at least for me) to derive an analytical representation of the equilibrium sequence. The problem goes much deeper than a simple matter of computability. While the general results contained in Harris (1985) may be used to prove that a perfect Nash equilibrium exists for the game defined in (2.2), this is of little help for our purposes. To carry out a meaningful analysis we would require the existence of an equilibrium representable by means of a stationary, continuous function of the state variables. This is a difficult mathematical issue, akin to those studied in Bernheim and Ray (1985) and Leininger (1986), and upon which I would rather avoid dwelling in these circumstances. The literature on the topic is quite extensive, I will refer the reader only to the recent elegant discussion of the matter contained in Chari and Kehoe (1990), and to Boldrin and Santos (1993) for a tentative attack.

Dropping the perfectness requisite is enough to deliver an analytically tractable notion of political equilibrium. In choosing its utility maximizing tax the median voter will simply take next period tax rate as a given number, and not as a function of the future state variables. In this way the maximization problem (2.2) collapses to the much simpler one

$$\max_{0 \leq \tau_t \leq 1} \; u_t(\tau_t, \tau_{t+1}) \tag{2.3}$$

$$\text{subject to : } \tau_{t+1} \text{ given in } [0, 1]$$

the solution of which is, in general some time-invariant mapping τ^* representing the equilibrium tax τ_t as a function of the current state (K_t, H_t) and the future tax rate τ_{t+1}. An equilibrium is then a sequence of tax rates $\{\tau_t^*\}_{t=0}^{\infty}$ such that τ_t^* solves (2.3) given τ_{t+1}^*.

The equilibrium tax is again the one chosen by the representative individual in the middle age group. In fact an agent born in period $t-2$, who belongs to the old generation during period t, will cast his vote by solving:

$$\max_{0\leq\tau\leq 1} \log\big((1-\tau)\pi_t s_{t-1}\big)$$

The solution to which is readily seen to be $\tau_t \equiv 0$. The old people have nothing to gain by investing in the education of the young generation: this only takes away current income from them and deliver a future increase in productivity which they cannot enjoy. A middle age individual, on the other hand, faces a more interesting tradeoff: by giving up some income today it will enjoy a higher return on capital tomorrow. An agent born in period $t-1$, solves the following problem when voting during period t:

$$\max_{0\leq\tau_t\leq 1} \log\big((1-\tau_t)\omega_t - s(\tau_t)\big) + \delta \log\big((1-\tau_{t+1})\pi_{t+1}(\tau_t)s(\tau_t)\big) \tag{2.4}$$

The solution to (2.4) may or may not be positive (see below). The outcome would not change if we had allowed young people to vote: their preferred tax rate is always equal to one. The only borderline case left is the one in which there is no population growth and the young agents do not vote: then $\tau_t = 0$ and the solution to (2.4) would each receive 50% of the vote and some ad-hoc mechanism would have to be introduced to break the tie.

2.3 Equilibrium Dynamics

Let us now go back to the maximization problem (2.4). Making use of the competitive equilibrium values of s_t and π_{t+1} and massaging the first order conditions yield:

$$\tau_t^* = \frac{\gamma\delta(1-\alpha)}{\gamma\delta(1-\alpha)+(1+\alpha\delta)} - \frac{\epsilon(1+\alpha\delta)}{\big(\gamma\delta(1-\alpha)+(1+\alpha\delta)\big)y_t} \tag{2.5}$$

which, after an obvious change of notation, can more parsimoniously be written as

$$\tau_t^* = \frac{a}{a+b} - \frac{\epsilon b}{(a+b)y_t} \tag{2.6}$$

Consider now the implications of the voting rule (2.6) for the equilibrium dynamics. Notice first that a "poverty trap" mechanism is always at work here. Whenever the current income level is not enough, or the marginal return to investing in education is low (from the view point of future capital holders) the approved tax will be zero. To help the intuition let us look at the equilibrium tax that obtains for a general, homothetic, utility function $u(c) + \delta u(c')$. This is

$$\tau = 1 - \frac{\pi(\tau)}{g(\pi(\tau)) \cdot \partial\pi/\partial\tau} \tag{2.7}$$

where $g(\pi)$ is the function satisfying $s = (1-\tau)\omega g(\pi)$. A specific prediction of the model is therefore that public funding for education should be an increasing function of the level of investment in physical capital or (more precisely) of the portion of current income that is invested in the future capital stock. Moreover under the fairly reasonable assumptions that $g(\pi)$ is less than unitary elastic and $\pi(\tau)$ is a concave function, (2.7) also suggests that the tax rate should be a decreasing function of the expected return on investment. A zero tax rate will be voted by the middle age group whenever the amount of income allocated to physical capital is not high enough, i.e. whenever

$$\frac{\partial\pi/\partial\tau}{\pi} \leq \frac{1}{g(\pi)}$$

The latter inequality also suggests that in a model with heterogeneous agents the of current savings among the middle age population should affect the political equilibrium. Middle age individuals with little or no physical capital would never vote in favor of an income tax to foster education. This points to a crucial interaction between the allocation of physical wealth and the level of public investment in education which ought to be more fully investigated.

In our model the no-taxes condition becomes:

$$y_t \leq \frac{\epsilon(1+\delta\alpha)}{\delta\gamma(1-\alpha)} = y_{min} \tag{2.8}$$

which is much simpler than the one for the general case, but retains most of its qualitative implications. Given the parameters of the utility and production functionsi, (2.8) boils down to a simple restriction on income levels: poor countries tend to vote against financing of education, from which the poverty trap. Furthermore, countries with a low saving rate will be less likely to invest in public education, and countries where the physical stock of capital receives a larger portion of national income will require a higher level of income per-capita to invest in public education.

When the median voter favors a positive tax the per-capita amount of resources devoted to education turns out to be equal to

$$z_t = \tau_t y_t = \frac{a y_t}{a+b} - \frac{\epsilon b}{a+b} \tag{2.9}$$

which is an increasing function of income as we would have expected. Plugging (2.9) into the law of motion of human capital delivers a simple growth condition

$$\{\frac{h_{t+1}}{h_t} > 1\} \quad \Leftrightarrow \quad \{y_t \; > \; (\frac{(a+b)(1+n)^{(1/\gamma)}}{a} - \epsilon) \equiv \underline{y} \; > \; y_{min}\} \tag{2.10}$$

which stresses the fact that some investment in education is not necessarily enough for growth. In order words the poverty trap extends to income level beyond those described by (2.8).

The qualitative properties of the equilibrium dynamics can now be derived. For all pairs of initial conditions (h_t, k_t) such that $k_t^\alpha h_t^{1-\alpha} \leq y_{min}$, one has

$$\begin{cases} k_{t+1} &= \frac{\delta(1-\alpha)}{(1+\delta)(1+n)} k_t^\alpha h_t^{1-\alpha} \\ h_{t+1} &= \frac{\epsilon^\gamma}{1+n} h_t \end{cases} \tag{2.11}$$

while for pairs of initial conditions (h_t, k_t) such that $k_t^\alpha h_t^{1-\alpha} > y_{min}$, the equilibrium dynamics is

$$\begin{cases} k_{t+1} &= \frac{\delta(1-\alpha)b}{(1+\delta)(a+b)(1+n)} \left(\epsilon + k_t^\alpha h_t^{1-\alpha}\right) \\ h_{t+1} &= \left(\frac{a(\epsilon + k_t^\alpha h_t^{1-\alpha})}{a+b}\right)^\gamma \frac{h_t}{1+n} \end{cases} \tag{2.12}$$

The dynamical system (2.11) has its only stationary state at the origin, which is obviously attracting for all orbits starting nearby. The system (2.12) has instead a unique interior stationary state that lies at the intersection between the curve

$$k^\alpha h^{1-\alpha} = \underline{y} \tag{2.13}$$

and

$$k = \frac{\delta b(1-\alpha)(1+n)^{(1-\gamma)/\gamma}}{a(1+\delta)} \tag{2.14}$$

where $\underline{y}$ is the minimum income value that allows for positive growth in per-capita human capital, defined in (2.10). The curve described by (2.13) in the (h,k) plane defines the border between the poverty trap area, in which too little resources are invested in education and income per capita decreases, and the growth area in which the amount invested in education is high enough to allow for persistent increase in income. This is represented in Figure 1.

2.4 Modelling School Attendance Rates

It is rather straightforward to replicate most of the previous results in a model where the members of the young generation are allowed to choose the amount of time they spend at school. Once again I will adopt a very simple functional form to allow for explicit calculations of the equilibrium values. To save on notation I will also set the growth rate of population $n = 0$ and pretend that, in front of an electoral tie, the will of the middle generation is enforced.

I start by assuming that, when young, a member of this society may either attend school or work in some productive activity that utilizes unskilled labor and requires no stock of physical capital (newspaper delivering, land mowing, babysitting, etc.). Working in this kind of "underground" economy pays a fixed wage rate β per unit of time, and I normalize β so that it is expressed in "utils". The life-time utility function of an individual born at $t-1$ can now be written as

$$U_{t-1} = \beta(1 - \ell_{t-1}) + \log c_t + \delta \log c_{t+1} \tag{2.15}$$

The description of the educational system needs to be modified accordingly to make the growth of human capital a function of the amount of the time invested in schooling. To keep the new model close to the initial one and tractable at the same time, the following functional form will be adopted:

$$h_t = h_{t-1}\left(\epsilon + \ell_{t-1}^{\theta} z_{t-1}\right)^{\gamma} \tag{2.16}$$

with the parameter θ restricted to $0 < \theta \leq 1$. Maximization of (2.15) under the budget constraints in (2.1) and the new constraints (2.16) and $0 \leq \ell_{t-1} \leq 1$, yields consumption-saving policies identical to those of the previous subsection, while the school attendance rate can be derived from the first order condition

$$\frac{\beta}{1+\delta} = \frac{\gamma\theta z_{t-1}\ell_{t-1}^{\theta-1}}{\epsilon + \ell_{t-1}^{\theta} z_{t-1}} \tag{2.17}$$

The latter implies that as long as expenditure on education is positive, school attendance is also positive and increasing until full scholarization is achieved at a level of expenditure per capita equal to

$$\overline{z} = \frac{\epsilon\beta}{(1+\delta)\gamma\theta - \beta}$$

To derive a closed form expression for the school attendance level we restrict our attention to the special case $\theta = 1$. In this case we have

$$\ell_{t-1} = \frac{\gamma(1+\delta)}{\beta} - \frac{\epsilon}{z_{t-1}} \tag{2.18}$$

Notice that $\ell_{t-1} = 0$ now becomes an equilibrium if public expenditure is too low. It is also a straightforward matter to verify that young agents still prefer a tax rate equal to 100%, so that the median voter is still the representative middle age individual.

By substituting (2.18) into the utility function (2.15), together with the usual consumption saving policies, and then solving the new version of the maximization problem (2.4) the equilibrium tax rate can be computed. It turns out to be always positive and, in fact, independent of the income level. Its value is

$$\tau^* = \frac{a}{a+b}$$

which corresponds to the maximum achievable tax rate under the exogenous school attendance regime of the previous subsections. Notice, though, that the poverty trap has not disappeared in this version of the model, as the level of school attendance will now be zero for all income levels satisfying

$$y_t \leq \frac{\epsilon}{a}\left(b + \frac{\beta(a+b)}{\gamma(1+\delta)}\right)$$

More generally the dynamical system describing the growth process when $\ell_t > 0$ is now

$$\begin{cases} k_{t+1} &= \frac{\delta(1-\alpha)b}{(1+\delta)(a+b)}\left(k_t^\alpha h_t^{1-\alpha}\right) \\ h_{t+1} &= \left(\frac{a\gamma(1+\delta)}{\beta(a+b)} k_t^\alpha h_t^{1-\alpha}\right)^\gamma h_t \end{cases} \tag{2.19}$$

which exhibits qualitative properties that are completely analogous to those of (2.12). In particular, for all initial conditions (h_t, k_t) such that

$$k_t^\alpha h_t^{1-\alpha} < \frac{\beta(a+b)}{a\gamma(1+\delta)}$$

the growth rate of the per-capita human capital stock will be negative and no growth will take place, while the opposite is true when the latter inequality is violated.

We can summmarize the predictions of the simple model introduced in this section along the following lines. For a given level of average education, support for public financing of schools will appear when the stock of physical capital reaches a critical level, before which persistent accumulation of human capital should not be observed. The portion of national income devoted to public education increases with income, but is bounded above by some number less than one. The same is true od the school attendance rate among young individuals. The latter can also be kept near zero if the amount of resources devoted to education is inadequate. This should be contrasted with the prediction of models where public education is motivated only by intragenerational redistribution purposes. In those cases expenditure for public education decreases there when average income increases.

While I have not attempted it here, it should be clear that all these predicitons can be appropriatyely quantified by applying the previous model to real world data.

3. Parental Altruism

In this section I test the performances of the basic model when parents are assumed to be altruistic, thereby providing a second motivation for the provision of schooling. Indeed the introduction of altruism is a necessary requirement for studying the relation between public and private school financing, as the latter seems to be understandable only on the ground of parental generosity. The introduction of enough altruism in our model can, by itself, explain the existence of schooling and persistent growth. Still it would not explain the widespread adoption of publicly supported schools as an instrument to increase society's average human capital.

To avoid collapsing the model into one of an infinitely lived dinasty I will assume that parental altruism expresses itself in the following form

$$U_{t-1} = \log c_t + \delta \log c_{t+1} + \log h_{t+1} \tag{3.1}$$

Parents care about their children well being only insofar as this has to do with their education. They will provide for schooling, but will not leave any other kind of physical bequests to their offspring.

3.1 Private versus Public School Financing.

Denote with z_t^p the portion of income that parents will be privately willing to devote to support education. The total amount of per-capita resources available for human capital accumulation is then equal to

$$z_t = z_t^p + \lambda \tau_t y_t$$

where $0 < \lambda \leq 1$ is a parameter meant to capture administrative costs and other factors making public financing relatively more inefficient than private financing. While no definite conclusion seems to have been reached on this point, I should mention that the available evidence suggests that, *net* of the deadweight losses of taxation, the empirical value of λ may be pretty close to one (Levin (1991), West (1991)). It is also important to note that in this section, as in the previous one, I am still assuming that public education works under a voucher-type system and that actual provision of the service occurs through a competitive market. This allows parents to supplement public funds with private ones and justify the new definition of z_t. The case in which public education entails publicly provision of educational services will be considered in the next section.

To save us the burden of an excessive notation set $\gamma = 1$ in the human capital accumulation rule, this will not affect the final result in any significant manner. When the maximization of (3.1) under the budget constraints $c_t + z_t^p + s_t \le (1-\tau_t)\omega_t$ and $c_{t+1} \le \tilde{\pi}_{t+1} s_t$, results in a set of interior solutions one has

$$\begin{aligned} c_t &= \frac{1}{2+\delta}\big((1-\tau_t)\omega_t + \epsilon + \lambda\tau_t y_t\big) \\ s_t &= \frac{\delta}{2+\delta}\big((1-\tau_t)\omega_t + \epsilon + \lambda\tau_t y_t\big) \\ z_t^p &= \frac{1}{2+\delta}\big((1-\tau_t)\omega_t - \epsilon(1+\delta) - (1+\delta)\lambda\tau_t y_t\big) \end{aligned} \tag{3.2}$$

The latter formulas emphasizes the fact that the middle age generation's current disposable income is now greater than its after-tax wage payments. It includes the real value of government transfers for education ($\lambda\tau_t y_t$) and the fixed value ϵ. One will also notice that the non-negativity constraint on z_t^p will be binding when the tax rate is too high and/or current labor income too low, i.e. when

$$\left\{ z_t^p = 0 \right\} \;\Leftrightarrow\; \left\{ \tau_t \ge \frac{1-\alpha}{(1-\alpha)+\lambda(1+\delta)} - \frac{\epsilon(1+\delta)}{y_t((1-\alpha)+\lambda(1+\delta))} \equiv \overline{\tau}(y_t) \right\}$$

Notice first that a "crowding out" mechanism has already been introduced by means of taxation: if, given current income, the latter is higher than $\overline{\tau}(y)$ no private expenditure should be observed. Moreover a very simple poverty trap is still present, as the private investment in education may be zero even when the tax rate is zero. This occurs when the country is too poor and/or when the share of income going to capital owners is very low, i.e. when $y_t \le \epsilon(1+\delta)/(1-\alpha)$.

We should determine next the equilibrium level of taxation. I will assume that the median voter takes into account the non-negativity constraint on z^p when choosing his tax rate, i.e. he chooses the value of $\tau \in [0, \overline{\tau}(y)]$ that maximizes

$$\begin{aligned} &\log\left(\frac{(1-\tau)\omega + \epsilon + \lambda\tau y}{2+\delta}\right) + \delta \log\left(\frac{\delta}{2+\delta}\tilde{\pi}(\tau)\big((1-\tau)\omega + \epsilon + \lambda\tau y\big)\right) + \\ &\quad + \log\left(h\Big(\epsilon + \lambda\tau y + \frac{(1-\tau)\omega - \epsilon(1+\delta) - \lambda(1+\delta)\tau y}{2+\delta}\Big)\right) \end{aligned} \tag{3.3}$$

The intergenerational redistribution of income now becomes a crucial factor in the political decision process. One can verify in fact that the equilibrium tax rate will be determined according to the following simple rule

$$\tau_t^* = \begin{cases} 0 & \text{if } \lambda < 1-\alpha \\ \overline{\tau}(y_t) & \text{if } \lambda > 1-\alpha \end{cases} \tag{3.4}$$

When the public administration is so inefficient that is does not pay out to "confiscate" income from the old people, the middle age voters will decide to rely completely on the private education system. They will instead impose the maximum feasible tax when the redistributive factor dominates.

Quite obviously this bang-bang behavior depends on the simplifying assumption that the substitutability between public and private inputs is a fixed constant and on the special utility functionwe I have adopted. But the general message would not have changed had I chosen different functional forms: the choice between a private and a public school system depends on the trade off between their relative efficiency and the redistribution of income (away from old capital stock holders) that the second manages to accomplish.

The dynamic implications of this result are not very hard to figure out by adapting the logic I already used in the previous section. Under either one of the two regimes there will exist some upper bound delimiting a set of initial conditions for which persistent accumulation cannot be an equilibrium outcome, while the economy will grow unbounded if it starts above this set. All the predictions listed at the end of section two remain true in the presence of altruistic parents.

3.2 Public Schools as Shoe Laces

In this subsection I will argue that, even if inefficient, the public financing of education may be conducive of aggregate growth in situations in which the private altruistic motive would not suffice.

To show this I impose the restriction $\lambda < 1 - \alpha$, so that no public financing of schools would emerge in the political equilibrium if the income level were high enough to induce some positive private spending on education. Remember next that, even if $\tau_t = 0$, private expenditure will be zero when the income level is below $\epsilon(1+\delta)/(1-\alpha) = y_{min}$. In this circumstances the optimization problem faced by the median voter at time t is quite different from (3.3). In particular no one-to-one offset occurs between an increase in public and a decrease in private expenditure on schooling, for the very simple reason that the latter is already equal to zero. The median voter then maximizes

$$\log\Big(\frac{(1-\alpha)(1-\tau_t)y_t}{1+\delta}\Big) + \delta\log\Big(\frac{\tilde{\pi}_{t+1}(\tau_t)\delta(1-\alpha)(1-\tau_t)y_t}{1+\delta}\Big) + \log\big(h_t(\epsilon + \lambda\tau_t y_t)\big) \quad (3.5)$$

Expression (3.5) is an increasing function of τ at $\tau = 0$ when

$$y_t > \frac{\epsilon(1+\alpha\delta)}{(1+\delta(1-\alpha))\lambda} \quad (3.6)$$

Now the latter is smaller than y_{min} whenever λ i and δ are close to one. In other words when the public financing system is not utterly inefficient and the future is not dicounted too heavvily, it pays to have some public schooling even if private schooling could not be afforded. The equilibrium tax rate, in these circumstances turns out to be

$$\tau_t^* = \frac{1+\delta(1-\alpha)}{2+\delta} - \frac{\epsilon(1+\alpha\delta)}{\lambda y_t(2+\delta)} \tag{3.7}$$

It seems therefore possible to conclude that a very poor country may still pull itself up by its own shoe laces by taxing national income according to (3.7) and investing the revenues in the production of human capital. Still we need to check that the amount of resources collected through (3.7) is enough to make the stock of per-capita human capital grow. Plugging (3.7) in the law of motion for h_t and imposing $h_{t+1}/h_t > 1$ yields

$$y_t > \frac{(2+\delta)\lambda(1-\epsilon)+\epsilon(1+\alpha\delta)}{\lambda\big(1+\delta(1-\alpha)\big)} \tag{3.8}$$

The latter is a much more stringent requirement than (3.6) as the level of income on its right-hand-side may be larger than y_{min}, in which case our optimistic conclusions would not longer hold. The crucial parameter is now ϵ, i.e the (inverse of the) speed at which average human capital depreciates from one generation to the next. If the latter is close enough to one the condition (3.8) reduces to (3.6) which, as we have seen, can always be satisfied. The empirical relevance of all this obviously needs to be determined.

When all the pieces of the story I have been telling in this section are glued together one has the phase diagram represented in figure 2. A poor country with an income level above the minimum given by (3.8), may start growing thanks to the introduction of a publicly financed education system. When its income is high enough it becomes possible to switch to a privately supported system if parents are sufficiently altruistic and the public system results more inefficient of a private one in spite of the redistributive role it may play.

4. Heterogeneous Agents Prefer Vouchers.

I have already insisted on the fat that the "publi" nature of the system I am onsidering, follows from its soure of finaning and not by the way in whih the servie is provided. In fat a number of assumptions I have made (see e.g. Setion 3) makes it resembles more an eduational vouhers system finaned by inome taxes than the publi shool system we are familiar with. In this last setion I move away from this assumption and introdue expliit restritions aimed at apturing an important feature of publily provided shool servies.

I onentrate my attention on the fat that provision of shooling involves a fundamental indivisibility: attending a given shool prevents a person from supplementing it with servies from different shools. More to the point, every shool typially provides a fixed amount of eduation on a "take-it-or-leave-it" basis. If more eduational servies are sought one would have to purhase *their totality* from a different soure. While this tehnologial restrition applies to private and publi shools alike, the latter are haraterized by the fat that it is very hard to inrease/derease the quality of eduation one reeives by moving from one publi shool to another. Within a given shool distrit a substantial uniformity exists and to move away from a distrit often involves very high transation osts.

It has been observed (e.g. Peltzman (1973)) that suh a mehanism tends to lower the total amount of eduation demanded relative to a system (suh as the one I onsidered earlier) in whih the government is transfering vouhers to families whih eventually purhase eduational servies in a ompetitive market.

I will argue here that in a dynami setting suh as the one I have elaborated publi provision of shooling will also tend to lower the amount of funding alloated to publi eduation and onsequently slow down the proess of apital aumulation. The intutive reason for this phenomenon is that, given a ertain amount of publily provided eduation there will always be families who are reeiving less than they onsider optimal. If they were allowed to supplement the government funding with private ones, they would simply do so and nothing essential would hange. When, instead, this is either impossible or very ostly, those parents seeking a high level of expenditure on their hildren eduation will be fored to give up the whole amount of funding oming from publi soure and bear the *full* ost of private eduation. From the point of view of these individuals the amount paid in taxes looses most of the utility they would otherwise have attahed to it. When it omes time for voting they will be willing to support either a much higher or a much smaller tax rate: under the first they will demand only public school services, while in the second case they will continue to use the private sector.

While simple to state, this mechanism is much harder to analyze as it involves a number of strategic subtleties that (as pointed out by Stiglitz (1974)) may easily lead to a pletora of different equilibria. As I am not interested in pursuing this line of reasoning in this istance, I will force the argument through by means of a number of simplifying (but not necessarily unrealistic) hypotheses.

Assume first that the growth rate of the population is near zero: this implies that, in order to be approved, a positive tax rate would require the support of the near totality of the members of the middle age group. Second, and most important, I make the hypothesis that, when faced with a large number of possible equilibria of the type often encountered in the "self-fulfilling prophecies" literature, the voting agents will adopt as focal points the simple ones.

In this context the crucial voters are those who might choose the public system if the latter could provide them with a very high level of services, but that would opt for a private school otherwise. This reduces the set of possible equilibria to the following: one in which everybody take advantage of the public system and another in which a certain portion of the society shift to the private system. I claim that the second equilibrium will obtain whenever the initial distribution of human capital displays more than a critical amount of dispersion. At this equilibrium the median voter is the one supporting the *smallest* level of taxation to finance public schools. I also show that, contrary what would be clearly true in the voucher-models of sections two and three, the median voter belongs now to the upper tail of the income distribution (how far in the tail turns out to depend on the magnitude of n, the population growth rate).

Let us assume that agents are heterogeneous in human capital levels, and impose the "either public or private" restriction on the educational technology. Assume then that each generation is still composed of a continuum of agents of size $(1+n)^t$, with type $i \in [0,1]$ reproducing itself from one generation to the next at a uniform rate $1+n$. The assumption that parents care about their children only insofar as their human capital is concerned but do not leave physical bequests, turns out to be very useful in this context as it breaks down the intergenerational linkage due to the physical stock of capital, which would have enormeously complicated our analysis.

As I do not intend to examine here the dynamic evolution of the allocation of income and human capital, I will make no special assumptions on the initial distribution of types $\mu(i)$. The aggregate state variables are defined as:

$$k_t = \int_0^1 k_t^i \mu(di); \qquad h_t = \int_0^1 h_t^i \mu(di); \qquad y_t = k_t^\alpha h_t^{1-\alpha}.$$

The life-cycle optimization problem of an individual of type i, born in period $t-1$ is now

$$\begin{aligned} &\max \left\{\log c_t^i + \delta \log c_{t+1}^i + \log h_{t+1}^i\right\} \\ &\text{subject to} : c_t^i + s_t^i + z_t^{ip} \leq (1-\tau_t)\omega_t^i \\ &\qquad c_{t+1}^i \leq \tilde{\pi}_{t+1} s_t^i \\ &\qquad h_{t+1}^i = h_t^i \cdot \left(\max\{h(z_t^{ip}), h(z_t)\}\right) \end{aligned} \tag{4.1}$$

where $h(x) = (\epsilon + x)^\gamma$, and $z_t = (\tau_t y_t)/m_t$ with $m_t \in [0,1]$ denoting that portion of the population currently attending public schools, something we should determine in equilibrium.

The "max" in the law of motion for human capital captures the exclusionary mechanism I discussed earlier. Notice that the hypothesis of "relative inefficiency" of public schools has been dropped to save on notation.

Manipulation of the first order conditions yields $s_t^i = \delta c_t^i$ as usual. The individual demand for private education requires a more detailed analysis. Begin by observing that given the commonality of preferences, those individuals demanding a higher total level of educational spending are also the wealthiest among the members of the middle age group. Given m_t and τ_t and by re-ordering types so that higher indices always correspond to higher incomes we have:

$$z_t^{ip} = 0 \quad \Leftrightarrow \quad \frac{h'(z_t)}{h(z_t)} \leq \frac{1}{(1-\tau_t)\omega_t^i - s_t^i - z_t} \quad \text{for } i \in [0, m_t] \tag{4.2}$$

and

$$z_t^{ip} > z_t \quad \Leftrightarrow \quad \frac{h'(z_t^{ip})}{h(z_t^{ip})} = \frac{1}{(1-\tau_t)\omega_t^i - s_t^i - z_t^{ip}} \quad \text{for } i \in [m_t, 1] \tag{4.3}$$

Notice the curious consumption pattern implied by (4.2), (4.3) and the "take-it-or-leave-it" provision. Under appropriately chosen parameter values there will exist an intermediate group of agents (the "not so rich" among those choosing to buy private education) that will have a consumption level lower than those inviduals immediately below them in the income scale that are opting for the public school system. A small amount of personal introspection supports the theoretical prediction.

The individual consumption-saving rules are the following.

Public School Families, $i \in [0, m_t]$:

$$\begin{aligned} c_t^i &= \frac{1-\tau_t}{1+\delta}\omega_t^i \\ s_t^i &= \frac{\delta(1-\tau_t)}{1+\delta}\omega_t^i \\ c_{t+1}^i &= \frac{\tilde{\pi}_{t+1}\delta(1-\tau_t)}{1+\delta}\omega_t^i \end{aligned}$$

Private School Families, $i \in [m_t, 1]$:

$$\begin{aligned} c_t^i &= \frac{(1-\tau_t)\omega_t^i + \epsilon}{1+\delta+\gamma} \\ s_t^i &= \frac{\delta\big((1-\tau_t)\omega_t^i + \epsilon\big)}{1+\delta+\gamma} \\ z_t^{ip} &= \frac{\gamma(1-\tau_t)\omega_t^i - \epsilon(1+\delta)}{1+\delta+\gamma} \\ c_{t+1}^i &= \frac{\tilde{\pi}_{t+1}\delta\big((1-\tau_t)\omega_t^i + \epsilon\big)}{1+\delta+\gamma} \end{aligned}$$

The equilibrium levels of m_t and τ_t need to be determined simultaneously. Consider first the possibility of an equilibrium where $m_t = 1$. The tax rate will then emerge from the following maximization problems

$$\max_{0\le\tau\le 1} \log\left(\frac{(1-\tau)\omega^i}{1+\delta}\right) + \delta\log\left(\frac{(1-\tau)\delta\tilde{\pi}\omega^i}{1+\delta}\right) + \log\big(h^i(\epsilon+\tau y)^\gamma\big)$$

which yields the following first order condition for an interior solution

$$\frac{1+\delta}{1-\tau} = \delta\frac{\partial\pi}{\partial\tau}\cdot\frac{1}{\pi(\tau)} + \frac{\gamma y}{\epsilon+\tau y} \tag{4.4}$$

Denote the unique solution to (4.4) with τ_t^*. Note that our choice of utility function makes it independent of the individual type: at the "good" equilibrium there is unanimity about the level of taxation. This will not be true for more general utility functions. From (4.2) an equilibrium pair (m_t, τ_t) has to satisfy:

$$m_t \le \frac{(1+\gamma)\tau_t y_t}{\gamma(1-\tau_t)\omega_t^i - \gamma s_t^i - \epsilon} \tag{4.5}$$

for all indices $i \in [0, m_t]$. Hence $(1, \tau_t^*)$ is *not* an equilibrium if there exists an agent $j \in [0,1]$ for which

$$\gamma(1-\tau_t^*)\omega_t^j - \gamma s_t^j - \epsilon \; > \; (1+\gamma)\tau_t^* y_t \tag{4.6}$$

The latter defines the maximum amount of income dispersion which is consistent with an equilibrium in which only public schools exist. I will assume that the threshold defined in (4.6) is violated for some non-negligible set ν_t of agents in $[0,1]$.

When this is true all agents will realize that (4.4) cannot be an equilibrium. The latter wiil instead be determined by a new pair (m_t, τ_t), with $m_t < 1$ satisfying (4.5). Those $i \in [0, m_t]$ for which (4.5) is satisfied will then vote according to

$$\frac{1+\delta}{1-\tau} = \delta \frac{\partial \pi}{\partial \tau} \cdot \frac{1}{\pi(\tau)} + \frac{\gamma y}{\epsilon m_t + \tau y} \tag{4.7}$$

which, under our assumptions, still has a unique solution $\overline{\tau}(m_t) > \tau_t^*$. Unfortunately the latter, while an equilibrium "choice" for all the individuals in $[0, m_t]$, is not going to be the equilibrium tax rate. As long as $m_t < 1$ (i.e. $\nu_t > 0$) the median voter is the *richest* $j \in \nu_t$. He chooses his vote according to

$$\max_{0 \leq \tau \leq 1} \; \log\left(\frac{(1-\tau)\omega^j + \epsilon}{1+\delta+\gamma}\right) + \delta \log\left(\frac{\tilde{\pi}\delta\big((1-\tau)\omega^j + \epsilon\big)}{1+\delta+\gamma}\right)$$
$$+ \log\left(h^j\left(\epsilon + \frac{\gamma(1-\tau)\omega^j - \epsilon(1+\delta)}{1+\delta+\gamma}\right)^{\gamma}\right)$$

which yields the first order condition

$$\frac{(1+\delta+\gamma)\omega^j}{(1-\tau)\omega^j + \epsilon} = \delta \frac{\partial \pi}{\partial \tau} \cdot \frac{1}{\pi} \tag{4.8}$$

A comparison of (4.7) and (4.8) shows that the solution to the latter is *always* strictly smaller than the solution to the former for some large enough level of income dispersion, i.e. for some appropriately chosen ω^j. The political equilibrium will therefore be established around the tax rate level proposed by the rich portion of the population, which is *smaller* than the one proposed by the other members of the middle age group.

An important implication of the present exercise is the following: when growth in average income is accompanied (as it seems to be in the real world) by a reduction of income inequalities, we should observe a correlation between increases in per-capita income and the amount of public financing of education. This aapears to be consistent with the empirical work of James (1992) who observes more private schools in poor countries. Furthermore, if higher growth rates are (at least partially) the outcome of a higher investment in education then less inequality means more economic growth.

The implications for the dynamic behavior of our model economy is straightforward: under the "subsidy-in-kind" system the equilibrium amount of public education provided is strictly less than the one that obtains under a voucher system. This impacts negatively on the overall process of human and physical capital accumulation which in turn leads to a slower growth rate of national income.

It is worth stressing that, while the arithmetics of the previous argument is greatly simplified by my choice of utility and production functions, the crucial point would be preserved by more general functional forms. Once again the empirical relevance of the phenomenon I have pointed out and its actual impact on the growth process of our economies may be quantified by making appropriate use of the model developed here.

The general result seems to be quite consistent with what we observe in reality and provides strenght to the argument claiming that the adoption of a market approach to the *provision* of education will in fact increase the equilibrium amount of resources devoted to it.

In fact one may push the argument further and claim that the introduction of monetary subsidies to education and the opening of a competitive market on the supply side, may help reduce the high amount of segregation we observe in many american communities. If we allow ourselves the freedom to extend the model outside its present boundaries, the first best for the rich portion of the population would be to try to create neighborhoods segregated along income lines with school financing to be provided locally. Something, indeed, we seem to observe extensively in the United States and which is the object of a recent interesting study by Fernandez and Rogerson (1992).

While I am unable to deliver a precise result with respect to the dynamic evolution of the distribution of income, I find it reasonable to conjecture that one should observe a larger increase in income inequalities under the "subsidy-in-kind" system than under teh vouchers system considered in earlier sections.

5. Conclusions.

I have proposed a model of schooling based on the fundamental idea that publicly subsidized education solves a free-rider problem in economies in which markets for financing of human capital investment are lacking. If human capital accumulation is one of the engines of growth, then public schools will tend to foster growth and will be introduced in those economies that have a high enough stock of physical capital to make the investment in education affordable and profitable at the same time.

When the amount of resources devoted to public education is decided by majority voting it becomes unavoidable to use it also as an intrument for intergenerational income redistribution. In my model, though, income redistribution runs from grandparents to children while the parents stand in between equalizing marginal costs and marginal gains. This aspect becomes even more important when parental altruism is introduced: parents will then finance some of their children education by taxing the grandparents' income. The incentive to do so is reduced, or even eliminated, when the public system is particularly inefficient relative to the private one and when the portion of income going to the elderly owners of the stock of capital is small.

Nevertheless there exist circumstances under which even an inefficient public school system may be useful to bootstrap economic development: in a political equilibrium with majority voting, public financing of schooling may be introduced when private financing would not emerge in equilibrium. This transferring of resources to education may be enough to start a growth process which, by increasing income per-capita past a critical level, may eventually lead to the dismissal of the public system in favor of a (supposedly more efficient) private one.

Finally I have argued that public provision of schooling, when practiced in the "take-it-or-leave-it" form which is the rule almost everywhere, will cause a decrease of the equilibrium amount of resources devoted to public education and a run away it and toward private schools on the part of the richest segments of the population. This, in turn will result in a reduction of the aggregate growth rates of human and physical stocks of capital. The implications it may also have on the income distribution dynamics ought to be investigated further.

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