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SOVEREIGN DEBT: FORGIVING AND FORGETTING RECONSIDERED

by

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Abstract

In this note we show that even if after default a sovereign can make deposits on a Swiss bank account, the exclusion from future debt is sufficient to deter a patient country from default. In contrast to the work by Bulow and Rogoff (1989) we assume that there is a fixed set of standard assets on the world financial markets in which the country can trade. In this setting the exclusion from future borrowing may seriously limit a country's ability to smooth consumption. Therefore the traditional reputational logic can be applied.

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1 Introduction

What ensures that sovereign countries do not always default on their loans to private creditors? Since in the case of sovereign debt legal sanctions are of little practical use, the creditor has to rely on a self enforcing mechanism. The obvious mechanism is to exclude borrowers who repudiate their debt from future borrowing. But does this threat deter a country from defaulting?

According to a paper by Bulow and Rogoff (1989), (in the following abbreviated by BR), the answer is no, if after default the country is allowed to purchase assets on the world capital markets. BR refute the claims of preceding work that the threat of no further borrowing after default is sufficient to give incentives to a "patient" country to honor its debt[1].

In this body of research it had generally been assumed that after default a sovereign suffers from total exclusion from world capital markets for at least some period of time. Hence after default the country can no longer smooth consumption and there is a clear incentive to avoid default. If, however, the sovereign can save after default it is not obvious that default does any damage to the ability to smooth consumption. In fact, in the framework analyzed by BR, at some date there is no loss in the country's smoothing ability and hence default always occurs. When the sovereign faces an uncertain future national income, the argument given by BR requires that in every period *every* asset with nonnegative payoffs in the following period is available. It should be noted that this condition is much stronger than the assumption of a complete set of securities in each period: Since any positive payoff vector in $t+1$ can be purchased in period t, the agents in this world do not have to assemble a portfolio for their insurance needs. They merely call a "Swiss Banker" who will deliver the custom made contract.

In this paper we will allow the sovereign to take its savings to a Swiss bank account even after default, but we will not make *every* insurance contract available to a defaulting country. In particular, we want to examine situations in which, although the sovereign is allowed to invest in assets after default, it cannot eliminate

[1]See, for example, Eaton, Gersovitz and Stiglitz (1986); Grossman and Van Huyck (1988), Manuelli (1987) and Cole and English)

all the risk in its future income. This limitation may come about for the following reasons:

- Suppose there is a complete set but a limited number of securities available. In this case every agent has to assemble an optimal portfolio for his or her insurance needs. In general this optimal portfolio will contain some negative positions. In particular, if the sovereign cannot find financial instruments with a payoff vector that is strongly negatively correlated with his income process he will want to hold negative positions of instruments that are positively correlated with his income. After default negative positions are no longer possible and hence the country might face a situation that is similar to market incompleteness: Although it is possible to hold any positive amount of any available asset, the "no-debt" restriction effectively makes the desired payoff stream unavailable.

- A second source of limited insurance possibilities after default is a reduced set of available assets: Certain assets may require the buyer to give up his anonymity and thereby bear the danger of seizure by the former creditor. Hence those assets may no longer be feasible choices for the country. This can lead to a situation of incomplete asset markets from the perspective of the defaulting country, e.g., if the only asset available is the riskless Swiss bank account.

We show that if a country's ability to insure against shocks in the income-process is limited after default then a patient country will have an incentive to repay its debt. We will give examples for a situation where even though before and after default there is a complete set of assets available for the country the restriction of "no-negative positions" is sufficient to give the country an incentive to repay its debt.

2 Sustainable Debt

We present a simple framework of a small country facing competitve, risk neutral foreign investors. The country is small in the sense that it cannot affect the world interest rate r. Let y_t be a random variable desribing the national income in period t and let θ be an exogenuous, identically and independently distributed random variable with finite support, $\theta \in \Theta = (1, ..., S)$. Then y_θ is the income if the realization of the disturbance is θ. Let π_θ be the probability that θ occurs in any given period.

Each period the sovereign can trade assets on the world financial markets that give some return in the next period. There are the same A assets available each period, indexed by $a = 1, ..., A$. If one unit of asset a is purchased in period t, then R^a_θ is the return of this asset in period $t+1$. Assume that $R^a_\theta \geq 0, \forall \theta$. Given a portfolio of assets $q \in \mathbf{R}^A$, the return can be described as the product of the return matrix (R^a_θ) and the portfolio q; $(R \cdot q)_\theta$ is the return if state θ occurs.

Let p^a_t denote the price of asset a in period t in terms of period t consumption. We assume that there is a complete set of assets, i.e. the span of the vectors R^a is $\mathbf{R}^S$.

Total income x_t in period t consists of national income y_t plus the returns on foreign investment $R \cdot q_{t-1}$. Each period the sovereign has to decide how much of its total income to spend on consumption c and how much to spend on each of the A assets available.

Let $\{c_t\}_{t=0}^{\infty}$ be a sequence of random variables describing a consumption plan. The sovereign's utility of this consumption is given by:

$$U(c) = E \sum_{t=1}^{\infty} \beta^t u(c_t) \tag{1}$$

where $u(\cdot)$ is strictly increasing, strictly concave and bounded, $|u| \leq K$, and $0 < \beta < 1$. We also normalize u so that $u(0) = 0$.

2.1 Reputation Contracts

Given the above structure of the decision problem, the representative agent may want to "borrow" on the world financial markets (i.e., hold negative positions). Suppose the country is allowed to hold debt up to an amount l. Suppose further that if a country ever defaults on its debt then it will no longer be able to hold short positions but it will still be able to make some cash-in-advance purchases on the world financial markets. Can such a reputation contract sustain positive debt?

Note that for this arrangement to be sustainable, it must be in the country's interest to honor its debt in every possible state of nature as long as the debt does not exceed some $l < 0$.

The sovereign can find itself in one of the following two environments:

(i) If in the past all debts have been honored, the country can hold negative positions up to a limit $l > 0$.The set of possible portfolios is $Y^l = \{q \in \mathbf{R}^A : (R^a \cdot q^a)_\theta \geq l, \forall a, \forall \theta\}$.

(ii) If at some point in the past default occured the sovereign is precluded from holding negative positions. Moreover, it can only purchase assets in a subset $A' \subset A$. In this case the set of possible portfolios is $Y^d = \mathbf{R}_+^{A'}$.

It is our goal to show that if the set of assets A' that are available for investment after default satisfies a certain restriction, then the country has no incentive to default in environment (i).

2.2 A Sufficient Condition for Sustainable Debt

To simplify the following analysis we make strong assumptions on asset prices and interest rates. We assume that asset prices are such that the sovereign can insure its risky income at fair prices and that interest rates do not change over time.

$$p_t^a = p^a = \frac{1}{1+r} \sum_\theta R_\theta^a \cdot \pi_\theta \tag{2}$$

It should be noted that the basic argument of this paper can also be applied to a more general setting. Under this simplifying assumption the sovereign's maximization problem in environment (i), and assuming that there is no default, can be written as:

$$v^1(x) = \max_{q \in Y^l} \{u(x - p \cdot q) + \beta E v^l(x'_\theta)\} \tag{3}$$

where

$$x'_\theta = y_\theta + (R \cdot q)_\theta$$

Similarly in environment (ii) the dynamic program is:

$$v^2(x) = \max_{q \in Y^d} \{u(x - p \cdot q) + \beta E v^2(x'_\theta)\} \tag{4}$$

where

$$x'_\theta = y_\theta + (R \cdot q)_\theta$$

After default the country is restricted in its trading opportunities on the world financial markets. We will impose the following condition on the effect of these restrictions:

Condition 1 *There does not exist a $q \in Y^d$ with $y_\theta + (Ry)_\theta = \lambda$ for all $\theta \in \Theta$, for some $\lambda > 0$.*

Condition 1 says that after default the agent is no longer able to purchase a portfolio that gives him full insurance, i.e. a sure income in the following period.

The weakest form of restricted acess to the capital markets after default corresponds to the idea that default does not lead to a restriction in the "savings" opportunities of a country. In that case all the assets that were available prior to default are available after default, with the restriction that only purchases are possible. Example 1 demonstrates that Condition 1 can be satisfied even if *all* assets can be purchased after default.

Note that this restriction is different from the restriction in BR in the following sense: BR assume that after default the total payoff vector received by the country in any future state has to be nonnegative, whereas we require the country not to hold any negative position in its portfolio. The idea here is that individual sellers of assets cannot keep track of the overall portfolio the sovereign is holding and hence the sovereign cannot go short in any single position.

Example 1 The first example shows how Condition 1 can be satisfied when the only restriction on asset purchases after default is "no negative position". Suppose there are 2 states and 2 assets. $y_{\theta_1} = 1, y_{\theta_2} = 2$.

$$R^1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} R^2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$Y = R^2_+$. Then Condition 1 is satisfied. Figure 1 shows the feasible state space after default for this example.

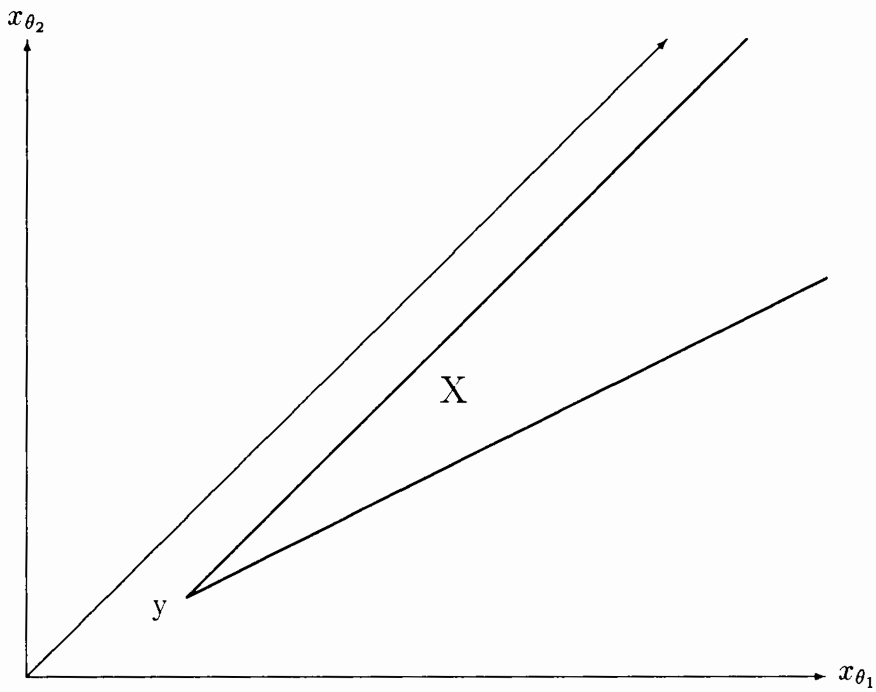

Figure 1

X is the feasible state space after default. Note that in this example even after default there is a complete set of assets available to the agent. The fact that short selling is prohibited precludes perfect insurance. The sovereign would like to purchase a portfolio that pays off more in the first state (when the income is low) than in the second state. The only way this can be done is by going short in the second asset. Since this is not possible after default, the sovereing cannot fully insure its future income.

In the second example we illustrate the effect of "loosing a market" after default. The following example illustrates the case where after default the only possible investment is an anonymous swiss bank account.

Example 2. Again there are 2 states, $y_{\theta_1} = 1, y_{\theta_2} = 2$.

$$R^1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} R^2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$Y = \{q \in R^2_+ : q^2 = 0\}$

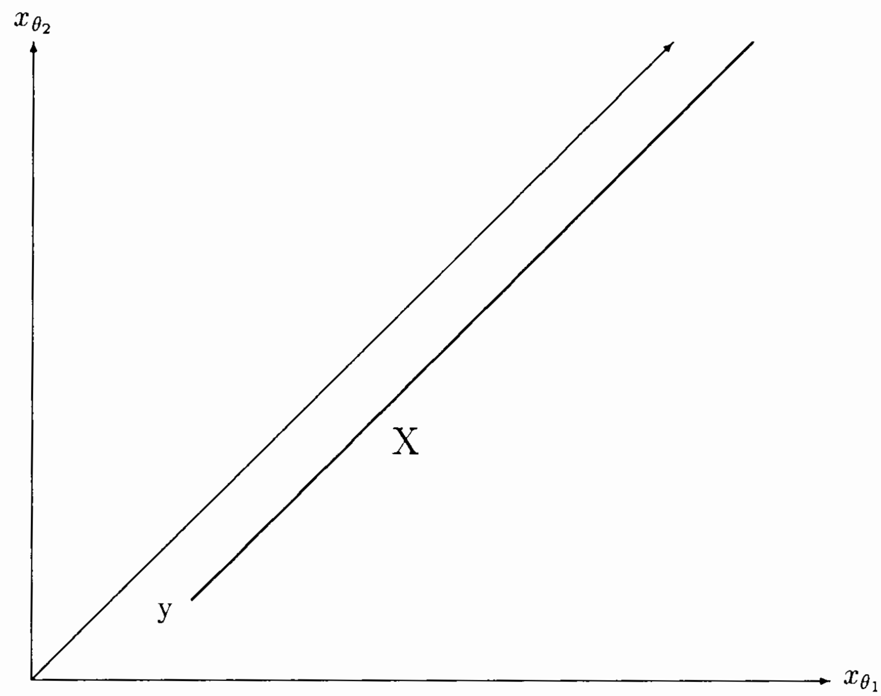

Figure 2

X is the feasible state space after default. In this case the sovereign can only hold a riskless asset after default. Again the desired portfolio would pay off more in state 1 than in state 2, but the exclusion from investing in market 2 makes it impossible for the sovereign to attain a riskless future income stream.

For simplicity we assume in the following that there is a riskless asset $\bar{a}$ that is available in both environments. Thus the sovereign can invest in a riskless asset after default.

Assumption 1 $\beta(1 + r) \leq 1$.

The following Proposition says that if Condition 1 and Assumption 1 hold then a patient country will repay its debt.

Proposition 1 *Suppose Assumption 1 and Condition 1 hold. Let $l < 0$ be given. Then if $\beta > \bar{\beta}$ for some $\bar{\beta} < 1$, the sovereign will honor every debt smaller (in absolute value) than l.*

The proof of the Proposition is given in the Appendix. The intuition behind the result can be summarized as follows. Even if the country can save after default, it cannot eliminate all the risk in its future income stream. Hence the sovereign can follow one of two possible programms: One possibility is that the country decides to self insure by accumulating larger and larger amounts of saving. In this case Condition 1 guarantees that the amount of savings needed for perfect self insurance goes to infinity. Hence self insurance through saving is very costly. In particular, it is more costly than repaying any given amount of debt, if the country is patient. For this argument to work Assumtpion 1 is needed. Assumption 1 guarantees that along the optimal investment policy income does not grow too fast. Otherwise, if (optimally) income converges to infinity in every state, there will be no cost to self insurance. The second possibility for the sovereign is to endure some risk in the consumption process. In this case default implies some utility loss *every* period. Assumption 1 guarantees that this loss will be bounded away from zero since the optimal consumption stays bounded with positive probability. Clearly a patient sovereign will repay its debt rather than occur a positive loss in utility every period.

3 Conclusion

We have shown that even if after default a country can make deposits on a Swiss bank account, the exclusion from holding negative positions can be sufficient to successfully deter default.

In contrast to the work by Bulow and Rogoff (1989) we assume that there is a fixed set of standard contracts (standard assets) on the world financial markets in which the country can trade. The fact that contracts will not be specifically designed for a country's needs after it defaulted makes negative positions more valuable than in Bulow and Rogoff's framework. Without debt the country's ability to smooth future income may be seriously damaged. Thus the traditional reputational logic can be applied. If the country is sufficiently patient then default does not pay since the loss of future smoothing ability outweighs the present gains from default.

The impossibility of these reputational contracts in BR is not caused by the innocuous assumption that every country can make deposits on a bank accout in Switzerland. Rather, the driving force behind their result is tha *any* state contingent contract with nonnegative future payoffs is available for purchase today, even after a country defaulted.

References

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Appendix

Proof of the Proposition: First without loss of generality let l be such that in environment 1 full insurance of the income stream is possible, i.e. $l \leq \bar{l}$ for some $0 < \bar{l} < \infty$ We want to show that for $x \geq 0$, $v^1(x+l) - v^2(x) > 0$, for $\beta > \bar{\beta}$, for some $\bar{\beta}$.

$$v^1(x+l) - v^2(x) = [v^1(x+l) - v^1(\bar{y}) + v^2(\bar{y}) - v^2(x)] + [v^1(\bar{y}) - v^2(\bar{y})]$$

Step 1: First we show that for any $l < 0$ there is a $L < \infty$ such that $[v^1(\bar{y}) - v^1(x+l) + v^2(x) - v^2(\bar{y})] \leq L \leq \infty$ for all $\beta \in [0,1]$. Let $\underline{y}$ denote $\min_\theta y_\theta$. To see Step 1 note that by the assumption of a riskless asset $\bar{a}$ we know that for $\underline{y} \cdot \tau \geq |l| + |\bar{y} - x|$ we have

$$v^1(x+l) \geq \sum_{t=0}^{\tau} \beta^t u(0) + \beta^{\tau+1} v^1(\bar{y})$$

and

$$v^2(x) \geq \sum_{t=0}^{\tau} \beta^t u(0) + \beta^{\tau+1} v^2(\bar{y})$$

Since $u(.)$ is bounded this implies that

$$v^1(\bar{y}) - v^1(x+l) + v^2(\bar{y}) - v^2(x) \leq 2(\tau+1)K$$

(Where K is the upper bound on u).

Step 2: We want to show that $v^1(\bar{y}) - v^2(\bar{y}) > L$ for β sufficiently close to one. Assume that r is such that $\beta(1+r) = 1$. Note that by raising r to this level we will *decrease* $v^1(\bar{y})$ (weakly) and *increase* $v^2(\bar{y})$. The latter follows from the fact that the agent cannot go short in environment 2. The optimal policy in environment 1 when $1/(1+r) = \beta$ implies a constant consumption of $\bar{y}$. Since this is also feasible for smaller (but positive) r, raising r to the indicated level can only decrease $v^1(\bar{y})$. Hence there is no loss of generality to assume $\beta(1+r) = 1$. In this case the optimal policy in both environments is independent of β.

Step 3: Let c be the optimal consumption path in environment 2 leading to $v^2(\bar{y})$. Then for every $C > \bar{y}$ there is an $\epsilon > 0$ such $T^1 = \{t : Prob\{c_t < C\} > \epsilon\}$ contains an infinite number of periods. To see this claim note that an upper bound for the utility of the agent in environment 2 is given by

$$v^2(\bar{y}) < \frac{u(\bar{y})}{1-\beta}$$

This upper bound follows from the fact that if complete insurance were available then the agent would optimally consume $\bar{y}$ in every period. Let $\mu = u(C) - u(\bar{y})$ and let $\epsilon = \mu/2u(C)$. Now suppose T^1 is finite, say of cardinality $B < \infty$. Then for β close to one we have

$$v^2(x) \geq \frac{u(\bar{y}) + \mu/2}{1-\beta} - B \cdot K > \frac{u(\bar{y})}{1-\beta}$$

which is a contradiction.

Step 4: For every $M < \infty$, if $y_t < M$, then for some τ, α and $\epsilon > 0$, whe have $c_{t'} < \bar{y} - \alpha$ with probability greater than ϵ for some $t \leq t' \leq \tau$. To see this claim note that Condition 1 and fair prices imply that there is a state θ such that

$$y_\theta - \bar{y} + (R \cdot q)_\theta < (1+r)p \cdot q - \eta$$

for all q that satisfy $p \cdot q \leq M$. By choosing β close to one we also choose r close to zero and hence we get

$$y_\theta - \bar{y} + (R \cdot q)_\theta < p \cdot q - \eta/2$$

for all q that satisfy $p \cdot q \leq M$. Thus if θ occurs sufficiently often (τ times) then $c_t < \bar{y} - \alpha$ with probability greater than π_θ^τ within the next τ periods.

Step 5 Using Step 3 and Step 4 we can distinguish 2 cases: Case 1 is when there is a subsequence $\{t_k\} \in T^1$ such that $prob\{c_{t_k} > C\} > \epsilon$ and $prob\{c_{t_k} < C - \eta\} > \epsilon$ for some $C < 2\bar{y}$, $\epsilon > 0, \eta > 0$ for all t_k. Case 2 is when $c_t \rightarrow \bar{y}$ a.s. and $y_t \rightarrow \infty$ a.s. In case 1 the agent can (in environment 1) imitate the consumption of environment 2 and in addition he can partially insure each periods consumption. The utility gain of this partial insurance is bounded away from zero in all periods in the subsequence $\{t_k\}$ and hence we have

$$v^1(\bar{y}) - v^2(\bar{y}) \rightarrow \infty$$

as $\beta \to 1$.

In case 2 choose M so that

$$T(\cdot u(\bar{y} + M/T) - \bar{u}(\bar{y})) > 2L$$

There is a τ such that $y_\tau > M$ with probability greater than 3/4. Now suppose in environment 1 the sovereign imitates the optimal policy of environment 2 and in period τ switches to the following policy: Consume $\bar{y} + M/T$ for T periods and $\bar{y}$ thereafter if $y_\tau > K$; If $y_\tau < M$ keep imitating the policy of environment 2. Since full insurance of the income is possible in environment 1 this is a feasible consumption plan. Clearly if β is sufficiently close to 1 then

$$v^1(\bar{y}) - v^2(\bar{y}) > L$$

Therefore a patient sovereign wishes to repay any debt smaller than $|l|$. For β sufficiently large we have $r \cdot |l| < \underline{y}$ in which case repaying a debt of l is feasible. This completes the proof of the proposition.