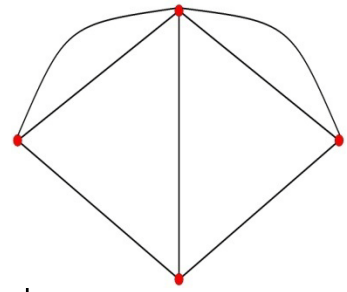


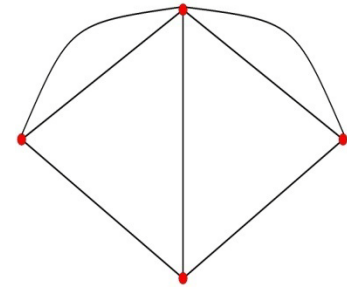
Network Formation and Behavioral Implications



Matthew O. Jackson
Lectures Northwestern
November 2009

**(please do not circulate or post
without author's permission)**

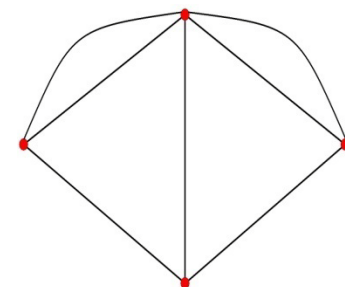
Why Study Networks?



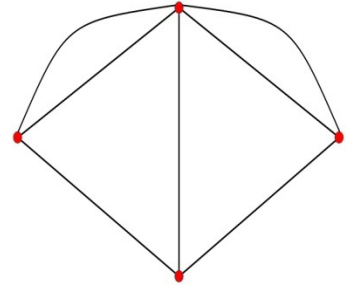
- **Many economic, political, and social interactions are shaped by the local structure of relationships:**
 - trade of goods and services, most markets are not centralized!...
 - sharing of information, favors, risk, ...
 - transmission of viruses, opinions...
 - choices of behavior, education, ...
 - political alliances, trade alliances...
- **Social networks influence behavior**
 - crime, employment, human capital, voting, smoking,...
 - networks exhibit heterogeneity, but also have enough underlying structure to model
- **Pure interest in social structure**
 - understand social network structure

SOCIAL AND ECONOMIC NETWORKS

Matthew O. Jackson

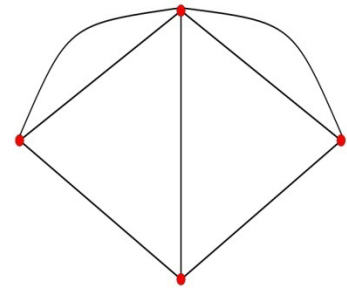


Primary Questions:



- How do networks form?
- How do networks influence behavior?

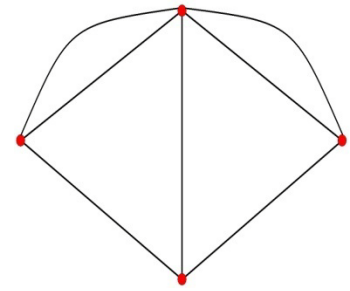
Outline



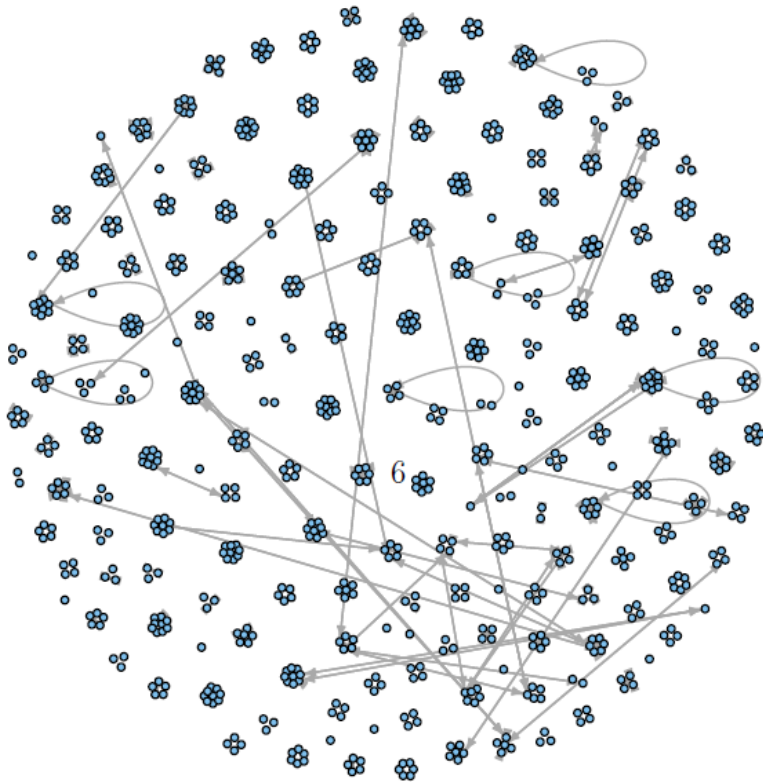
3 Examples of models and the questions they can answer:

- Random graph models
contagion/diffusion
- Game theoretic/strategic model
efficiency versus stability
- A hybrid model
estimating friendship formation

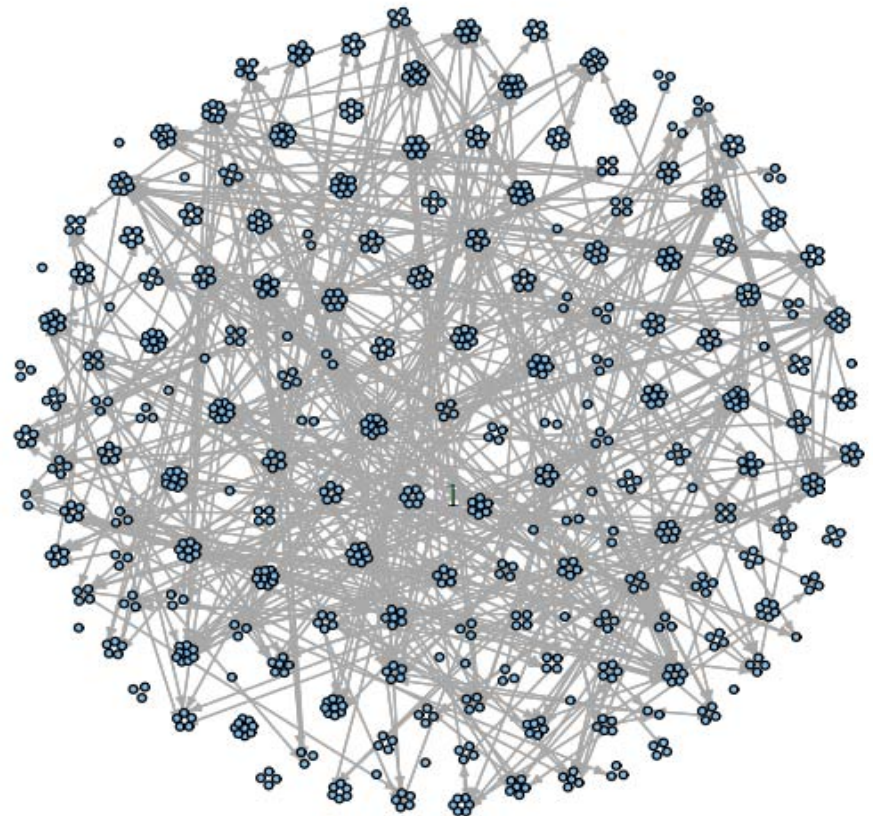
Village 24:



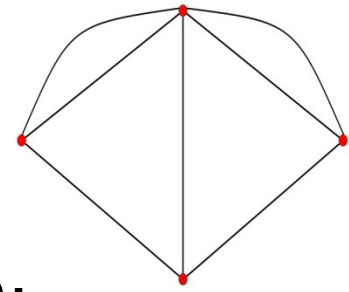
Temple



Borrow&Lend:



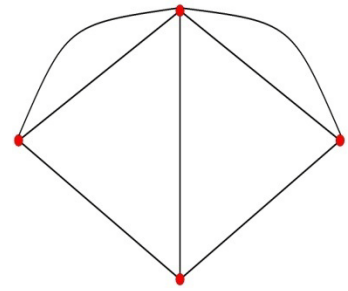
Outline



3 Examples of models and the questions they can answer:

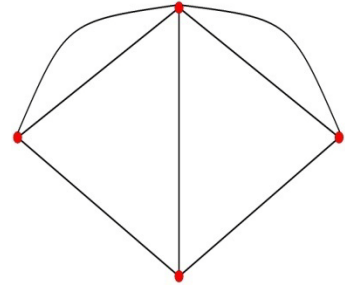
- Random graph models
contagion/diffusion
- Game theoretic/strategic model
efficiency versus stability
- A hybrid model
empirical estimation of friendship
formation

Random Network Models



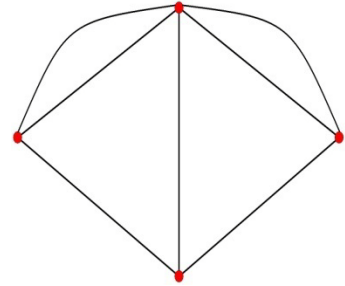
- Provide some insight into structure
 - How will a disease diffuse?
 - How do link patterns affect diffusion speed?

Questions:



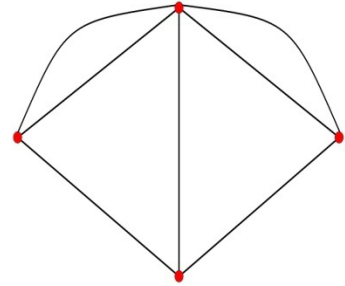
- Consider a disease, an idea that spreads by contact
- When do we get diffusion?
- What is the extent of diffusion?
- How fast is diffusion?

Model - “SI”:



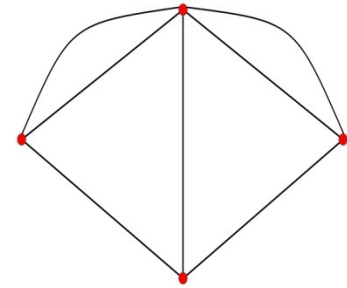
- Society is described by a random network
- Some node is initially infected
- That node infects its neighbors
- They infect their neighbors, and so forth

Extent of Diffusion



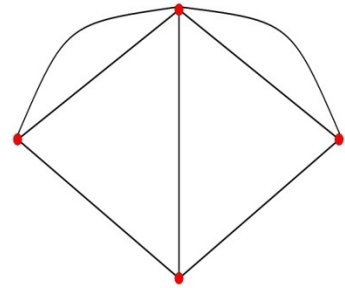
- Get nontrivial diffusion if someone in the giant component is infected/adopts
- Size of the giant component determines likelihood of diffusion and its extent
- Random network models allow for giant component calculations

Representing Networks



- $N = \{1, \dots, n\}$ nodes, vertices, players
- $g \in \{0, 1\}^{n \times n}$ represents the relationships
- $g_{ij} = 1$ indicates a link or edge between i and j
- Notation: $ij \in g$ indicates a link between i and j
- Network (N, g)

Basic Definitions

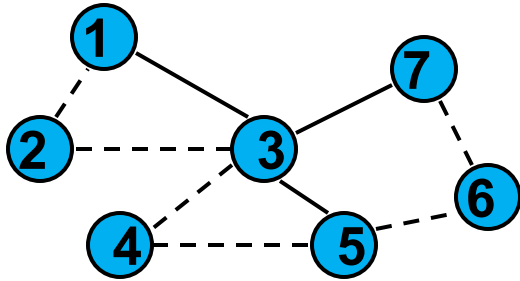


- Walk from i_1 to i_k : sequence of links $(i_1i_2, i_2i_3, \dots, i_{k-1}i_k)$

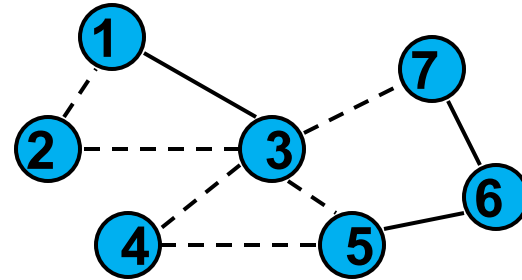
Often convenient simply to represent it as a sequence of nodes (i_1, i_2, \dots, i_k) such that $i_{k-1}i_k \in g$ for each k

- Path: a walk (i_1, i_2, \dots, i_k) with each node i_k distinct
- Cycle: a walk where $i_1 = i_k$
- Geodesic: a shortest path between two nodes

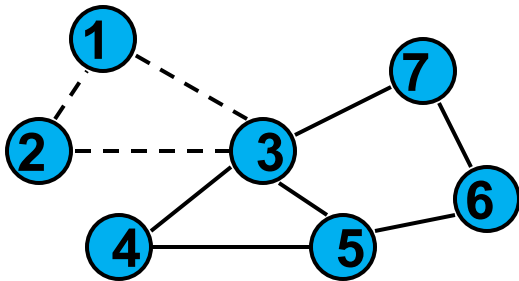
Paths, Walks, Cycles...



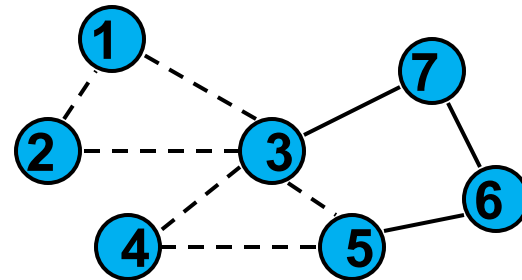
Path (and a walk) from 1 to 7:
1, 2, 3, 4, 5, 6, 7



Walk from 1 to 7 that is not a path:
1, 2, 3, 4, 5, 3, 7

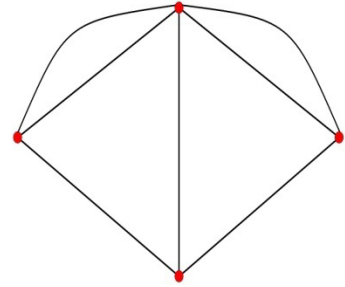


Simple Cycle (and a walk) from 1 to 1:
1, 2, 3, 1



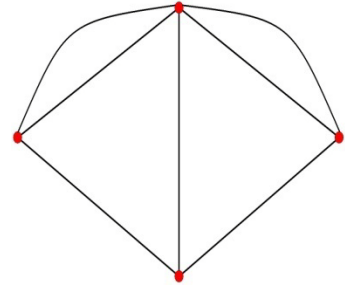
Cycle (and a walk) from 1 to 1:
1, 2, 3, 4, 5, 3, 1

Components



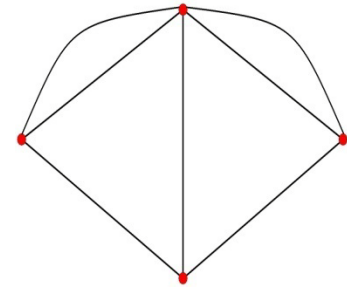
- (N, g) is connected if there is a path between every two nodes
- Component: maximal connected subgraph
 - $(N', g') \sqsubseteq (N, g)$
 - (N', g') is connected
 - $i \in N'$ and $ij \in g$ implies $j \in N'$ and $ij \in g'$

Diameter



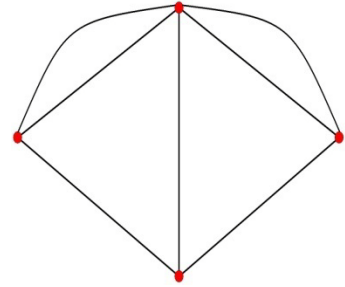
- Diameter – largest geodesic
 - if unconnected, of largest component...
- Average path length (less prone to outliers)

Erdos-Renyi Random Networks



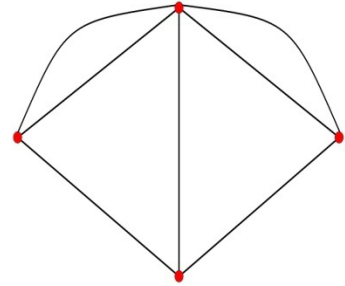
- Each link is formed with an independent probability p
- Look at large n : properties as society becomes large

Calculating the Size of the Giant Component



- q is fraction of nodes in largest component on $n-1$ node network (roughly the fraction on n nodes too)
- add node n and connect it
- chance that this node is outside of the giant component is $(1-q)^d$ where d is this node's degree

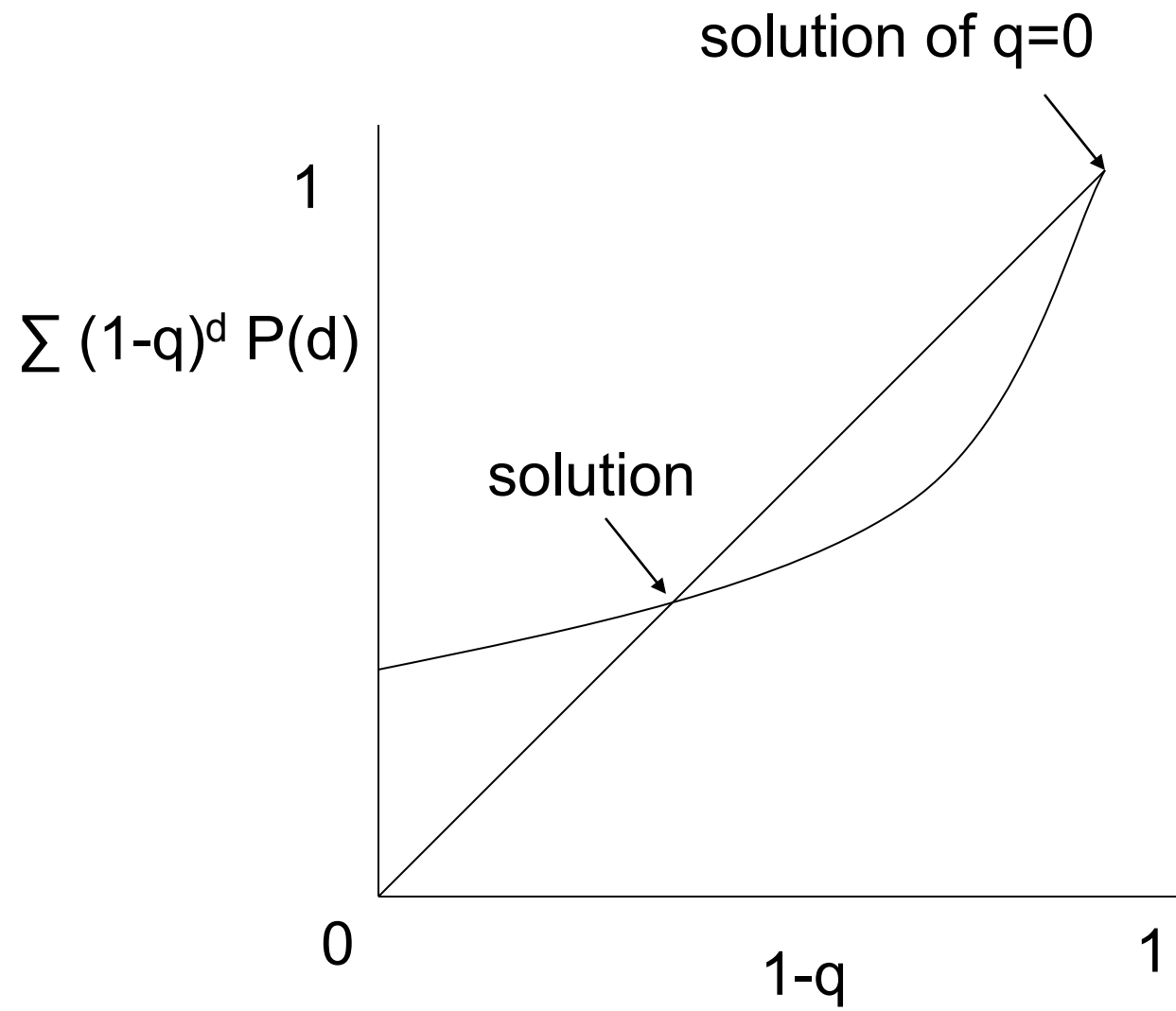
Giant Component Size



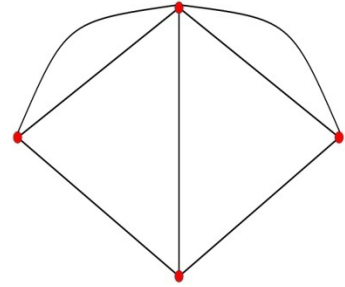
- So, probability $1-q$ that a node is outside of the giant component is

$$1-q = \sum (1-q)^d P(d)$$

- Solve for q ...



Degree Distribution:



- probability that node has d links is **binomial**

$$P(d) = [(n-1)! / (d!(n-d-1)!)] p^d (1-p)^{n-d-1}$$

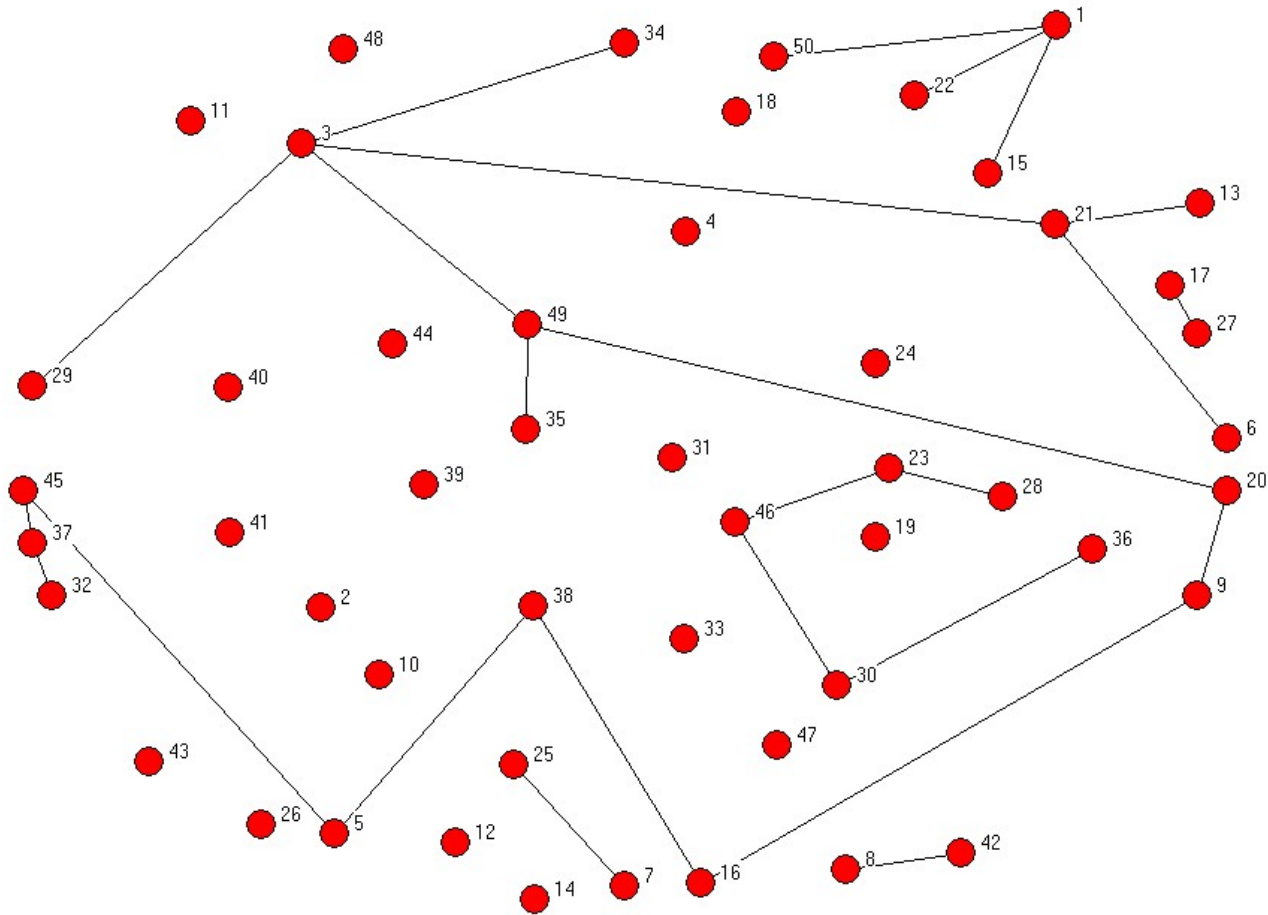
- Large n , small p , this is approximately a **Poisson** distribution:

$$P(d) = [(n-1)^d / d!] p^d e^{-(n-1)p}$$

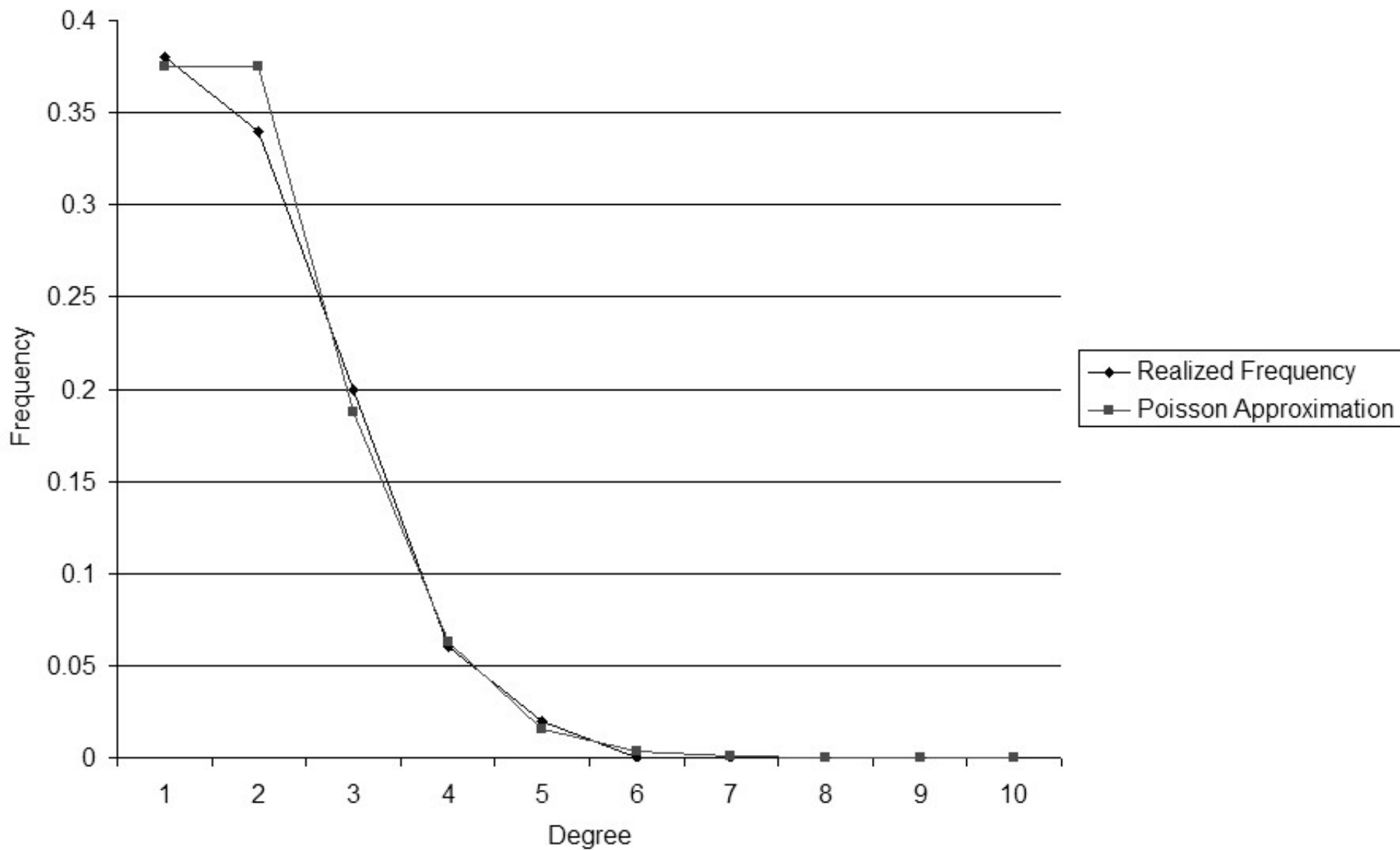
- hence name ``**Poisson random graphs**''

Random network

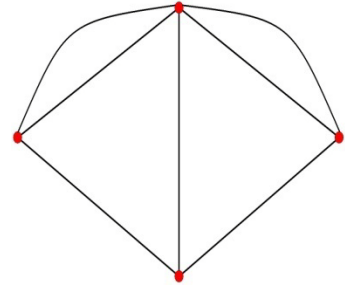
$p=.02$, 50 nodes



Degree Distribution $p=.02$



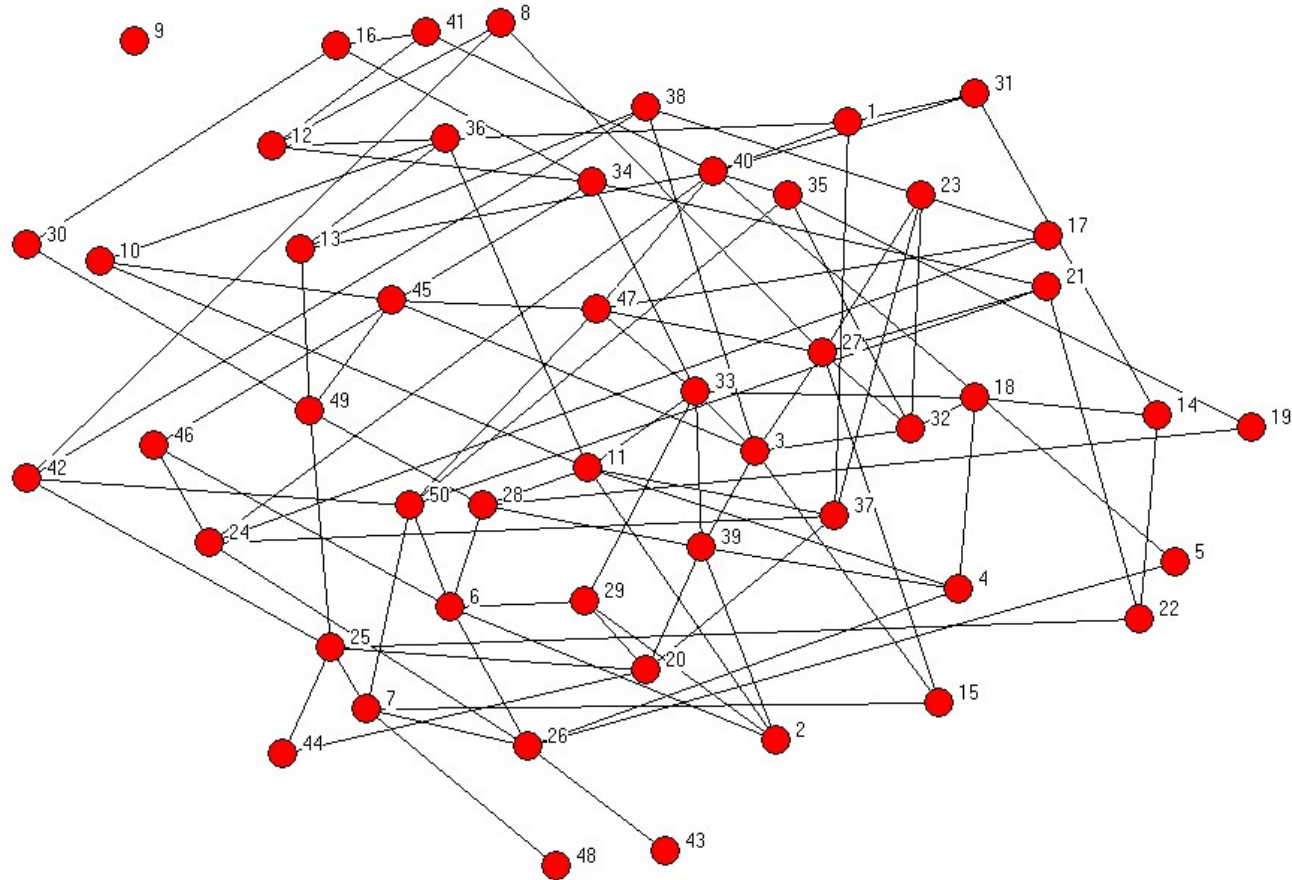
Note



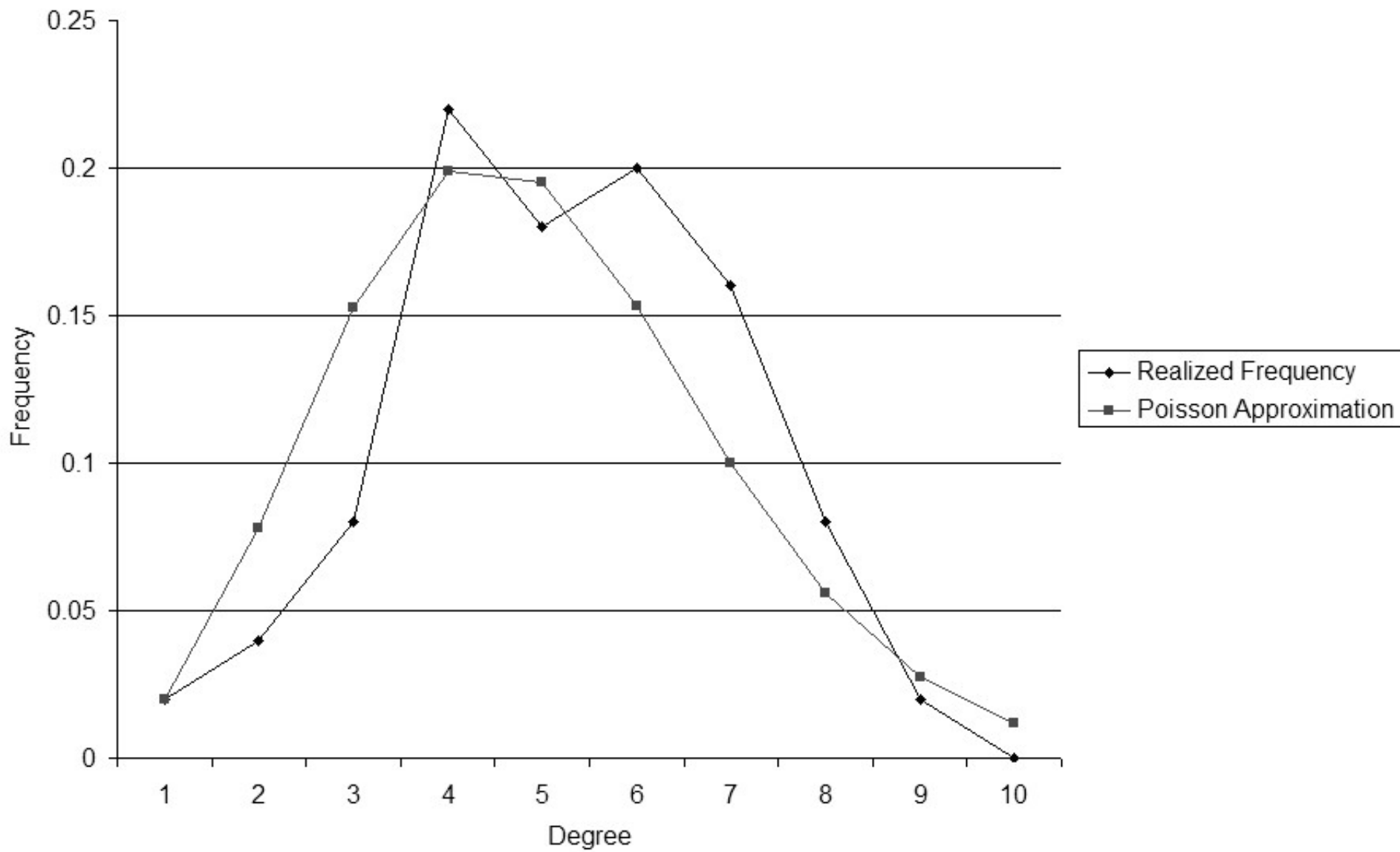
- many isolated nodes
- several components
- no component has more than a small fraction of the nodes, just starting to see one large one emerge

Random Network

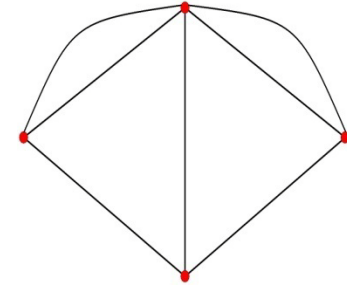
$p=.08$, 50 nodes



Degree Distribution $\rho=.08$



Giant Component Size: Poisson Case



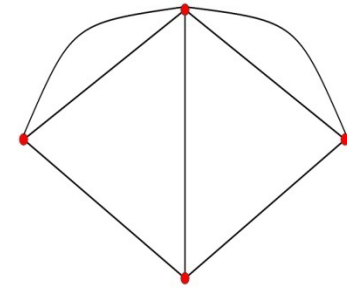
Solve $1-q = \sum (1-q)^d P(d)$

when $P(d) = [(n-1)^d / d!] p^d e^{-(n-1)p}$

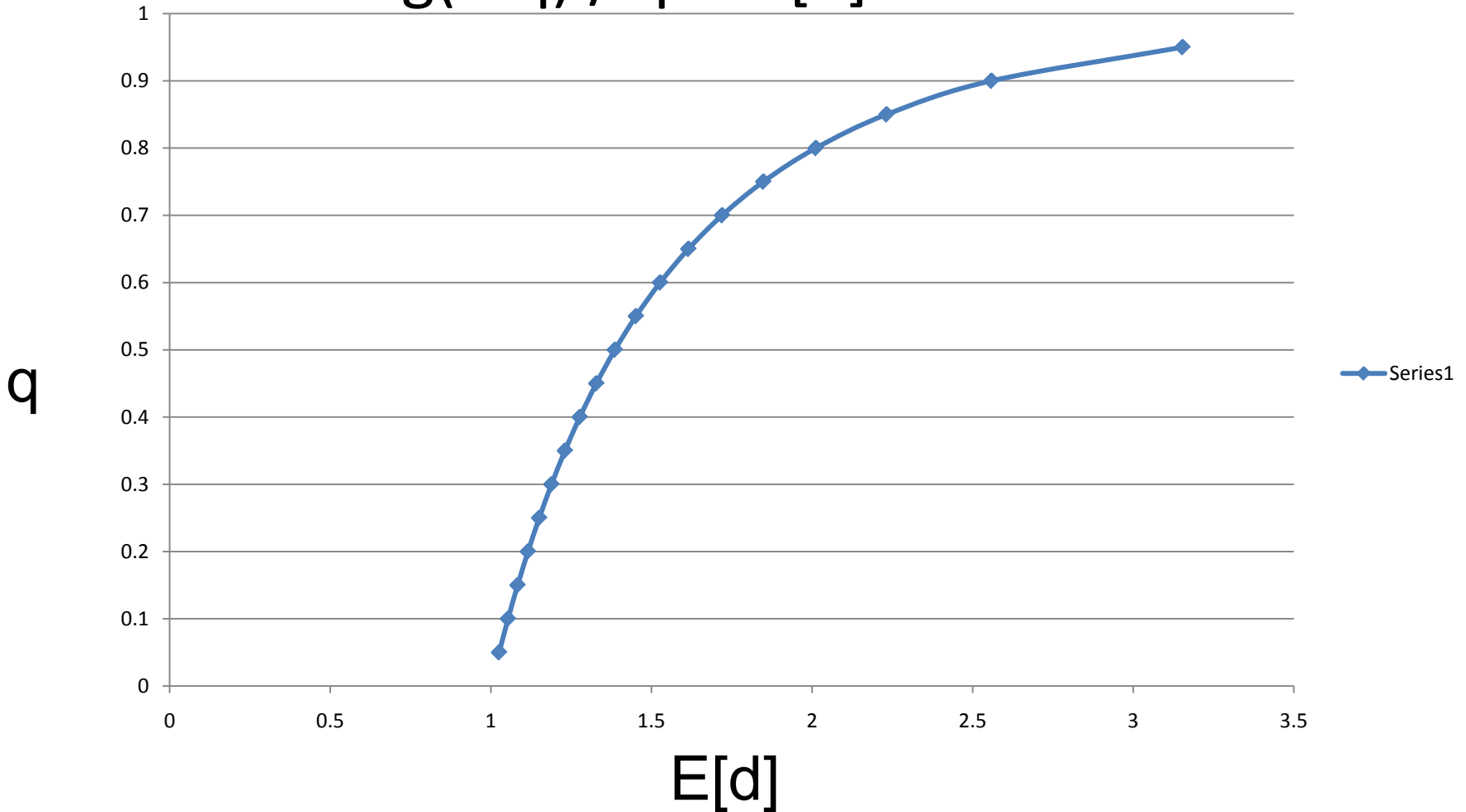
so
$$\begin{aligned} 1-q &= e^{-(n-1)p} \sum [(1-q) (n-1)p]^d / d! \\ &= e^{-(n-1)p} e^{(n-1)p(1-q)} \\ &= e^{-q(n-1)p} \end{aligned}$$

or $-\log(1-q) / q = E[d]$

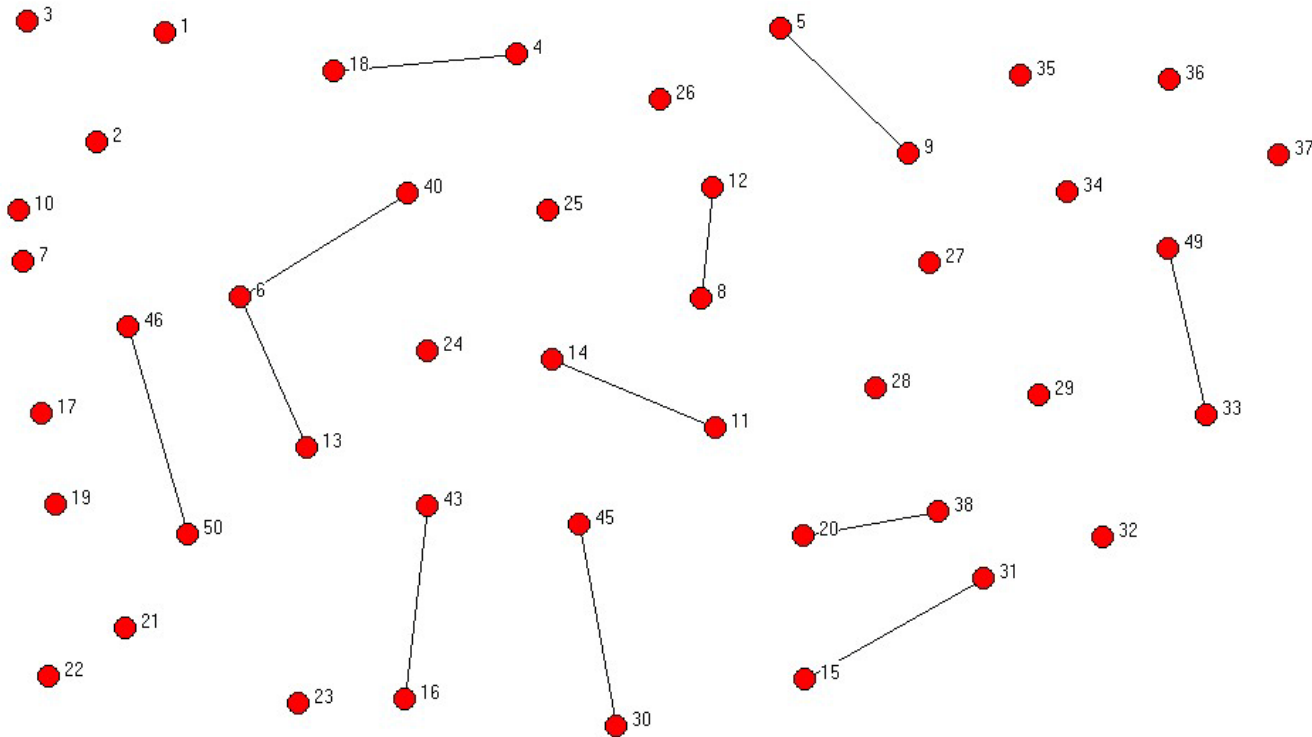
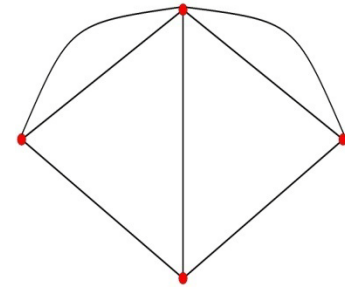
Giant Component Size:



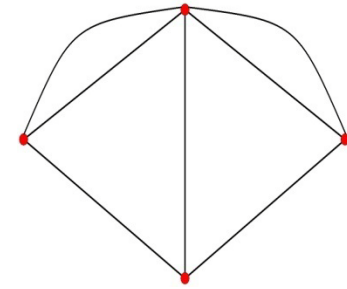
$$-\log(1-q) / q = E[d]$$



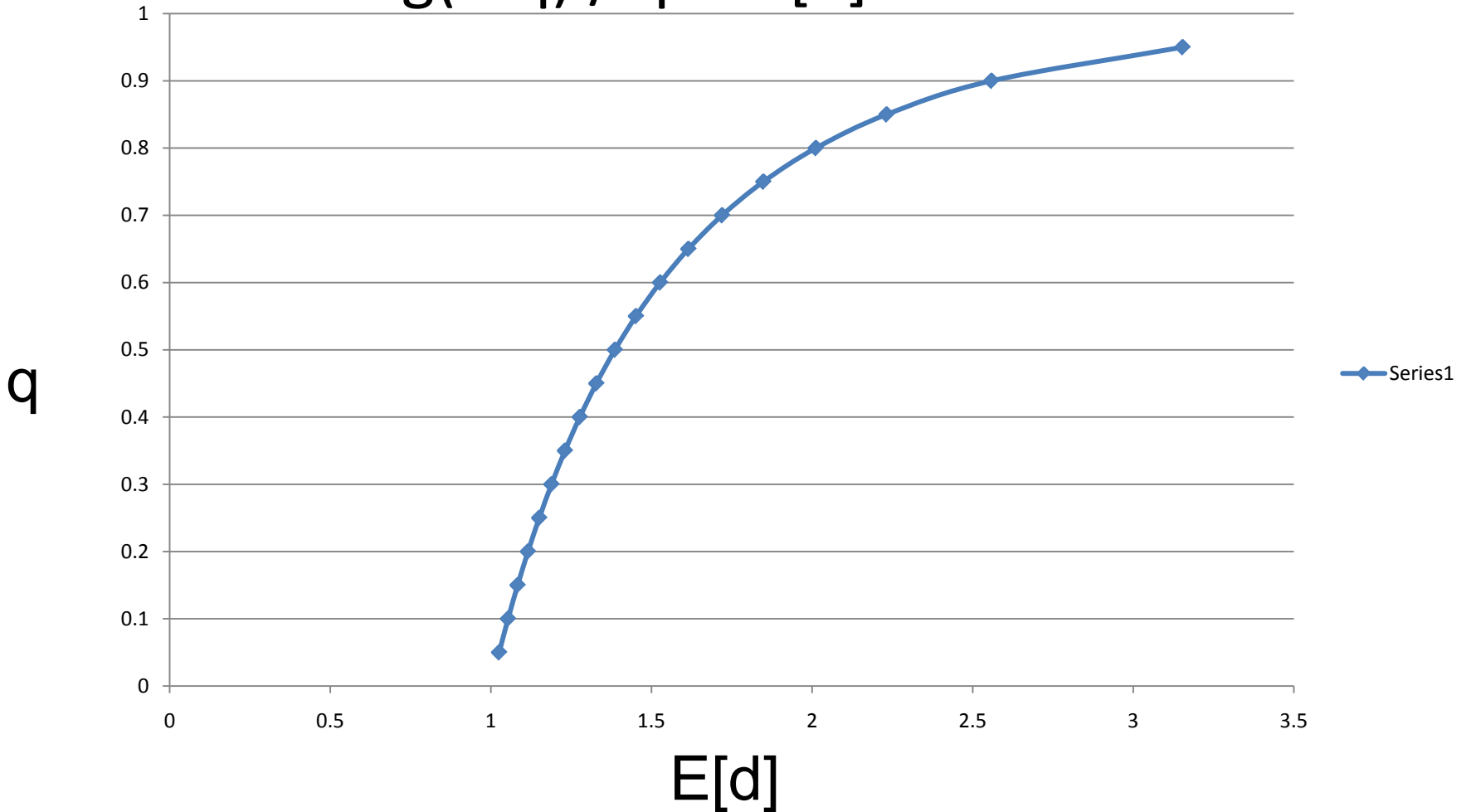
$E[d]=.5$, 50 nodes



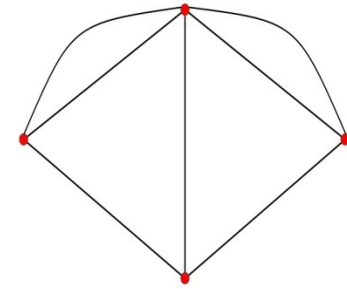
Giant Component Size:



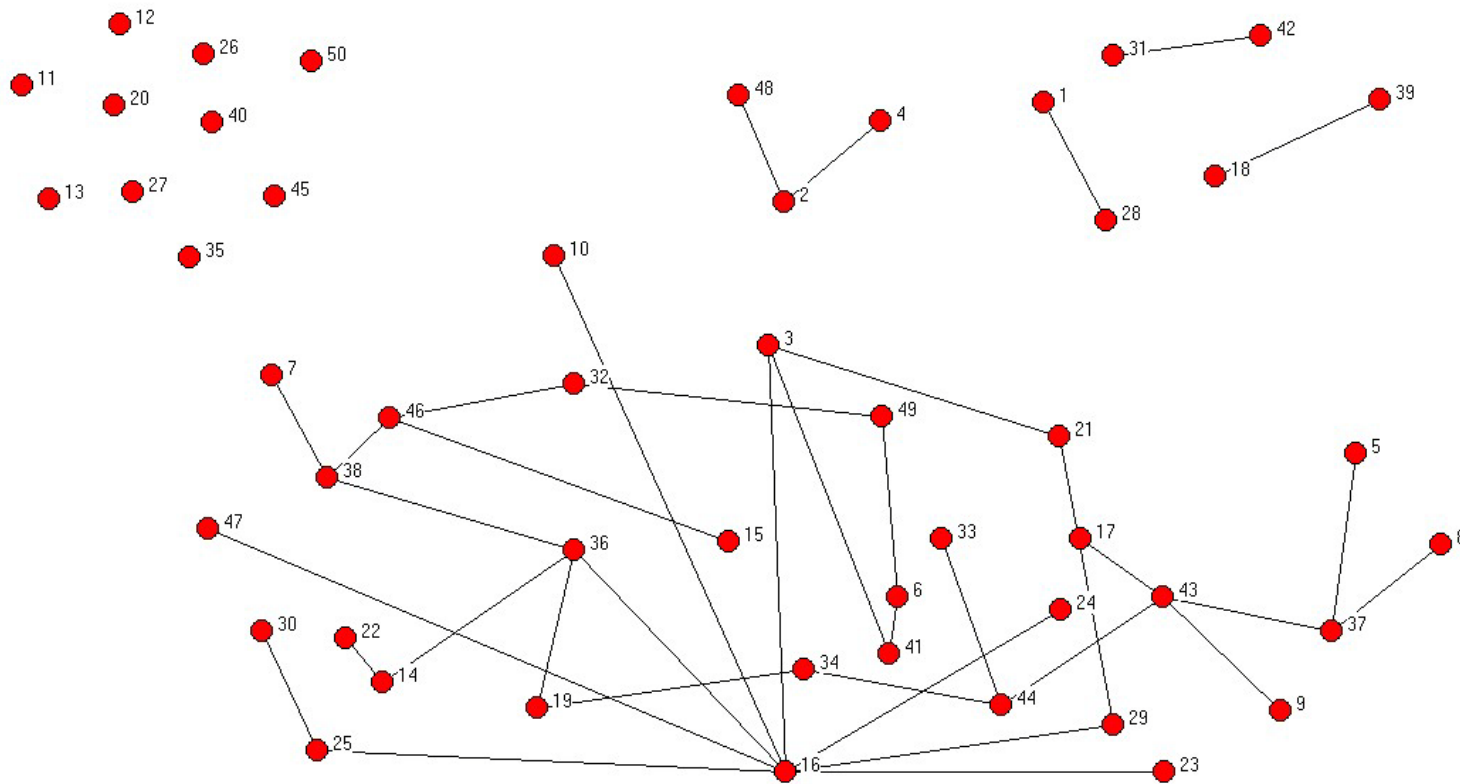
$$-\log(1-q) / q = E[d]$$



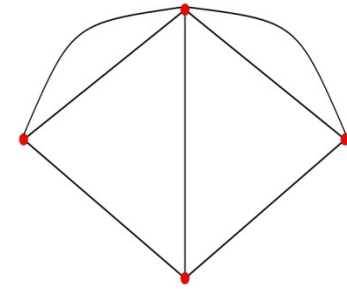
$E[d]=1.5$, 50 nodes



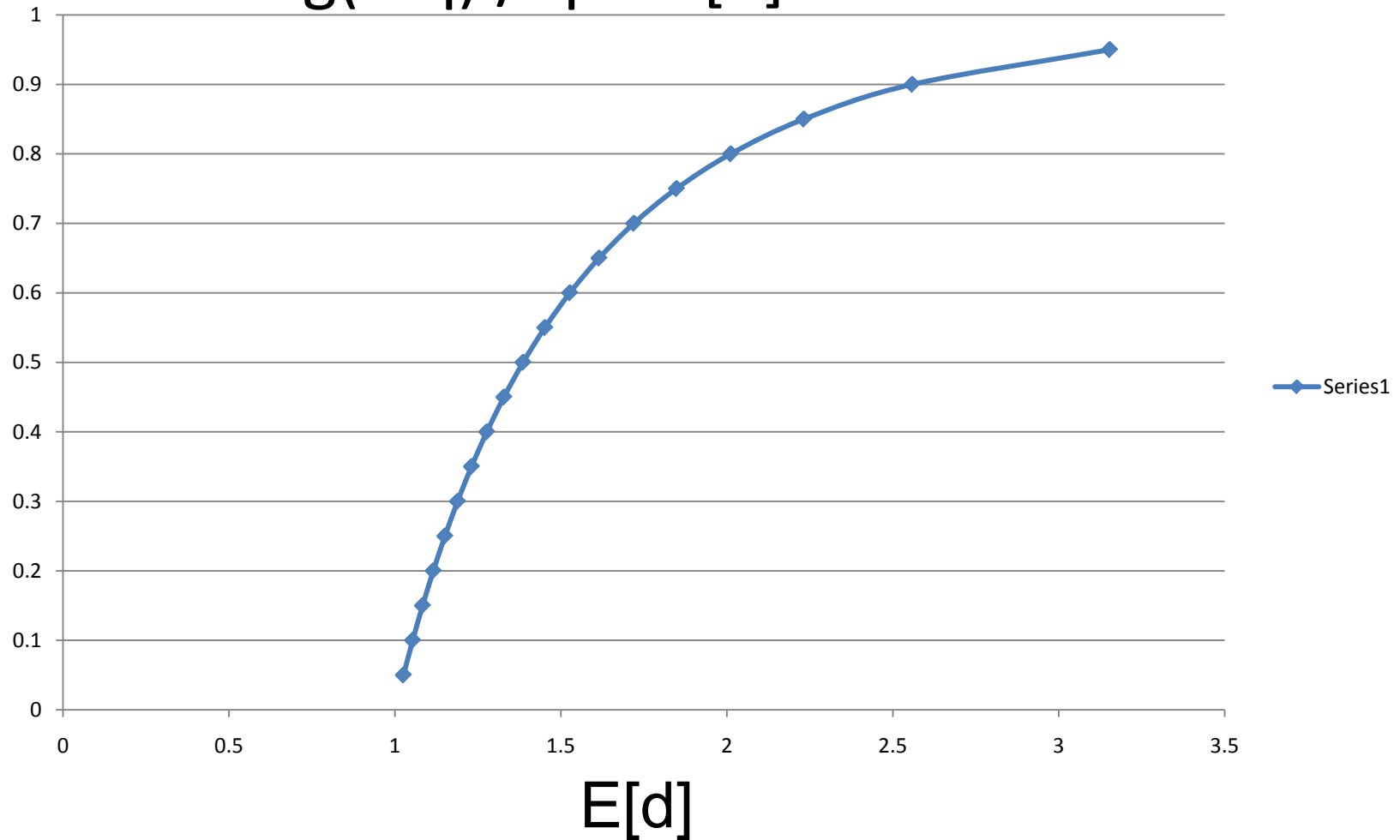
$E[d]=1$ is the threshold for emergence of cycles and a giant component



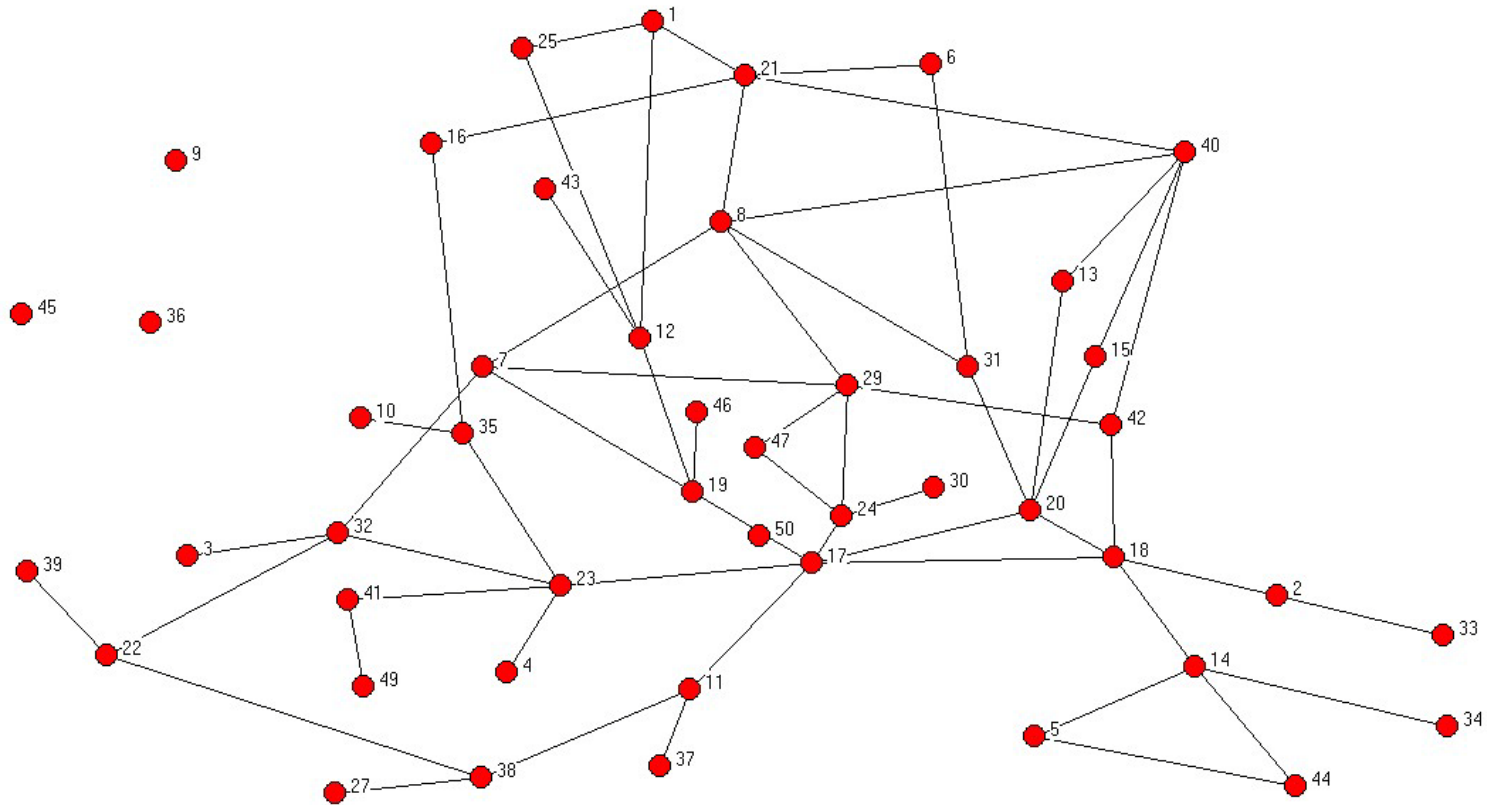
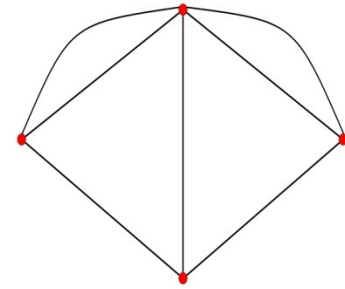
Giant Component Size:



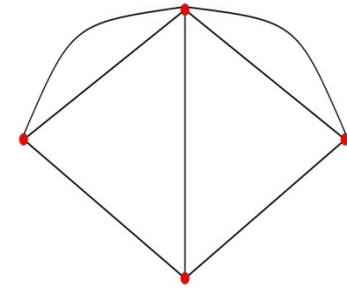
$$-\log(1-q) / q = E[d]$$



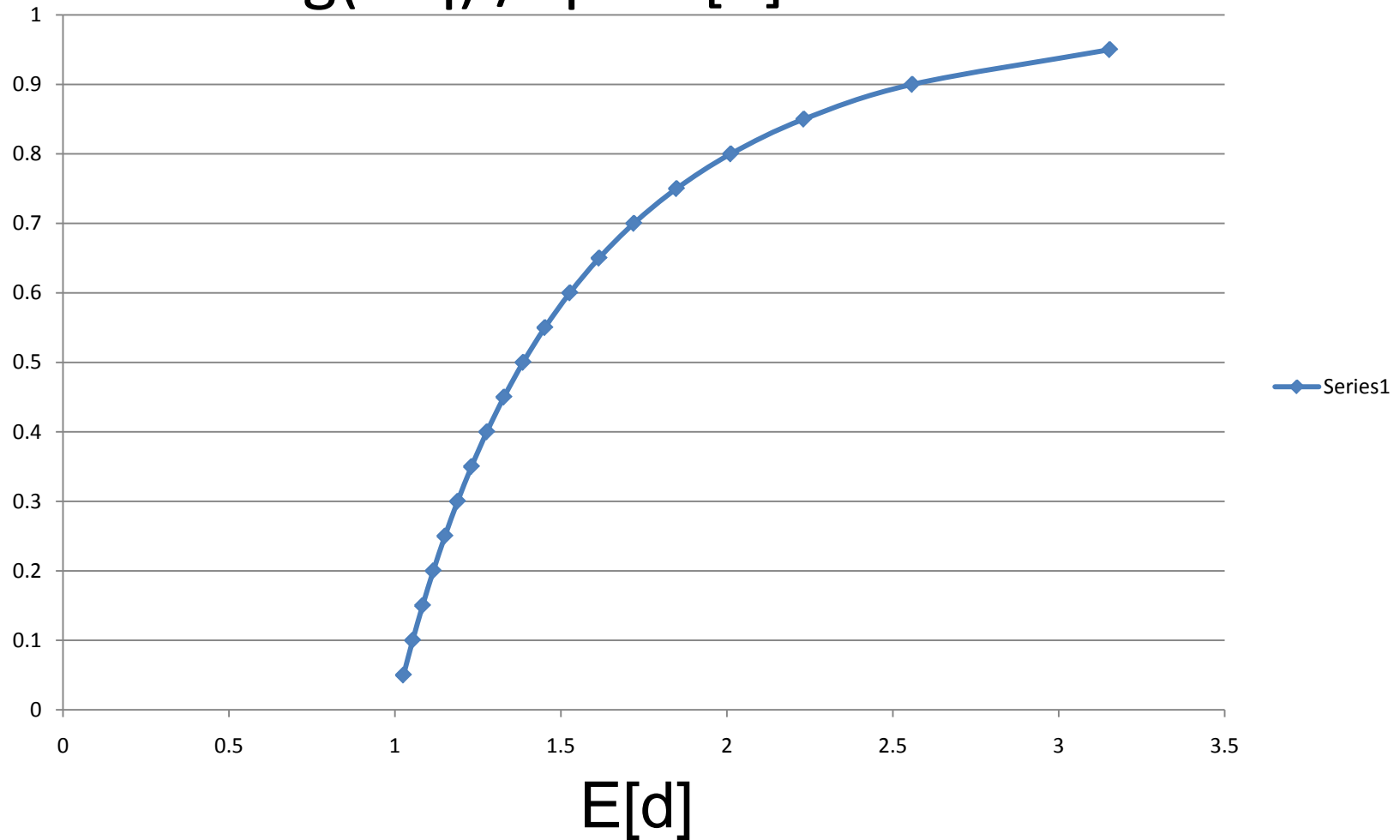
$E[d]=2.5$, 50 nodes



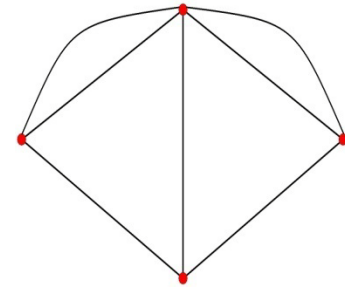
Giant Component Size:



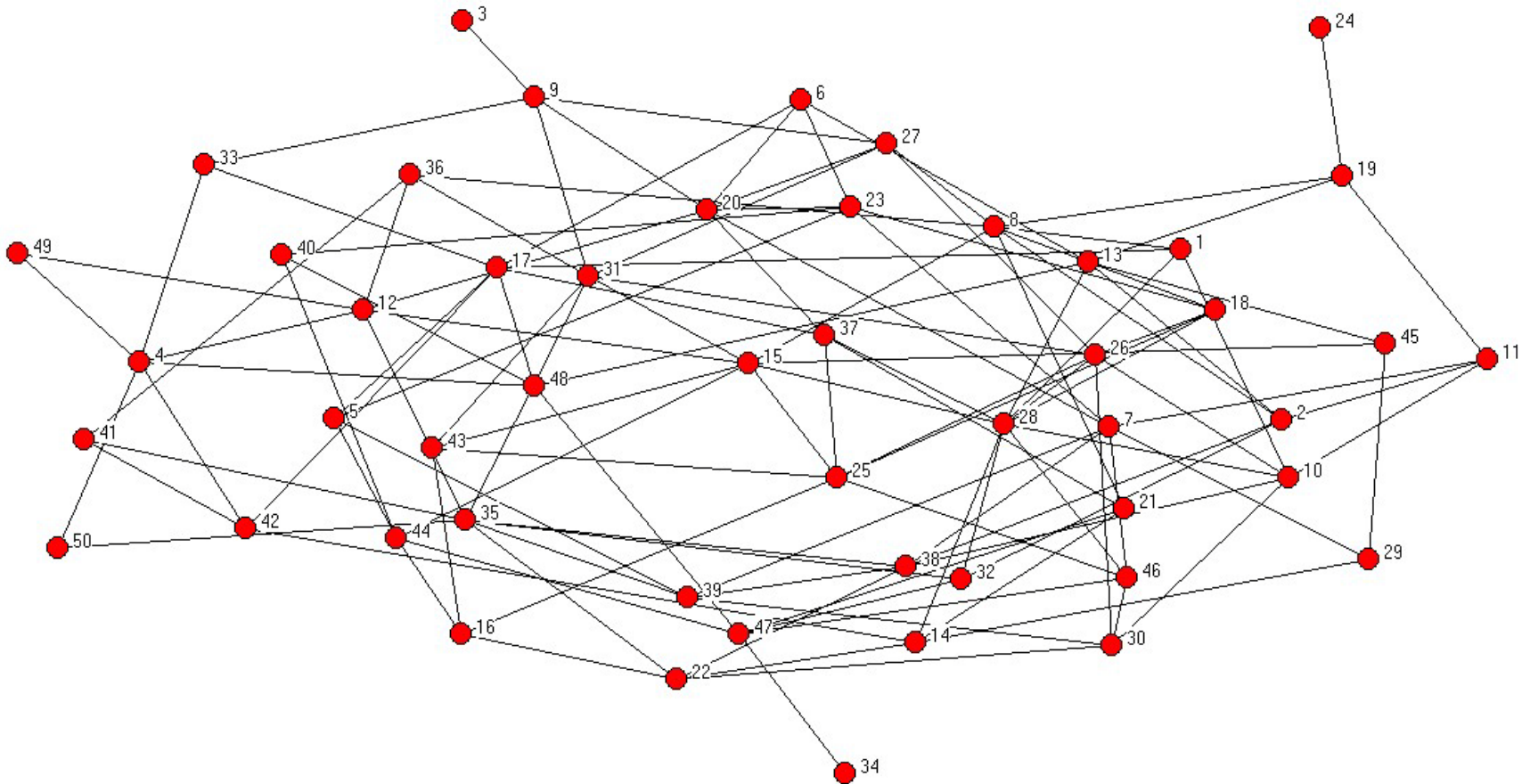
$$-\log(1-q) / q = E[d]$$



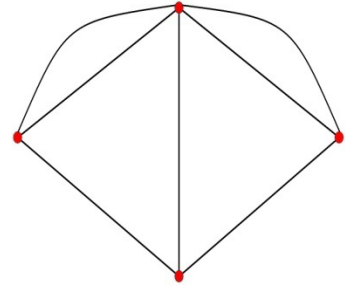
$E[d]=5$, 50 nodes



$E[d]=4$ leads to high probability of connection



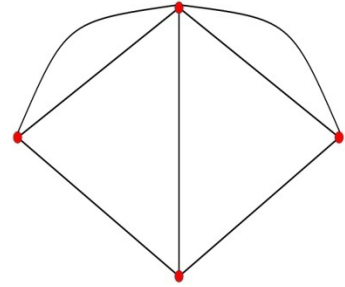
Who is infected?



- Probability of being in the giant component:
- $1-(1-q)^d$ increasing in d
- More connected, more likely to be infected

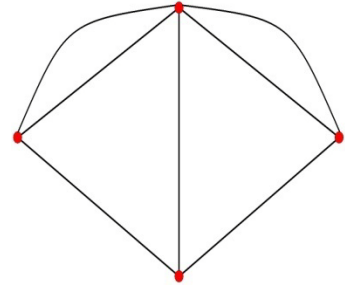
(more likely to be infected at any point in time...)

Lessons:



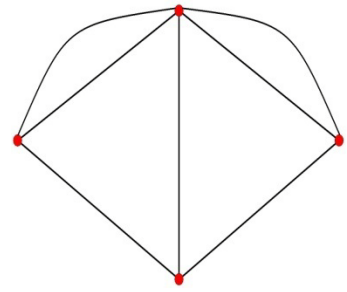
- Thresholds/“Phase Transitions”:
 - low density no contagion
 - middle density some probability of infection, part of population infected
 - high density sure infection and all infected
- Degree affects who is infected and when

Extensions:



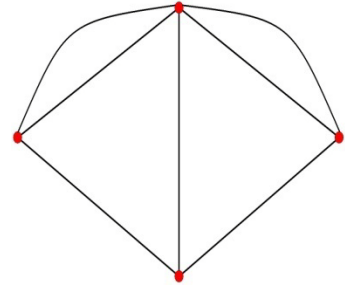
- Immunity: delete a fraction of nodes and study the giant component on remaining nodes
- Probabilistic infection
 - Random infection: have some links fail, just lower p

Speed of Diffusion?



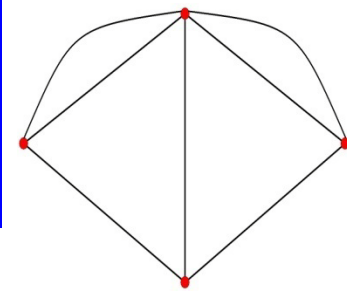
- How do shortest paths in a network depend on the size of the society and the connectedness of the society?

Networks



- Networks differ in their link density
- Networks differ in how links are spread across nodes: **Homophily**
 - Bias of relationships towards own type
- Technology and globalization are changing networks:
 - More relationships??
 - more/less homophily??

Density: Average Degree (# links)



HS Friendships (CJP 09) 6.5

Romances (BMS 03) 0.8

Borrowing (BDJ...) 3.2

Co-authors (Newman 01, GLM 06)

Bio 15.5

Econ 1.7

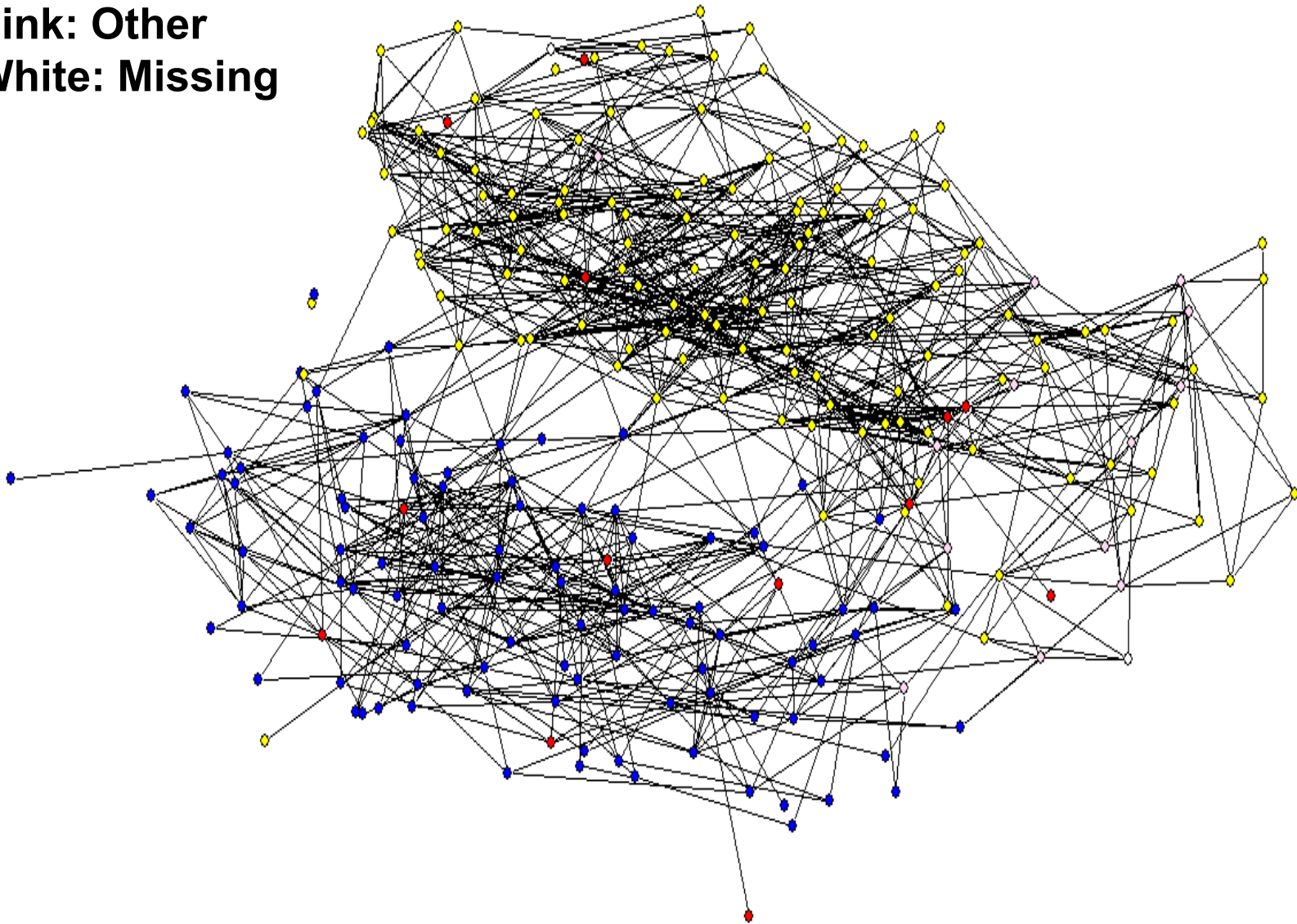
Math 3.9

Physics 9.3

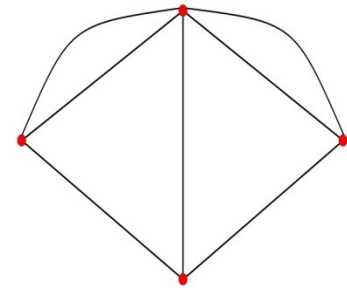
Facebook (Marlow 09) 120

Yellow: Whites
Blue: Blacks
Reds: Hispanics
Green: Asian
Pink: Other
White: Missing

CJP (2009)

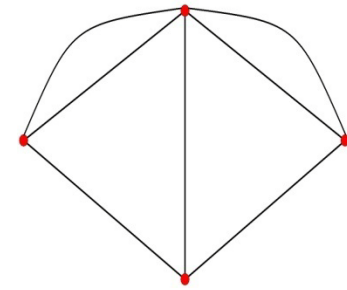


Homophily:



- Tendency to associate with others with similar characteristics: age, race, gender, religion, profession....
 - Lazarsfeld and Merton (1954) “Homophily”
 - Shrum (gender, ethnic, 1988...), Blau (professional 1974, 1977), Burt, Marsden (variety, 1987, 1988), Moody (grade, racial, 2001...), McPherson (variety, 1991...)...
 - Add Health: Moody (2001), CJP (2007), Goodreau, Kitts, Morris (2009), Currarini, Jackson, Pin (2009)

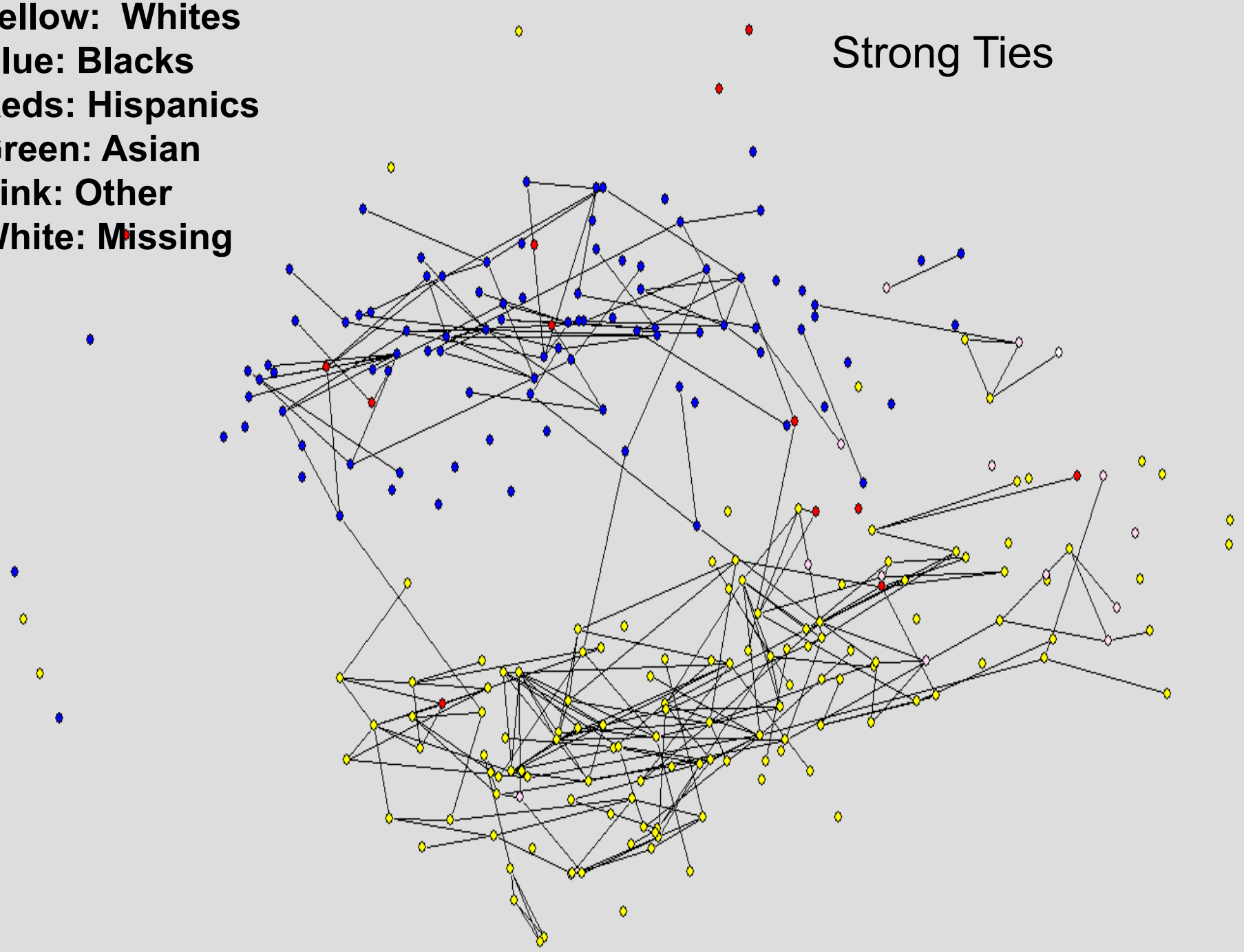
Adolescent Health, High School in US:



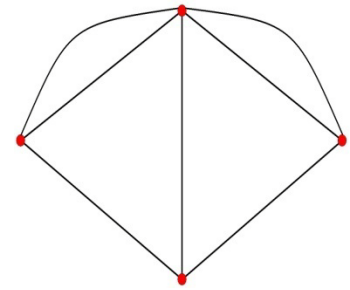
Percent:	52	38	5	5
	White	Black	Hispanic	Other
White	86	7	47	74
Black	4	85	46	13
Hispanic	4	6	2	4
Other	6	2	5	9
	100	100	100	100

Yellow: Whites
Blue: Blacks
Reds: Hispanics
Green: Asian
Pink: Other
White: Missing

Strong Ties

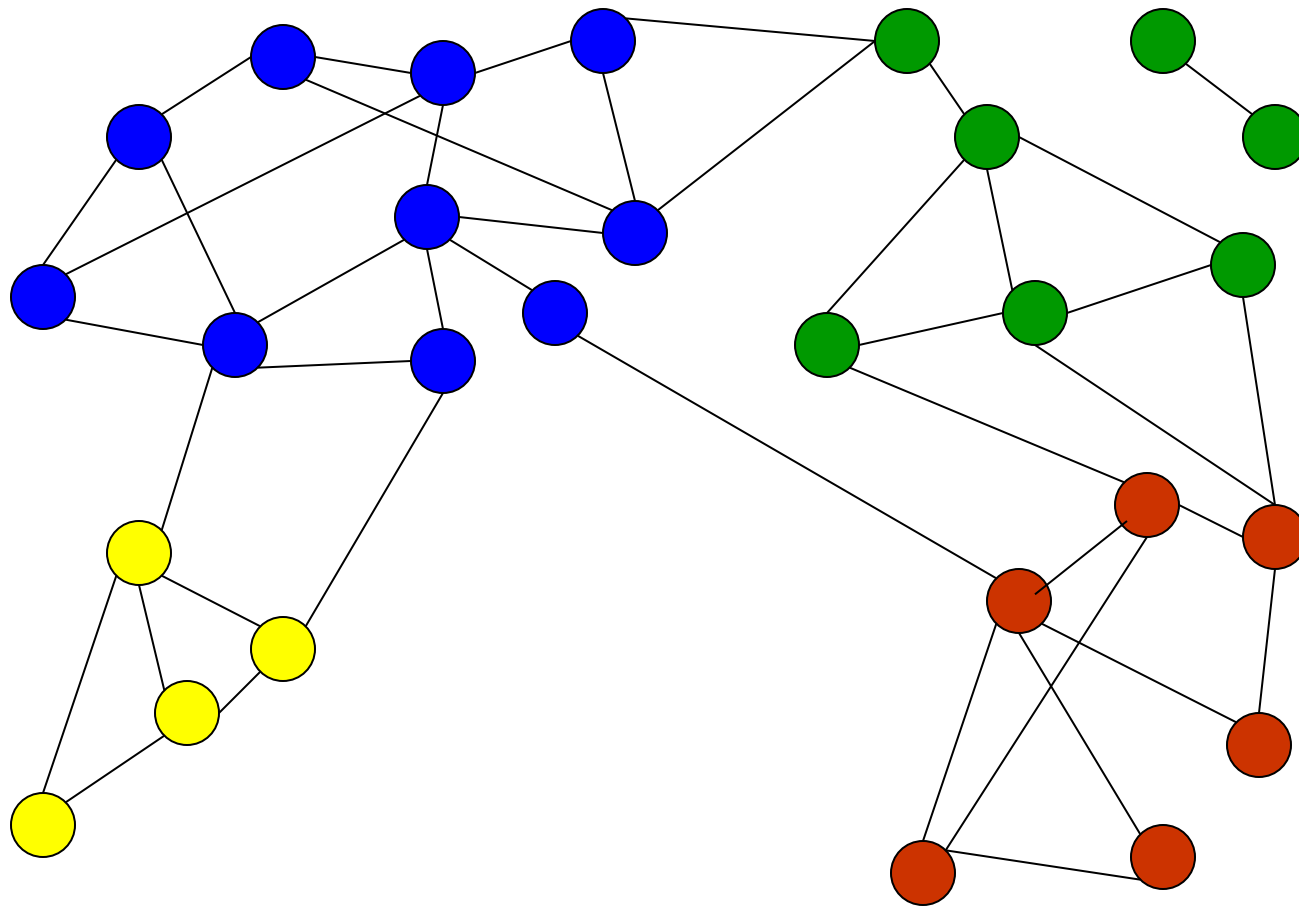


Multi-Type Random Network Model

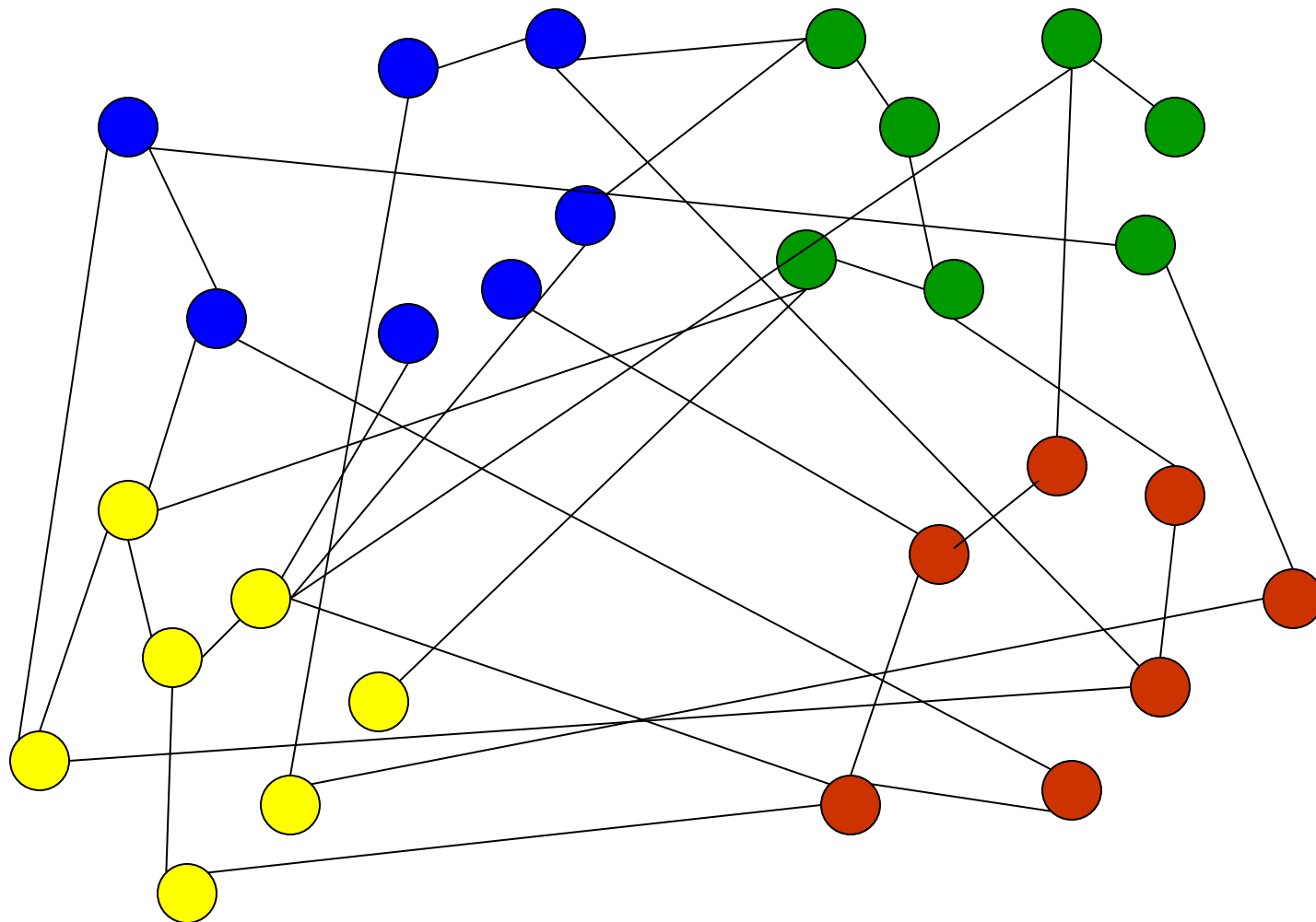


- $\{1, \dots, n\}$ agents/nodes
- Partitioned into groups N_1, \dots, N_K
- Node i in group k is linked to a node j in group k' with probability $P_{kk'}$ (undirected)
- Homophily: $P_{kk} > P_{kk'}$ for $k' \neq k$

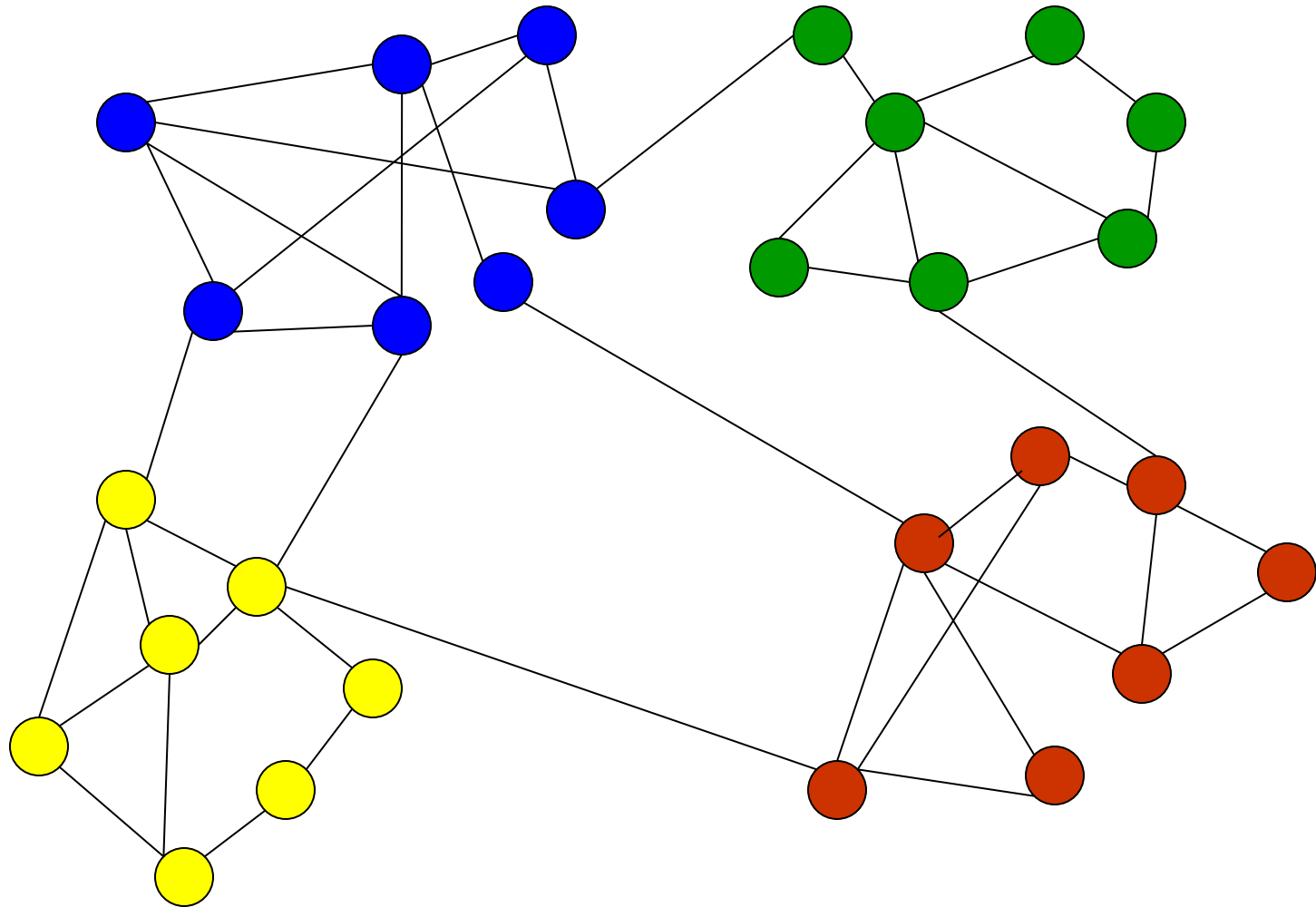
Multi-Type Random network



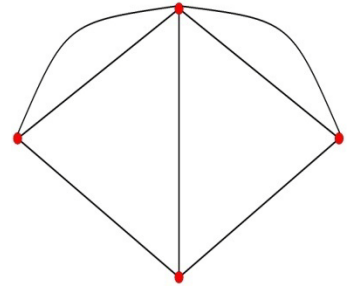
Example Low Homophily



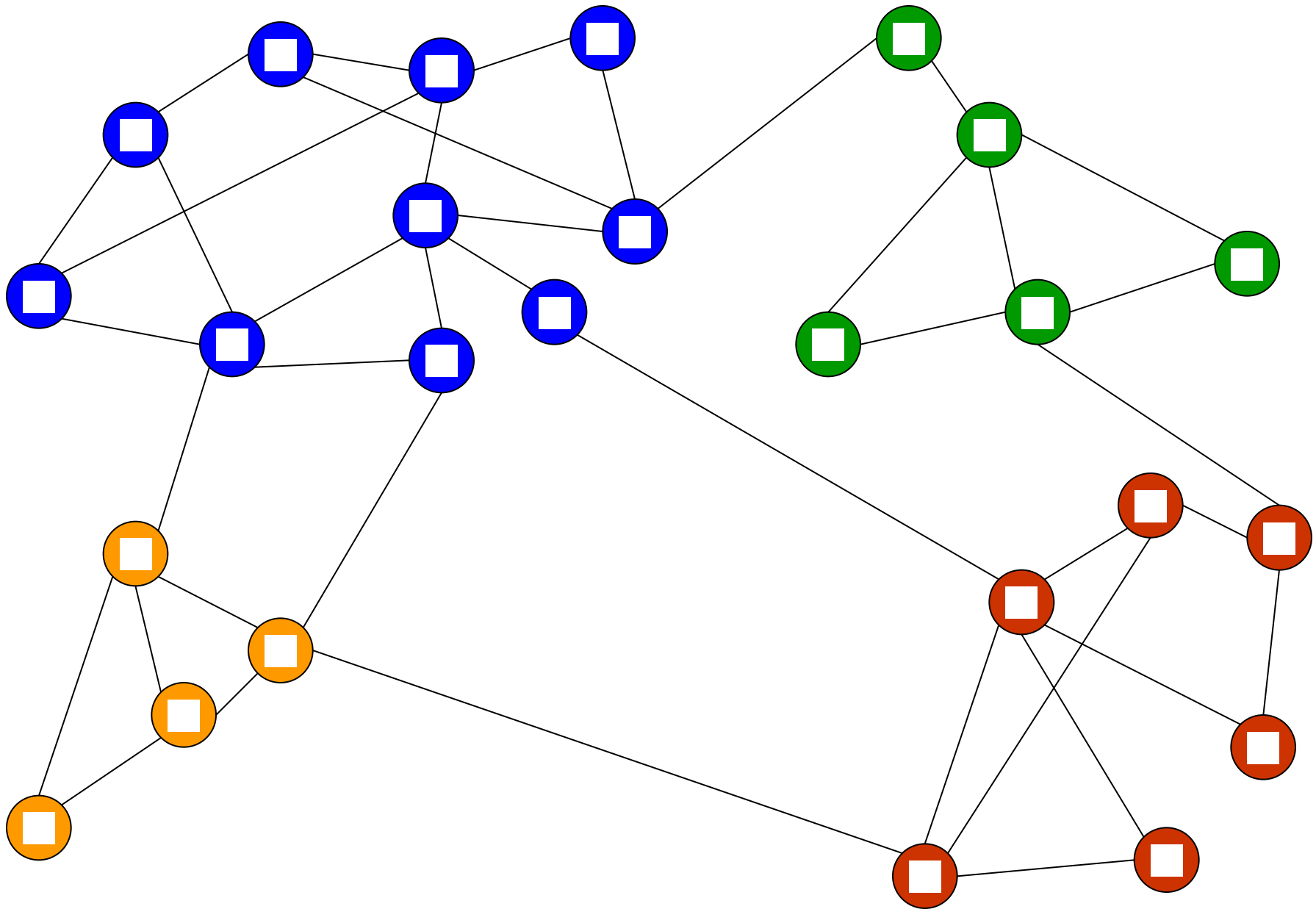
Example High Homophily

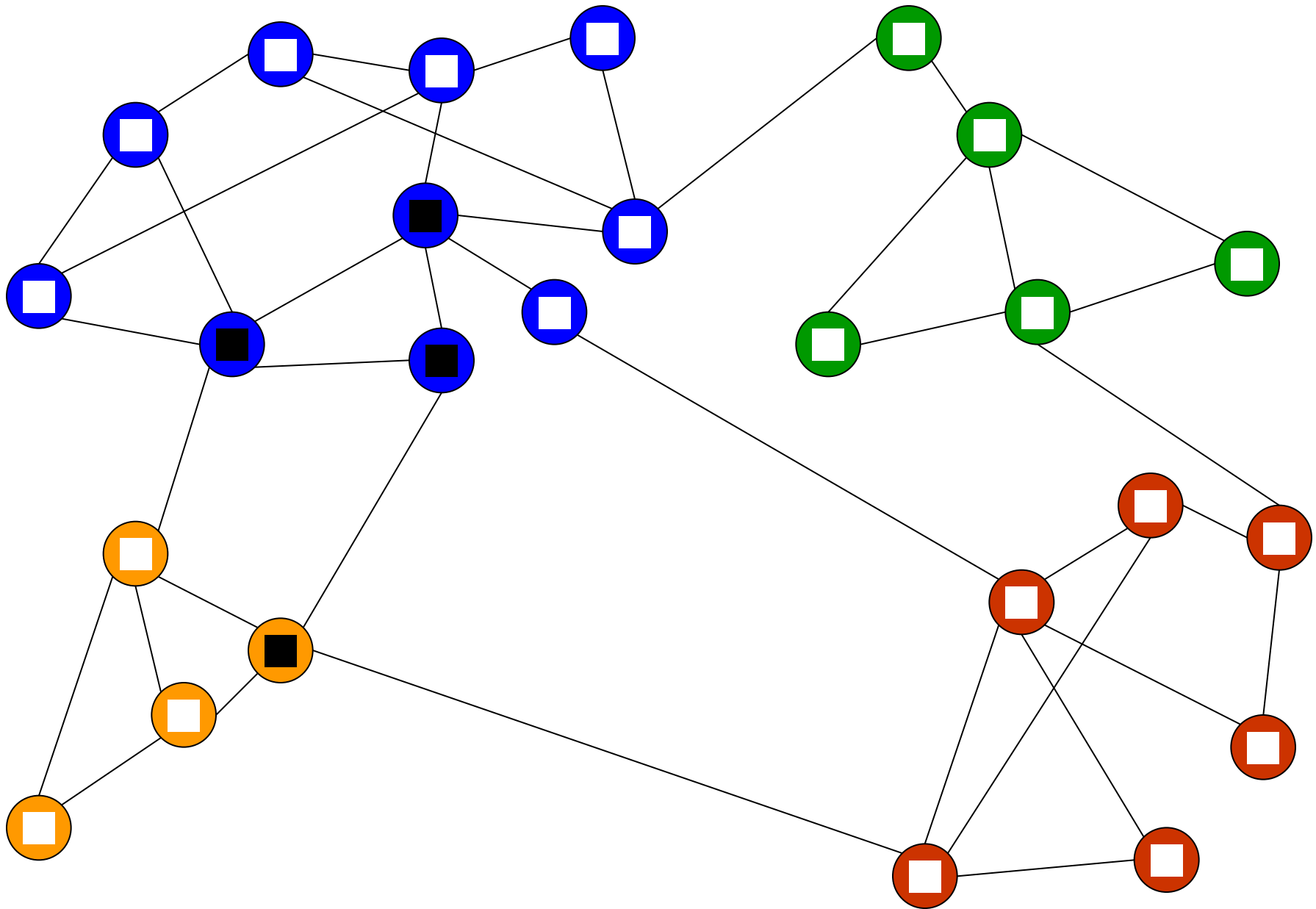


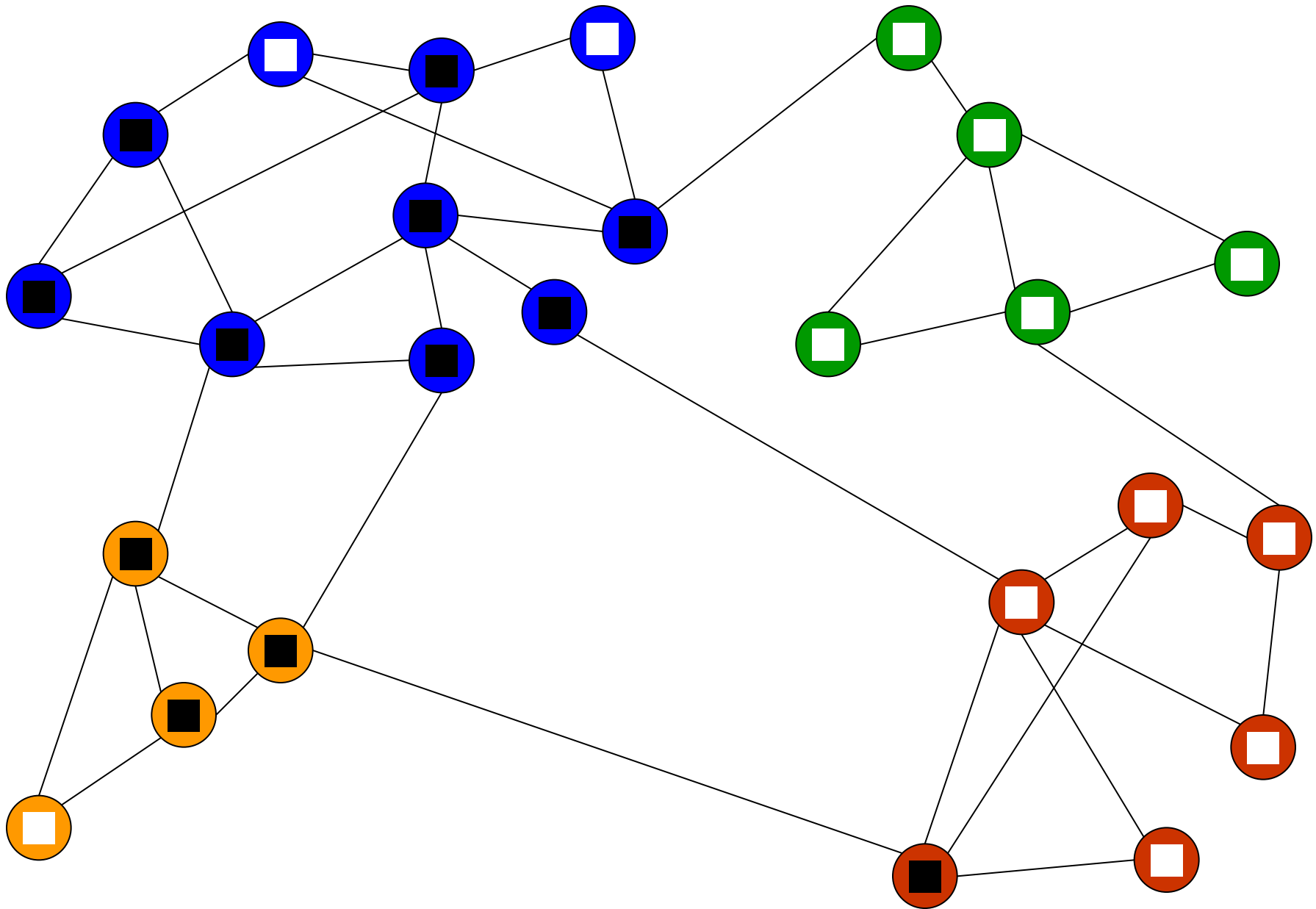
Why do we care: Diffusion

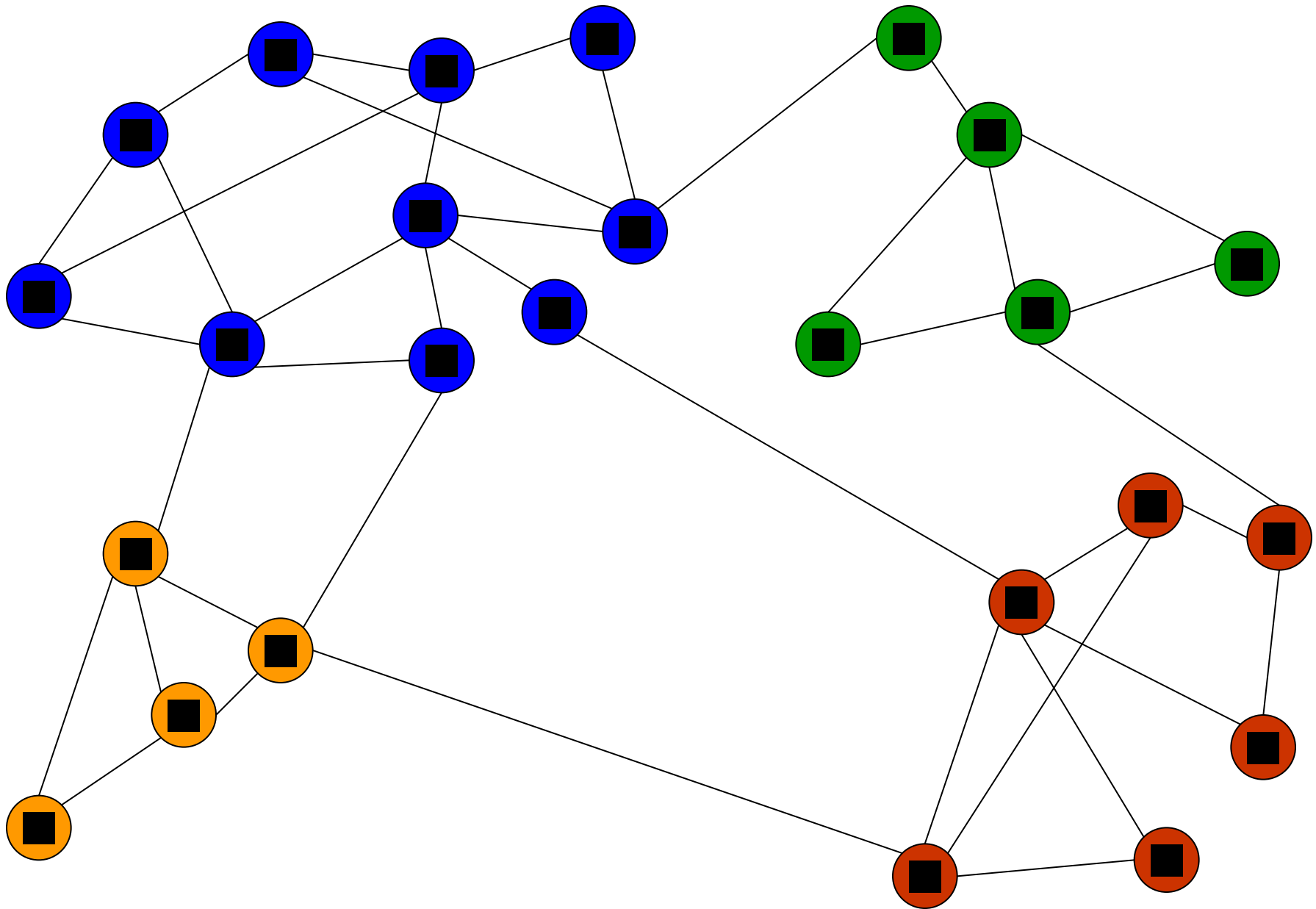


- Characterize how shortest paths are affected by density and homophily
- How will things diffuse?

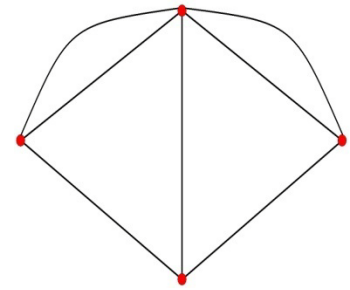






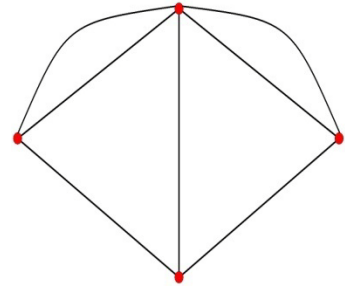


Multi-Type Random Network Model



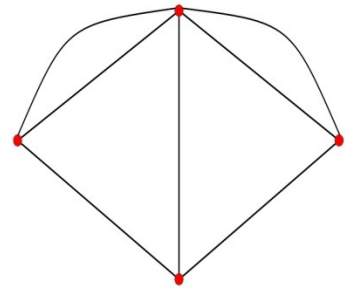
- $\{1, \dots, n\}$ agents/nodes
- Partitioned into groups N_1, \dots, N_K
- Node i in group k is linked to a node j in group k' with probability $P_{kk'}$ (undirected)
- Homophily: $P_{kk} > P_{kk'}$ for $k' \neq k$

Sequences of Networks



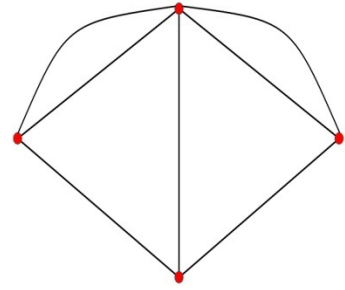
- $(n, K(n), N_1(n), \dots, N_{K(n)}(n), \{P_{kk'}(n)\}_{kk'})$
- $d(n) = \sum_{kk'} P_{kk'}(n) n_k(n) n_{k'}(n) / n^2$ overall avg degree

Sequences of Networks



- Links are dense enough so that network is connected:
$$d(n) \geq (1+\varepsilon) \log(n) \text{ some } \varepsilon > 0$$
- Some non-vanishing proportion of links are across groups so that network does not split:
$$P_{kk'}(n) \geq \varepsilon P_{kk} \text{ for some } \varepsilon > 0 \text{ and all } kk'$$
- $d(n)/n \square 0$ network is not too complete

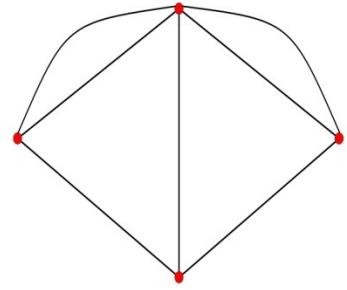
Theorem on Network Structure (Jackson 08)



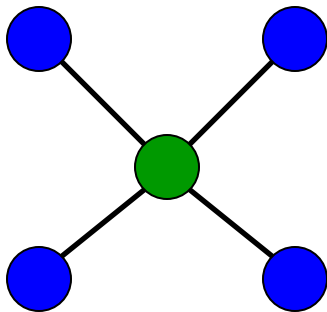
$$\frac{\text{AvgDist}(n)}{\log(n)/\log(d)} \rightarrow^P 1$$

link density matters but not homophily

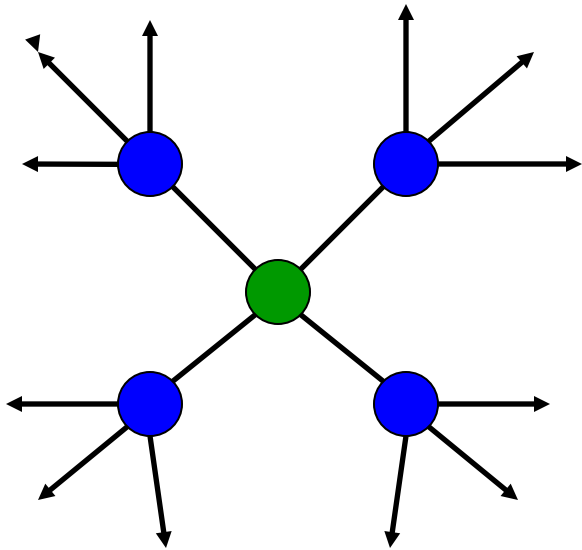
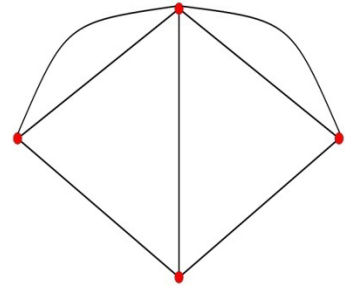
Intuition:



1 step: Reach d nodes,

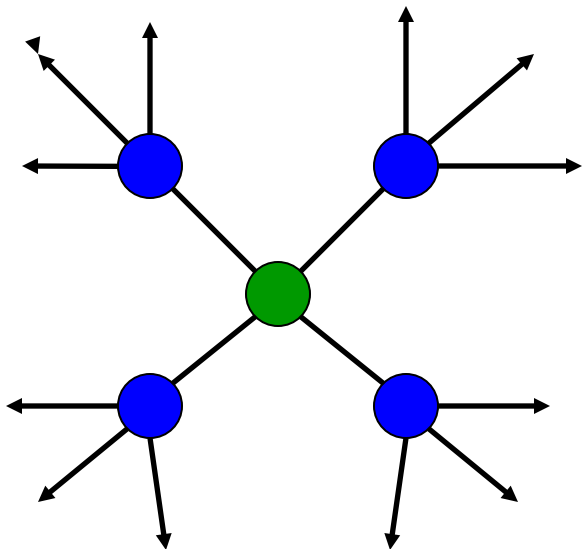
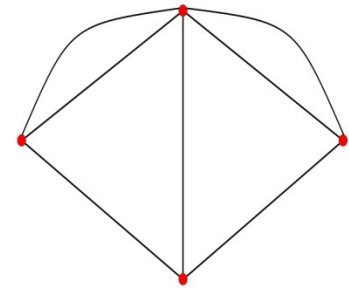


Ideas:



1 step: Reach d nodes,
then $d(d-1)$,

Ideas:

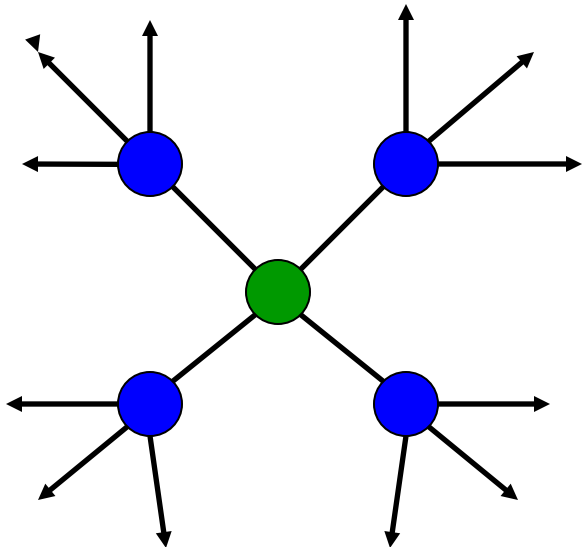
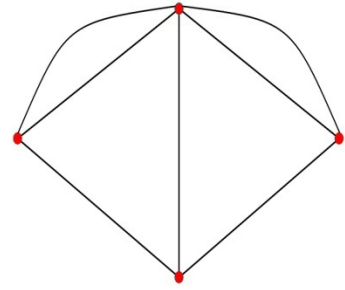


1 step: Reach d nodes,

then $d(d-1)$,

then $d(d-1)^2$,

Ideas:



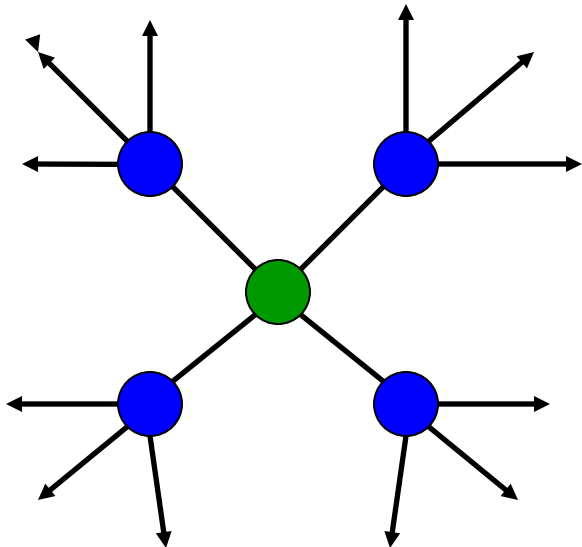
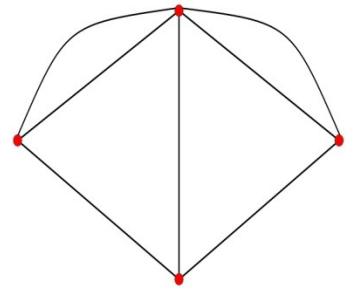
1 step: Reach d nodes,

then $d(d-1)$,

then $d(d-1)^2$, $d(d-1)^3$, ...

After k steps, totals roughly d^k

Ideas:

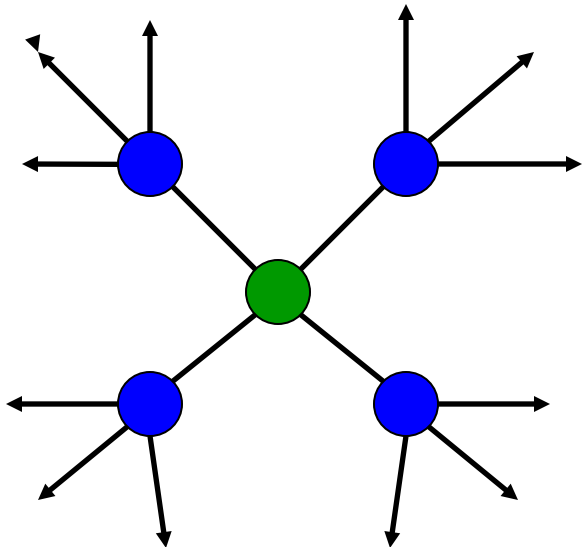
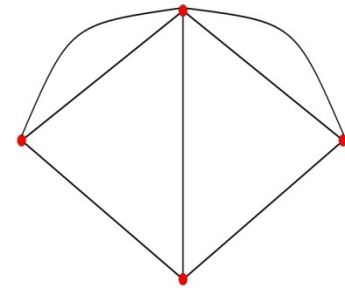


After k steps, reach d^k

When do we reach all n ?

$$d^k = n \text{ or } k = \log(n)/\log(d)$$

Ideas:



After k steps, reach d^k

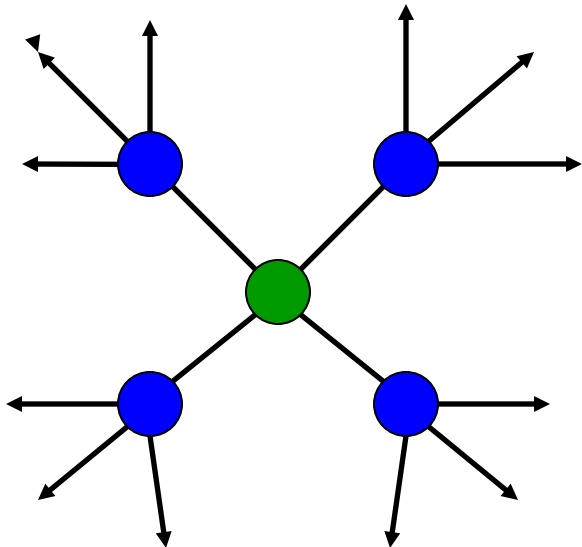
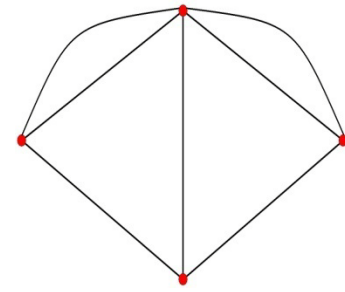
$$d^k = n \text{ or } k = \log(n)/\log(d)$$

suppose reach at least/at most fd at each step

need at least/at most $(fd)^k = \log(n)/[\log(d) + \log(f)]$

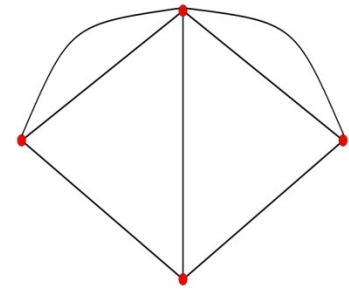
bound f

Ideas:



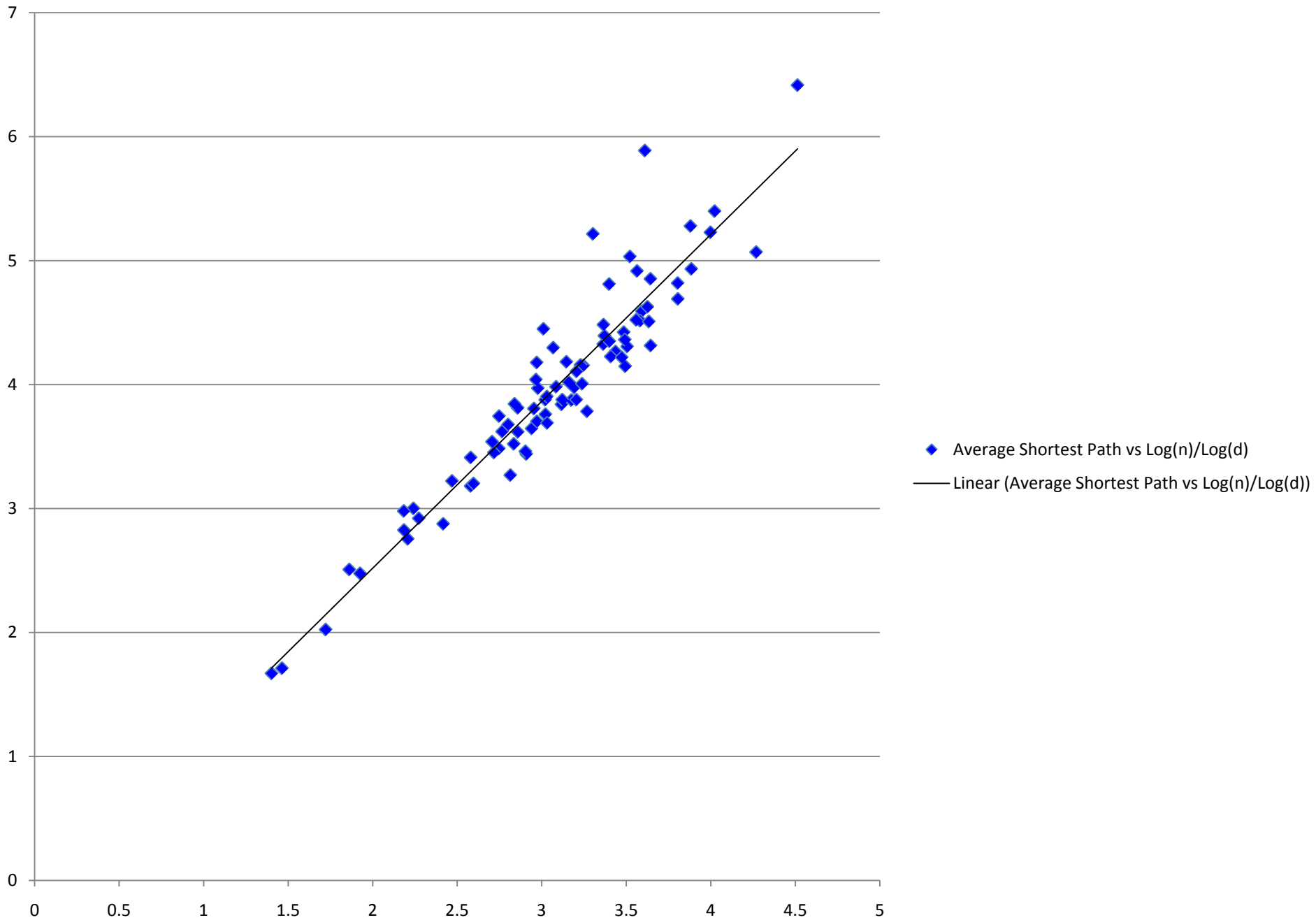
Most at maximum distance
(10, 100, 1000, 10000...)

Small Worlds/Six Degrees of Separation

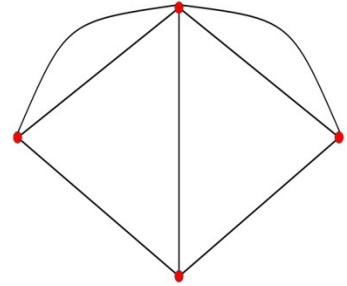


- $n = 6.7$ billion (world population)
- $d = 50$ (friends, relatives...)
- $\log(n)/\log(d)$ is about 6 !!

Average Shortest Path vs $\log(n)/\log(d)$



Diffusion

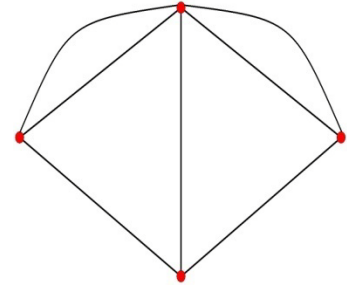


Network structure affects diffusion:

- probability of infection/contagion
- extent of infection
- who becomes infected
- speed of diffusion

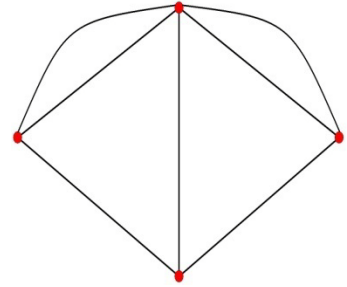
Technology is changing the world!

Diffusion



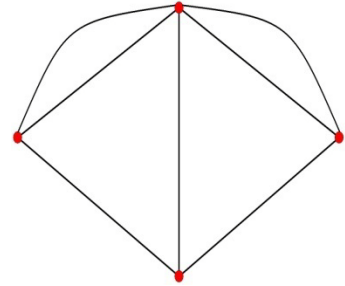
- Network structure matters
- Tractable, and simulations can go a long way to offering predictions
- experiment with changes in network structure, immunization, etc...

Implications



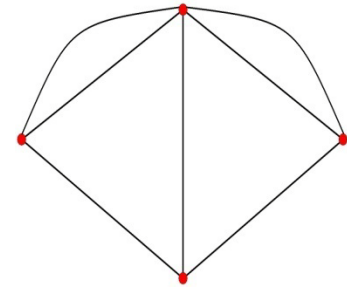
- For education/immunization
- Targeting nodes for deletion/infection...
- Endogenizing network?

Games on Networks



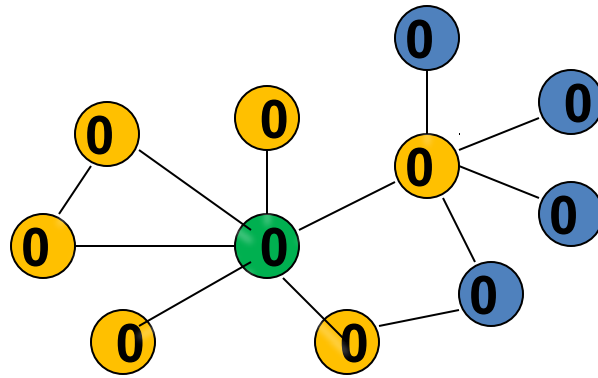
- Decisions to be made each chooses 0 or 1
 - not just diffusion
 - not just updating
- Local Complementarities - payoffs depend on neighbors' actions...
- ``Strategic'' Interplay
 - Inter-dependencies

Definitions



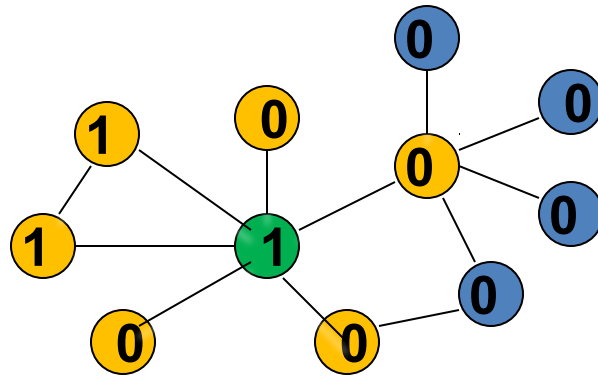
- Each player chooses action x_i in $\{0,1\}$
- $u_i(x_i, x_{N_i(g)})$ payoff to i
- Often will examine cases where i 's payoff depends only on $d_i(g)$ and $m_{N_i(g)}$ - the number of neighbors of i choosing 1

Example:



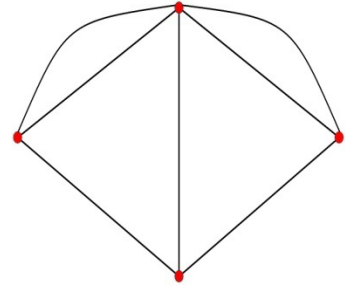
- Agent prefers to take action 1 if and only if at least two neighbors do

Example:



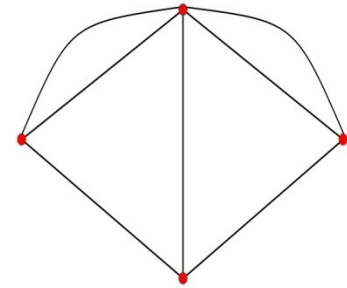
- An agent is willing to take action 1 if and only if at least two neighbors do

Strategic Setting there are multiple equilibria



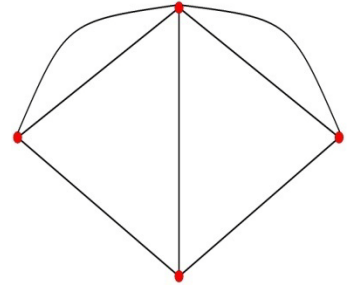
- When can both actions be sustained in an equilibrium?
- What happens to diffusion in such settings?

When can multiple actions be sustained:

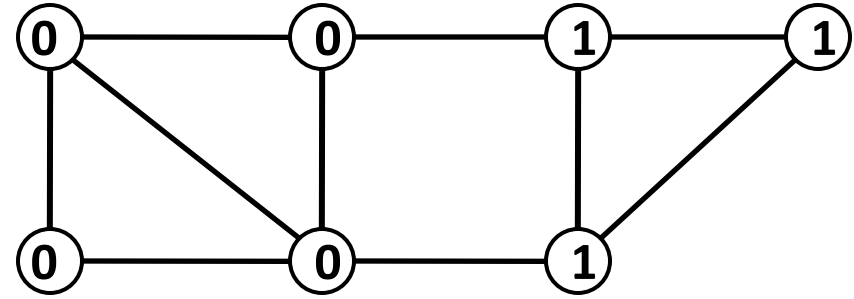
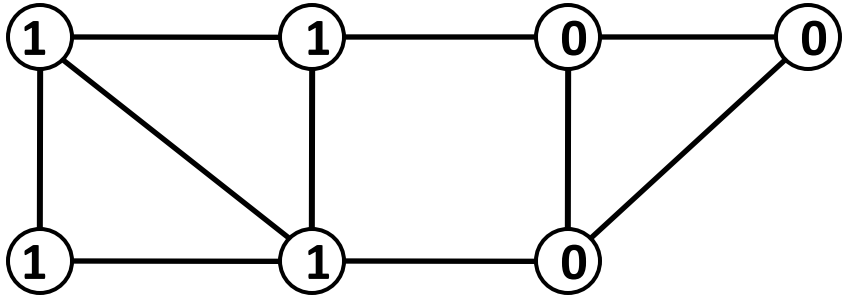
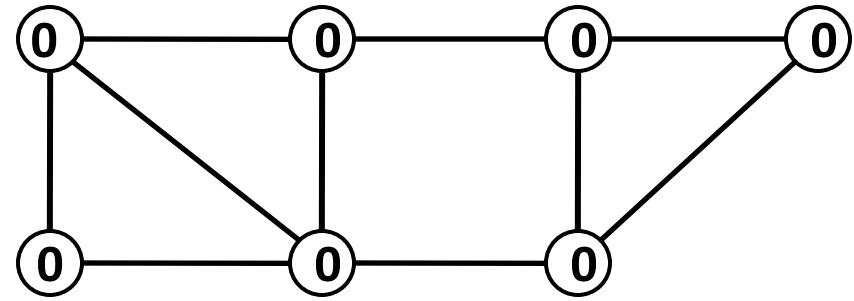
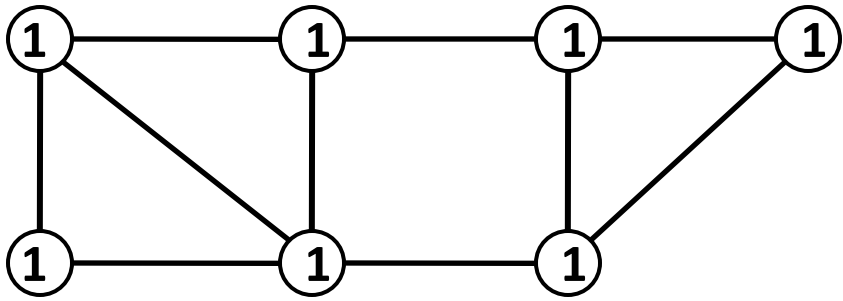


- Example: Morris (2000) Coordination game
- prefer to take action 1 if and only if more than a fraction q of neighbors take action 1

Pure Strategy Equilibrium Structure

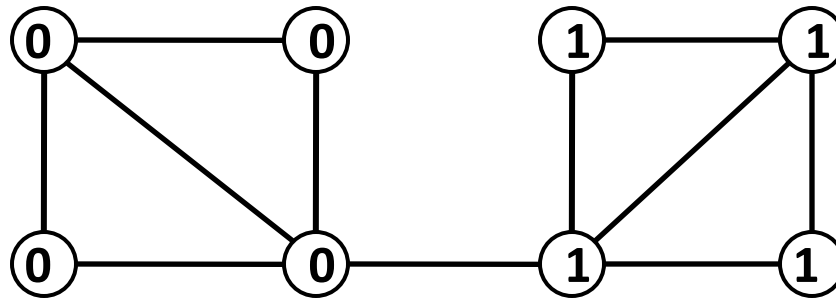
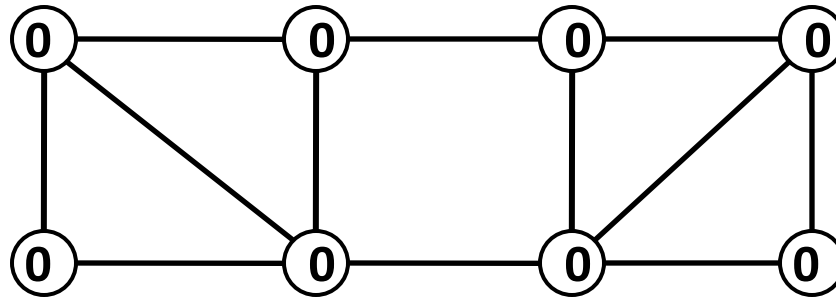


- Let S be the group that take action 1
- Each i in S must have fraction of at least q neighbors in S
- Each i not in S must have less than a fraction of q neighbors in S



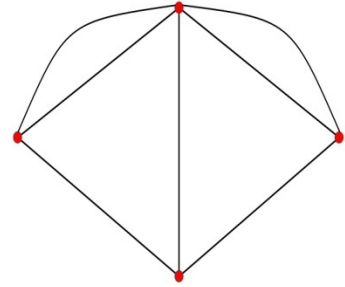
Equilibria when agents are willing to take action 1 if and only if more than half of their neighbors do

Agents will play 1 if and only if at least 70% of their neighbors do



In the top network all agents must play the same action
In the bottom network, both actions can be sustained

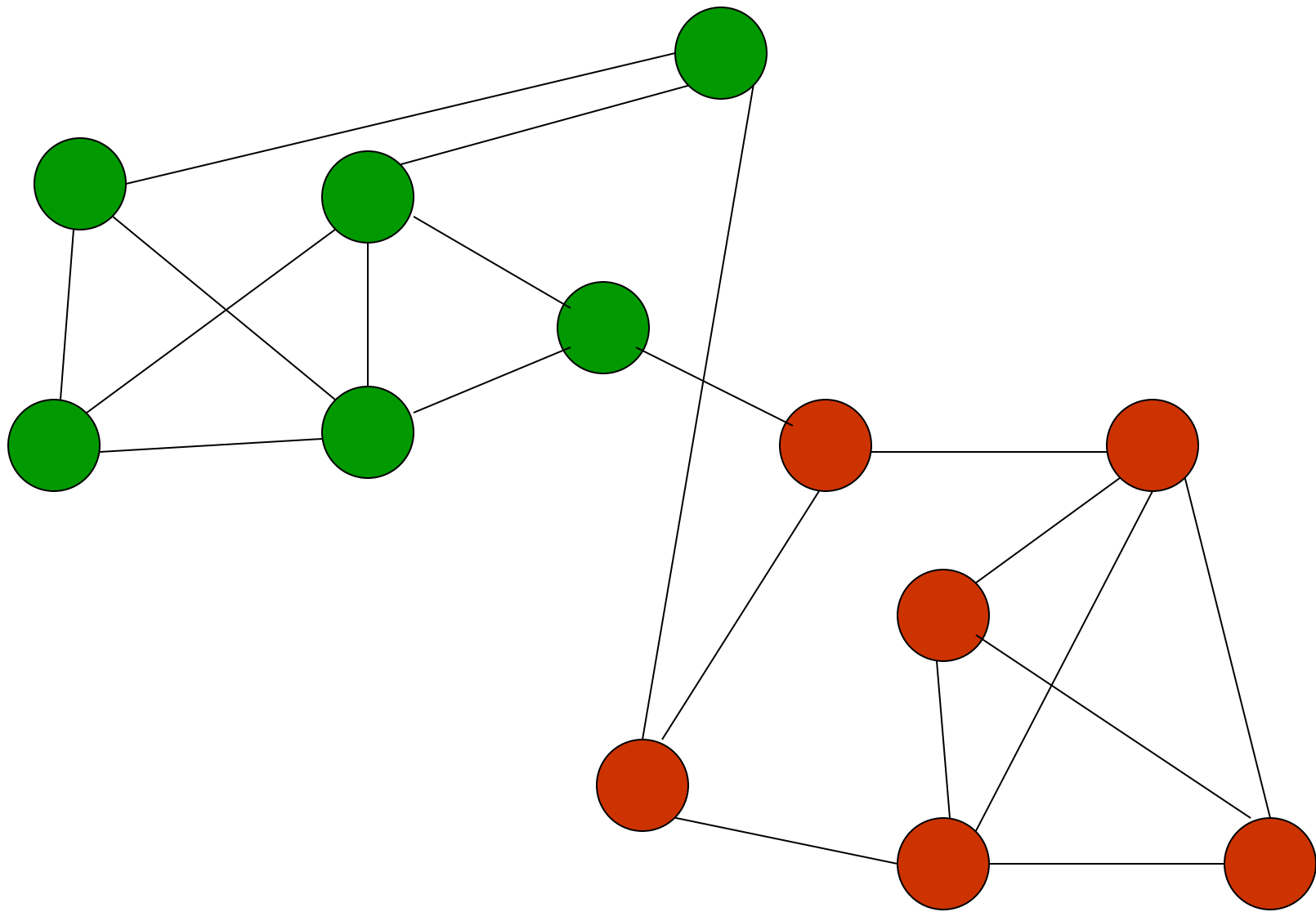
Cohesion



- A group S is r -cohesive relative to g if

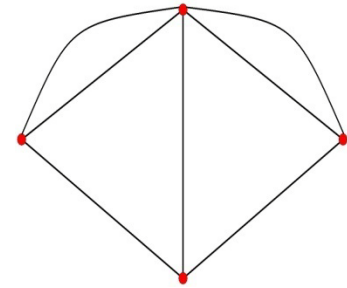
$$\min_{i \in S} |N_i(g) \cap S| / d_i(g) \geq r$$

Cohesiveness of S is $\min_{i \in S} |N_i(g) \cap S| / d_i(g)$



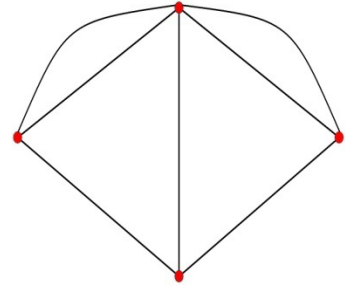
Both groups are $2/3$ cohesive

Equilibria where both strategies are played:



Morris (2000): there exists a pure strategy equilibrium where both actions are played if and only if there is a group S that is at least q cohesive and such that its complement is at least $1-q$ cohesive.

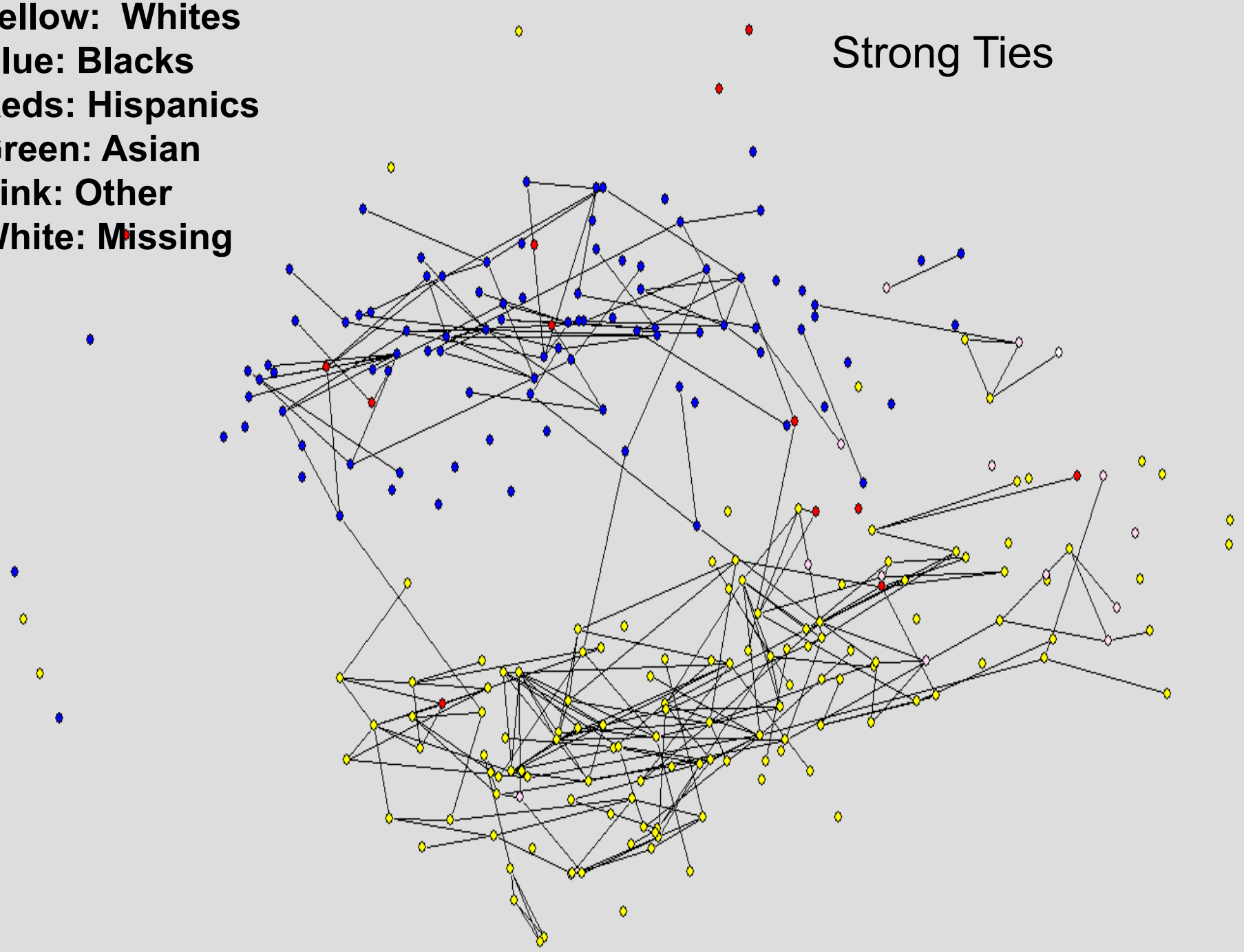
Homophily?



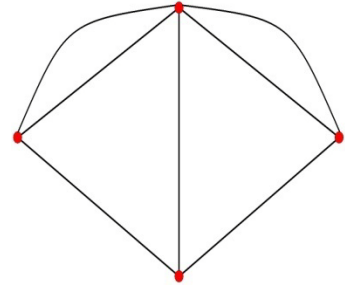
- If $q=1/2$ – so want to match majority
- Then two groups that have more self-ties than cross-ties suffices
- As q goes up, need more homophilous behavior between the groups

Yellow: Whites
Blue: Blacks
Reds: Hispanics
Green: Asian
Pink: Other
White: Missing

Strong Ties

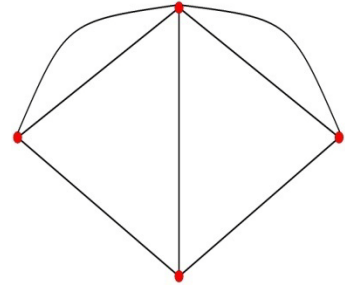


Contagion/Diffusion



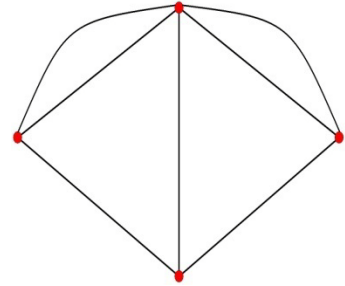
- Start with some group of m nodes taking action 1 – fix their action
- Iterate on best replies for the rest of the population (break ties to 1)
- When does action 1 diffuse to the whole society?

Proposition (Morris (2000))



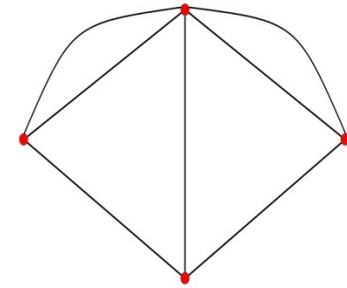
Contagion from m nodes occurs if and only if there is no subset of the remaining nodes that is more than $1-q$ cohesive.

Proof



- If there is a group S that is more than $1-q$ cohesive, then no member of that group has a fraction of at least q of its friends outside of S
- No member of that group changes to 1.
- If there is no such group, then some member of the complement of m has at least a fraction of q of its friends in m .
- At every iteration, some agent among those not yet taking 1, has a fraction of at least q of his or her friends taking action 1; otherwise the remaining group would be more than $1-q$ cohesive

Application:



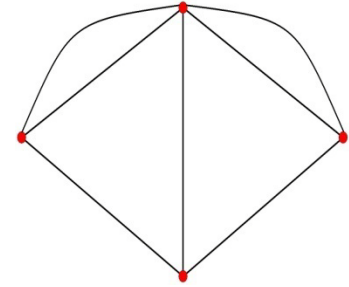
- Drop out decisions
- Strategic complements

Drop-Out Rates

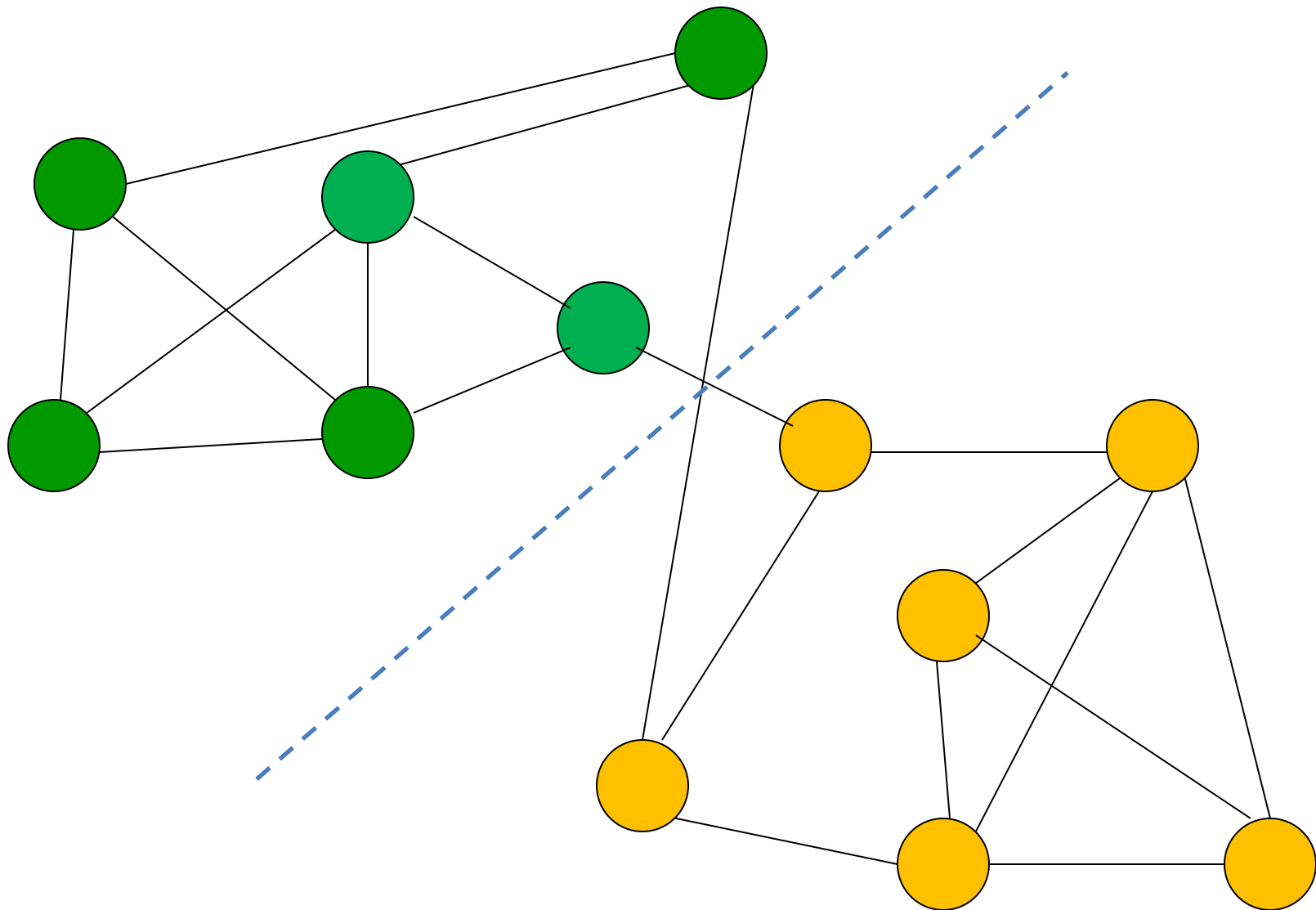
- Chandra (2000) Census – males 25 to 55

	1940	1950	1960	1970	1980	1990
whites	3.3	4.2	3.0	3.5	4.8	4.9
blacks	4.2	7.5	6.9	8.9	12.7	12.7

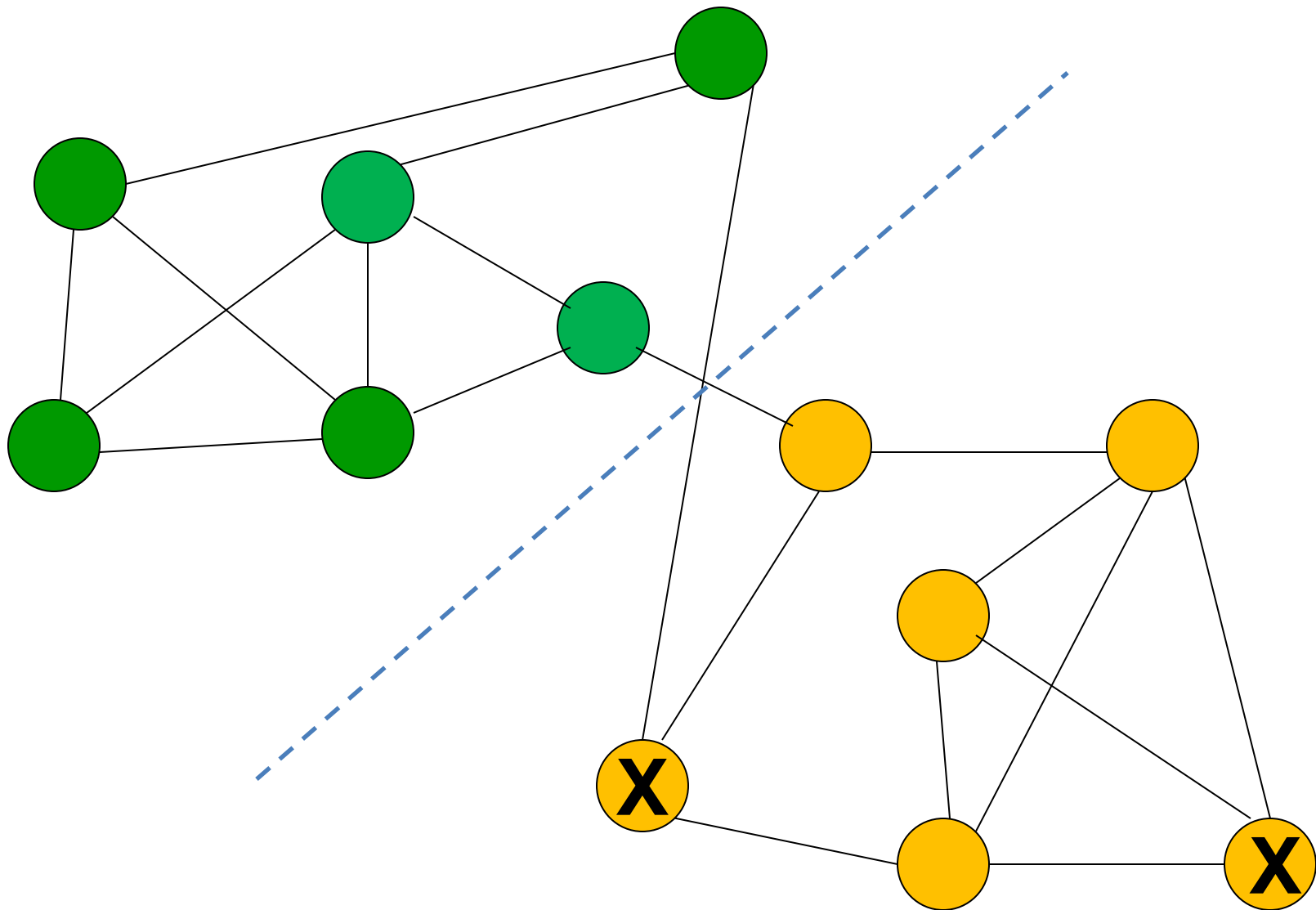
Drop-Out Decisions



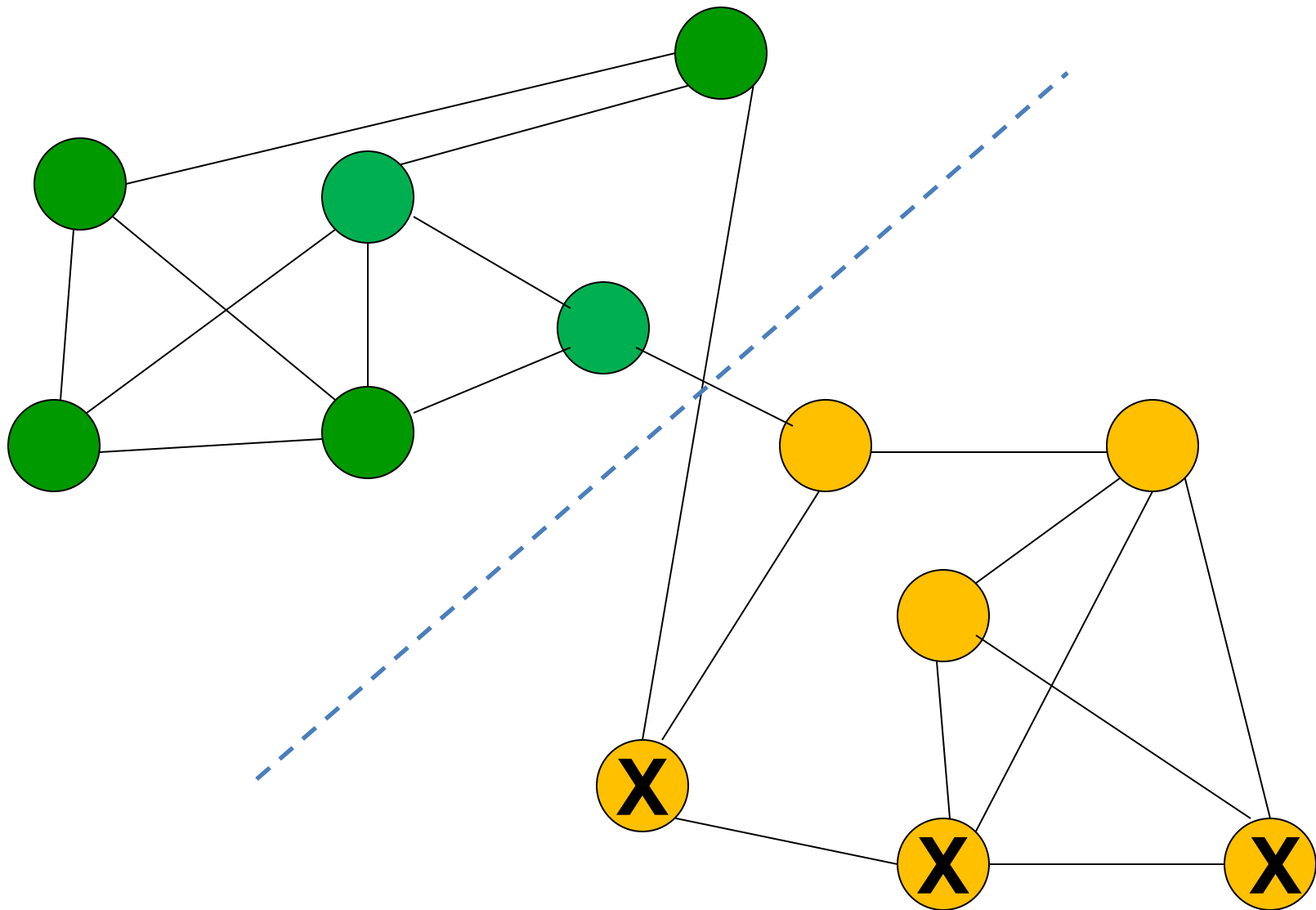
- Value to being in the labor market depends on number of friends in labor force
- Drop out if some number of friends drop out
- Some heterogeneity in threshold (different costs, natural abilities...)
- Homophily – segregation in network
- Different starting conditions: history...



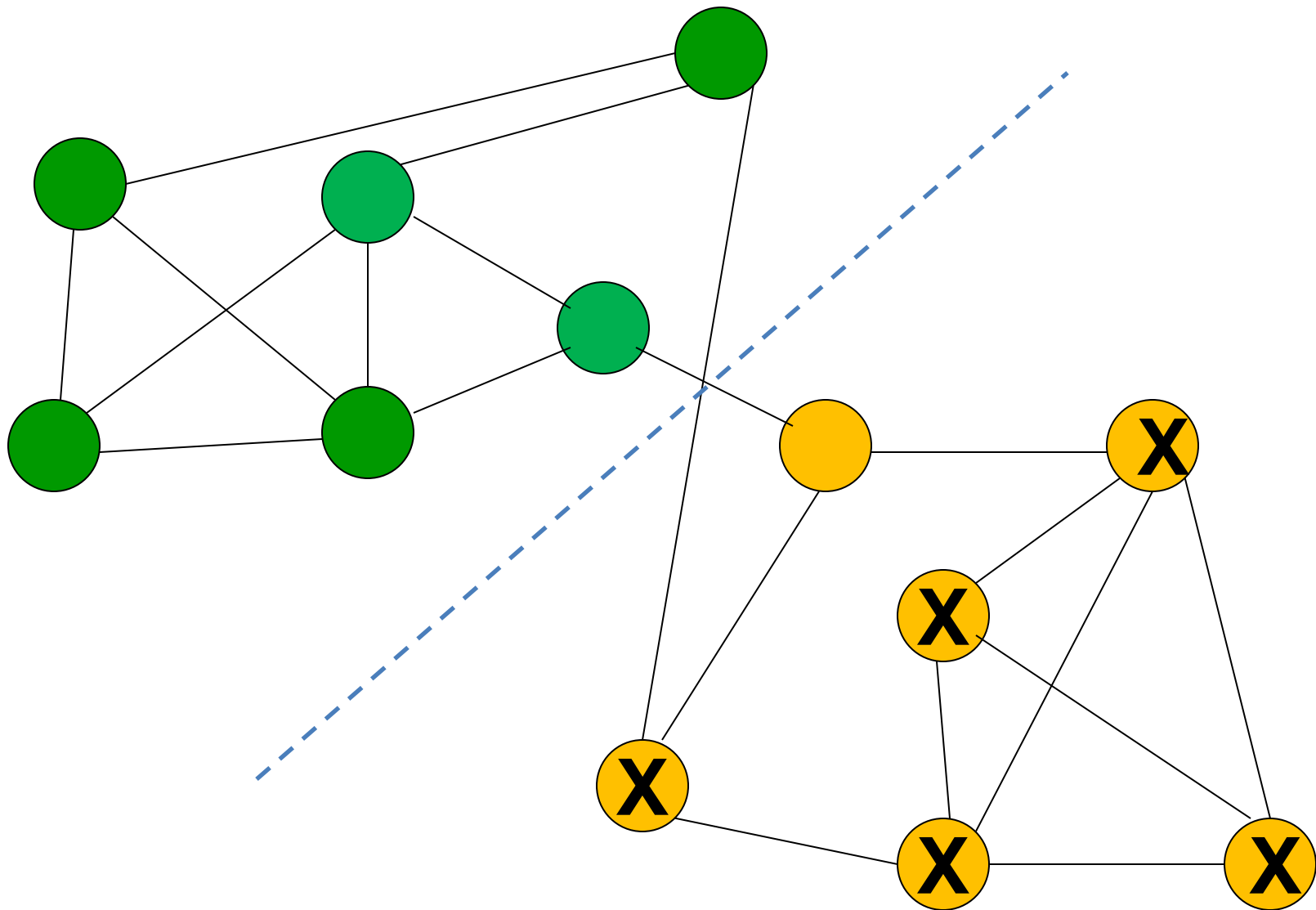
Two groups exhibit homophily...



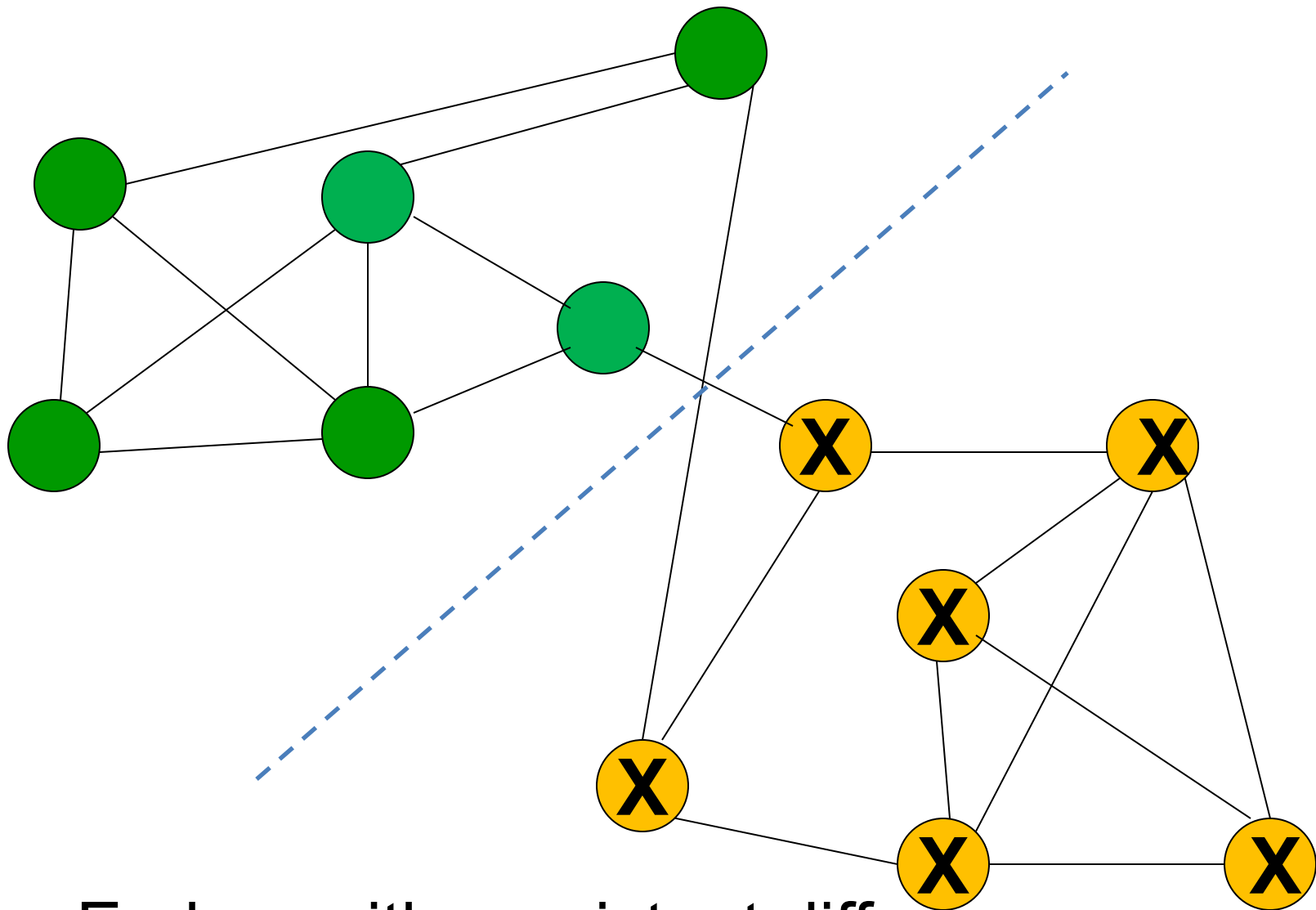
Drop-out if at least half of neighbors do -- begin with two initial dropouts...



Drop-out if at least half of neighbors
do...

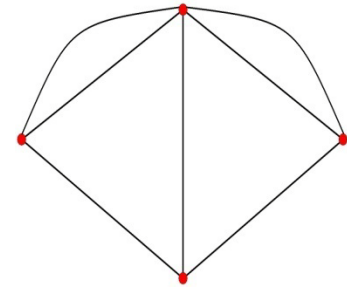


Drop-out if at least half of neighbors
do...



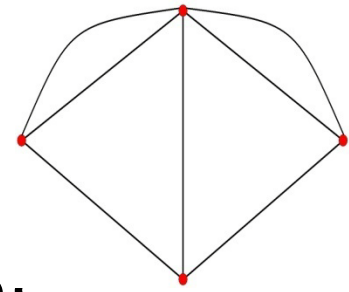
End up with persistent differences across groups... Applications to social mobility, wage inequality, etc.

Summary – Games on Networks:



- **Structure matters:**
 - Multiplicity of equilibria
 - Multiple actions can emerge depending on cohesion/homophily patterns...
- **Diffusion:**
 - Dynamics are more complicated than pure diffusion case, depend on homophily, thresholds, heterogeneity...

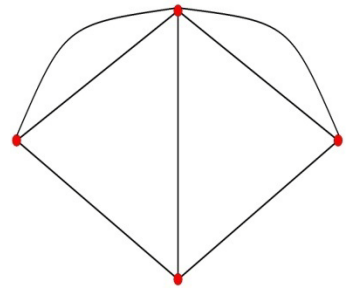
Outline



3 Examples of models and the questions they can answer:

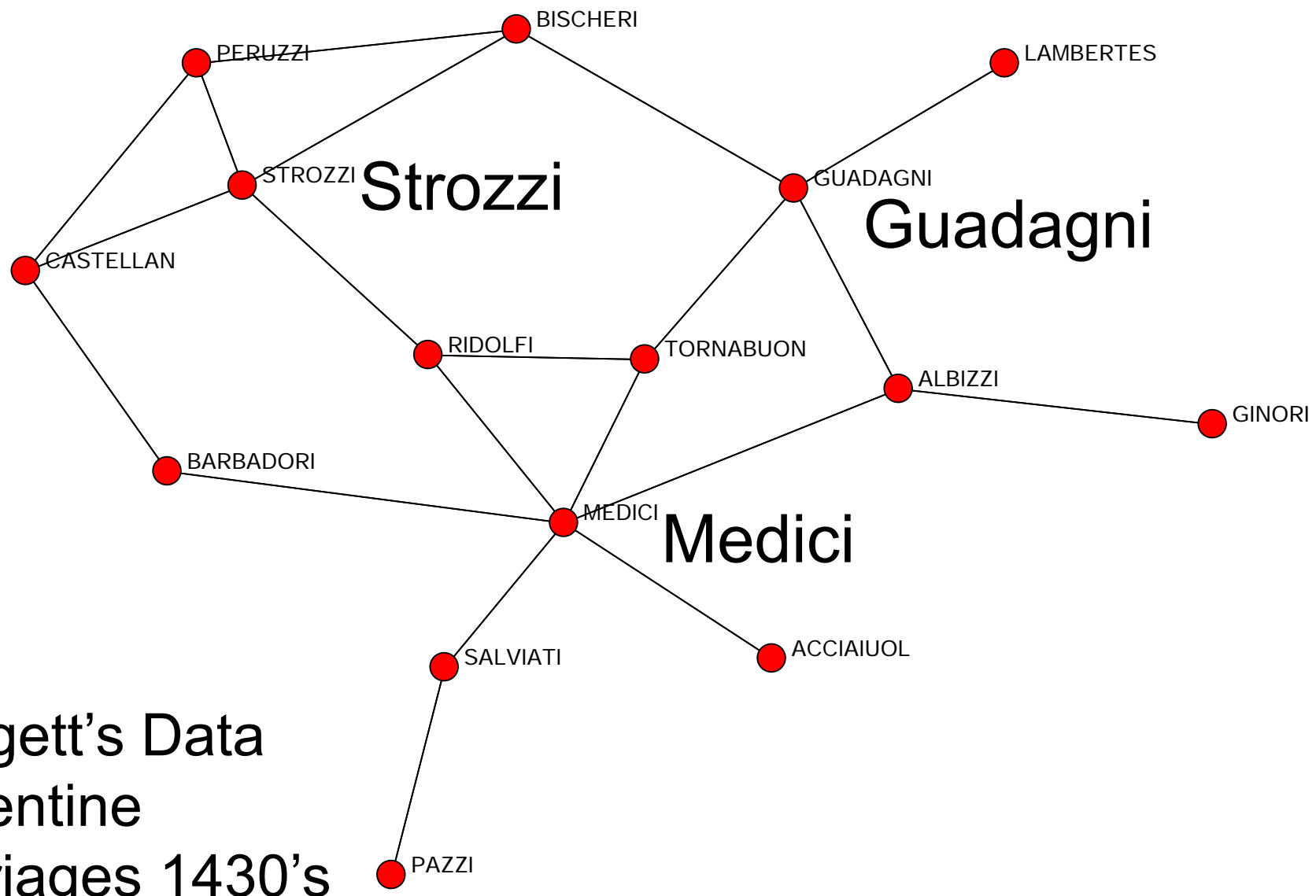
- Random graph models
contagion/diffusion
- **Game theoretic/strategic model**
efficiency versus stability
- A hybrid model
empirical estimation of friendship
formation

Strategic Models



- Help answer “why” networks take certain form (why the P_{kk} 's?)
- Do “right” networks form?
- Welfare measures

● PUCCI



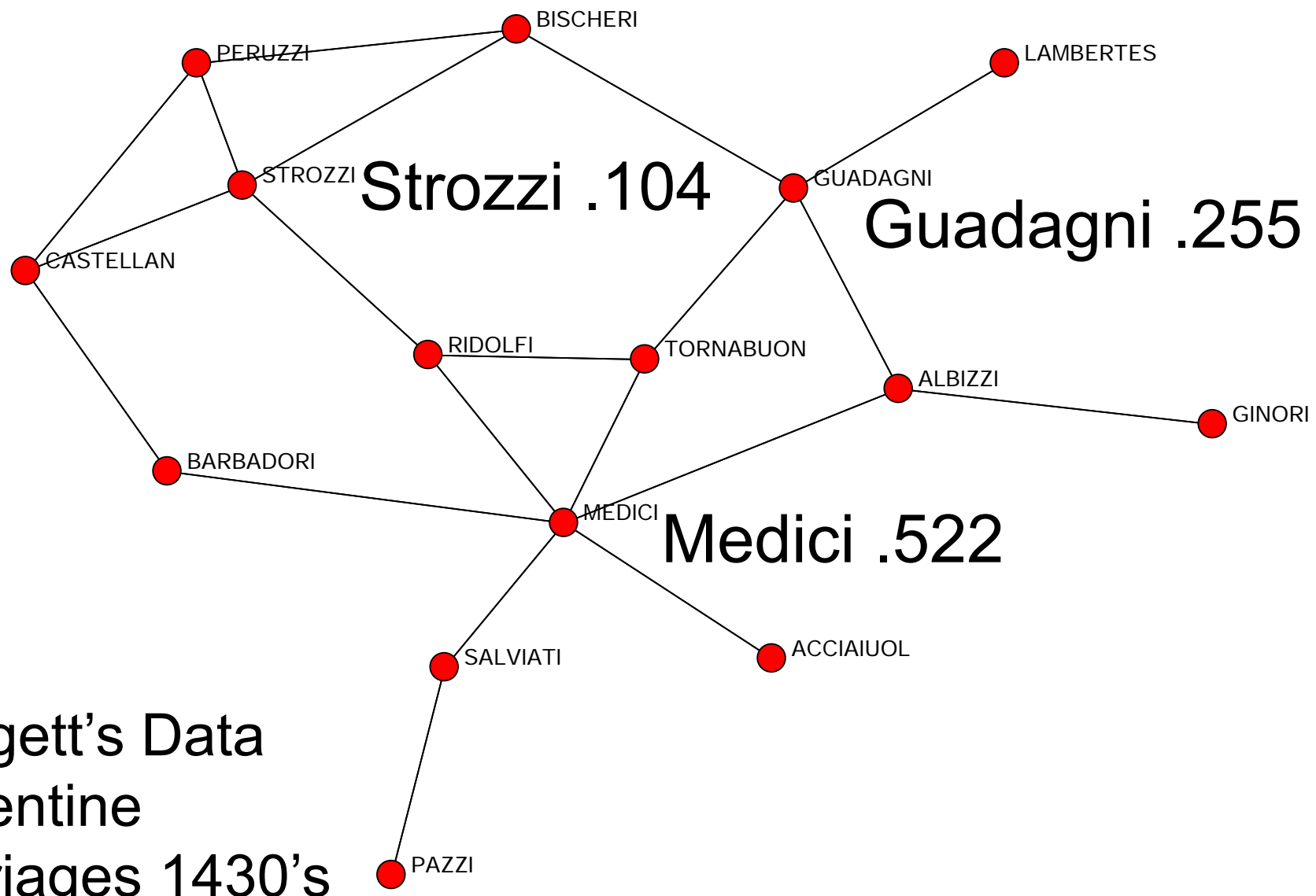
Strozzi

Guadagni

Medici

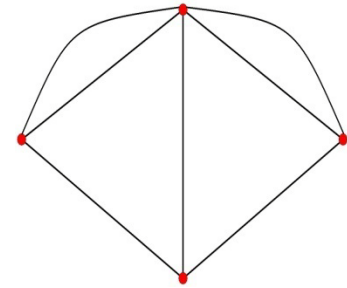
Padgett's Data
Florentine
Marriages 1430's

● PUCCI



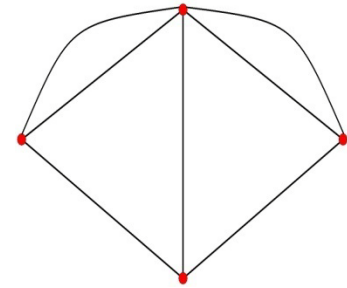
Padgett's Data
Florentine
Marriages 1430's

Representing Networks



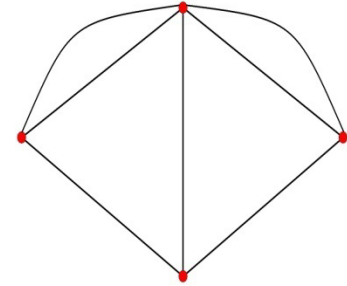
- $N = \{1, \dots, n\}$ nodes, vertices, players
- $g \in \{0, 1\}^{n \times n}$ represents the relationships
- $g_{ij} = 1$ indicates a link or edge between i and j
- Notation: $ij \in g$ indicates a link between i and j

An Economic Analysis: Jackson Wolinsky (1996)



- $u_i(g)$ - payoff to i if the network is g
- undirected network formation

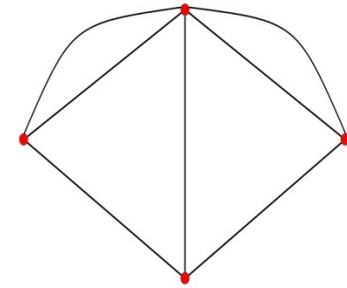
Connections Model JW96



- $0 \leq \delta \leq 1$ a benefit parameter for i from connection between i and j
- $0 \leq c_{ij}$ cost to i of link to j
- $\ell(i,j)$ shortest path length between i,j

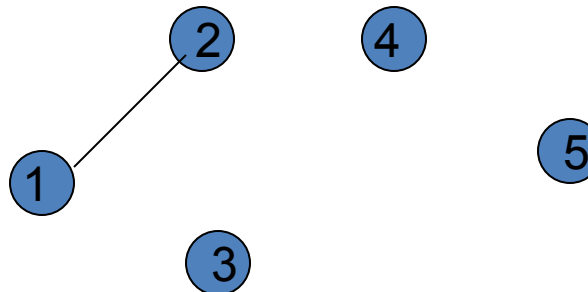
$$u_i(g) = \sum_j \delta^{\ell(i,j)} - \sum_{j \text{ in } N_i(g)} c_{ij}$$

Symmetric Version:



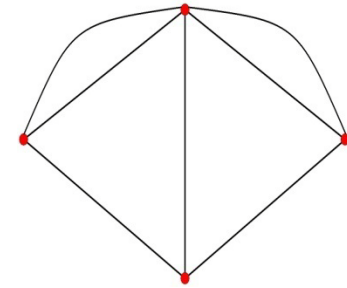
- benefit from a friend is $\delta < 1$
- benefit from a friend of a friend is δ^2, \dots
- cost of a link is $c > 0$

$$u_1 = \delta - c$$

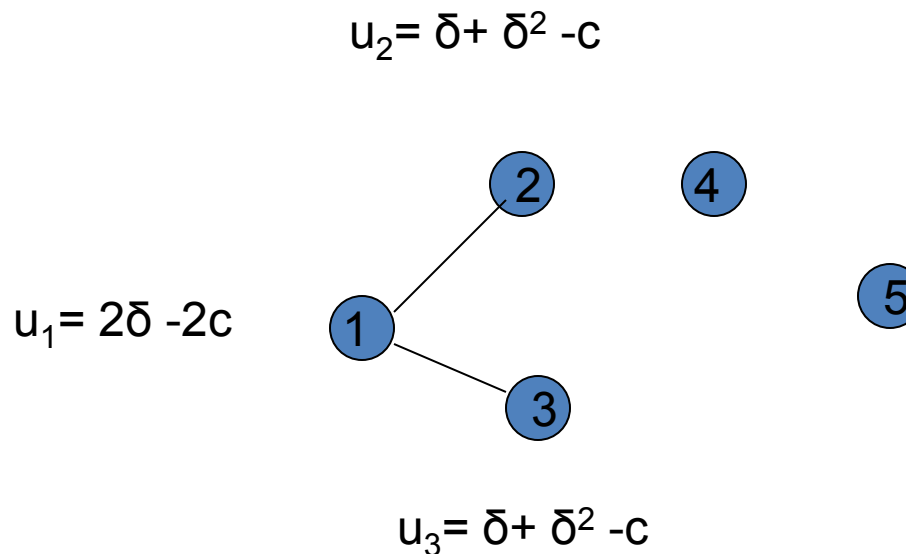


$$u_2 = \delta - c$$

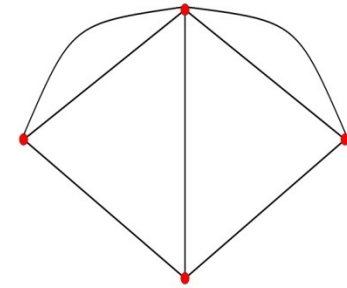
Symmetric Version:



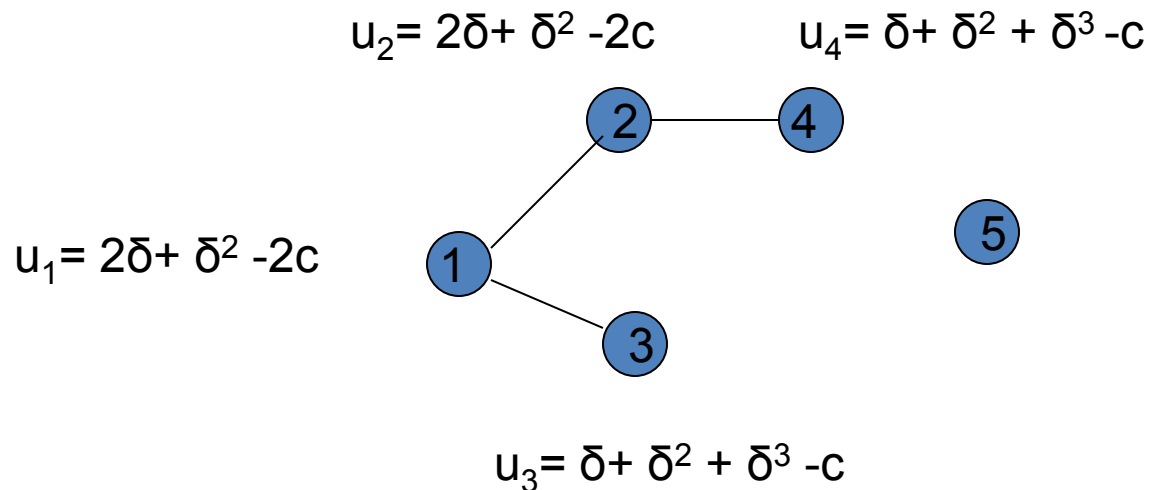
- benefit from a friend is $\delta < 1$
- benefit from a friend of a friend is δ^2, \dots
- cost of a link is $c > 0$



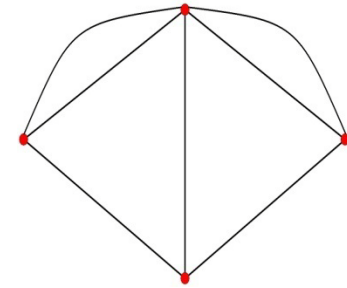
Symmetric Version:



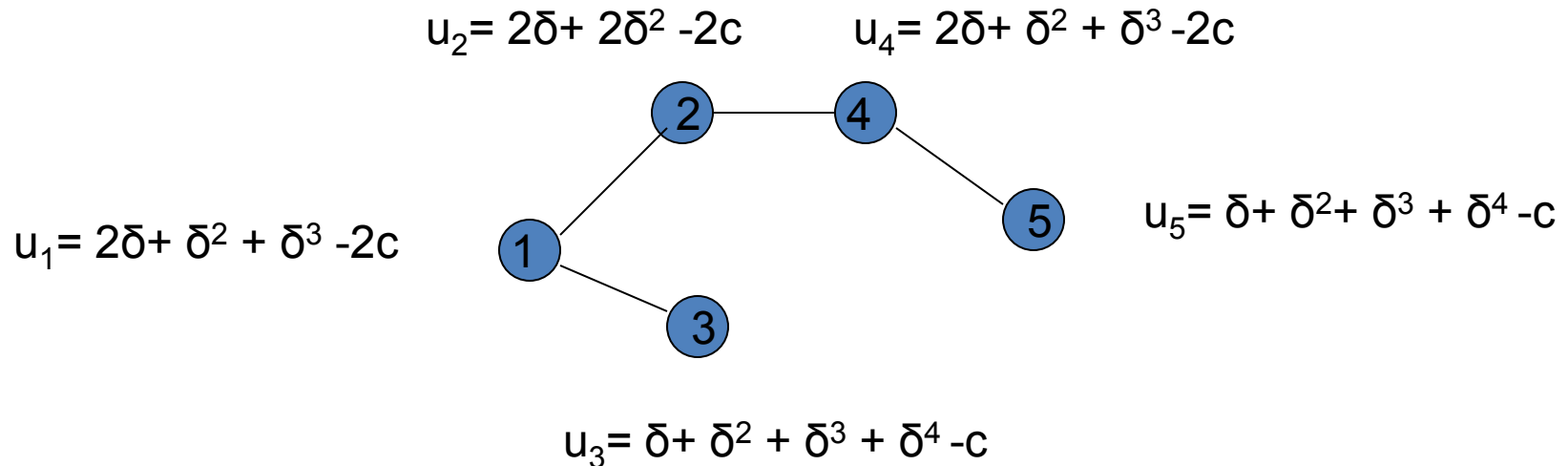
- benefit from a friend is $\delta < 1$
- benefit from a friend of a friend is δ^2, \dots
- cost of a link is $c > 0$



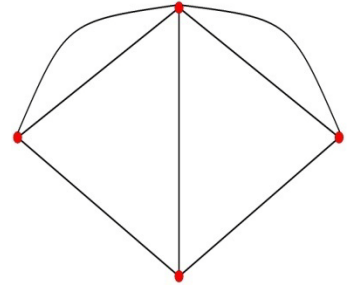
Symmetric Version:



- benefit from a friend is $\delta < 1$
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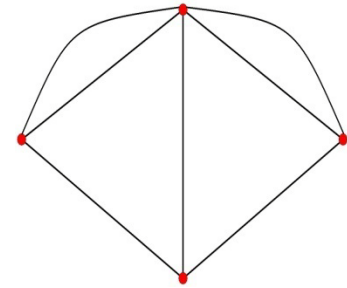


Questions:



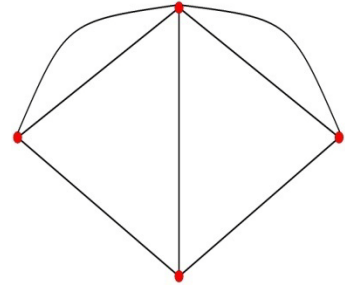
- Which network are best for society?
- Which networks are formed by the agents?

Modeling Incentives: Pairwise Stability

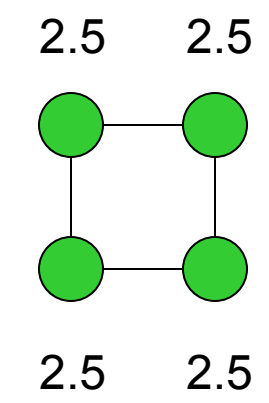
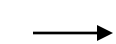
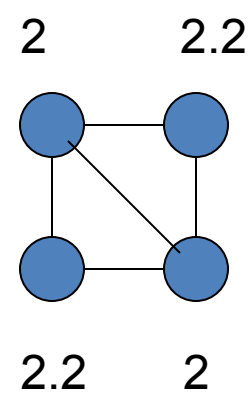
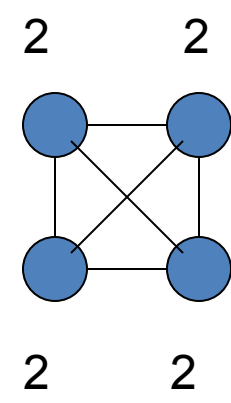
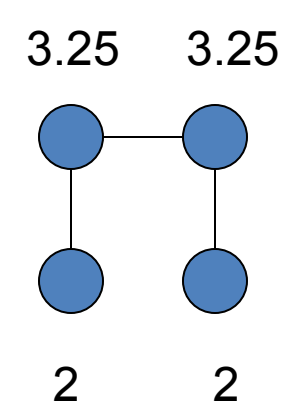
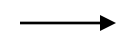
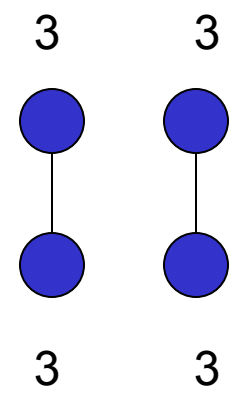
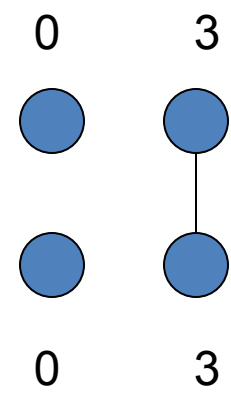
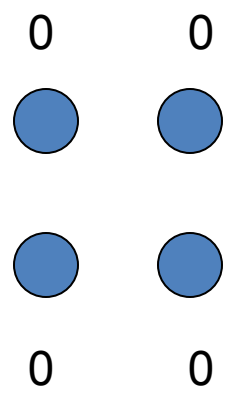


- no agent gains from severing a link – relationships must be beneficial to be maintained
- no two agents both gain from adding a link (at least one strictly) – beneficial relationships are pursued when available

Pairwise Stability

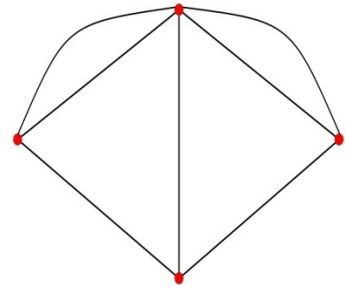


- $u_i(g) \geq u_i(g-ij)$ for i and $ij \in g$
 - no agent gains from severing a link
- $u_i(g+ij) > u_i(g)$ implies $u_j(g+ij) < u_j(g)$ for $ij \notin g$
 - no two agents both gain from adding a link (at least one strictly)
- a 'weak' concept, but often narrows things down



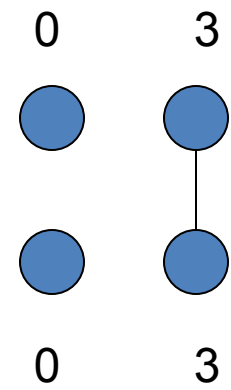
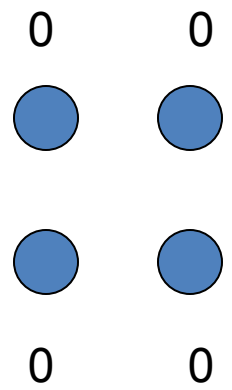
Pairwise Stable

Efficiency

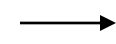
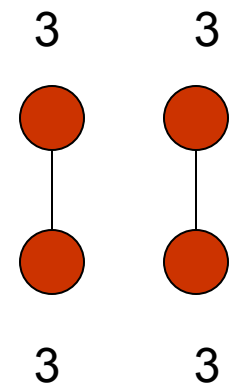


- **Pareto efficient** g : there does not exist g' s.t.
 - $u_i(g') \geq u_i(g)$ for all i , strict for some

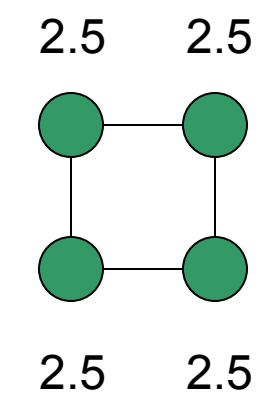
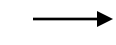
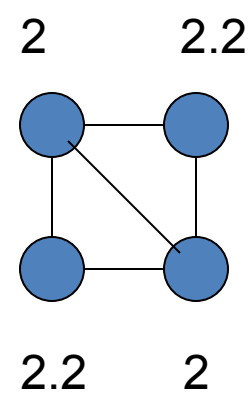
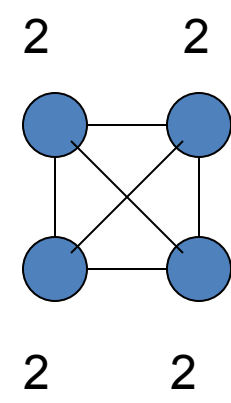
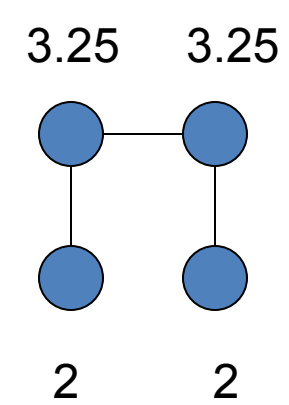
- **Efficient** g (Pareto if transfers):
 - g maximizes $\sum u_i(g')$



**Efficient and
Pareto Efficient**

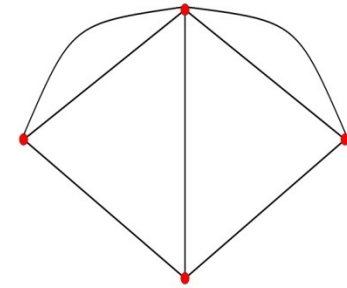


Pareto Efficient



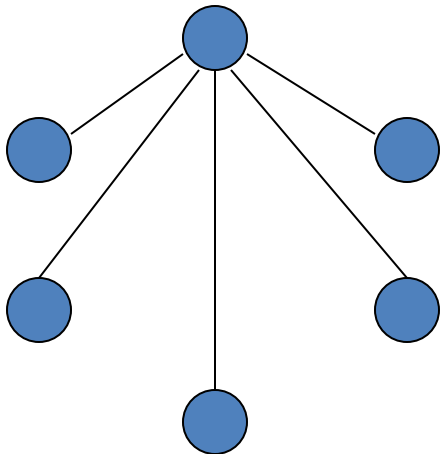
**Pairwise
Stable**

Example: Pairwise stable and inefficient

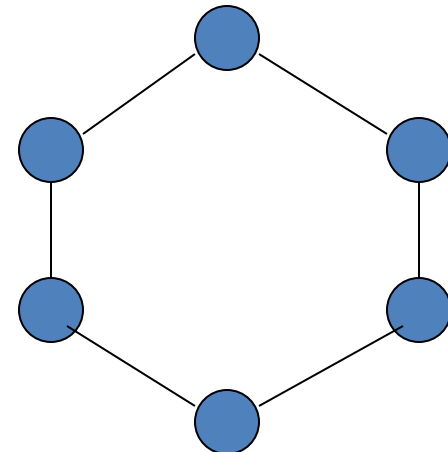


- $\delta < c < (\delta + \delta^2)(1 - \delta^3)$ $n = 6$

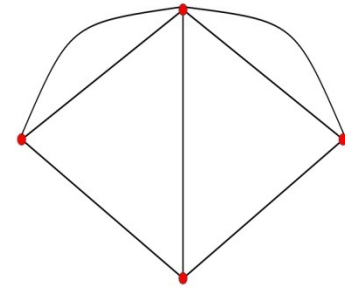
- efficient: (not ps)



ps:

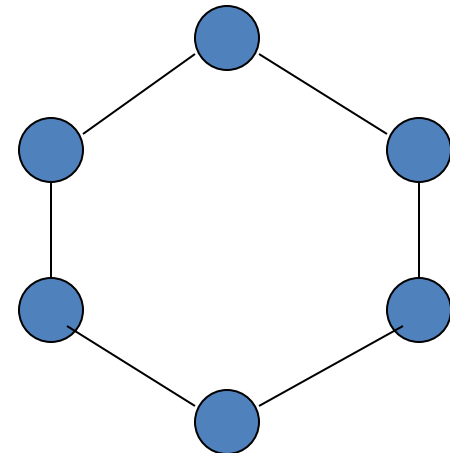
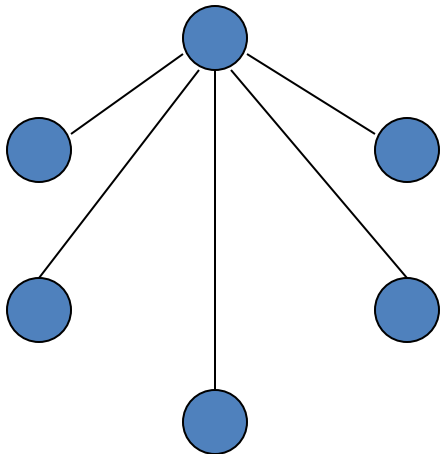


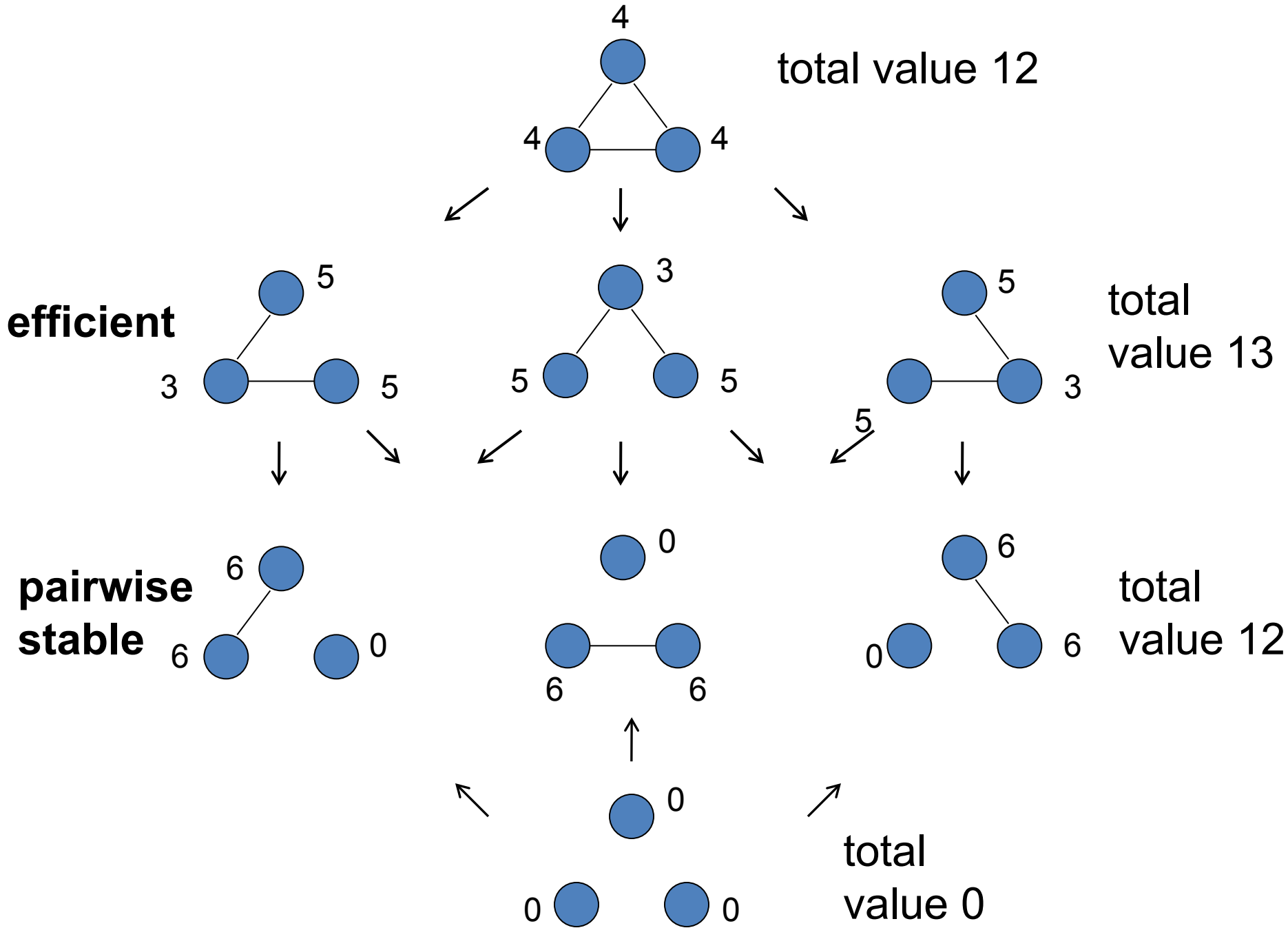
Transfers?



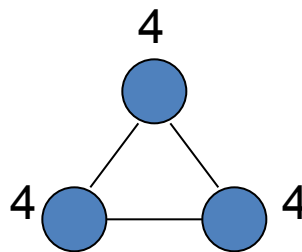
Inefficiency due to fact that center won't sustain links

- Pay center to equilibrate values
 - Does this always work?

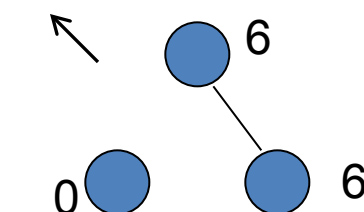
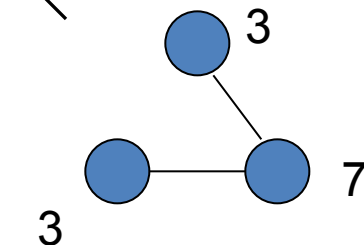
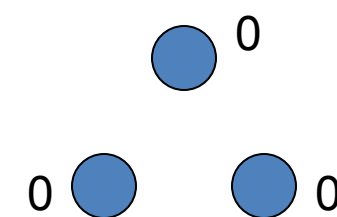
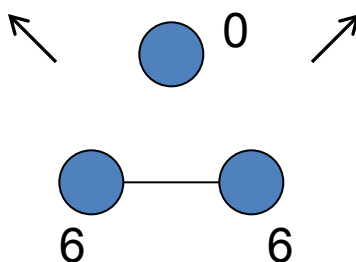
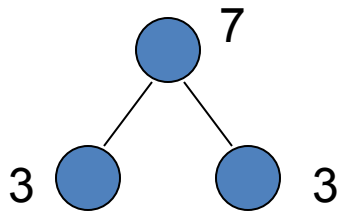




**pairwise
stable**



total value 12

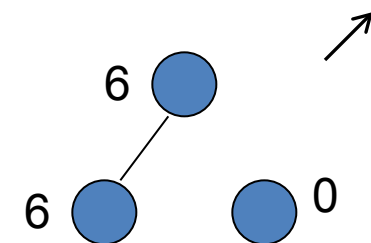
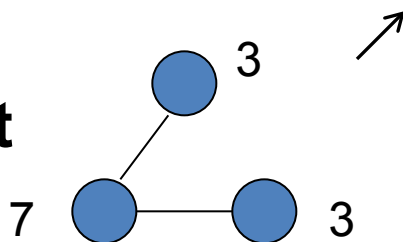


total
value 13

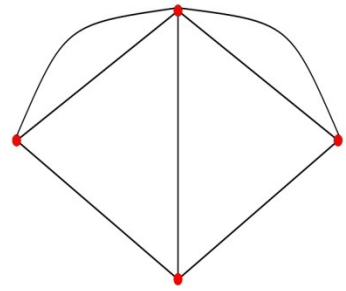
total
value 12

total
value 0

efficient

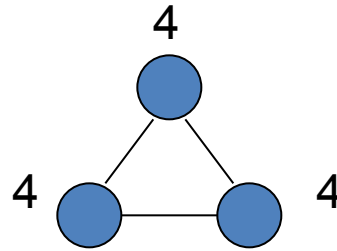


transfers



- $t_i(g)$ such that
 - if $d_i(g) = 0$ then $t_i(g) = 0$
 - if $N_i(g) \setminus \{j\} = N_j(g) \setminus \{i\}$ then $t_i(g) = t_j(g)$

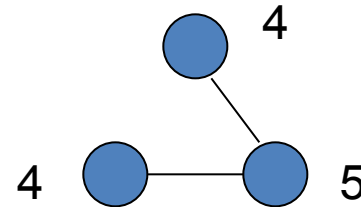
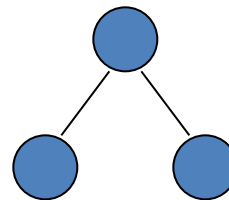
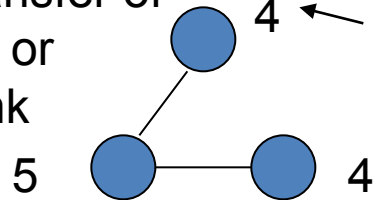
Transfers cannot always help



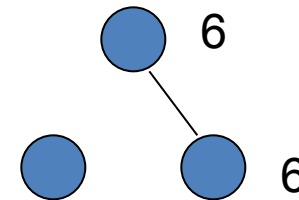
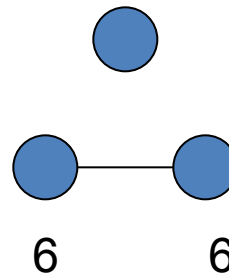
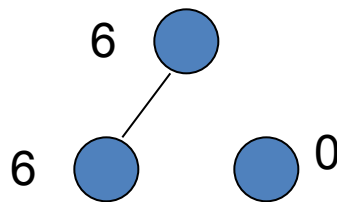
value 12

need nonnegative transfers or add link

needs transfer of at least 1 or severs link

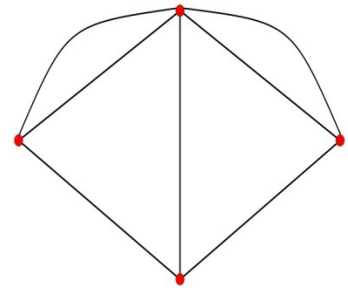


value 13
efficient



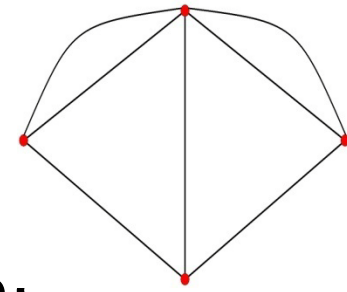
value 12

Economic Network Models



- Highlight tension between selfish formation and efficiency
- Understand externalities
- Policy predictions....

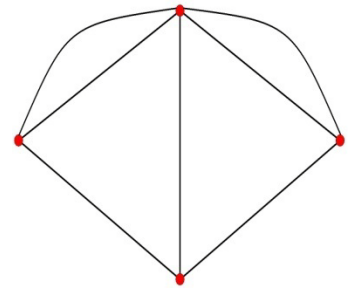
Outline



3 Examples of models and the questions they can answer:

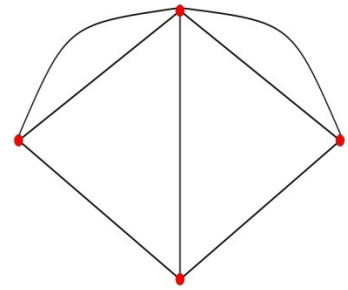
- Random graph models
contagion/diffusion
- Game theoretic/strategic model
efficiency versus stability
- **A hybrid model**
empirical estimation of friendship formation

Hybrid Network Models



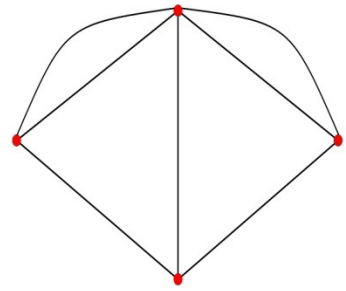
- Most networks involve both choice and chance in formation
- What are the relative roles?
- Random/Strategic models can be too extreme
- Can we see relative roles in homophily?

Homophily:



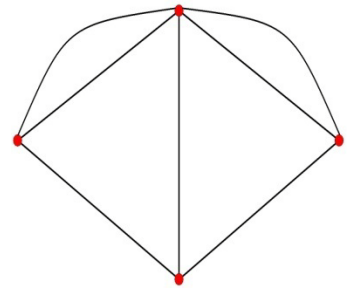
- Group A and Group B form fewer cross race friendships than would be expected given population mix
 - Is it due to structure: few meetings?
 - Is it due to preferences of group A?
 - Is it due to preferences of group B?
- Extend CJP model to answer this
- Compare across races in data on high school friendships

Revealed Preference Theory



- Common to Consumer Theory
- Use it in mapping social/friendship choices too!
- Different information than surveys on racial attitudes

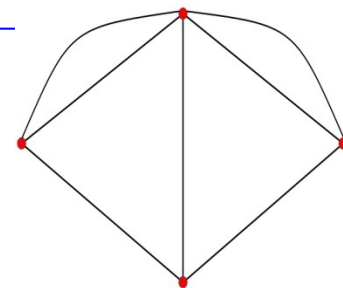
Model to incorporate both



- Utilities specified as a function of friendships
- Meeting process that incorporates randomness
- Allow both utilities and meeting process to depend on types

II. Model

Currarini, Jackson, Pin 2009ab:



Types: $i \in \{1, \dots, K\}$

s_i = # same-type friends

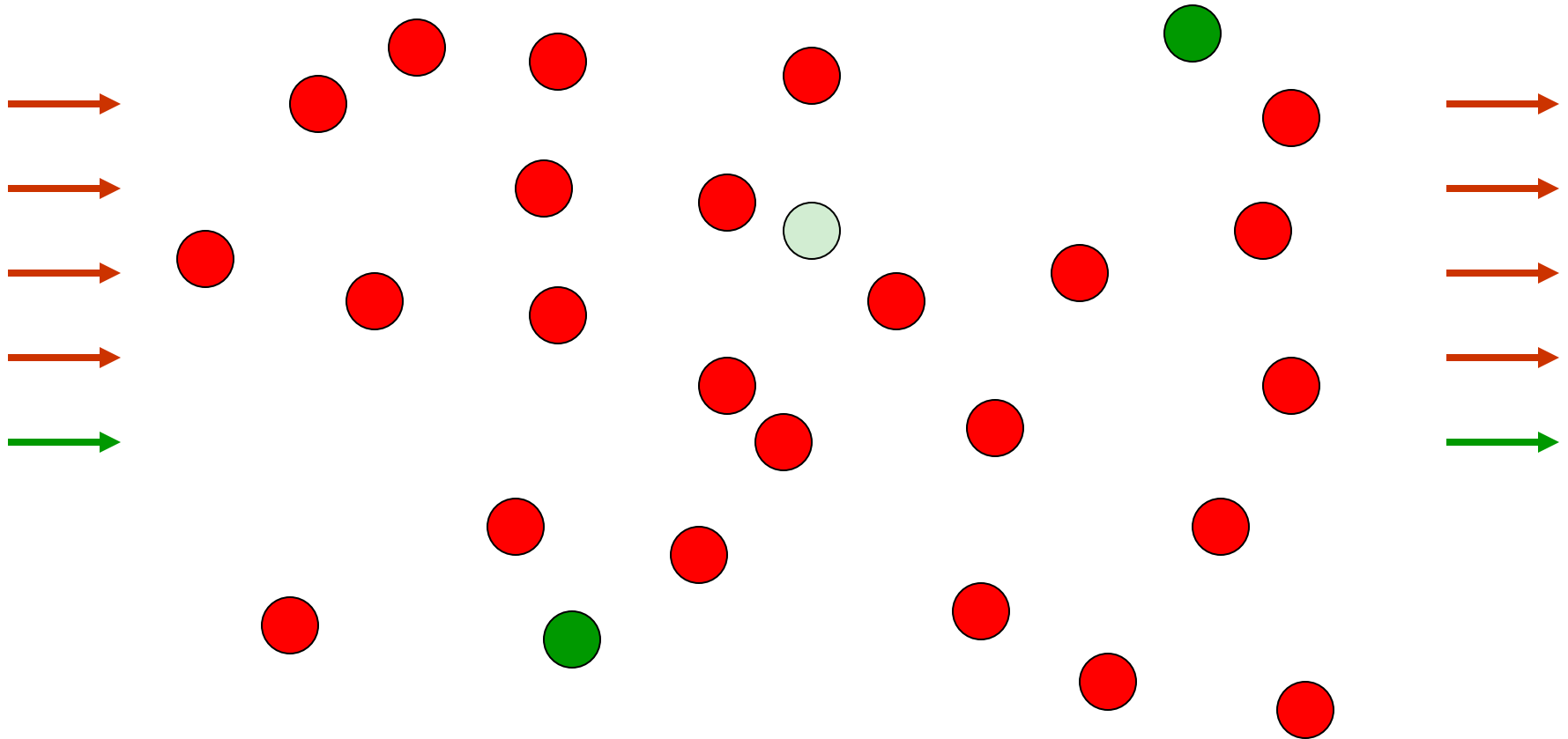
d_i = # different-type friends

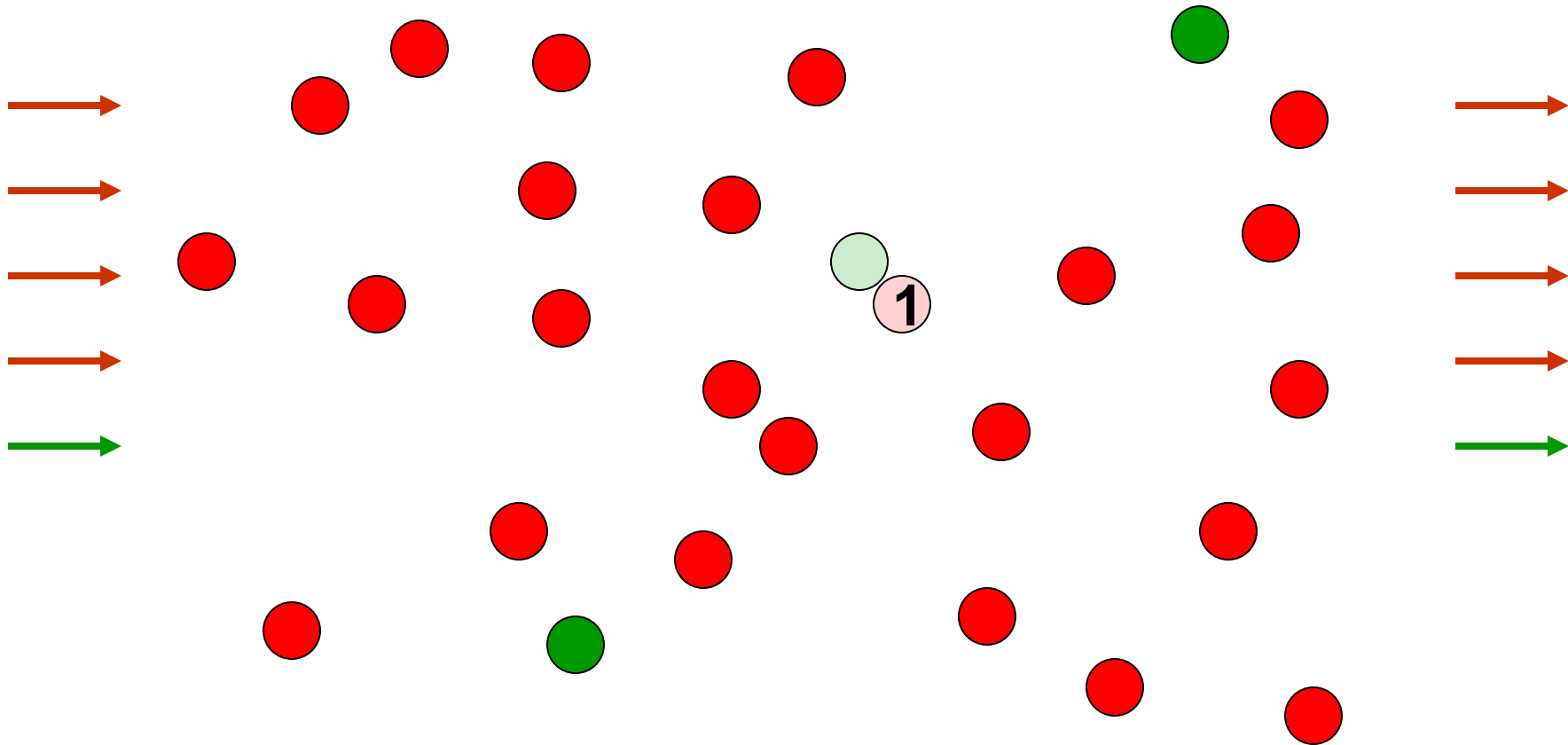
$U_i = (s_i + \gamma_i d_i)^\alpha$ utility to type i

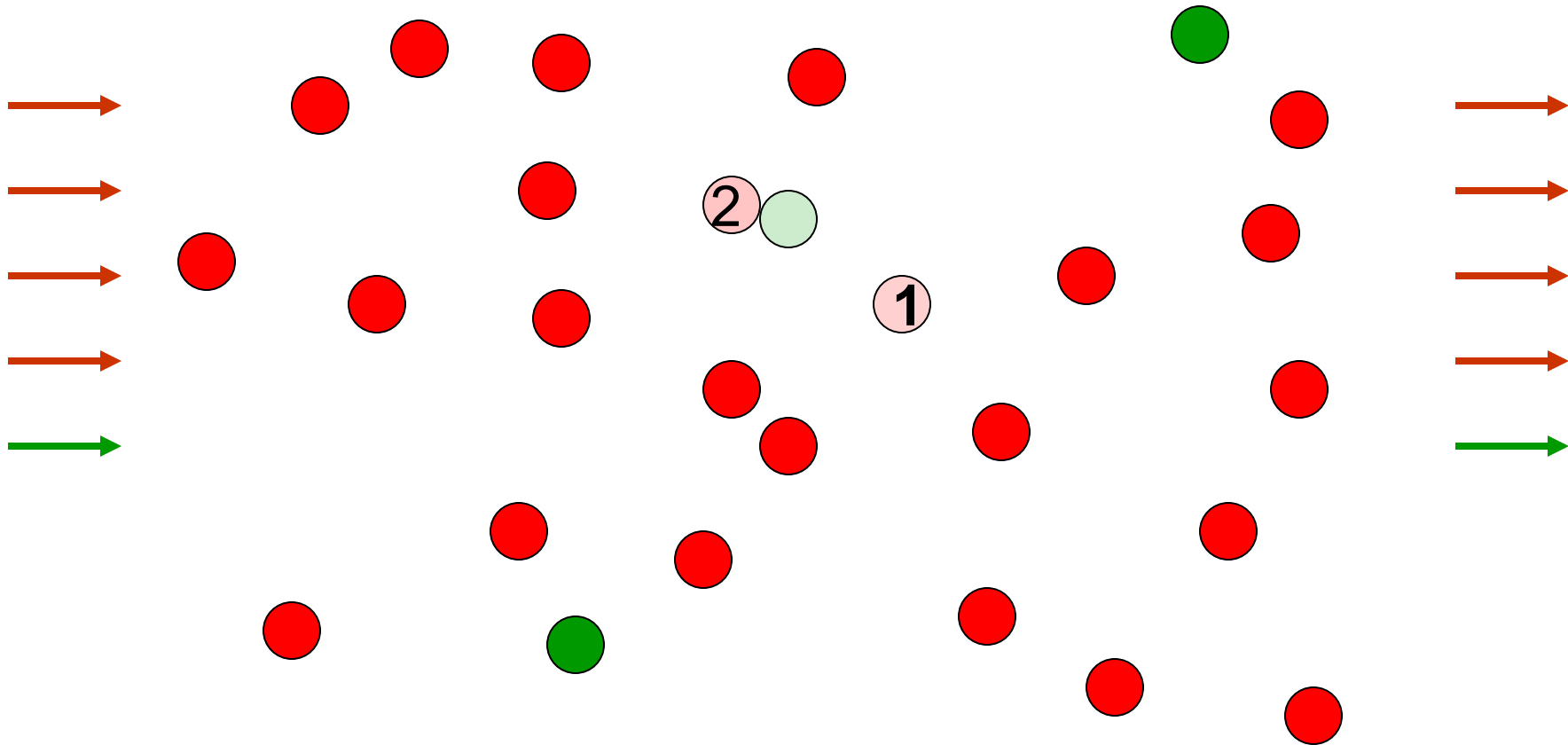
γ_i is the preference bias

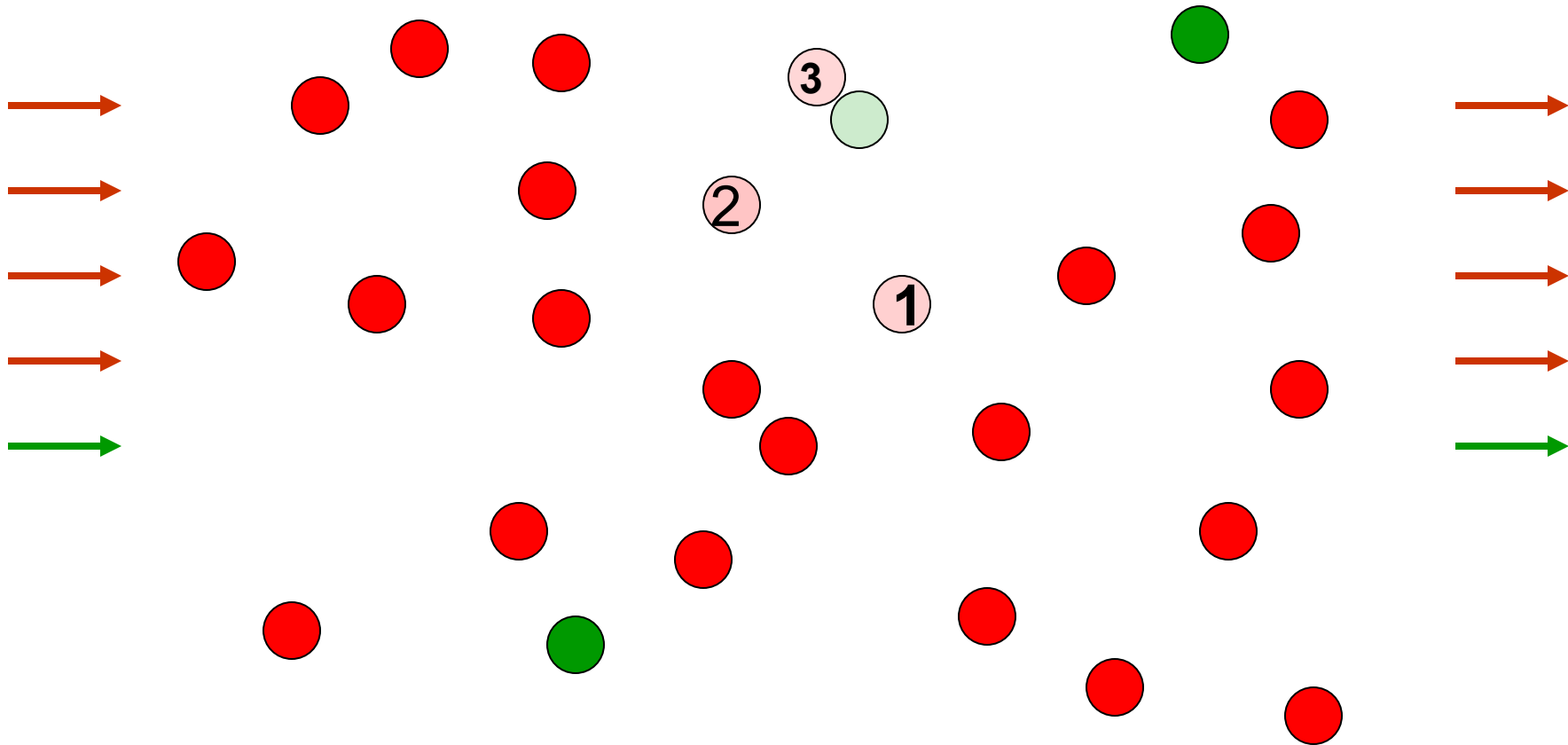
α captures diminishing returns

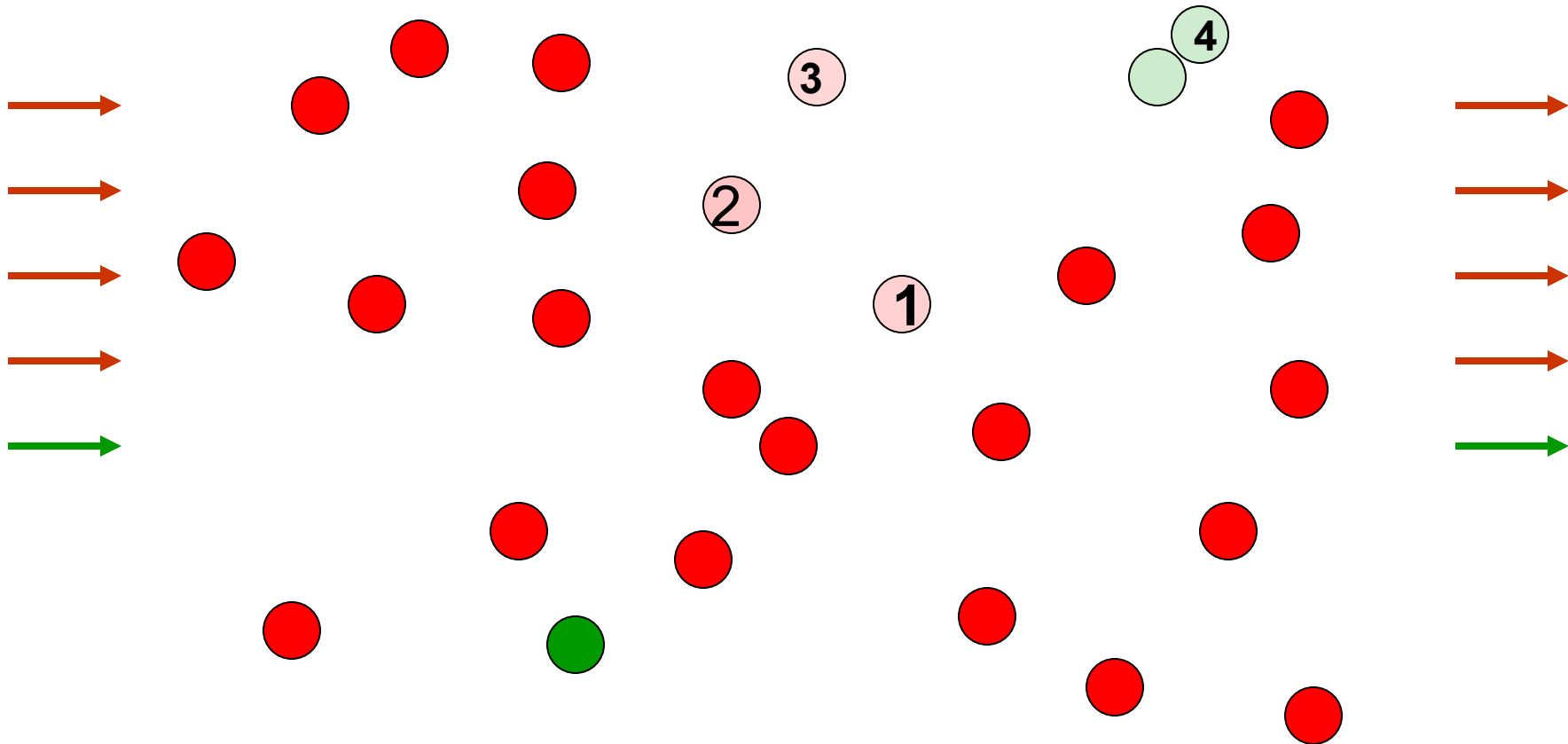
Meeting Process: “Party”

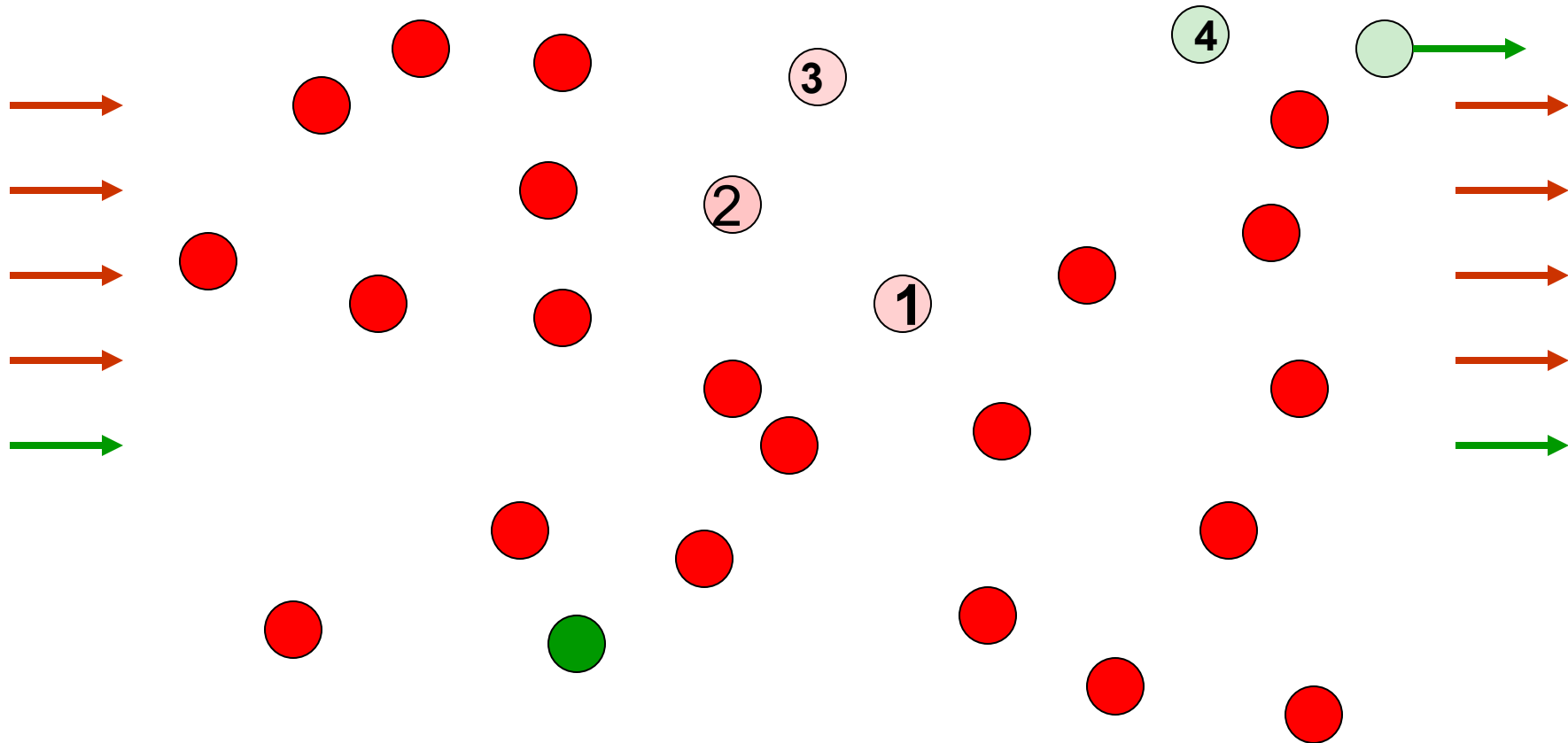




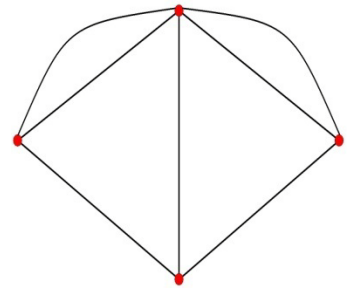








Meeting Process



q_i rate at which type i meets type i ,
 $1-q_i$ rate at which type i meets other types

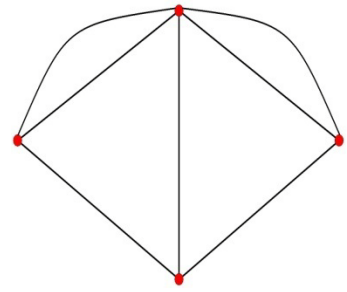
$$q_i = (\text{stock}_i)^{1/\beta_i}$$

$$\sum q_i \beta_i = 1$$

$\beta_i = 1$ “unbiased”: $q_i = \text{stock}_i$

$\beta_i > 1$ meet own types faster than stocks

Meeting Process



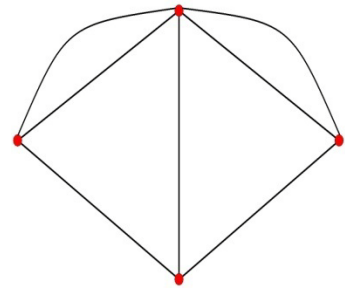
$$q_i = (\text{stock}_i)^{1/\beta_i}$$

$$\beta_i = 1 \quad \text{if } \text{stock}_i = 1/2 \text{ then } q_i = (1/2)^{1/1} = 1/2$$

$$\beta_i = 2 \quad \text{if } \text{stock}_i = 1/2 \text{ then } q_i = (1/2)^{1/2} = .707$$

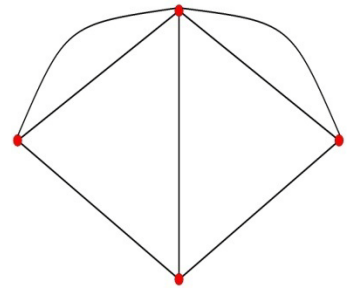
$$\beta_i = 7 \quad \text{if } \text{stock}_i = 1/2 \text{ then } q_i = (1/2)^{1/7} = .906$$

Equilibrium Conditions:



- t_i maximizes $(q_i t_i + \gamma_i (1-q_i)t_i)^\alpha - ct_i$
- $\text{stock}_i = w_i t_i / \sum w_j t_j$ fraction of type i in the matching
- $q = q_i = (\text{stock}_i)^{1/\beta_i}$ meetings determined by stocks;
- $q_{ij} \text{stock}_i = q_{ji} \text{stock}_j$ (balanced meetings)
- atomless population

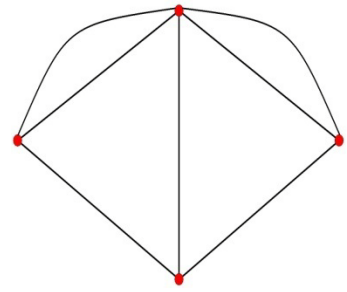
Fitting: Equilibrium Conditions



$$\max_{t_i} (q_i t_i + \gamma_i (1 - q_i) t_i)^\alpha - c t_i$$

$$\sum_i q_i^{\beta_i} = 1$$

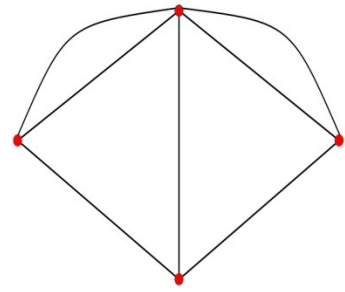
Equations Characterizing Equilibrium:



$$t_i (\gamma_j + (1 - \gamma_j)q_j)^{\frac{\alpha}{1-\alpha}} = t_j (\gamma_i + (1 - \gamma_i)q_i)^{\frac{\alpha}{1-\alpha}}$$

$$\sum_i q_i^{\beta_i} = 1$$

Fitting Technique:



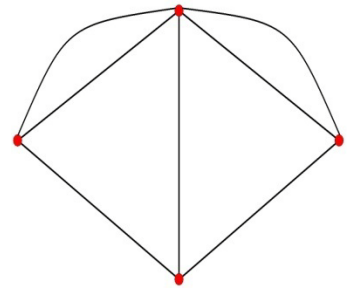
Search on grid of biases in preferences and meetings

For each network (school) and specification of biases, calculate an error in terms of total deviation from fitting equations

Sum squared errors across networks (schools)

Choose biases to minimize (weighted) sum of squared errors

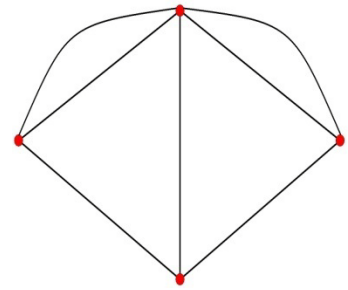
Fitted Values



ALPHA = .55

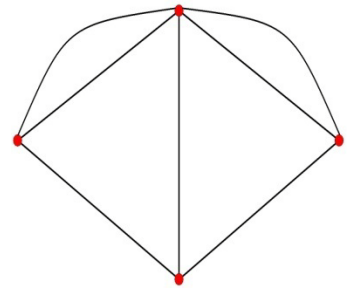
	A	B	H	W	O
GAMMA =	0.9	0.55	0.65	0.75	0.9
BETA =	7	7.5	2.5	1	1

Summary



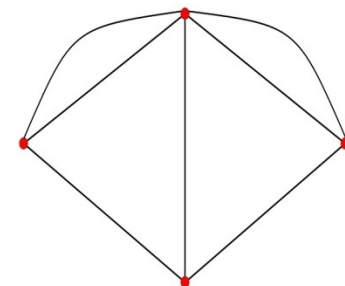
- **Highly significant biases in both preferences and meetings**
- **Highly significant differences across races:**
 - Preference bias ranges from 0.55 to 0.90; sig diffs
 - Meeting bias ranges from 1 to 7.5, sig diffs
 - Blacks, Asians: high meeting bias
 - Whites: no meeting bias, Hispanics: int meeting bias
 - Blacks, Hispanics more preference bias, Asians least
- **School size affects biases dramatically, but not preferences(?)**

Conclusions



- **Model allows identification:**
 - numbers of friends- identifies preference bias
 - profile of mix of friends- identifies meeting bias
- **Significant differences across Races**
 - What drives racial differences?
 - Still see effect when incorporate school size,
- **Why do large schools have larger preference biases?**
- **Other correlates, attributes, wealth...?**

Frontiers and Future



- Bridging random/economic models of formation
- Furthering existing random/economic models
- Relate Networks to outcomes –
 - Applications: labor, knowledge, mobility, voting, trade, collaboration, crime, www, ...
 - general game structures
 - markets...
- Co-evolution networks and behavior
- Empirical/Experimental
 - many case studies lack economic variables that would tie networks to outcomes
 - enrich modeling of social interactions from a structural perspective - fit network models to data, test network models
- Foundations and Tools– centrality, power, transfers, community structures and homophily, ...