Cadet-Branching at U.S. Army Programs

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Based on:

Matching with (Branch-of-Choice) Contracts at United States Military Academy

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and

Bidding for Army Career Specialties: Improving the ROTC Branching Mechanism

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A Fruitful Decade for Matching Markets

- In the last decade there has been a lot of activity and excitement among economists working on matching markets.
- Theory, pioneered by Gale and Shapley (1962), matured to a point where matching theorists could make policy suggestions in key areas including education and health care.

  Highlights:

  ✓ Reforms of student assignment mechanisms in major school districts such as Boston and New York City.

  ✓ Establishment of regional and national kidney exchange programs in the U.S. and U.K.
- In his June 2011 Congress testimony, Dr. Myron Gutmann (Assistant Director, SBE, NSF) emphasized that research on matching markets has resulted in measurable gains for the U.S. taxpayer.

A Fruitful Decade for Matching Markets

- Recipe for success: Discovery of important practical applications backed by solid theory.
- Contributions of the Current Project:
  - ✓ Introduction and analysis of a brand-new matching problem: Cadet-branch matching at U.S. Army Programs.
  - ✓ More generally, development of model where part of the allocation is done based on priorities, and the rest is handled by the markets.
  - ✓ Improved mechanisms for USMA and ROTC.
  - ✓ A new perspective to a recent debate on the scope of Hatfield and Milgrom (2005) Matching with Contracts model.

Army's Difficulty of Junior Officer Retention

- There are two main programs the U.S. Army relies on to recruit officers:
  - United States Military Academy (USMA)
  - Reserve Officer Training Corps (ROTC)
- Graduates of USMA and ROTC enter active duty for an initial period of obligatory service upon completing their programs.
- The Active Duty Service Obligation (ADSO) is
  - 5 years for USMA graduates,
  - 4 years for ROTC scholarship graduates, and
  - 3 years for ROTC non-scholarship graduates.

Army's Difficulty of Junior Officer Retention

- Upon completion of this obligation, an officer may apply for voluntary separation or continue on active duty.
- The low retention rate of these junior officers has been a major issue for the U.S. Army since the late 1980s.
- In the last few years, the Army has responded to this challenge with unprecedented retention incentives, including branch-for-service incentives programs offered by both USMA and ROTC (Wardynski, Lyle, and Colarusso 2010).

Army Branches

- During the fall semester of their senior year, USMA and ROTC cadets "compete" for a slot from the following 16 branches:

  - Adjutant General's Corps
  - Air Defense Artillery
  - Armor
  - Aviation
  - Chemical Corps
  - Corps of Engineers
  - Field Artillery
  - Finance Corps
  - Infantry
  - Medical Service Corps
  - Military Intelligence
  - Military Police Corps
  - Ordnance Corps
  - Quartermaster Corps
  - Signal Corps
  - Transportation Corps

- Important Decision! Career advancement possibilities vary widely across different branches.

Cadet-Branching Prior to 2006

- There has been a long tradition of assigning branches to cadets based on their preferences and their merit ranking.
- This merit ranking is known as the order-of-merit list (OML) in the military and is based on a weighted average of academic performance, physical fitness test scores, and military performance.

Cadet-Branching Reform in 2006

- In 2006, both programs changed their mechanisms that year in response to historically low retention rates of their graduates.
- The idea behind this change was simple: Since branch choice is essential for most cadets, why not allow them to bid an additional period of obligatory sevice for their desired branches?
- The fraction of slots up for bidding is
  - 25 % for USMA, and
  - 50 % for ROTC.

Cadet-Branch Matching Problem

A **cadet-branch matching problem** consists of

1. a finite set of cadets $I = \{i_1, i_2, \ldots, i_n\}$,
2. a finite set of branches $B = \{b_1, b_2, \ldots, b_m\}$,
3. a vector of branch capacities $q = (q_b)_{b \in B}$,
4. a set of "terms" or "prices" $T = \{t_1, \ldots, t_k\} \in \mathbb{R}^k_+$ where t_1 is the cheapest, ..., and t_k is the most expensive term,
5. a list of cadet preferences $P = (P_i)_{i \in I}$ over $(B \times T) \cup \{\emptyset\}$, and
6. a list of base priority rankings $\pi = (\pi_b)_{b \in B}$.

- $\pi_b : I \to \{1, \ldots, n\}$: The function that represents the base priority ranking of cadets for branch b
- $\pi_b(i) < \pi_b(j)$ means that cadet i has higher claims to a slot at branch b than cadet j, other things being equal.

Cadet Preferences

Cadet Preferences over branch-price pairs are:

- Strict, and
- monotonically decreasing in length of service:

$$\forall i \in I, \forall b \in B, \forall t, t' \in T, \qquad t < t' \Leftrightarrow (b,t)P_i(b,t').$$

- Moreover cadet preferences over branches are independent of the price and thus each cadet has well-defined preferences over branches.

$\succ_i$: Cadet preferences over branches alone
$\mathcal{P}$: The set of all preferences over $(B \times T) \cup \{\emptyset\}$
$\mathcal{Q}$: The set of all preferences over B

Outcome of the Problem

- A **contract** $x = (i, b, t) \in I \times B \times T$ specifies a cadet i, a branch b, and the terms of their match.

  $X \equiv I \times B \times T$: The set of all contracts

- An **allocation** $X' \subset X$ is a set of contracts such that each cadet appears in at most one contract and no branch appears in more contracts than its capacity.

$\mathcal{X}$:	The set of all allocations
$X'(i) = (b, t)$:	The **assignment** of cadet i under allocation X'
$X'(i) = \emptyset$:	Cadet i remains unmatched under X'

Fairness

- For a given problem, an allocation X' is **fair** if

$$\forall i, j \in I, \qquad \underbrace{X'(j)}_{=(b,t)} \; P_i \; X'(i) \Rightarrow \pi_b(j) < \pi_b(i).$$

  That is, a higher-priority cadet can never envy the *assignment* of a lower-priority cadet under a fair allocation.

- Remark: It is still possible for a higher-priority cadet to envy the branch assigned to a lower-priority cadet under a fair allocation:

  A lower-priority cadet may be able to get a more preferred branch, because he is willing to pay a higher price for it.

Mechanisms

- A mechanism is a strategy space S_i for each cadet i along with an outcome function $\varphi : \prod_{i \in I} S_i \to \mathcal{X}$ that selects an allocation for each strategy vector $(s_1, s_2, \ldots, s_n) \in \prod_{i \in I} S_i$.
- A direct mechanism is a mechanism where the strategy space is simply the set of preferences $\mathcal{P}$ for each cadet i.

Desiderata for Mechanisms

- A direct mechanism is fair if it always selects a fair allocation.
- A direct mechanism φ is strategy-proof if

$$\forall i \in I,\ \forall P_{-i} \in \mathcal{P}^{n-1},\ \forall P_i, P_i' \in \mathcal{P} \qquad \varphi(P_i, P_{-i})\ P_i\ \varphi(P_i', P_{-i}).$$

That is,

✓ no matter which cadet i we consider,
✓ no matter what his true preferences P_i are,
✓ no matter which preferences P_{-i} the rest of the cadets report (true or not),
✓ and no matter which potential "misrepresentation" P_i' cadet i considers,

truthful preference revelation is in his best interests.

Desiderata for Mechanisms

- Given two lists of base priority rankings π^1, π^2, we will say that π^1 is an unambiguous improvement for cadet i over π^2 if
  1. the standing of cadet i is at least as good under π^1_b as π^2_b for any branch b,
  2. the standing of cadet i strictly better under π^1_b than π^2_b for some branch b, and
  3. the relative priority between all other cadets remain the same between π^1_b and π^2_b for any branch b.
- A direct mechanism respects improvements if a cadet never receives a strictly worse assignment as a result of an unambiguous improvement.

- Remark: The failure of this property hurts the mechanism not only from a normative perspective, but also via the adverse incentives it creates in case cadet effort plays any role in calculation of the base priorities.

The USMA Mechanism

- All cadets receive an assignment under the USMA mechanism.

  $\mathcal{P}$: Set of preferences over $B \times T$

- Since 2006, $T = \{t_1, t_2\}$.

  t_1: Base price

  t_2: Increased price

- We refer any contract with increased price t_2 as a branch-of-choice contract.

Strategy Space under the USMA Mechanism

- Each cadet is asked to choose
  1. a ranking of branches alone, and
  2. a number of branches (possibly none) for which the cadet is asked to sign a branch-of-choice contract.

  Hence $S_i = \mathcal{Q} \times 2^B$ for each cadet i.

- Let $(\succ'_i, B_i)$ be the strategy choice of cadet i under the USMA mechanism for a given problem.

  Interpretation of B_i:
  - For each branch $b \in B_i$, cadet i is willing to pay the increased price t_2 in exchange for favorable treatment for the last 25 percent of slots.
  - Cadet i will need to pay the increased price only if he receives one of the last 25 percent of the slots for which he is favored.

Strategy Space under the USMA Mechanism

- For each branch b,
  - while the priority for the top 75 percent of slots is determined by the order-of-merit list $\pi_b = \pi^{OML}$,
  - cadets who sign a branch-of-choice contract for branch b receive favorable treatment for the last 25 percent of slots.
- That is, priority for the last 25 percent of slots is based on the following adjusted priority ranking π_b^+:

  For any $i, j \in I$,
  - if $b \in B_i$ and $b \notin B_j$, then $\pi_b^+(i) < \pi_b^+(j)$,
  - if $b \in B_i$ and $b \in B_j$, then $\pi_b^+(i) < \pi_b^+(j) \;\Leftrightarrow\; \pi_b(i) < \pi_(j)$,
  - if $b \notin B_i$ and $b \notin B_j$, then $\pi_b^+(i) < \pi_b^+(j) \;\Leftrightarrow\; \pi_b(i) < \pi_b(j)$.

The Outcome Function under the USMA Mechanism

For a given strategy profile $(\succ'_i, B_i)_{i \in I}$, the USMA mechanism determines the final outcome with the following USMA algorithm:

Step 1: Each cadet i "applies" to his top-choice under $\succ'_i$.

* Each branch b *holds* the top $0.75q_b$ candidates based on π_b.
* Among the *remaining applicants* it *holds* the top $0.25q_b$ candidates based on the adjusted priorities π^+_b.

Any remaining applicants are rejected.

In general, at

Step k: Each cadet i who is rejected at Step (k-1) "applies" to his next-choice under $\succ'_i$.

* Each branch b reviews the new applicants along with those held from Step (k-1), and *holds* the top $0.75q_b$ based on π_b.
* For the remaining slots, branch b considers all remaining applicants and *holds* the top $0.25q_b$ of them based on the adjusted priorities π^+_b.

Any remaining applicants are rejected.

The Outcome Function under the USMA Mechanism

The algorithm terminates when no applicant is rejected. All tentative assignments are finalized at that point.

For any branch b,

- any cadet who is assigned one of the top 75 percent of slots is charged the base price t_1,
- any cadet who is assigned one of the last 25 percent of slots is charged
  - the increased price t_2 if he has signed a branch-of-choice contract for branch b, and
  - the base price t_1 if he has not signed a branch-of-choice contract for branch b.

$\psi^{WP}(s)$: The outcome of USMA mechanism under $s = (\succ'_i, B_i)_{i \in I}$

Preliminary Observations on the USMA Mechanism

- When $\lambda = 0$:
  - The USMA mechanism reduces to the simple serial dictatorship induced by the order-of-merit list.
  - The USMA algorithm can be interpreted as a special case of the celebrated agent-proposing deferred acceptance algorithm (Gale and Shapley 1962), which allows for a different priority ranking at each branch.
  - Both of these mechanisms are very well-behaved: Not only do they always result in a fair allocation, but truthful preference revelation is a dominant strategy for all cadets under either mechanism.

Preliminary Observations on the USMA Mechanism

- When $\lambda > 0$:
  - The analysis of the USMA mechanism is somewhat more delicate.
  - That is because not only may truthful preference revelation be suboptimal under the USMA mechanism, but also the optimal choice of branch-of-choice contracts is a challenging task.
  - Crucial shortcoming: The mechanism tries to infer cadet preferences over branch-price pairs from their submitted preferences over branches alone and signed branch-of-choice contracts.

    The strategy-space provided by the USMA mechanism is not nearly rich enough to reasonably represent cadet preferences.

Preliminary Observations on the USMA Mechanism

Proposition: Truth-telling may not be an optimal strategy under the USMA mechanism. Furthermore, a Nash equilibrium outcome of the USMA mechanism can be unfair, Pareto inferior to a fair allocation, and may penalize cadets for unambiguous improvements.

Remark: We will later show that, all these shortcomings can be overcome with a slight modification, upon correcting the above mentioned crucial shortcoming.

This will require relating cadet-branch matching problem to matching with contracts model introduced by Hatfield and Milgrom (2005) and further developed by Hatfield and Kojima (2010).

The ROTC Mechanism

- About 10-20 % of the slots are reserved and only the remaining slots are assigned by the ROTC mechanism. Hence being unassigned is a serious possibility under the ROTC mechanism.

  $\mathcal{P}$: Set of preferences over $(B \times T) \cup \{\emptyset\}$

- The assignments of unmatched cadets are manually determined by the Department of the Army Branching Board.

- As in the case of the USMA, $T = \{t_1, t_2\}$.
- Similarly, as in the case of the USMA, each cadet is asked to choose
  1. a ranking of branches alone, and
  2. a number of branches (possibly none) for which the cadet is asked to sign a branch-of-choice contract.

  Hence $S_i = \mathcal{Q} \times 2^B$ for each cadet i.

The Outcome Function of the ROTC Mechanism

- While the strategy space is the same as USMA mechanism, the outcome function is very different.
- For a given order-of-merit list π^{OML} and a strategy-profile $(\succ_i', B_i)_{i \in I}$, the outcome of the ROTC mechanism is obtained as follows:
- Consider each cadet one at a time, following the order-of-merit list. The treatment of cadets at the top 50 percent of the OML is different than those at the bottom 50 percent.

The Outcome Function of the ROTC Mechanism

- For each cadet at the top 50 percent of the OML, consider the following six options in the given order, and if none of them works, leave the cadet unassigned.
  1. Assign the cadet his first-choice branch at base price t_1, if less than 50 percent of the slots are full.
  2. Assign the cadet his first-choice branch at increased price t_2, if he signed a branch-of-choice contract and less than 65 percent of the slots are full.
  3. Assign the cadet his second-choice branch at base price t_1, if less than 50 percent of the slots are full.
  4. Assign the cadet his second-choice branch at increased price t_2, if he signed a branch-of-choice contract and less than 65 percent of the slots are full.
  5. Assign the cadet his third-choice branch at base price t_1, if less than 50 percent of the slots are full.
  6. Assign the cadet his third-choice branch at increased price t_2, if he signed a branch-of-choice contract and less than 65 percent of the slots are full.

The Outcome Function of the ROTC Mechanism

- For each cadet at the bottom 50 percent of the OML, consider the following six options in the given order, and if none of them works, leave the cadet unassigned.
  1. Assign the cadet his first-choice branch at base price t_1, if less than 50 percent of the slots are full.
  2. Assign the cadet his first-choice branch at increased price t_2, if he signed a branch-of-choice contract and not all slots are full.
  3. Assign the cadet his second-choice branch at base price t_1, if less than 50 percent of the slots are full.
  4. Assign the cadet his second-choice branch increased higher price t_2, if he signed a branch-of-choice contract and not all slots are full.
  5. Assign the cadet his third-choice branch at base price t_1, if less than 50 percent of the slots are full.
  6. Assign the cadet his third-choice branch at increased price t_2, if he signed a branch-of-choice contract and not all slots are full.

Observations on the ROTC Mechanism

- ROTC mechanism only uses the top three choices. Hence truth-telling can clearly be sub-optimal. However "truncation" is not the only reason for the lack of incentive compatibility. Another reason is, the expensive option is always considered right after the cheap option for each branch (as in the case of the USMA mechanism).
- For each branch b, ROTC branch priorities are given as follows:
  - ✓ For the top 50 percent of the slots, the priority is based on cadet OML.
  - ✓ The next 15 percent of the slots are reserved for cadets who have signed a branch-of-choice contract for branch b, and among them priority is based on cadet OML.
  - ?? The last 35 percent of the slots are reserved for cadets who are at the bottom 50 percent of the OML who have signed a branch-of-choice contract for branch b. Among them priority is based on cadet OML.

Observations on the ROTC Mechanism

- There is an affirmative action constraint for the last 35 percent of the slots at each branch, and cadets at the upper half of the OML ranking are denied access to these slots whether they are willing to pay the increased price or not.

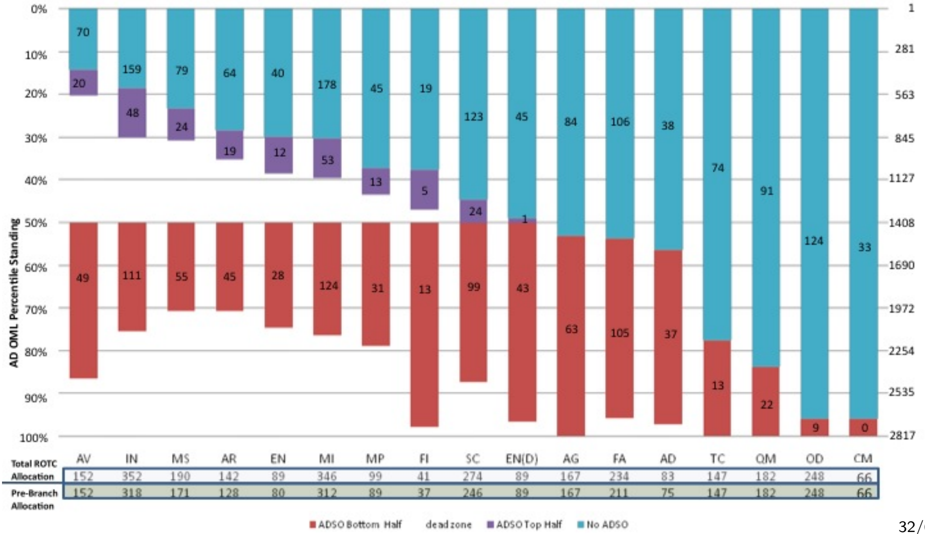
  Proposition: Truth-telling may not be an optimal strategy under the ROTC mechanism. Furthermore, a Nash equilibrium outcome of the ROTC mechanism can be unfair, Pareto inferior to a fair allocation, and may penalize cadets for unambiguous improvements.

- At first sight the shortcomings of the ROTC mechanism and the USMA mechanism appear to be very similar. However, while the USMA mechanism can be fixed with a minor modification, a substantial “fix” is necessary for the ROTC mechanism.

The Main Difficulty: Dead Zones

- For a given branch, the range of the OML where higher-ranking cadets lose priority to cadets in the lower-half of the OML is referred as the dead zone by the Army.
- In 2011, eight of the most popular branches had a dead zone. These branches and their dead zones are:
  1. Aviation with cadets between 20-50 percent of the OML,
  2. Infantry with cadets between 30-50 percent of the OML,
  3. Medical Service with cadets between 31-50 percent of the OML,
  4. Armor with cadets between 35-50 percent of the OML,
  5. Engineering with cadets between 38-50 percent of the OML,
  6. Military intelligence with cadets between 40-50 percent of the OML,
  7. Military police with cadets between 43-50 percent of the OML, and
  8. Finance with cadets between 47-50 percent of the OML.

2011 ROTC Cadet-Branch Matching Results



	AV	IN	MS	AR	EN	MI	MP	FI	SC	EN(D)	AG	FA	AD	TC	QM	OD	CM
Total ROTC Allocation	152	352	190	142	89	346	99	41	274	89	167	234	83	147	182	248	66
Pre-Branch Allocation	152	318	171	128	80	312	89	37	246	89	167	211	75	147	182	248	66

Are Cadets Worried About Dead Zones?

Google

rotc dead zone

Advanced search

Search

About 121,000 results (0.06 seconds)

Everything
Images
Maps
Videos
News
Shopping
Books
More

Newton, MA
Change location

All results
Related searches
More search tools

[PDF] Bidding for Army Career Specialties: Improving the **ROTC** Branch...
economics.uchicago.edu/workshops/Sonmez1_10-4-11.pdf
File Format: PDF/Adobe Acrobat - View as HTML
Sep 8, 2011 – the role of the order-of-merit system in cadet branching. We believe that is not the case. The creation of **dead zones** by **ROTC** priorities already **...**

[PPT] Accessions Process
www.purdue.edu/army**rotc**/_.../accessions_briefing_11_Apr_2011.p...
File Format: Microsoft Powerpoint - Quick View
Apr 10, 2011 – "**Dead Zone**" – the area on the branch bar graph where it is impossible for a cadet to receive a certain branch. 4. FY 11 **ROTC** OML Model **...**

The **dead zone** - Google Books Result
books.google.com/books?isbn=0451155750...
Stephen King, Stephen King - 1980 - Fiction - 402 pages
Vietnam and the fight over **ROTC** and Kent State have activated more students than ever before. I doubt if there's ever been a time when there were so few **...**

[PDF] **ROTC** Branch of Choice
www.career-satisfaction.army.mil/.../**ROTC**_BRADSO_Briefing_slide...
File Format: PDF/Adobe Acrobat - Quick View
Branch Process Slides Provided by Cadet Command G1. **ROTC**. Branch of Choice **...** **ROTC** Still Branches According to the Cadet OML. (See OMS **...** **dead zone** **...**

What percentage of the **ROTC** OML will I be around with 80.109 for **...**
answers.yahoo.com › ... › Politics & Government › Military
Sep 17, 2011 – **ROTC** accessions OMS points How many points did yo... **...** so you may find yourself in the **Dead Zone** when it comes to Branch choice. **...**

Are Cadets Worried About Dead Zones?

Matching with Contracts

- Fortunately it is possible fix the deficiencies of the USMA mechanism, and even the ROTC mechanism. This requires relating cadet-branch matching to a recent model which has received a lot of attention.
- The cadet-branch matching problem can be modeled as a special case of the matching with contracts model (Hatfield and Milgrom 2005) that subsumes and unifies the Gale and Shapley (1962) college admissions model and the Kelso and Crawford (1982) labor market model, among others.
- In the original Hatfield-Milgrom model, each branch (hospitals in their framework) has preferences over sets of agent-term pairs. These hospital preferences induce a choice set from each set of contracts, and it is this choice set (rather than hospital preferences) that relevant for our model.

Representation of Priorities via Choice Sets

- In the present framework, branches are not agents and they do not have preferences. However, branches have priorities over cadet-price pairs, and these priorities also induce choice sets.
- In general, the choice set of branch b from a set of contacts X' depends on the policy on who has higher claims for slots in branch b. We can represent the current USMA priorities, ROTC priorities, or any other priorities by adequate construction of choice sets.
- For a given priority structure for branch b,

$C_b(X')$:	The set of contracts chosen from $X' \subseteq X$
$R_b(X') \equiv X' \setminus C_b(X')$:	The rejected set

USMA Choice Set

Phase 0: Remove all contracts that involve another branch b' and add them all to rejected set $R_b(X')$. Hence each contract that survives Phase 0 involves branch b.

Phase 1: For the first $0.75q_b$ potential elements of $C_b(X')$, choose the contracts with highest-OML cadets one at a time. When two contracts of the same cadet are available, choose the contract with the base price t_1 and reject the other one. Continue until either all contracts are considered or $0.75q_b$ elements are chosen for $C_b(X')$. If the former happens, terminate the procedure and if the latter happens proceed with Phase 2.1.

USMA Choice Set

Phase 2.1: For the last $0.25q_b$ potential elements of $C_b(X')$, give priority to contracts with increased price t_2. Hence in this phase only consider branch-of-choice contracts and among them include in $C_b(X')$ the contracts with highest-OML cadets. If any cadet covered in Phase 2.1 has two contracts in X' reject the contract with the base price t_1. Continue until either all branch-of-choice contracts are considered in X' or $C_b(X')$ fills all q_b elements. For the latter case, reject all remaining contracts, and terminate the procedure. For the former case, terminate the procedure if all contracts in X' are considered and proceed with the Phase 2.2 otherwise.

Phase 2.2: By construction, all remaining contracts in X' have the base price t_1. Include in $C_b(X')$ the contracts with highest-priority cadets one at a time until either all contracts in X' are considered or $C_b(X')$ fills all q_b elements. Reject any remaining contracts.

ROTC Choice Set

Phase 0: Remove all contracts that involve another branch b' and add them all to the rejected set $R_b(X')$. Hence each contract that survives Phase 0 involves branch b.

Phase 1.1: For the first $0.5q_b$ potential elements of $C_b(X')$, simply choose the contracts with highest OML-priority cadets one at a time. When two contracts of the same cadet are available, choose the contract with the base price t_1 and reject the other one. Continue until either all contracts are considered or $0.5q_b$ elements are chosen for $C_b(X')$. If the former happens, terminate the procedure, and if the latter happens, proceed with Phase 1.2.

Phase 1.2: Remove all surviving contracts with base price t_1. Proceed with Phase 2.1 if there is at least one surviving contract and terminate the procedure otherwise.

ROTC Choice Set

Phase 2.1: All remaining contracts have increased price t_2. Among them include in $C_b(X')$ the contracts with highest OML-priority cadets for the next $0.15q_b$ potential elements of $C_b(X')$. Continue until either all contracts are considered in X' or $0.65q_b$ elements are chosen for $C_b(X')$. For the former case terminate the procedure. For the latter case, terminate the procedure if all contracts in X' are considered, and proceed with Phase 2.2 otherwise.

Phase 2.2: Remove all surviving contracts that belong to cadets from the upper half of the OML list. Proceed with Phase 3 if there is at least one surviving contract and terminate the procedure otherwise.

Phase 3: All remaining contracts have increased cost t_2 and belong to cadets from the lower half of the OML list. Among them include in $C_b(X')$ the contracts with highest OML-priority cadets for the last $0.35q_b$ potential elements of $C_b(X')$. Reject all remaining contracts and terminate the procedure.

Stability

- Since the seminal paper of Gale and Shapley (1962), a condition known as *stability* has been central to the analysis of two-sided matching markets.
- In the context of cadet-branch matching, an allocation X' is stable if
  1. no cadet or branch is imposed an unacceptable contract, and
  2. there exists no cadet i, branch b, and contract $x = (i, b, t) \in X \setminus X'$ such that

$$(b, t) \; P_i \; X'(i) \quad \text{and} \quad x \in C_b\big(X' \cup \{x\}\big).$$

- In the context of cadet-branch matching, the only plausible allocations are the stable ones: If the first condition fails then the outcome is not individually rational, and if the second requirement fails then there exists an unselected contract (i, b, t) where not only cadet i prefers pair (b, t) to his assignment, but also contract x has sufficiently high priority to be selected by branch b.

The Law of Aggregate Demand

- Two properties of choice sets, or equivalently branch priorities in our context, have played an important role in the analysis of matching with contracts.
- Priorities satisfy the law of aggregate demand for branch b if

$$X' \subset X'' \Rightarrow |C_b(X')| \leq |C_b(X'')|$$

That is, the size of the chosen set never shrinks as the set of contracts grows under the law of aggregate demand.

Lemma: The USMA priorities and the ROTC priorities both satisfy the law of aggregate demand.

The Substitutes Condition

- Of the two conditions, the second one plays an especially important role in two-sided matching literature.
- Elements of X are substitutes for branch b if for all $X' \subset X'' \subseteq X$ we have $R_b(X') \subseteq R_b(X'')$.

  That is, contracts are substitutes if any contract that is rejected from a set X' is also rejected from any set X'' that contains X'.

  Equivalently, a contract x that is chosen from a set X'' shall also be chosen from any of its subsets $X' \subset X''$, provided that $x \in X'$.
- If elements of X are substitutes, then the set of stable allocations is non-empty (Hatfield and Milgrom 2005).

The Substitutes Condition

- The substitutes condition has been very “handy” in analysis of matching with contracts: Fixed-point techniques in lattice theory has strong implications under the substitutes condition.

  Hatfield and Milgrom (2005) builds their model around this condition and it is assumed in much of the subsequent literature as well.
- Indeed, until Hatfield and Kojima (2008) showed otherwise, the substitutes condition was thought to be a necessary condition for the guaranteed existence of a stable allocation.
- A recent paper by Echenique (2011) questions the value added of the matching with contracts model.

An Unexpected Isomorphism

Theorem (Echenique 2011): The matching with contracts model can be embedded within the Kelso and Crawford (1982) labor market model under the substitutes condition.

- Kominers (2011) extends this isomorphism to a many-to-many matching provided that the two sides of the market can sign at most one contract.
- The substitutes condition is key for both results to hold. Indeed Echenique (2011) emphasizes that a recent theory paper by Hatfield and Kojima (2010) analyzes matching with contracts under weaker conditions, and his embedding does not work under their conditions.

The Unilateral Substitutes Condition

- One of the conditions offered in Hatfield and Kojima (2010) is the following:

  Elements of X are unilateral substitutes for branch b if, whenever a contract $x = (i, b, t)$ is rejected from a smaller set X' even though x is the only contract in X' that includes cadet i, contract x is also rejected from a larger set X'' that includes X'.

- While the lattice structure of the set of stable outcomes no longer persists under the unilateral substitutes condition, Hatfield and Kojima (2010) shows that a number of important results survives this weakening of the substitutes condition.

The Unilateral Substitutes Condition

- Remarkably the unilateral substitutes condition plays a key role in our context:

  Lemma: While neither the USMA priorities nor the ROTC priorities satisfy the substitutes condition, they both satisfy the unilateral substitutes condition.

- This observation begs the following question: What exactly Hatfield and Kojima (2010) have shown under the unilateral substitutes condition?

  In order to answer this question, we need to present an extension of the celebrated Gale and Shapley (1962) agent-optimal stable mechanism.

Cumulative Offer Algorithm and COSM

- We refer the agent-optimal stable mechanism as cadet-optimal stable mechanism (COSM) in the present context.
- The strategy space of each cadet is $\mathcal{P}$ under the COSM, and hence it is a direct mechanism.
- Fix branch priorities (and thus the choices sets). Given a preference profile $P \in \mathcal{P}$, the following cumulative offer algorithm (COA) (Hatfield and Milgrom 2005) can be used to find the outcome of COSM.

  Step 1: Start the offer process with the highest-merit-score cadet $\pi(1) = i(1)$. Cadet $i(1)$ offers his first-choice contract $x_1 = (i(1), b(1), t)$ to branch $b(1)$ that is involved in this contract. Branch $b(1)$ holds the contract if $x_1 \in C_{b(1)}(\{x_1\})$ and rejects it otherwise. Let $A_{b(1)}(1) = \{x_1\}$ and $A_b(1) = \emptyset$ for all $b \in B \setminus \{b(1)\}$.

Cumulative Offer Algorithm and COSM

In general, at

Step k: Let $i(k)$ be the highest-merit-score cadet for whom no contract is currently held by any branch. Cadet $i(k)$ offers his most-preferred contract $x_k = (i(k), b(k), t)$ that has not been rejected in previous steps to branch $b(k)$. Branch $b(k)$ holds the contract if $x_k \in C_{b(k)}(A_{b(k)}(k-1) \cup \{x_k\})$ and rejects it otherwise. Let $A_{b(k)}(k) = A_{b(k)}(k-1) \cup \{x_k\}$ and $A_b(k) = A_b(k-1)$ for all $b \in B \setminus \{b(k-1)\}$.

The algorithm terminates when all cadets have an offer that is on hold by a branch. Since there are a finite number of contracts, the algorithm terminates after a finite number T of steps. All contracts held at this final Step T are finalized and the final allocation is $\bigcup_{b \in B} C_b(A_T)$.

COSM and Unilateral Substitutes

We will built on the following results to fix the deficiencies of the USMA mechanism and the ROTC mechanism.

Theorem (Hatfield and Kojima 2010): Suppose the priorities satisfy the unilateral substitutes condition. Then the COA produces a stable allocation. Moreover, this allocation is weakly preferred by any cadet to any stable allocation.

Theorem (Hatfield and Kojima 2010): Suppose the priorities satisfy the unilateral substitutes condition and the law of aggregate demand. Then the induced COSM is strategy-proof.

- Remark: Echenique (2011) embedding does not work in our framework. Hence, the cadet-branch matching problem is an application of matching with contracts that is beyond the scope of Kelso and Crawford (1982) labor market model!

Improving the USMA Mechanism

φ^{USMA}: COSM induced by USMA priorities

- COSM induced by USMA priorities fixes all previously mentioned deficiencies of the USMA mechanism.

  Proposition The outcome of φ^{USMA} is stable under USMA priorities and it is weakly preferred by any cadet to any stable allocation. Moreover φ^{USMA} is strategy-proof, fair, and respects improvements.

- Indeed USMA mechanism can be interpreted as an "approximation" of the COSM. Recall that cadet preferences over branch-price pairs are never asked but rather "approximated" under the USMA mechanism.

USMA Mechanism vs. COSM

- Fix a cadet-branch problem and let $s_i = (\succ'_i, B_i)$ be the strategy choice of cadet i under the USMA mechanism. For each cadet i construct the proxy preference relation P_i^* by simply
  - ranking of the cheaper options of each branch based on $\succ'_i$, and
  - simply inserting the expensive option (b, t_2) right after the cheap option (b, t_1) for each branch b for which cadet i has signed a branch-of-choice contract.

Proposition *Let $s = (\succ_i, B_i)_{i \in I}$ be a Nash equilibrium strategy profile under the USMA mechanism and $P^* = (P_i^*)_{i \in I}$ be the resulting proxy preferences as defined above. Then*

$$\psi^{WP}(s) = \varphi^{USMA}(P^*).$$

- Hence a modest modification of West Point's design, provides major benefits to cadets and the Army.

Improving the ROTC Mechanism

φ^{ROTC}: COSM induced by ROTC priorities

- Why not just using the same trick for the ROTC?

  Proposition: The outcome of φ^{ROTC} is stable under ROTC priorities and it is weakly preferred by any cadet to any stable allocation. Moreover φ^{ROTC} is strategy-proof. However φ^{ROTC} is neither fair nor it respects improvements.

- Hence COSM under ROTC priorities only partially fixes the deficiencies of the ROTC mechanism.

  In contrast to USMA priorities, ROTC priorities are not compatible with the design of a fully satisfactory mechanism. We next formalize this point.

Fairness and Priorities

- Priorities are fair if for any branch b the induced choice function C_b is such that, for any set of contracts X' and any pair of contracts $x, y \in X'$ with $x_B = y_B = b$,

$$\left.\begin{array}{l} \pi_b(y_I) < \pi_b(x_I), \\ y_T = x_T, \text{ and} \\ x \in C_b(X') \end{array}\right\} \implies \exists z \in C_b(X') \text{ such that } z_I = y_I.$$

  That is, if a contract x of a lower-priority cadet is chosen, then a contract z of a higher-priority cadet who is willing to pay as much under a reference contract y shall also be chosen under fair priorities.

  Here the chosen contract of cadet y_I can be the reference contract y or an alternative contract z.

Fairness and Priorities

- While USMA priorities are fair, ROTC priorities are not. Cadets from the upper half of the OML are simply denied for the last 35 percent of slots at each branch. That is what creates the dead zones!

  Proposition: Suppose that the priorities satisfy the unilateral substitutes condition and the law of aggregate demand. Then the COSM is fair if and only if the priorities are fair.

- Hence it is necessary to seek an alternative priority structure in order to design a satisfactory mechanism for ROTC branching.

Bidding for Priorities

- There is only one reason for this unusual choice of ROTC priorities. The Army desires to allocate skill somewhat evenly across its branches.
- Could it be possible to reach the Army's distributional goal without creating a dead zone?

  Yes ✓
- Under our proposed mechanism cadets are able to bid more than three years. In particular, we need the highest price to be large enough, so that only the most motivated cadets will be willing to pay the highest price.
- This will decrease the role of the OML and increase the role of willingness to serve in branch priorities.

Bidding for Priorities

- Another factor that will shift the balance in favor of willingness to serve is increasing the fraction of slots up for bidding.
- The idea is that the Army's distributional goal of can be achieved if the role of willingness to serve is sufficiently increased and the role of the OML is sufficiently decreased in branch priorities.

Bidding for Priorities

For a given λ and set of terms $T = \{t_1, \ldots, t_k\}$, the choice of branch b from a set of contracts X' is obtained as follows under Bid-for-Your-Career (BfYC) priorities.

Phase 0: Remove all contracts that involve another branch b' and add them all to the rejected set $R_b(X')$.

Phase 1: For the first λ percent potential elements of $C_b(X')$, choose the contracts with highest π_b priority cadets one at a time. When multiple contracts of the same cadet are available, choose the contract with the lowest cost. Continue until either all contracts are considered or λ percent of the capacity is full. If the former happens, terminate the procedure, and if the latter happens, proceed with Phase 2.

Bidding for Priorities

Phase 2: For the last $(1 - \lambda)$ percent potential elements of $C_b(X')$, choose the contracts with highest costs while using the base priorities π_b to break ties. When multiple contracts of the same cadet are available, choose the contract with the highest cost. Continue until either all contracts are considered or the capacity is full. Reject any remaining contracts.

- Our next Lemma shows that BfYC priorities are compatible with the design of a satisfactory mechanism.

  Lemma BfYC priorities satisfy the unilateral substitutes condition, the law of aggregate demand, and they are fair.

- This lemma implies that COSM is well-defined under BfYC priorities.

An Improved Mechanism for ROTC

φ^{BfYC}: COSM induced by BfYC priorities.

- φ^{BfYC} fixes all previously mentioned shortcomings of the ROTC mechanism:

  Proposition: The outcome of φ^{BfYC} is stable under BfYC priorities and it is weakly preferred by any cadet to any stable allocation. Moreover φ^{BfYC} is strategy-proof, fair, and respects improvements.

- Indeed,

  Proposition: Given BfyC priorities, φ^{BfYC} is the only mechanism that is stable and strategy-proof.

Policy Implications

- We have shown that the potential adoption of the COSM induced by BfYC priorities benefits cadets in numerous ways. Most notably
  - ✓ the dead zone is eliminated,
  - ✓ more generally the fairness of the mechanism is restored, and
  - ✓ the vulnerability of the mechanism to gaming either through preference manipulation or through effort reduction is fully eliminated.
- We next explain why cadets are not the only beneficiaries of this potential branching reform.
- From a mechanism design perspective, the ROTC mechanism is a severely deficient mechanism. This is not only a matter of theoretical aesthetics and the elimination of these shortcomings mitigates several policy problems that the Army has identified.
- Several of these points are valid for the replacement of the USMA mechanism as well.

Better Utilization of Branch-for-Service Incentives Program

- Restricting cadet bids to only a one-time bid of three additional years reduces the potential impact of the mechanism.
- Moreover, ROTC cadets between 20-50 percent of the OML are to a large extent shut off from the branch-for-service program because of the dead zones they face.
- Favoring low-performing cadets at the expense of these cadets not only undermines the order-of-merit system, but also potentially aggravates their attrition rate.
- The adoption of φ^{BfYC} will not only allow all cadets to bid more than three years for their desired career specialties, it will also allow the Army to distribute talent across branches based on cadet willingness to serve rather than artificially created dead zones.
- Instead of favoring arbitrary low-performing cadets, our proposed mechanism favors cadets who are most eager to serve in the Army.

Branch Choice and Diversity among Senior Military Officers

- In 2006 while minorities made up 31 percent of the enlisted ranks of the military, they made up 16 percent of all officers, and only 5 percent of all Generals (Lim et al. 2009). This is cause for major concern, and significant resources have been devoted to understanding this phenomenon.
- In a recent Rand Corporation report prepared for the Office of the Secretary of Defense, Lim et al. (2009) conclude that the relative scarcity of minorities in combat arms branches of the Army is a potential barrier to improving demographic diversity in the senior officer ranks.

  In 2006, 80 percent of all Generals were from combat arm branches.
- Using 2007 Army ROTC data, Lim et al. (2009) show that while 58 percent of white cadets' submitted first choices were in combat arms, only 31 percent of African American cadets' first choices were in combat arms.

Branch Choice and Diversity among Senior Military Officers

- They also report that minorities tend to rank lower on the OML and conclude that these numbers may not truly reflect a lack of interest on the part of minorities for combat arms.
- The authors are unable to interpret ROTC preference data because they do not know to what extent minorities strategically avoided more competitive career fields (to avoid a forced assignment): The vulnerability of the ROTC mechanism to preference manipulation thus has adversely affected the authors' ability to prescribe an adequate policy recommendation in this important analysis.
- These and numerous similar studies show that the adoption of a strategy-proof mechanism is highly valuable to ROTC. Hence even if ROTC is persistent in keeping its current priority structure that relies on dead zones, adoption of COSM will eliminate the difficulties the Army faces in preference data interpretation and allow it to adopt adequate policies to combat minority underrepresentation in its senior ranks.

Flexibility to Accommodate Branch-Specific Priorities

- ROTC leadership currently distributes talent across branches by shutting off the upper-half of the OML from the last 35 percent of slots at each branch.
- This direct approach heavily relies on the use of a common base priority ranking across all branches.
- Leadership at some of the branches has been critical of this practice (eg. Military Intelligence).
- Many also believe that ROTC-OML is overly subjective.
- COSM, unlike the ROTC mechanism, is fully flexible on the choice of base priorities.

Avoiding the Risk of Cadets Intentionally Lowering OML

- Since the ROTC mechanism severely penalizes cadets from the 20th to 50th percentiles of the OML, it gives strong incentives to these cadets to reduce their efforts in their studies so that they can be ranked below the median.
- This incentive is especially strong for cadets just above the median cadet, since they can avoid losing access to essentially all career branches with a relatively small “compromise” in their OML.
- Indeed manipulating ROTC mechanism through effort reduction is rather easy: The Army provides all the necessary data that is needed in the following link:

  `http://www.career-satisfaction.army.mil/pdfs/Order_of_Merit_Score_Calculations.pdf`
- A mechanism that promotes such behavior can clearly compromise the Army's efforts in investing its future.
- COSM under BfYC priorities fully aligns cadets' interests with those of the Army.

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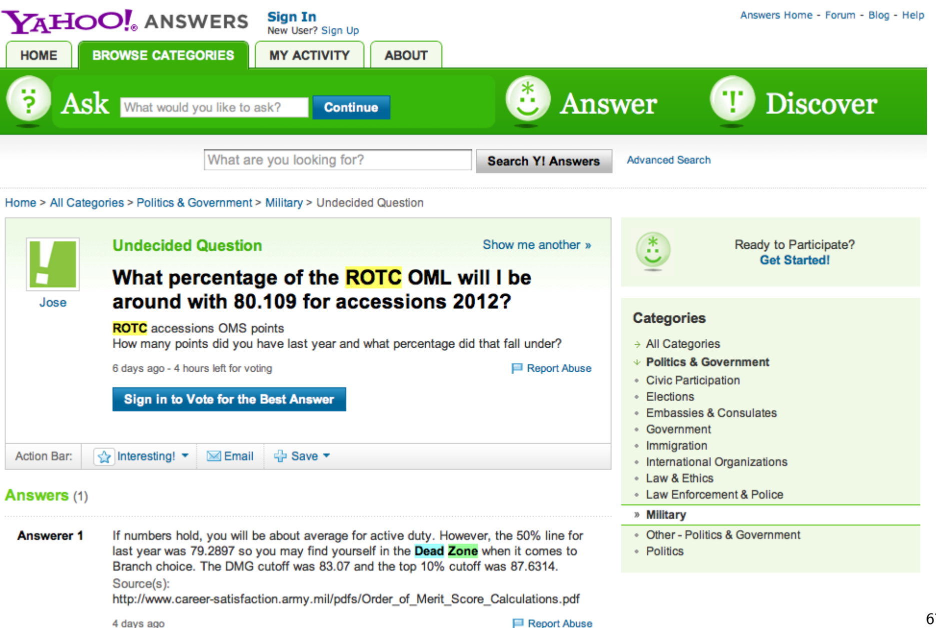

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What percentage of the ROTC OML will I be around with 80.109 for accessions 2012?

ROTC accessions OMS points
How many points did you have last year and what percentage did that fall under?

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Answers (1)

Answerer 1 If numbers hold, you will be about average for active duty. However, the 50% line for last year was 79.2897 so you may find yourself in the Dead Zone when it comes to Branch choice. The DMG cutoff was 83.07 and the top 10% cutoff was 87.6314.
Source(s):
http://www.career-satisfaction.army.mil/pdfs/Order_of_Merit_Score_Calculations.pdf

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Conclusion

- ✓ We introduced a brand-new matching problem, one with significant practical relevance.
- ✓ Our proposed mechanisms benefit cadets in a number of ways and mitigates several problems the Army has identified.
- ✓ While our focus has been on Army branching mechanisms, our intention is also introducing a resource allocation model where part of the allocation is based on priorities and market principles take over the rest. Some examples include school admissions and parking space allocation.

  The model easily extends in a number of directions.
- ✓ We have shown that matching with contracts model has important implications for domains beyond the traditional ones that satisfy the substitutes condition.