Kidney Exchange: Two Basic Models

Tayfun Sönmez

Based on mostly:
Roth, Sönmez, & Ünver (QJE 2004) and Roth, Sönmez, & Ünver (JET 2005)
There are close to 90,000 patients on the waiting list for cadaver kidneys in the U.S. as of October 2011.

In 2010:

* About 34,400 patients were added while 27,800 patients were removed from the waiting list.
* There were over 10,600 transplants of cadaver kidneys performed in the U.S.
* About 4,650 patients died while on the waiting list and 2,100 were removed from the list due to being too sick to receive a transplant.
* There were about 6,300 transplants of kidneys from living donors.

Often living donors are incompatible with their intended patient.
Institutional Constraint: No Money

- The shortage of kidney increases by about 3,500 kidneys each year in the U.S.
- The 1984 National Organ Transplant Act (and in many states the Uniform Anatomical Gift Act) makes paying for an organ for transplantation a felony.

Section 301, National Organ Transplant Act (NOTA), 42 U.S.C. 274e 1984:

“it shall be unlawful for any person to knowingly acquire, receive or otherwise transfer any human organ for valuable consideration for use in human transplantation.”

- There is a rich literature on whether the ban on buying and selling of kidneys be repealed (ex: Becker & Elias 2002).
Medical Constraint: Blood Type Compatibility

- There are four blood types: A, B, AB and O.
- In the absence of other complications:
  * Type O kidneys can be transplanted into any patient;
  * type A kidneys can be transplanted into type A or type AB patients;
  * type B kidneys can be transplanted into type B or type AB patients; and
  * type AB kidneys can only be transplanted into type AB patients.
Medical Constraint: Tissue Type Compatibility

- Tissue type or **Human Leukocyte Antigen (HLA)** type: Combination of six proteins.
- Prior to transplantation, the potential recipient is tested for the presence of preformed antibodies against donor HLA. If there is a **positive crossmatch**, the transplantation cannot be carried out.
Allocation of Cadaver Kidneys in the U.S.

- U.S. Congress views cadaveric kidneys offered for transplantation as a national resource, and the National Organ Transplant Act of 1984 established the **Organ Procurement and Transplantation Network (OPTN)**.
- Run by the **United Network for Organ Sharing (UNOS)**, it has developed a centralized priority mechanism for the allocation of cadaveric kidneys.
Live Donor Transplants: Much Less Organized Until 2004

- A patient identifies a willing donor and, if the transplant is feasible, it is carried out.
- Otherwise, the patient remains on the queue for a cadaver kidney, while the donor returns home.
- Recently, however, in a small number of cases, additional possibilities have been utilized:
  - **Paired exchanges**: Exchanges between two incompatible pairs.
  - **Indirect exchanges**: An exchange between an incompatible pair and the cadaver queue.
Paired Kidney Exchange

- First proposed by Rapaport (Transplantation Proceedings 1986).
- The first kidney exchanges were carried out in South Korea in early 1990s.
- Renewed interest in the U.S. with Ross et al. (NEJM 1997) on “Ethics of Kidney Exchange.”
In 2000 the transplantation community issued a consensus statement declaring it as “ethically acceptable.”

The consensus statement also specified the following Incentives Constraint: All four operations shall be carried out simultaneously!

The first kidney exchange in the U.S. was carried out in Rhode Island in 2000.

Widespread concern in transplantation community: Indirect exchanges can harm type O patients with no living donors.

Nevertheless, many transplant centers have started pilot indirect exchange programs since 2000 (ex: Johns Hopkins Comprehensive Transplant Center, New England Medical Center.)
Kidney Exchange as a Market Design Problem

- In the early 2000s, we observed that the two main types of kidney exchanges conducted in the U.S. correspond to the most basic forms of exchanges in *house allocation with existing tenants* model of Abdulkadirioğlu & Sönmez (JET 1999).

- Inspired by this observation and building on the existing practices in kidney transplantation, we analyzed in Roth, Sönmez, & Ünver (QJE 2004) how an efficient and incentive-compatible system of exchanges might be organized, and what its welfare implications might be.
Even in the absence of more elaborate exchanges, merely organizing the paired-exchanges may result in increased efficiency.
Additional live-donor transplants may be possible through three-way, four-way, . . . , exchanges.
Value-Added of Structured Exchange: More Efficient Indirect Exchanges

- Additional benefits from more elaborate indirect exchanges.
Prior to our interaction with the transplantation community, three assumptions shaped our initial modeling of kidney exchange:

1. Patient preferences over compatible kidneys.
   a. The “European” view: The graft survival rate increases as the tissue type mismatch decreases (Opelz Transplantation 1997).
   b. The “American” view: The graft survival rate is the same for all compatible kidneys (Gjertson & Cecka Kidney International 2000, Delmonico NEJM 2004).

2. The number of simultaneous transplants.
3. Feasibility of indirect exchanges.

In subsequent analysis, a few other factors also proved to be important:

4. Integration of good-samaritan donors (a.k.a. altruistic donors). Sequential implementation of good-samaritan chains.
5. Participation by compatible pairs.
6. Center Incentives.
7. Dynamic aspects.

- **Assumption 1:** The graft survival rate increases as the tissue type mismatch decreases (i.e., the European view).
- **Assumption 2:** There is no constraint on the number of transplants that can be simultaneously carried out.
- **Assumption 3:** Indirect exchanges are feasible.

This first kidney exchange model builds on house allocation with existing tenants model of Abdulkadiroğlu & Sönmez (JET 1999).

**Other Related Literature:**
* Shapley & Scarf (J. Math. Econ 1974)
* Roth & Postlewaite (J. Math. Econ 1977)
* Roth (Economics Letters 1982)
Kidney Exchange Problem

- Elements of the problem:
  
  \( (k_i, t_i) \): A donor-patient pair.
  
  \( K_i \): Living donor kidneys compatible with patient \( t_i \).
  
  \( w \): Priority in the waitlist in exchange for a live kidney.
  
  \( P_i \): Strict preferences over \( K_i \cup \{k_i, w\} \).

- The outcome: Matching of kidneys/waitlist option to patients such that:
  
  1. each patient is either assigned a compatible kidney, or her donor’s kidney, or the waitlist option, and
  2. no kidney can be assigned to more than one patient although the waitlist option \( w \) can be assigned to several patients.
A **kidney exchange mechanism** is a systematic procedure to select a matching for each kidney exchange problem.

**Top Trading Cycles and Chains (TTCC)** mechanism relies on an algorithm consisting of several rounds. In each round:

* each patient $t_i$ points either towards a kidney in $K_i \cup \{k_i\}$ or towards $w$, and
* each kidney $k_i$ points to its paired recipient $t_i$. 
Cycles

- Cycles represent direct exchanges.
- No two cycles can intersect.
w-chains

- w-chains are associated with indirect exchanges.

- w-chains represent more elaborate versions of indirect exchanges.
w-chains can intersect!

- A kidney-patient pair can be part of several w-chains.
- **Important Design Consideration:** Choice of a plausible chain-selection rule.
- **Remark:** Choice of the chain-selection rule has efficiency and incentive-compatibility implications.
The following Lemma is the backbone of the TTCC mechanism:

**Lemma 1:** Consider a graph in which both the patient and the kidney of each pair are distinct nodes, as is the waitlist option w. Suppose each patient points either towards a kidney or w, and each kidney points to its paired recipient. Then

* either there exists a cycle or,
* each pair initiates a w-chain.
The Exchange

Fix a chain-selection rule. The TTCC mechanism determines the exchanges as follows:

1. Initially all kidneys are available and all agents are active. At each stage
   * each remaining active patient $t_i$ points to the best remaining unassigned kidney or to the waitlist option $w$, whichever is more preferred,
   * each remaining passive patient continues to point to his assignment, and
   * each remaining kidney $k_i$ points to its paired recipient $t_i$. 
2. By Lemma 1, there is either a cycle, or a w-chain, or both.
   a. Proceed to Step 3 if there are no cycles. Otherwise locate each cycle and carry out the corresponding exchange. Remove all patients in a cycle together with their assignments.
   b. Each remaining patient points to its top choice among remaining choices and each kidney points to its paired recipient. Proceed to Step 3 if there are no cycles. Otherwise locate all cycles, carry out the corresponding exchanges, and remove them.

Repeat this step until no cycle exists.
The Exchange

3. If there are no pairs left, then we are done. Otherwise by Lemma 1, each remaining pair initiates a w-chain.

Select only one of the chains with the chain selection rule.

The assignment is final for the patients in the selected w-chain. In addition to selecting a w-chain, the chain selection rule also determines

a. whether the selected w-chain is removed, or
b. the selected w-chain remains in the procedure although each patient in it is passive henceforth.

4. Each time a w-chain is selected, a new series of cycles may form. Repeat Steps 2 and 3 with the remaining active patients and unassigned kidneys until no patient is left.
Examples of chain-selection rules

a. Choose the longest w-chain and remove it.

b. Choose the longest w-chain and keep it.

c. Prioritize patient-donor pairs in a single list. Choose the w-chain starting with the highest priority pair and remove it.

d. Prioritize patient-donor pairs in a single list. Choose the w-chain starting with the highest priority pair and keep it.
Efficiency

- **Theorem:** Consider a chain-selection rule where any w-chain selected at a non-terminal round remains in the procedure and thus the kidney at its tail remains available for the next round. The TTCC mechanism, implemented with any such chain-selection rule, is efficient.

- **Two examples:**
  1. the rule that chooses the longest w-chain and keeps it, and
  2. the priority based rule that selects the w-chain starting with the highest priority pair and keeps it.
Incentive Compatibility and Relation with YRMH-IGYT

- **Theorem:** Consider the priority based chain-selection rules c and d. The TTCC mechanism, implemented with either of these chain selection rules is strategy-proof.

- **Corollary:** The TTCC mechanism, implemented with chain selection rule d is efficient and strategy-proof.

TTCC is motivated by the you request my house - I get your turn (YRMH-IGYT) algorithm of Abdulkadiroğlu and Sönmez (1999). Krishna & Wang (JET 2007) formalize the relation between the two algorithms.

**Theorem (Krishna & Wang JET 2007):** The TTCC algorithm executed with the chain-selection rule d is equivalent to the YRMH-IGYT algorithm.
## Simulations on Welfare Gains

### Table III

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<th>Pop. size</th>
<th>Pref.</th>
<th>Exchange regime</th>
<th>Total trans. %</th>
<th>Own donor trans. %</th>
<th>Trade %</th>
<th>Wait-list upgrade %</th>
<th>HLA mis.</th>
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<td>68.50 (9.90)</td>
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<td>73.59 (4.97)</td>
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<td>18.80 (3.81)</td>
<td>10.24 (3.07)</td>
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</table>
Initial Reactions of the Transplantation Community

- Following RSÜ (2004), we entered into discussions with New England transplant surgeons and their colleagues in the transplant community.
- In the course of those discussions it became clear that a likely first step will be to implement logistically simpler pairwise exchanges.
- Furthermore, doctors indicated that they would be more comfortable with a model where patient preferences are dichotomous: As a first approximation, patients can be assumed to be indifferent among all compatible kidneys.
- Finally doctors showed less interest in indirect exchanges due to concerns over blood-type O patients w/o living donors.
- This motivated Roth, Sönmez, & Ünver (JET 2005), “Pairwise Kidney Exchange.”

- **Assumption 1**: The graft survival rate is the same for all compatible kidneys (i.e. the American view).
- **Assumption 2**: No more than two transplants can be carried out simultaneously.
- **Assumption 3**: Indirect exchanges are not allowed.

**Related Literature in Operations Research and Economics:**
- Gallai (MTAMKIK 1963, 1964)
- Edmonds (Can. J. of Math. 1965)
- Bogomolnaia & Moulin (Econometrica 2004)
Pairwise Kidney Exchange Problem

$N$: Set of patients (each with one or more incompatible donors).

$r_{i,j}$: Indicates mutual compatibility between patients $i$ and $j$ ($r_{i,j} = 1$ if compatible, $r_{i,j} = 0$ otherwise).

$R$: Mutual compatibility matrix for all patient pairs.

- Pairwise kidney exchange problem can be represented with an undirected graph:

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![Graph representation of the problem and a subproblem for the set {1, 2, 6, 7, 8}.](image)

*Problem:* 

*Subproblem for* $\{1, 2, 6, 7, 8\}$:

- Even component
- Odd component
Deterministic and Stochastic Outcomes

- The deterministic outcome (a pairwise matching): A function $\mu : N \to N$ such that
  1. if $\mu(i) = j$ then $\mu(j) = i$ (i.e. only pairwise exchanges are possible), and
  2. if $\mu(i) = j$ then $r_{i,j} = 1$ unless $i = j$ (i.e. only mutually beneficial exchanges are possible).

- The stochastic outcome: A lottery $\lambda$ among matchings.

  $a_{i,j}(\lambda)$: The probability that patients $i$ and $j$ are matched with each other under $\lambda$.

  $u_i(\lambda)$: Utility of patient $i$ under $\lambda$.

  ($u_i(\lambda) = \sum_{j \in N \setminus \{i\}} a_{i,j}(\lambda)$ specifies the odds for a transplant.)
Efficiency

- A matching is **Pareto efficient** if there is no other matching that makes every patient weakly better off and some patient strictly better off.
- A lottery is **ex-post efficient** if it gives positive weight to only Pareto efficient matchings.
- A lottery is **ex-ante efficient** if there is no other lottery that makes every patient weakly better off and some patient strictly better off.
- The following is a well-known result in combinatorial optimization literature:
  
  **Lemma 2:** The same number of patients are matched at each Pareto efficient matching.

  **Remark:** Lemma 2 would not hold if exchange was possible among three or more patients.
Equivalence of Ex-ante and Ex-post Efficiency

- In general
  \[\text{Ex-ante Efficiency} \Rightarrow \text{Ex-post Efficiency}\]

- But in the context of pairwise kidney exchange, Lemma 2 implies:
  \[\text{Ex-ante Efficiency} \iff \text{Ex-post Efficiency}\]
Priority Mechanisms

For a given priority ordering of $|N|$ patients, the induced priority mechanism selects a matching in the below described set $\mathcal{E}^{|N|}$, constructed in $|N| + 1$ iterations as follows:

- $\mathcal{E}^0$ is the set of all matchings.
- $\mathcal{E}^1 = \mathcal{E}^0$ if there is no matching that matches the highest priority patient, and it is the set of all matchings which matches the highest priority patient otherwise.

For each $k \leq |N|$, 

- $\mathcal{E}^k = \mathcal{E}^{k-1}$ if there is no matching in $\mathcal{E}^{k-1}$ that matches the $k^{th}$ priority patient, and it is the set of all matchings in $\mathcal{E}^{k-1}$ which matches the $k^{th}$ priority patient otherwise.
Efficiency & Incentive Compatibility of Priority Mechanisms

**Theorem:** The priority mechanism is not only Pareto efficient but also it makes it a dominant strategy for a patient to reveal both

a. her full set of compatible kidneys, and

b. her full set of available donors.

**Remark:** Not only it is straight-forward to extend priority mechanisms to a model that allows larger exchanges, but also a counterpart of the above result directly holds for such extensions. (See, for example, Hatfield JET 2005).
Underdemanded, Overdemanded, and Perfectly-Matched Patients

$N^U$: Patients unmatched at least at some efficient matching

$N^O$: “Neighbors” of $N^U$

$N^P$: Others
Gallai-Edmonds Decomposition

Theorem (Gallai-Edmonds Decomposition): Let \( \mu \) be any Pareto efficient matching for the original problem \((N, R)\) and \((I, R_I)\) be the subproblem for \( I = N \setminus N^O \).

1. Any overdemanded patient is matched with an underdemanded patient under \( \mu \).
2. \( J \subseteq N^P \) for any even component \( J \) of the subproblem \((I, R_I)\) and all patients in \( J \) are matched with each other under \( \mu \).
3. \( J \subseteq N^U \) for any odd component \( J \) of the subproblem \((I, R_I)\) and for any patient \( i \in J \), it is possible to match all remaining patients with each other under \( \mu \). Moreover under \( \mu \)
   a. either one patient in \( J \) is matched with an overdemanded patient and all others are matched with each other,
   b. or one patient in \( J \) remains unmatched while the others are matched with each other.
Competition Among Odd Components

- \( D = \{D_1, \ldots, D_p\} \): Set of odd components.
- Based on GED Lemma, Pareto efficient matchings each leave unmatched \(|D| - |N^O|\) patients, each one in a distinct odd component.
- Competition at two levels:
  1. Competition among odd components for overdemanded patients.
  2. Competition among members of each odd component that does not secure an overdemanded patient.
Equity

There is a very natural utility function in the context of pairwise kidney exchange:

- **Utility**: The probability of receiving a transplant.
- In this context equilizing utilities as much as possible may be considered very plausible from an equity perspective.
Useful Intellectual Exercise

Let

- $\mathcal{J} \subseteq \mathcal{D}$ be an arbitrary set of odd components,
- $I \subseteq N^O$ be an arbitrary set of overdemanded patients, and
- $C(\mathcal{J}, I)$ denote the “neighbors” of $\mathcal{J}$ among $I$.

**Question:** Suppose only overdemanded patients in $I$ are available to be matched with underdemanded patients in $|\bigcup_{J \in \mathcal{J}} J|$. Can we give an upper-bound for the utility that can be received by the least fortunate patient in $|\bigcup_{J \in \mathcal{J}} J|$?

**Answer:**

$$f(\mathcal{J}, I) = \frac{|\bigcup_{J \in \mathcal{J}} J| - (|\mathcal{J}| - |C(\mathcal{J}, I)|)}{|\bigcup_{J \in \mathcal{J}} J|}$$
The Egalitarian Mechanism

- This upper-bound can be received only if:
  1. all underdemanded patients in $\bigcup_{J \in J} J$ receive the same utility, and
  2. all overdemanded patients in $C(J, I)$ are committed for patients in $\bigcup_{J \in J} J$.

- So partition $D$ as $D_1, D_2, \ldots$ and $N^O$ as $N^O_1, N^O_2, \ldots$ as follows:

  **Step 1:**
  
  $$D_1 = \arg \min_{J \subseteq D} f \left( J, N^O \right) \quad N^O_1 = C \left( D_1, N^O \right)$$

  **Step k:**
  
  $$D_k = \arg \min_{J \subseteq D \setminus \bigcup_{\ell=1}^{k-1} D_\ell} f \left( J, N^O \setminus \bigcup_{\ell=1}^{k-1} N^O_\ell \right)$$
  
  $$N^O_k = C \left( D_k, N^O \setminus \bigcup_{\ell=1}^{k-1} N^O_\ell \right)$$
The Egalitarian Utility

- Construct the vector $u^E = (u^E_i)_{i \in N}$ as follows:
  1. For any overdemanded patient and perfectly-matched patient $i \in N \setminus N^U$,
     $$u^E_i = 1.$$
  2. For any underdemanded patient $i$ whose odd component left the above procedure at Step $k(i)$,
     $$u^E_i = f(D_{k(i)}, N^O_{k(i)}).$$

**Theorem:** The vector $u^E$ is feasible.

- Two major challenges in the proof:
  1. Construction of an allocation matrix that yields the egalitarian utilities.
  2. Construction of a lottery that yields this allocation matrix.
Lorenz Domination

- **Notation:** For any utility profile $u$, re-order individual utilities in an increasing order $(u^{(t)})_{t \in \{1,...,n\}}$ such that

  \[ u^{(1)} \leq u^{(2)} \leq \cdots \leq u^{(n)} \]

- Utility profile $u$ **Lorenz dominates** utility profile $v$ if
  1. $\sum_{s=1}^{t} u^{(s)} \geq \sum_{s=1}^{t} v^{(s)}$ for all $t$, and
  2. $\sum_{s=1}^{t} u^{(s)} > \sum_{s=1}^{t} v^{(s)}$ for some $t$.

- **Theorem:** The utility profile $u^E$ Lorenz dominates any other feasible utility profile (efficient or not).
Efficiency & Incentive Compatibility of the Egalitarian Mechanism

**Theorem:** The egalitarian mechanism is not only ex-ante Pareto efficient but also it makes it a dominant strategy for a patient to reveal both

- a. her full set of compatible kidneys, and
- b. her full set of available donors.
Subsequent Research on Kidney Exchange

- Despite the elegance of the underlying math and the presence of well-behaved mechanisms for pairwise kidney exchange, there is significant welfare gap between TTCC and efficient pairwise kidney exchange mechanisms.

- Two important factors in this welfare difference are:
  1. the loss of compatible pairs under pairwise exchange with dichotomous preferences; and
  2. the two-way exchange constraint.
Subsequent Research on Kidney Exchange

Hence we focused on increasing welfare in subsequent research:

- Roth, Sönmez, & Ünver (AER 2007): Welfare gains from 3-way exchange is especially important.
- Roth, Sönmez, Ünver, Saidman, & Delmonico (AJT 2006): “Simultaneous transplant” constraint can be relaxed for good-samaritan donor chains (a.k.a. nondirected-donor chains), and thus substantially larger exchanges can be conducted.

While the transplantation community was initially hesitant about each of these design proposals, the first two became standard all around the world within only few years.

As for the third, so far only Columbia University has adopted a program with compatible pairs. However not only the welfare gains from inclusion of compatible pairs is by far the largest of all, but also it restores the elegant mathematical structure of kidney exchange.
Collaboration with Transplantation Community

- **New England Program for Kidney Exchange (NEPKE):** Together with New England surgeons and tissue typing experts, especially Frank Delmonico and Susan Saidman, we have launched centralized kidney exchange in New England to cover all six states (and 14 transplant centers) in 2004.

  NEPKE became the first kidney exchange program to use optimization.

- **Alliance for Paired Donation (APD):** We have also been running matches for Drs. Steve Woodle and Michael Rees and their colleagues in the Paired Donation Consortium they started in midwest, and more recently for the Alliance for Paired Donation.

  APD currently has more than 80 transplant centers.
Welcome

A Life-Saving Option
The New England Program for Kidney Exchange offers new life-saving options to those seeking a kidney transplant, but whose potential living donor is not a good biological "match" due to either blood type incompatibility or cross-match incompatibility. This option is known as kidney exchange, kidney paired donation, or kidney swap.

NEPKE uses a computer program to find cases where the donor in an incompatible pair can be matched to a recipient in another pair. By exchanging donors, a compatible match for both recipients may be found. You can learn more about the program HERE and read our newsletter here.

NEPKE can also find potential kidney recipients for those generous people who seek to become non-directed living donors (otherwise known as Good Samaritan Donors or Altruistic Donors). Information about that process is available HERE.
Alliance for Paired Donation

More than 84,000 people in America are waiting for a kidney transplant; sadly, about 12 of these patients die every day because there aren’t enough donors. Many kidney patients have someone who is willing to donate, but because of immune system or blood type incompatibilities, they are not able to give a kidney to their loved one.

The Alliance for Paired Donation can help. Kidney paired donation matches one incompatible donor/recipient pair to another pair in the same situation, so that the donor of the first pair gives to the recipient of the second, and vice versa. In other words, the two pairs swap kidneys. APD has also pioneered a new way of using altruistic, or good Samaritan, donors, so that the transplants no longer have to be performed simultaneously. Non-simultaneous Extended Altruistic Donor Chains (NEAD Chains) allow donors to “pay it forward” after their loved one receives a transplant.
Based on findings of RSÜ (2007) and Roth et al. (2006), NEPKE and APD both adopted 3-way exchanges as well as (sequential) nondirected-donor chains.
When we initially helped found NEPKE, it was unclear whether kidney exchange is in violation of NOTA.

In particular, it was unclear whether kidney exchange was considered to involve transfer of a human organ for valuable consideration.

In Dec 2007, an amendment of NOTA has passed in the U.S. Senate, clarifying that kidney exchange is legal.

Charlie W. Norwood Living Organ Donation Act, opened the doorway for national kidney exchange in the U.S.
2009: RSÜ (2005, 2007) provided the basis for national kidney exchange in UK where a group of computer scientists at U. of Glasgow helped design the National Matching Scheme for Paired Donation. Their algorithm finds an optimal matching under 2-way + 3-way exchanges.
U.S. National Kidney Paired Donation Pilot Program

2010: A pilot national kidney exchange program in U.S. is launched, also adopting an optimal mechanism under 2-way + 3-way exchanges.

As of December 2011, NEPKE is part of the national kidney exchange pilot program.