

# From Psychological to Perceptual Expected Utility Theory

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# Introduction

- ▶ Four talks detail trip through psychology and economics
- ▶ Starting point as purely conceptual
  - ▶ Psychological forces impact decisions
  - ▶ So incorporate them!
- ▶ Along the way became obsessed with testability
  - ▶ Saw uncontrollable data growth
  - ▶ A methodological challenge
- ▶ Last two lectures address this challenge
  - ▶ Last one first time presentation
  - ▶ Purest statement yet
  - ▶ Your invitation forced the pace: thanks!

# Introduction

- ▶ Behavioral economics starts with “paradoxes”
- ▶ If psychology systematically important for choices, on main road
  - ▶ If not, why bother?
- ▶ Goal: model psychological factors and systematic impact on contingent behaviors
- ▶ Why not ask? Because don't want know!

# PEU

- ▶ Motivational rewards separate in time from physical rewards important to decisions. Feelings of living with uncertainty include:
  - ▶ Anticipation of future pleasures
  - ▶ Anxiety and dread
  - ▶ Love of suspense
  - ▶ Curiosity
- ▶ For one aware of these feelings, it is reasonable to take them into account.
  - ▶ Curiosity and drive to learn
  - ▶ Boosting esteem of loved ones
- ▶ Market relevance?
  - ▶ Is “Equity Premium” due to living with uncertainty?
- ▶ Policy relevance
  - ▶ Manipulate future orientation

# PEU

- ▶ To model, change domain
  - ▶ From objective prizes to subjective
  - ▶ PEU of CL is general EU with psychological prizes.
  - ▶ Includes production function for relevant inner states.
  - ▶ Substitution axiom as reasonable as ever
- ▶ General feature is time inconsistency
  - ▶ Pay to heighten savoring (e.g. betting on favorite)
- ▶ Worked examples collapse time for simplicity

# PEU

- ▶ To collapse time, add belief over final state to the prize space,

$$Z = \{(p, \theta) \mid 0 \leq p \leq 1, \theta = A, B\},$$

where  $p \in [0, 1]$  is the probability of state  $A$  and  $\theta$  is the outcome that eventuates.

- ▶ Example is  $(0.5, A)$  a belief that states  $A$  and  $B$  are equally likely ( $p = 0.5$ ), and an outcome in which  $A$  in fact occurs ( $\theta = A$ ).
- ▶ The substitution axiom is applied to preferences on  $X$ , the space of lotteries over these “belief-state” prizes.
- ▶ Conclude that there exists  $u : Z \rightarrow \mathbb{R}$  such that, given any two elements  $H, J \in X$ ,

$$H \succsim J \text{ if and only if } E^H(u) \geq E^J(u).$$

- ▶ Generic element  $F \in X$  lists  $K$  belief-outcome lotteries  $(p_k^F, \theta_k^F)$  and  $q_k^F \geq 0$ ; with  $(p_k^F, \theta_k^F) \in Z$  all  $k$  and with  $\sum q_k^F = 1$ . Write,

$$F = [(p_1^F, \theta_1^F) \circ q_1^F; \dots; (p_k^F, \theta_k^F) \circ q_k^F; \dots; (p_K^F, \theta_K^F) \circ q_K^F].$$

# PEU

- ▶ The space  $X$  is intricate. Some easy to understand such as:  
 $[(0.5, A) \circ 0.5; (0.5, B) \circ 0.5] = L(0.5) \in X$ .
  - ▶ Let  $L = \{(p, A) \circ p; (p, B) \circ 1 - p \mid 0 \leq p \leq 1\} \subset X$  be the set of such lotteries over “belief-state” prizes.
- ▶ Also interest in  $L^2 \subset X$ , lotteries over  $L$ .
  - ▶ To describe  $H \in L^2$  list possible lotteries  $L(p_k^H)$ , and their probabilities  $q_k^H \geq 0$ ; with  $L(p_k^H) \in L$  all  $k$  and with  $\sum q_k^H = 1$ . Write,

$$H = [L(p_1^H) \circ q_1^H; \dots; L(p_k^H) \circ q_k^H; \dots; L(p_K^H) \circ q_K^H].$$

- ▶ Other members of  $X$  not personally feasible, such as:  
 $[(0.5, A) \circ 0.9; (0.5, B) \circ 0.1] \in X$ .
  - ▶ May be strategically feasible
  - ▶ Thought experiment preferences in the spirit of Savage

# Anxiety and Information

- ▶ Medical example:  $B$  incurable degenerative disease onset 10 years from now,  $A$  not
- ▶ Prior probability that do not have is  $\pi$ .
- ▶ Assume best prize is good news early, worst is bad news early,
- ▶ Natural monotonicity in the case of the good outcome. Simplest case linear,

$$u^{ANX}(p, A) = \alpha^{ANX} p + (1 - \alpha^{ANX}),$$

where  $\alpha^{ANX} \in (0, 1)$  gives the weight of prior beliefs relative to ultimate reality.

- ▶ Even with bad outcome assume better to have lived in hope,

$$u^{ANX}(p, B) = \beta^{ANX} p.$$

where again  $\beta^{ANX} \in (0, 1)$  gives the weight of prior beliefs when ultimate reality is bad.

- ▶ In PEU, natural to have same parameter on optimism and pessimism regardless of ultimate state (conceptual separation of period utilities).



# Anxiety and Information

- ▶ Study preferences over the signal set,

$$S = \{s(\delta) | \delta \in [0, 1 - \pi]\}.$$

- ▶ Quality of signal is  $\delta \in [0, 1 - \pi]$ : ex ante signal equally likely to raise or lower the probability of state A by  $\delta$ .

- ▶ Post-signal belief that enters the utility function.

- ▶ With uninformative signal  $s(0)$ , get belief-state lottery  $L(\pi) \in L$  for sure,

$$L(\pi) = [(\pi, A) \circ \pi; (\pi, B) \circ 1 - \pi] \in L.$$

- ▶ Signal  $s(\delta)$  ends up producing a lottery over such lotteries,

$$L(\pi + \delta) \circ \frac{1}{2} \oplus L(\pi - \delta) \circ \frac{1}{2} \in L^2.$$

# Anxiety and Information

- ▶ We define a single function  $K^{ANX} : [0, 1] \rightarrow R$  to summarize choice of signal,

$$\begin{aligned}K^{ANX}(p) &\equiv pu^{ANX}(p, A) + (1 - p)u^{ANX}(p, B) \\ &= p(\alpha^{ANX}p + (1 - \alpha^{ANX})) + (1 - p)(\beta^{ANX}p) \\ &\equiv \Delta^{ANX}p^2 + (1 - \Delta^{ANX})p,\end{aligned}$$

where  $\Delta^{ANX} = \alpha^{ANX} - \beta^{ANX}$ .

- ▶ For signals  $s(\delta) \in S$ ,  $s(\delta) \succcurlyeq s(\tilde{\delta})$  iff,

$$\begin{aligned} \frac{K^{ANX}(\pi + \delta)}{2} + \frac{K^{ANX}(\pi - \delta)}{2} &\geq \frac{K^{ANX}(\pi + \tilde{\delta})}{2} + \frac{K^{ANX}(\pi - \tilde{\delta})}{2} \\ \Delta^{ANX} [(\pi + \delta)^2 + (\pi - \delta)^2] + (1 - \Delta^{ANX})2\pi &\geq \Delta^{ANX} [(\pi + \tilde{\delta})^2 + (\pi - \tilde{\delta})^2] + (1 - \Delta^{ANX})2\pi \\ \Delta^{ANX} [2\pi^2 + 2\delta^2] &\geq \Delta^{ANX} [2\pi^2 + 2\tilde{\delta}^2] \\ \delta^2 \Delta^{ANX} &\geq \tilde{\delta}^2 \Delta^{ANX} \end{aligned}$$

- ▶ Higher values of  $\delta$  (more information) is chosen if and only if  $\Delta^{ANX} > 0$ , or  $\alpha^{ANX} > \beta^{ANX}$ .
- ▶ Note that sign of  $\Delta^{ANX}$  determines shape of  $K^{ANX}(p)$ .
  - ▶ Strictly concave if  $\Delta^{ANX} < 0$ : no information chosen in this case.
  - ▶ Strictly convex if  $\Delta^{ANX} > 0$ : information chosen in this case.
- ▶ Intuition: with better signal, more likely to have optimistic anticipation in good than in bad event.
- ▶ Gain positive anticipatory feelings in state of good outcome, lose an equivalent amount in case of bad outcome.
- ▶ Worthwhile if optimism is ex ante more important a benefit when good state eventuates than when bad state eventuates.

- ▶ With PEU, one can essentially treat  $K^{ANX}(p)$  as primitive
- ▶ Now different interpretation of the shape of  $K^{ANX}(p)$ : it is non-linearities of anticipatory utility in beliefs.
- ▶ In medical cases, often think of extreme pessimism as tough: can live with average beliefs, not with bad truths.
- ▶ So stay ignorant!
- ▶ Simple argument shows that is as before:
  - ▶ concavity of utility says prefer middle to extremes so no information;
  - ▶ convexity says prefer extremes

# Anxiety and Information

- ▶ Kim Witte's proposes that a fear appeal either triggers additional danger control through prevention, or instead promotes inattention and avoidance. Perceived efficacy is the key.
- ▶ Costs of preventive measure  $C > 0$ : lowers the probability of bad health in period 2 from  $b_N$  to  $b_P$  with utility advantage of health in period 2 of  $H$ .
- ▶ Period 1 experience of fear  $F > 0$ , associated with the health threat. Prevention will be undertaken if and only if,

$$(b_N - b_P)H + (F_N - F_P) \geq C.$$

- ▶ The "fear differential" represents the difference in the level of fear depending on whether or not the preventive act is undertaken.

## Anxiety and Information

- ▶ Measure danger resulting from action  $P$  is assumed to be  $b_P H$ , the higher danger from action  $N$  is  $b_N H$ . Allow attentional multipliers,  $A_P$  and  $A_N$ , both positive,

$$F_P = A_P b_P H;$$

$$F_N = A_N b_N H;$$

- ▶ Let  $A_P(m, H)$  and  $A_N(m, H)$  reflect attention given to a health threat of type  $H$  given a message of intensity  $m$ , conditional respectively on undertaking and on not undertaking the preventive act.
- ▶ Suppose the preventive act has a fixed proportionate impact  $\lambda > 0$  on the attention,

$$A_P(m, H) = (1 + \lambda)A_N(m, H).$$

the condition for prevention to raise the level of fear is,

$$\lambda > \frac{b_N - b_P}{b_P}.$$

# Anxiety and Information

- ▶ Captures efficacy with natural measure  $\frac{b_N - b_P}{b_P}$ .
- ▶ With high efficacy, fear is reduced if the preventive act is undertaken, and more intense message transmission serves to expand this fear-based differential.
- ▶ With low efficacy, prevention raises fear, and intense message transmission serves only to further discourage prevention.
- ▶ Variations can create different information-action interactions.



# Where Next

- ▶ Suggests a progressive agenda to health-related choices
  - ▶ Genetic testing
  - ▶ Psychological incentives in insurance contracts
- ▶ Certification policies for communicable diseases
  - ▶ Work with Kfir Eliaz

# Other Applications

- ▶ Other applications of monitoring/avoidance
  - ▶ How often one checks assets in relation to stock market
  - ▶ Failure to plan for retirement due to stress?
- ▶ The impacts of attentional interventions
  - ▶ Reminders that force issues to mind
- ▶ Similar framework for other emotions.
- ▶ Curiosity and learning
  - ▶ How can one induce further search and learning due to desire to know?
  - ▶ "Library science"

# Empirical Advance

- ▶ To implement PEU fit psychological production function to get around “Lucas Critique”
- ▶ Standard choice data of possible value in fitting production function
  - ▶ Becker and Rubinstein study demand for "fear-related" goods after various attacks
  - ▶ Another study in the Jerusalem Housing Market

# Non-Standard Data on Emotions

- ▶ Use of non-choice “psychological” data is challenging
- ▶ What are the relevant states? What produces them? How can they be measured?
  - ▶ Data on time use?
  - ▶ Eye tracking?
  - ▶ Self reports on affect?
  - ▶ Physiological measures and manipulations?
- ▶ Next time: how the challenge of testing and the flood of new data drove me to methodology