From Psychological to Perceptual Expected Utility Theory

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Introduction

- Four talks detail trip through psychology and economics
- Starting point as purely conceptual
  - Psychological forces impact decisions
  - So incorporate them!
- Along the way became obsessed with testability
  - Saw uncontrollable data growth
  - A methodological challenge
- Last two lectures address this challenge
  - Last one first time presentation
  - Purest statement yet
  - Your invitation forced the pace: thanks!

Introduction

- Behavioral economics starts with "paradoxes"
- If psychology systematically important for choices, on main road
  - If not, why bother?
- Goal: model psychological factors and systematic impact on contingent behaviors
- Why not ask? Because don't want know!

PEU

- Motivational rewards separate in time from physical rewards important to decisions. Feelings of living with uncertainty include:
  - Anticipation of future pleasures
  - Anxiety and dread
  - Love of suspense
  - Curiosity
- For one aware of these feelings, it is reasonable to take them into account.
  - Curiosity and drive to learn
  - Boosting esteem of loved ones
- Market relevance?
  - Is "Equity Premium" due to living with uncertainty?
- Policy relevance
  - Manipulate future orientation

PEU

- To model, change domain
  - From objective prizes to subjective
  - PEU of CL is general EU with psychological prizes.
  - Includes production function for relevant inner states.
  - Substitution axiom as reasonable as ever
- General feature is time inconsistency
  - Pay to heighten savoring (e.g. betting on favorite)
- Worked examples collapse time for simplicity

PEU

- To collapse time, add belief over final state to the prize space,

$$Z = \{(p, \theta) | 0 \leq p \leq 1, \ \theta = A, B\},$$

  where $p \in [0, 1]$ is the probability of state A and θ is the outcome that eventuates.
  - Example is $(0.5, A)$ a belief that states A and B are equally likely ($p = 0.5$), and an outcome in which A in fact occurs ($\theta = A$).
- The substitution axiom is applied to preferences on X, the space of lotteries over these "belief-state" prizes.
- Conclude that there exists $u : Z \longrightarrow \mathbb{R}$ such that, given any two elements $H, J \in X$,

$$H \succsim J \text{ if and only if } E^H(u) \geq E^J(u).$$

- Generic element $F \in X$ lists K belief-outcome lotteries (p_k^F, θ_k^F) and $q_k^F \geq 0$; with $(p_k^F, \theta_k^F) \in Z$ all k and with $\sum q_k^F = 1$. Write,

$$F = [(p_1^F, \theta_1^F) \circ q_1^F; ..; (p_k^F, \theta_k^F) \circ q_k^F; ..; (p_K^F, \theta_K^F) \circ q_K^F].$$

PEU

- The space X is intricate. Some easy to understand such as: $[(0.5, A) \circ 0.5; (0.5, B) \circ 0.5] = L(0.5) \in X$.
  - Let $L = \{(p, A) \circ p; (p, B) \circ 1 - p | 0 \leq p \leq 1\} \subset X$ be the set of such lotteries over "belief-state" prizes.
- Also interest in $L^2 \subset X$, lotteries over L.
  - To describe $H \in L^2$ list possible lotteries $L(p_k^H)$, and their probabilities $q_k^H \geq 0$; with $L(p_k^H) \in L$ all k and with $\sum q_k^H = 1$. Write,

$$H = [L(p_1^H) \circ q_1^H; ..; L(p_k^H) \circ q_k^H; ..; L(p_K^H) \circ q_K^H].$$

- Other members of X not personally feasible, such as: $[(0.5, A) \circ 0.9; (0.5, B) \circ 0.1] \in X$.
  - May be strategically feasible
  - Thought experiment preferences in the spirit of Savage

Anxiety and Information

- Medical example: B incurable degenerative disease onset 10 years from now, A not
- Prior probability that do not have is π.
- Assume best prize is good news early, worst is bad news early,
- Natural monotonicity in the case of the good outcome. Simplest case linear,

$$u^{ANX}(p, A) = \alpha^{ANX} p + (1 - \alpha^{ANX}),$$

  where $\alpha^{ANX} \in (0, 1)$ gives the weight of prior beliefs relative to ultimate reality.
- Even with bad outcome assume better to have lived in hope,

$$u^{ANX}(p, B) = \beta^{ANX} p.$$

  where again $\beta^{ANX} \in (0, 1)$ gives the weight of prior beliefs when ultimate reality is bad.
- In PEU, natural to have same parameter on optimism and pessimism regardless of ultimate state (conceptual separation of period utilities).

Anxiety and Information

- Study preferences over the signal set,

$$S = \{s(\delta) | \delta \in [0, 1 - \pi]\}.$$

- Quality of signal is $\delta \in [0, 1 - \pi]$: ex ante signal equally likely to raise or lower the probability of state A by δ.
  - Post-signal belief that enters the utility function.
- With uninformative signal $s(0)$, get belief-state lottery $L(\pi) \in L$ for sure,

$$L(\pi) = [(\pi, A) \circ \pi; (\pi, B) \circ 1 - \pi] \in L.$$

- Signal $s(\delta)$ ends up producing a lottery over such lotteries,

$$L(\pi + \delta) \circ \frac{1}{2} \oplus L(\pi - \delta) \circ \frac{1}{2} \in L^2.$$

Anxiety and Information

- We define a single function $K^{ANX} : [0,1] \rightarrow R$ to summarize choice of signal,

$$
\begin{aligned}
K^{ANX}(p) &\equiv pu^{ANX}(p, A) + (1-p)u^{ANX}(p, B) \\
&= p(\alpha^{ANX} p + (1-\alpha^{ANX})) + (1-p)(\beta^{ANX} p) \\
&\equiv \Delta^{ANX} p^2 + (1-\Delta^{ANX})p,
\end{aligned}
$$

where $\Delta^{ANX} = \alpha^{ANX} - \beta^{ANX}$.

- For signals $s(\delta) \in S$, $s(\delta) \succsim s(\tilde{\delta})$ iff,

$$\begin{aligned}
\frac{K^{ANX}(\pi+\delta)}{2} + \frac{K^{ANX}(\pi-\delta)}{2} &\geq \frac{K^{ANX}(\pi+\tilde{\delta})}{2} + \frac{K \\
\Delta^{ANX}\left[(\pi+\delta)^2 + (\pi-\delta)^2\right] + (1-\Delta^{ANX})2\pi &\geq \Delta^{ANX}\left[(\pi+\tilde{\delta})^2 - \right. \\
\Delta^{ANX}\left[2\pi^2 + 2\delta^2\right] &\geq \Delta^{ANX}\left[2\pi^2 + 2\tilde{\delta}^2\right] \\
\delta^2\Delta^{ANX} &\geq \tilde{\delta}^2\Delta^{ANX}
\end{aligned}$$

- Higher values of δ (more information) is chosen if and only if $\Delta^{ANX} > 0$, or $\alpha^{ANX} > \beta^{ANX}$.
- Note that sign of Δ^{ANX} determines shape of $K^{ANX}(p)$.
  - Strictly concave if $\Delta^{ANX} < 0$: no information chosen in this case.
  - Strictly convex if $\Delta^{ANX} > 0$: information chosen in this case.
- Intuition: with better signal, more likely to have optimistic anticipation in good than in bad event.
- Gain positive anticipatory feelings in state of good outcome, lose an equivalent amount in case of bad outcome.
- Worthwhile if optimism is ex ante more important a benefit when good state eventuates than when bad state eventuates.

- With PEU, one can essentially treat $K^{ANX}(p)$ as primitive
- Now different interpretation of the shape of $K^{ANX}(p)$: it is non-linearities of anticipatory utility in beliefs.
- In medical cases, often think of extreme pessimism as tough: can live with average beliefs, not with bad truths.
- So stay ignorant!
- Simple argument shows that is as before:
  - concavity of utility says prefer middle to extremes so no information;
  - convexity says prefer extremes

Anxiety and Information

- Kim Witte's proposes that a fear appeal either triggers additional danger control through prevention, or instead promotes inattention and avoidance. Perceived efficacy is the key.
- Costs of preventive measure $C > 0$: lowers the probability of bad health in period 2 from b_N to b_P with utility advantage of health in period 2 of H.
- Period 1 experience of fear $F > 0$, associated with the health threat. Prevention will be undertaken if and only if,

$$(b_N - b_P)H + (F_N - F_P) \geq C.$$

- The "fear differential" represents the difference in the level of fear depending on whether or not the preventive act is undertaken.

Anxiety and Information

- Measure danger resulting from action P is assumed to be $b_P H$, the higher danger from action N is $b_N H$. Allow attentional multipliers, A_P and A_N, both positive,

$$\begin{aligned} F_P &= A_P b_P H; \\ F_N &= A_N b_N H; \end{aligned}$$

- Let $A_P(m, H)$ and $A_N(m, H)$ reflect attention given to a health threat of type H given a message of intensity m, conditional respectively on undertaking and on not undertaking the preventive act.
- Suppose the preventive act has a fixed proportionate impact $\lambda > 0$ on the attention,

$$A_P(m, H) = (1 + \lambda) A_N(m, H).$$

the condition for prevention to raise the level of fear is,

$$\lambda > \frac{b_N - b_P}{b_P}.$$

Anxiety and Information

- Captures efficacy with natural measure $\frac{b_N - b_P}{b_P}$.
- With high efficacy, fear is reduced if the preventive act is undertaken, and more intense message transmission serves to expand this fear-based differential.
- With low efficacy, prevention raises fear, and intense message transmission serves only to further discourage prevention.
- Variations can create different information-action interactions.

Where Next

- Suggests a progressive agenda to health-related choices
  - Genetic testing
  - Psychological incentives in insurance contracts
- Certification policies for communicable diseases
  - Work with Kfir Eliaz

Other Applications

- Other applications of monitoring/avoidance
  - How often one checks assets in relation to stock market
  - Failure to plan for retirement due to stress?
- The impacts of attentional interventions
  - Reminders that force issues to mind
- Similar framework for other emotions.
- Curiosity and learning
  - How can one induce further search and learning due to desire to know?
  - "Library science"

Empirical Advance

- To implement PEU fit psychological production function to get around “Lucas Critique”
- Standard choice data of possible value in fitting production function
  - Becker and Rubinstein study demand for "fear-related" goods after various attacks
  - Another study in the Jerusalem Housing Market

Non-Standard Data on Emotions

- Use of non-choice "psychological" data is challenging
- What are the relevant states? What produces them? How can they be measured?
  - Data on time use?
  - Eye tracking?
  - Self reports on affect?
  - Physiological measures and manipulations?
- Next time: how the challenge of testing and the flood of new data drove me to methodology