A Testable Theory of Imperfect Perception

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May 15, 2011. Preliminary and Incomplete

Abstract

Psychologists have long recognized that perceptions do not simply mirror reality, but may differ in systematic fashion. Economists are increasingly interested in incorporating this insight. We present a model of optimal decision making that has strong testable implications without in any way restricting perception. To do this, we impose rational expectations: decision makers are assumed to understand how their perceptions relate to choices and their consequences. The signature of the resulting theory is that stochastic choices must be "unimprovable", with the precise improvements that can be ruled out depending on the data available for model testing. Our model captures two distinct forms of presentation effect whereby one and the same choice set gives rise to distinct patterns of choice. While restrictive, our model allows for reasonable forms of stochastic choice that are inconsistent with stochastic utility models.

Key Words: Stochastic choice, Bounded Rationality, Imperfect Perception, Rational Expectations, Framing Effects, Nudges

1 Introduction

Psychologists from the time of Weber [1834] on have investigated the gap between subjective perceptions and external reality (Glimcher [2010]). Somewhat belatedly, economists are starting to incorporate imperfect perception into the standard model of optimal decision making. One factor that has impeded such research is that it is hard, if not impossible, to observe perceptions. It is

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also highly challenging to derive testable implications of imperfect perception for observed choices.[1] Current economic models overcome this limitation by making distinct yet strong assumptions on the nature of perceptual errors as resulting from, e.g: from particular bounds on memory; incomplete and selective attention; sequential search; particular decision making protocols and procedures; algorithmic approximations, etc. (e.g. Mullainathan [2002], Wilson [2002], Sims [2003], Manzini and Mariotti [2007], Rabin and Weizsacker [2009], Masatlioglu and Nakajima [2009], Gennaioli and Shleifer [2010], Gottlieb [2010], Schwartzstein [2010], Ergin and Sarver [2010], Caplin and Dean [2011], Compte and Postlewaite [2011]).

We introduce a general model that allows an essentially unlimited array of perceptual limitations to be introduced into the standard economic model of choice under uncertainty. We neither restrict perceptions nor attempt to observe them directly. Rather, our theory derives its testable implications from the assumption of rational expectations. We consider decision makers (DMs) who have come to understand how their perceptions relate to choices and their consequences. In the "rational expectations perception based representations" (RE-PREP) that we study, DMs fully internalize how their perceptions, whatever they may be, relate to choices and to their consequences. The signature of the resulting theory is that stochastic choices must be "unimprovable", with the precise improvements that can be ruled out depending on the data available for model testing.

Our main characterization result involves a data set that includes the fine details of the decision making environment, in particular detailed information about how the decision was presented to the DM (in a sense to be made precise). In this context, existence of a RE-PREP representation is equivalent to a "No Improving Switches" (NIS) condition: there must be a utility function such that no action can be switched for an alternative in a manner that raises utility. These restrictions correspond to the non-emptiness of the feasible set for a data-defined linear program. The sharpest tests requires non-standard choice data: in that sense this paper continues a line of research on choice theory and non-standard choice data (Caplin [2008], Caplin and Dean [2011] and Caplin, Dean, and Martin [forthcoming]).

Our model of perception and our focus on experimental tests draws strongly from the psychological tradition. In particular, our formulation of the act of choice as separate from reception of

[1] This is related to the difficulties in identifying "mistaken" decisions from choice data, and the dangers of so doing without strong theoretical guidance: see Koszegi and Rabin [2008], Bernheim and Rangel [2008] and Gul and Pesendorfer [2008].

a prize was inspired by simple choice experiments of the drift-diffusion form (see Ratcliff [1978], Ratcliff and McKoon [2008], and Shadlen and Newsome [2001]). This same separation is explicit in the experimental study by Reutskaja, Nagel, Camerer and Rangel [2011] of choice among food options represented by icons positioned on a computer screen. Framing effects were uncovered: the stochastic structure of demand was impacted by the position of prizes on the screen (Rubinstein and Salant [2006], Salant and Rubinstein [2008], Bernheim and Rangel [2008]).

Our model allows for two quite distinct framing effects. Both the layout of the prizes (e.g. order in a list or position on a screen) and the information content of the environment (e.g. the extent to which the first in the list has turned out to be the best) impact choice behavior. Hence our model allows rich study of how informational "nudges" impact attention and choice (Madrian and Shea [2001], Benartzi and Thaler [2004], Thaler and Sunnstein [2008], and Choi, Laibson, Madrian, and Metrick [2009]). We outline an experimental design to explore these effects, focusing particular attention on how information from the decision making environment interacts with perceptual effort to produce decision making mistakes.

While allowing for framing effects, our model has implications for more standard data set on stochastic choices. In this case, as in other theories of choice, the data restrictions apply across choice problems. We show that the restrictions imposed by our theory are weaker than those imposed by standard models of stochastic choice (Block and Marschak [1960], Luce [1959], McFadden [1973], Falmange [1978], Gul and Pesendorfer [2006]). For example, our theory covers cases in which perceptual similarities between a good and a bad prize result in the selection of a moderately good but more distinctive prize when all three are available (Debreu [1960] and Natenzon [2011]).[2]

In section 2 we introduce our formal model and define "RE-PREP" representations. In sections 3 and 4 we provide the precise observable restrictions associated with the model in cases in which the decision making environment can be observed with precision: the NIS conditions. In section 5 we consider the various framing effects that our model encompasses. A theoretically-inspired experimental design for understanding the sources and the nature of imperfect imperception is presented in section 6, together with highly preliminary findings. In section 7 we characterize the restrictions our model places on standard stochastic choice data and contrast these with those

[2] In that sense our results relate to the question posed some sixty years ago by Block and Marschak [1960] concerning how to distinguish stochastic choices resulting from decision making errors from those resulting from stochastic utility.

deriving from random utility models. Concluding remarks outlining other applications of our model are in section 8.

2 Decision Making and Perceptions

We study decisions in which the DM may not fully understand all objects of choice. This may be because there are many available options, because these items are complex, or because the DM is not able or willing to expend the effort to be thoroughly familiar with the available options. This is the world envisaged in models of bounded rationality at least since the pioneering work of Simon [1955].

2.1 Prizes, Actions, and Layouts

We assume that there is an "ideal observer" (IO) who understands fully the choice environment and who is distinct from the decision maker (DM) whose choice behavior is being modeled. The IO knows not only the key ingredients of the decision making environment the DM is facing but also observes the choices and their results. To distinguish the choice from the results of that choice, we separate in the model the act of choice from receipt of a prize. For the DM's choice to be seen by the IO as based on misperception requires actions and consequences to be separately observable.

In formal terms, our model involves a finite prize set X with generic element x_n, for $1 \leq n \leq N$: this fixes the physical presentation of each option once and for all, including its packaging, etc. There is a separate set Y of choices that has the same cardinality as the prize set,

$$|Y| = |X| = N \geq 2.$$

Each $y \in Y$ is to be physically interpreted as e.g. locations on a screen or lines in a list. A **layout** is a 1-1 and hence onto function $f : Y \to X$, with $\mathcal{F}$ the set of all such layouts,

$$\mathcal{F} = \{f : Y \to X | f \text{ is } 1-1\}.$$

It is the layout that connects actions with consequences.

2.2 Choice Environment

If a given action choice always produced the same prize, one would expect the DM to learn this, making the question of misperception moot. It is primarily when there is doubt about how actions get translated into prizes that there is room for misperception. We study a DM making many independent choices in a stable environment identified with a specific probability measure over layouts, $\mu \in \Delta(\mathcal{F})$. We refer to this triple as (X, Y, μ) as the "**choice environment**."

We define $\mathcal{F}(\mu) \subset \mathcal{F}$ as the support of μ. In principle, the measure μ provides the subject a statistical sense of what tends to happen when a given action is selected. DM choices may be influenced both by this information and by information that they derive from their effort to accurately perceive the available options.

2.3 Experimental Data

Given $\mu \in \Delta(\mathcal{F})$ an **experimental data set**, P, identifies the probability distribution over feasible actions as it depends on the layout,

$$P : \mathcal{F} \longrightarrow \Delta(Y).$$

This entire function is assumed to be known to the IO and provides the data that a theory of choice must explain.[3] It is the quadruple (X, Y, μ, P) that fully identifies the objects upon which our theory of perception is built. We call such a quadruple an "**observed choice environment**" (OCE).

Our model allows the structure of observed choices to be dependent on the layout. For example, we allow for situations of search in which the first item in a list is most likely to be the best, which may induce default behavior of selecting this option. In such cases, the stochastic structure of choice is strongly impacted by which object is put first in the list. For that reason, a test of our theory requires that the set of choices Y is identifiable to the IO, and that the choice environment μ is also understood. Such knowledge is most naturally derived in an experiment, and plays a key role in the experimental design of section 6. In section 7 we consider cases in which the layout is unobservable, and the only available data relates to stochastic choice of prize.

[3] The domain of this data does not allow the order of the experimental runs to be recorded, hence the explanatory hypotheses we consider will not involve learning the structure of the experiment.

2.4 Subjective States and the Perception Function

We take as given an OCE sets (X, Y, μ, P) and look to structure perceptual theories of choice based on three elements: a **subjective state space**, S; a **perception function**, $\rho : \mathcal{F}(\mu)\rightarrow\Delta(S)$; and a **choice function**, $C : S(\rho) \rightarrow Y$, where $S(\rho) \subset S$ is the union across $f \in \mathcal{F}(\mu)$ of the supports of $\rho(f)$.

The set S specifies all possible subjective mental states of the DM, and is intended as a sufficient statistic for the information that the DM has extracted from the choice environment and the specific layout in front of them. In general, S may encode characteristics or facets of the good, state of a finite automaton, etc. In the current context it will be sufficient to consider the set S to comprise the space of "subjective prize lotteries" with finite support $S = \Delta(X)^M$,

$$S = \{s \in \mathbb{R}_+^{MN} | \sum_{n=1}^{N} s_{mn} = 1 \text{ for } 1 \leq m \leq M\}.$$

The reason we can so limit the subjective state space is that the behavior we are characterizing is based on expected utility theory, for which lotteries over prizes are the appropriate objects of choice. One can array $s \in S$ as an $M \times N$ matrix with row m identifying the lottery over prizes associated with action y_m.

The perception function maps the actual deterministic layout, as specified by $f \in \mathcal{F}(\mu)$, into the probability distributions over subjective states with finite support, $\Delta(S)$. Given the finite support, the set $S(\rho)$ itself, the set of lottery states that are perceived as possible in the given experiment, is finite, and each state $s \in S(\rho)$ has strictly positive probability.

2.5 RE-PREPs

For (ρ, C) to be a possible explanation (perception-based representation or PREP) of the observed data requires that, when combined with the perception of the evidence $\rho : E\rightarrow\Delta(S)$, C generates the observed experimental data set. We are interested only in such explanations in which the DM is a Bayesian EU maximizer with rational expectations. We consider whether or not there is some utility function $U : X \rightarrow \mathbb{R}$ that can rationalize the data as a result of optimization based on rational expectations. Note the insistence in the technical definition on strictness in the utility comparison of some pair of acts in some state of mind, to prevent the conditions from being trivially satisfied by a utility function in which all utilities are identical.

Definition 1 (ρ, C, U) *form a* ***rational expectations perception-based representation*** *(RE-PREP) of* (X, Y, μ, P) *if:*

1. *Data Matching:* $P^f(y) = \rho^f\{C^{-1}(y)\}$ *for all* $f \in F$ *and* $y \in Y$*.*

2. *Rational Expectations: For all* $1 \leq m \leq M$*,* $1 \leq n \leq N$*, and* $s \in S(\rho)$*,*
$$s_{mn} = \frac{\sum_{\{f \in \mathcal{F} | f(y_m) = x_n\}} \mu(f) \rho^f(s)}{\sum_{\{f \in \mathcal{F}\}} \mu(f) \rho^f(s)}.$$

3. *Optimization: If* $C(s) = y_m$*, then,*
$$\sum_{n=1}^{N} s_{mn} U(x_n) \geq \sum_{n=1}^{N} s_{kn} U(x_n) \text{ all } 1 \leq k \leq M,$$
with the inequality being strict for some pair k, m *with* $1 \leq k, m \leq M$*.*

Note that if there is only one action taken,

$$C(s) = y \text{ all } s \in S,$$

then it is trivial to find a non-trivial RE-PREP by setting y as the uniquely optimal choice. Hence we will consider only cases in which there are at least two distinct actions taken with strictly positive probability.

2.6 Rational Expectations and Experimentation

The most distinctive aspect of the RE-PREP is rational expectations, which, when combined with data matching, implies that the DM is aware of the consequences of all choices in each state of mind. In a RE-PREP, the statistical association between subjective perceptions, choices, and prizes is assumed to have been internalized by the DM. Implicitly, this is based on the idea that this is a familiar environment and that the DM has learned through a process of trial and error what the results are when any given choice is made in any particular state of mind. Having thus experimented, they select an optimal such action, and the consequences are then at least acceptable, in the sense that they do not perceive there to be any superior choice. The full formal confirmation of this intuitive description of the axioms is formalized in our representation theorem.

As is often the case, the application of rational expectations logic is easiest to justify as the end result of an unmodeled and unobserved process of experimentation. However, in the actual definition, a specific deterministic action is taken in each state of mind, and there is no experimentation whatever. The exact process of experimentation or decision making tremble that would produce rational expectations and thereby rationalize the strong informational assumption that the RE-PREP imbeds is not modeled.

3 Testability: the $2 \times 2 \times 2$ Case

The question we address in the next two sections is how to characterize conditions on the EDS equivalent to existence of a RE-PREP for a given μ. Before introducing the general characterization theorem in the next section, we provide in this section a thorough analysis of the $2 \times 2 \times 2$ case, with $X = \{x_1, x_2\}$; $Y = \{y_1, y_2\}$;and with two possible decision states, $\mathcal{F} = \{g, h\}$,

$$g(y_1) = h(y_2) = x_1; g(y_2) = h(y_1) = x_2.$$

In this setting, two parameters p_1^g, $p_1^h \in [0, 1]$ identify the EDS,

$$\begin{aligned} (P_1^g(y_1), P_1^g(y_2)) &= (p_1^g, 1 - p_1^g); \\ (P_1^h(y_1), P_1^h(y_2)) &= (p_1^h, 1 - p_1^h); \end{aligned}$$

Throughout this section, we will look for RE-PREPs using two perceptual states,

$$S = \{s^1, s^2\}$$

with,

$$s_{11}^i, s_{21}^i \in [0, 1], \; i = 1, 2,$$

denoting the probability in the subjective lottery states s^1, s^2 that actions (y_1, y_2) will yield prize x_1 rather than prize x_2, with,

$$s_{12}^i = 1 - s_{11}^i, \; s_{22}^i = 1 - s_{21}^i,$$

representing the corresponding probabilities for prize x_2.

Figures 1 to 3 illustrate the logic of this simple $2 \times 2 \times 2$ case. We prove in the next section that adding more states than actions does not expand feasibility, so that the analysis of this section is complete.

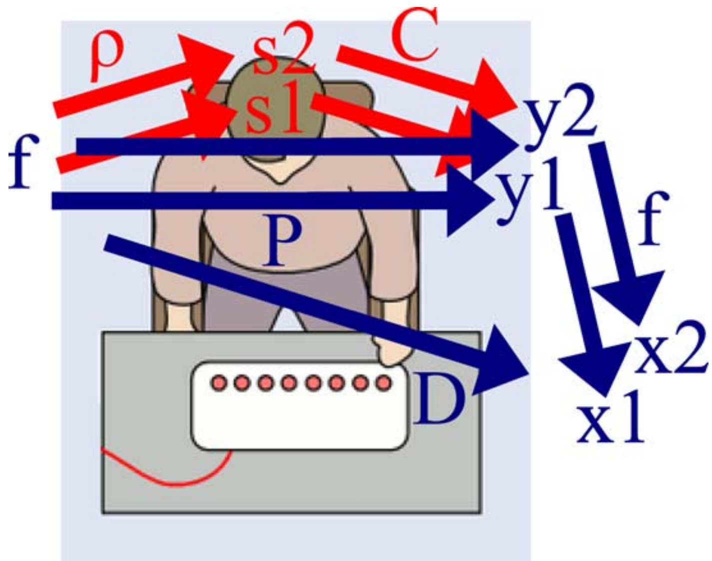

Figure 1: $2 \times 2 \times 2$ case

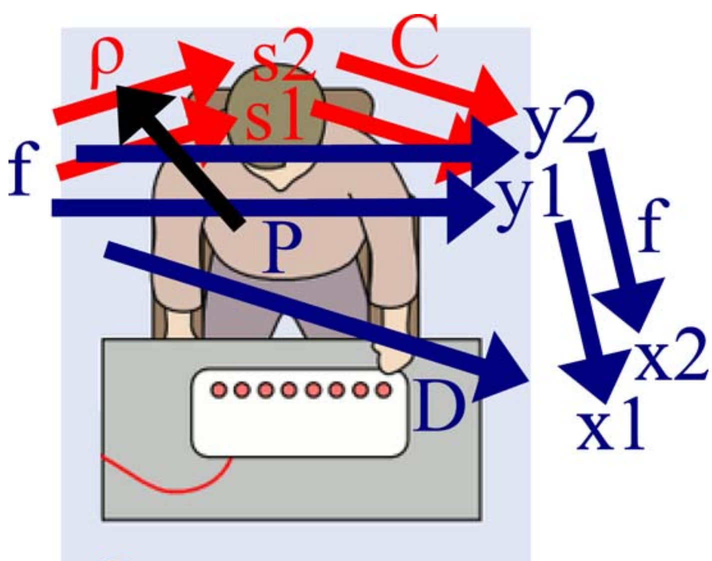

Figure 2: Data matching in $2 \times 2 \times 2$ case

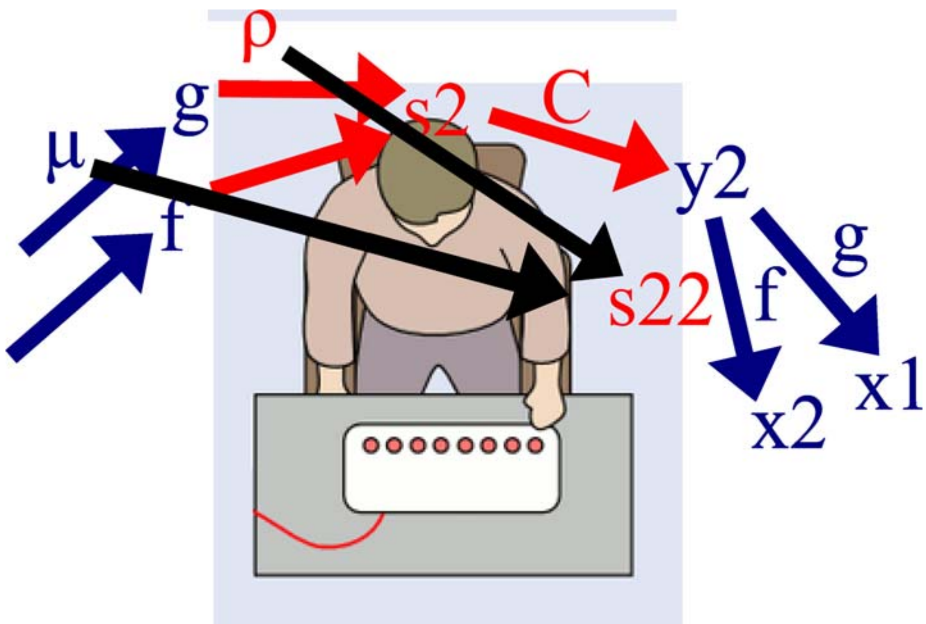

Figure 3: Rational expectations in $2 \times 2 \times 2$ case

3.1 A Symmetric Case

We start with a case in which each choice is equally likely to yield the better prize,

$$\mu(g) = \mu(h) = 0.5.$$

We suppose also that the EDS is frame-free and that y_1 is chosen strictly more frequently,

$$p_1^g = (1 - p_1^h) > 0.5.$$

1. We identify the more frequently chosen object, x_1, as having higher utility,

$$U(x_1) = 1 > U(x_2) = 0.$$

2. We specify subjective lottery states as follows:

$$\begin{aligned} s_{11}^1 &= s_{21}^2 = p_1^g > 0.5; \\ s_{21}^1 &= s_{11}^2 = 1 - p_1^g = p_1^h < 0.5. \end{aligned}$$

Intuitively, subjective state s^1 signals that action y_1 is more likely than is action y_2 to yield prize x_1 and vice versa for s^2.

3. We specify that the action is taken that is more likely to yield the better prize: y_1 in state s^1, y_2 in state s^2;

$$C(s^i) = y_i,\ \ i = 1, 2.$$

4. Finally we specify that the probability that when the act-prize function is g (so that action y_1 will indeed yield x_1) the probability of the state s^1, indicating that y_1 is more likely to yield x_1, precisely matches the true stochasticity associated with choice of y_1 revealed in the experimental data set. The corresponding statement holds for h and its association with state s^2,

$$\rho^g(s^1) = \rho^h(s^2) = p_1^g \Longrightarrow \rho^g(s^2) = \rho^h(s^1) = 1 - p_1^g = p_1^h.$$

It is straightforward to confirm that this represents a RE-PREP by checking the three conditions.

- Data Matching: By construction,

$$\begin{aligned} \rho^g\{C^{-1}(y_1)\} &= \rho^g(s^1) = \rho^h\{C^{-1}(y_2)\} = \rho^h(s^2) = p_1^g; \\ \rho^g\{C^{-1}(y_2)\} &= \rho^g(s^2) = \rho^h\{C^{-1}(y_1)\} = \rho^h(s^1) = p_1^h. \end{aligned}$$

- Rational Expectations: Note first that the states $s = \alpha, \beta$ are equiprobable

$$\mu(g)\rho^g(s^1) + \mu(h)\rho^h(s^1) = \mu(g)\rho^g(s^2) + \mu(h)\rho^h(s^2) = 0.5.$$

  Hence the rational expections conditions hold by construction:

$$\begin{aligned} s_{11}^1 &= 2\mu(g)\rho^g(s^1) = p_1^g; \\ s_{21}^1 &= 2\mu(h)\rho^g(s^1) = 1 - p_1^g; \\ s_{11}^2 &= 2\mu(h)\rho^h(s^2) = \rho^h(s^1) = 1 - p_1^g; \\ s_{21}^2 &= 2\mu(g)\rho^h(s^2) = p_1^g. \end{aligned}$$

- Optimization: In state s^1 it is strictly better to select y_1 than y_2, since this yields the lottery with the higher chance of yielding prize x,

$$s_{11}^1 U(x_1) + (1 - s_{11}^1)U(x_2) = p_1^g > 1 - p_1^g = s_{21}^1 U(x_1) + (1 - s_{21}^1)U(x_2).$$

  This same inequality implies that it is strictly better to select y_2 than y_1 in state s^2.

3.2 Asymmetric Cases

Given $p_1^g, p_1^h, \mu^g \in (0,1)$ we again consider conditions for a RE-PREP involving two perceptual states, $S = \{s^1, s^2\}$, with $C(s^i) = y_i$, $i = 1,2$ and with x_1 having higher utility,

$$U(x_1) = 1 > U(x_2) = 0.$$

We analyze necessary conditions for a RE-PREP in stages, and then establish sufficiency. Finally, we note the symmetric conditions that apply for RE-PREPs in which x_2 has higher utility.

- Data Matching: We list all four necessary conditions:

$$\begin{aligned}
\rho^g\{C^{-1}(y_1)\} &= \rho^g(s^1) = p_1^g;\\
\rho^g\{C^{-1}(y_2)\} &= \rho^g(s^2) = 1 - p_1^g;\\
\rho^h\{C^{-1}(y_1)\} &= \rho^h(s^1) = p_1^h;\\
\rho^h\{C^{-1}(y_2)\} &= \rho^g(s^2) = 1 - p_1^h.
\end{aligned}$$

- Rational Expectations: It is necessary that,

$$s_{11}^i = \frac{\mu^g \rho^g(s^i)}{\mu^g \rho^g(s^i) + (1-\mu^g)\rho^h(s^i)} = s_{22}^i, \; i = 1,2.$$

- Optimality yields,

$$s_{11}^1 \geq s_{21}^1 \text{ and } s_{21}^2 \geq s_{11}^2,$$

  with at least one strict.

Substitution of DM in RE yields

$$\begin{aligned}
s_{11}^1 &= \frac{\mu^g p_1^g}{\mu^g p_1^g + (1-\mu^g)p_1^h} = s_{22}^1;\\
s_{11}^2 &= \frac{\mu^g(1-p_1^g)}{\mu^g(1-p_1^g) + (1-\mu^g)(1-p_1^h).} = s_{22}^2
\end{aligned}$$

Substitution in Optimality yields,

$$s_{11}^1 \geq \frac{1}{2} \text{ and } s_{22}^2 \leq \frac{1}{2},$$

with at least one strict. Substitution for s_{11}^1 yields,

$$\begin{aligned}
2\mu^g p_1^g &\geq \mu^g p_1^g + (1-\mu^g)p_1^h;\\
p_1^g &\geq \frac{p_1^h(1-\mu^g)}{\mu^g} = \frac{p_1^h}{R};
\end{aligned}$$

where,

$$R \equiv \frac{\mu^g}{1-\mu^g}.$$

Substitution for s_{22}^2 yields,

$$\begin{aligned}
2\mu^g(1-p_1^g) &\leq \mu^g(1-p_1^g)+(1-\mu^g)(1-p_1^h); \\
1-p_1^g &\leq \frac{(1-p_1^h)(1-\mu^g)}{\mu^g}=\frac{1-p_1^h}{R}; \\
p_1^g &\geq \frac{R-1}{R}+\frac{p_1^h}{R}
\end{aligned}$$

Looking at these two necessary conditions, there are three cases depending on the value of R:

- If $R=1$ ($\mu^g=\frac{1}{2}$), then in order for both inequalities to hold, one strictly, it is necessary and sufficient that,

$$p_1^g > p_1^h.$$

- If $R>1$ ($\mu^g>\frac{1}{2}$), then the only constraint is,

$$p_1^g \geq \frac{R-1}{R}+\frac{p_1^h}{R}.$$

- If $R<1$ ($\mu^g<\frac{1}{2}$), the only constraint is,

$$p_1^g \geq \frac{p_1^h}{R}.$$

The conditions in the case in which x_2 has strictly higher utility are readily seen to be precisely the converse. Combining the two permissible utility functions, we arrive at the following necessary and sufficient conditions:

- If $R=1$ ($\mu^g=\frac{1}{2}$), then,

$$p_1^g \neq p_1^h.$$

- If $R>1$ ($\mu^g>\frac{1}{2}$), then the constraints are,

$$p_1^g \leq \frac{p_1^h}{R} \text{ or } p_1^g \geq \frac{R-1}{R}+\frac{p_1^h}{R}.$$

- If If $R<1$ ($\mu^g<\frac{1}{2}$), then the constraints are,

$$p_1^g \leq \frac{R-1}{R}+\frac{p_1^h}{R} \text{ or } p_1^g \geq \frac{p_1^h}{R}.$$

3.3 Three Examples

Example 1 *Assume* $\mu^g = \frac{1}{2}$, $U(x_1) = 1 > U(x_2) = 0$. *In this case the necessary and sufficient condition for a RE-PREP is that* $p_1^g > p_1^h$. *This implies that, in an unconditional sense, action* y_1 *is strictly more likely to yield prize* x_1 *than it is to yield prize* x_2. *Conversely, this also implies that* $p_2^h > p_2^g$, *so that action* y_2 *is also strictly more likely to yield prize* x_1 *than it is to yield prize* x_2. *The converse condition* $U(x_2) = 1 > U(x_1) = 0$ *has as a necessary and sufficient condition that* $p_1^g < p_1^h$. *This implies that, in an unconditional sense, action* y_1 *is strictly more likely to yield prize* x_2 *than it is to yield prize* x_1. *Conversely, this also implies that* $p_2^h < p_2^g$, *so that action* y_2 *is also strictly more likely to yield prize* x_2 *than it is to yield prize* x_1. *The only case that is ruled out is when each act yields each prize with equal probability. Here only complete ignorance can justify, so that any two states of mind would involve equal probability, reducing us to the trivial case that is removed by definition.*

Example 2 *Consider the case with* $\mu^g = 0.8$ *so that* $R = 4$. *If* $U(x_1) = 1 > U(x_2) = 0$, *the condition is,*

$$p_1^g \geq 0.75 + 0.25 p_1^h.$$

Conversely, if $U(x_2) = 1 > U(x_1) = 0$, *the condition is,*

$$p_1^g \leq 0.25 p_1^h.$$

We draw these constraint sets in figure 4.

Example 3 *The model can characterize a reversal of apparent preference, as when stochastic demand is,*

$$p_1^g = 0.9; p_1^h = 0.7,$$

so that x_1 *is chosen with probability 0.9 with* g, *yet only with probability 0.3 in* h. *To construct a RE-PREP in this case, set* $U(x_1) = 1 > U(x_2) = 0$ *and allow for a mental state* s^1 *in which* y_1 *is taken and state* s^2 *in which* y_2 *is taken. Since mental state 1 is equivalent to action 1 in the RE-PREP representation, rational expecations implies that,*

$$\rho^g(s^1) = 0.9 \text{ and } \rho^h(s^1) = 0.7,$$

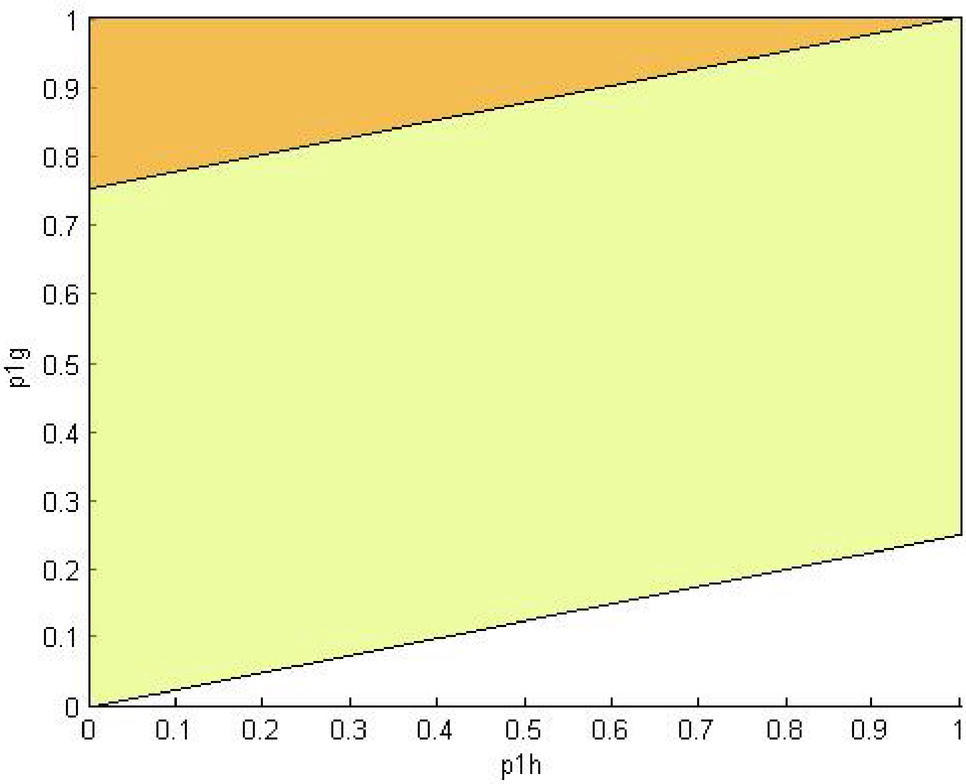

Figure 4: Constraints implied by R=4 (interior region is infeasible)

to reflect the empirically observed frequencies with which action y_1 is chosen. This implies that the lottery associated with choice of y_1 in s^1 is yields prize x_1 with probability,

$$\frac{0.9}{1.6} > 0.5,$$

Similarly, the prize lottery associated with action y_2 in s^2 yields prize 1 with probability,

$$\frac{\rho^h(s^2)}{\rho^g(s^2) + \rho^h(s^2)} = \frac{0.3}{0.4} > 0.5.$$

We can complete the specification of a RE-PREP by applying the rational expectations condition to set the probabilities for the untaken lotteries in line with what they would actually be,

$$\begin{aligned} s^1_{21} &= \frac{\rho^h(s^1)}{\rho^g(s^1) + \rho^h(s^1)} = \frac{0.7}{0.7 + 0.9} < 0.5; \\ s^2_{11} &= \frac{\rho^g(s^2)}{\rho^g(s^2) + \rho^h(s^2)} = \frac{0.1}{0.1 + 0.3} < 0.5; \end{aligned}$$

Hence it is strictly optimal to pick y_1 is s^1 and y_2 is s^2, completing the confirmation that this is a RE-PREP.

4 No Improving Switches

4.1 Action Switches

There is a simple idea that organizes the conditions identified in the $2 \times 2 \times 2$ case and that applies to the general case. The idea involves first switching attention from the action that is chosen to the prize that results. Consider in this light the third example above with $\mu^g = 0.8$ so that $R = 4$, and suppose that $p_1^g = 0.75$ and $p_1^h = 0$, so that $p_2^g = 0.25$ and $p_1^h = 0$. In this case, whenever action 1 is taken the layout is g, so that it yields prize 1 for sure. On the other hand, act y_2 is taken both when it yields prize x_1 and when it yields prize x_2. To understand the stochastic choice resulting from action y_2, note that this action is taken with probability

$$\mu^g p_2^g + (1 - \mu^g) p_2^h = 0.8(0.25) + 0.2 = 0.4,$$

yielding x_1 only in h, for an overall even chance of winning prize x_1 as opposed to prize x_2.

There are two general points that this example illustrates. First, note that the overall probability of getting the good prize is 1 in the 60% of cases in which action y_1 is taken, and 0.5 in the 40%

of cases in which action y_2 is taken, for a net probability of 0.8. It is intuitive that there cannot be a RE-PREP with a lower such probability, since the simple strategy of picking y_1 always yields this outcome. But there is one other observation that explains the constraints with more precision, which is that the chance of getting the better prize from action y_2 cannot be strictly less than 50%. Intuitively, if it were to be thus, then it would be strictly superior to switch action y_2 to y_1 and always to take the latter action. It is this insight that characterizes a RE-PREP in the general case.

First, note that the expected utility when action y_1 is taken in a setting with $U(x_1) = 1 > U(x_2) = 0$ is the probability that it is taken in state 1 as against the overall probability with which y_1 is taken,

$$\frac{\mu^g p_1^g}{\mu^g p_1^g + (1-\mu^g)p_1^h}.$$

The probability of receiving the good prize if action y_1 was replaced whenever it was chosen by action y_2 would be reversed,

$$\frac{(1-\mu^g)p_1^h}{\mu^g p_1^g + (1-\mu^g)p_1^h}.$$

Hence the condition that this not yield an improvement is precisely,

$$\mu^g p_1^g \geq (1-\mu^g)p_1^h;$$

or,

$$p_1^g \geq \frac{p_1^h}{R}.$$

In like manner, the condition that the expected utility when action y_2 is taken in a setting with $U(x_1) = 1 > U(x_2) = 0$ exceeds that from switching to action y_1 is,

$$(1-\mu^g)p_2^h \geq \mu^g p_2^g;$$

or,

$$(1-\mu^g)(1-p_1^h) \geq \mu^g(1-p_1^h),$$

and finally,

$$p_1^g \geq \frac{R-1}{R} + \frac{p_1^h}{R}.$$

The fact that these are identical constraints is not coincidental.

4.2 Characterization

The key observation is that a necessary condition for a RE-PREP is that there be some utility numbers such that it is better to take an action than it is to switch it to some fixed alternative in all situations in which it was used. We also establish sufficiency of this condition, which corresponds to non-emptiness of a set of $(N-1)^2$ linear inequalities.

Definition 2 *Given (X, Y, μ, P), the stochastic demand associated with choosing y_k in place of y_m, $\delta_{mk} \in \Delta(X)$ is defined as*

$$\delta_{mk}(x_n) = \frac{\sum_{\{f \in \mathcal{F} | f(y_k) = x_n\}} \mu(f) P^f(y_m)}{\sum_{f \in \mathcal{F}} \mu(f) P^f(y_m)}.$$

*We say that the **NIS inequalities** are satisfied by a utility function $U : X \to \mathbb{R}$ if, for all $1 \leq k, m \leq N$,*

$$\sum_{n=1}^{N} [\delta_{mm}(x_n) - \delta_{mk}(x_n)] U(x_n) \geq 0,$$

Note that $\delta_{mm} \in \Delta(X)$ is the stochastic demand associated with choice of y_m in the EDS.

Theorem 4 *(X, Y, μ, P) have a RE-PREP representation if and only if there exists $U : X \to \mathbb{R}$ that satisfies the NIA inequalities.*

Proof. Sufficiency. Supposs that there one state of mind s^j per action that is taken with strictly positive probability (**for now assume all acts**), with,

$$C(s^j) = y_j.$$

To satisfy data matching requires that,

$$\rho^f\{s^j\} = P^f(y_j),$$

all $f \in \mathcal{F}(\mu)$ and $1 \leq j \leq M$. We substitute this into the rational expectations condition to solve for all s^j_{mn}: for all $1 \leq m \leq M$, $1 \leq n \leq N$, and $s^j \in S$,

$$s^j_{mn} = \frac{\sum_{\{f \in \mathcal{F} | f(y_m) = x_n\}} \mu(f) P^f(y_j)}{\sum_{f \in \mathcal{F}} \mu(f) P^f(y_j)}.$$

We now pick $U : X \to \mathbb{R}$ such that, for all $1 \le k, m \le M$,

$$\sum_{n=1}^{N} [\delta_{mm}(x_n) - \delta_{mk}(x_n)] U(x_n) \ge 0.$$

By construction (ρ, C, U) satisfy data matching and rational expectations. To establish that they identify a RE-PREP it remains only to show that, for all $1 \le m \le M$,

$$\sum_{n=1}^{N} s_{mn}^{m} U(x_n) \ge \sum_{n=1}^{N} s_{kn}^{m} U(x_n) \text{ all } 1 \le k \le M,$$

with the inequality being strict for some pair k, m with $1 \le k, m \le M$. We substitute for s^m as in (*) and the required condition becomes,

$$\sum_{n=1}^{N} \frac{\sum_{\{f \in \mathcal{F} | f(y_m) = x_n\}} \mu(f) P^f(y_m)}{\sum_{f \in \mathcal{F}} \mu(f) P^f(y_m)} U(x_n) \ge \sum_{n=1}^{N} \frac{\sum_{\{f \in \mathcal{F} | f(y_k) = x_n\}} \mu(f) P^f(y_m)}{\sum_{f \in \mathcal{F}} \mu(f) P^f(y_m)} U(x_n),$$

or,

$$\sum_{n=1}^{N} \delta_{mm}(x_n) U(x_n) \ge \sum_{n=1}^{N} \delta_{mk}(x_n) U(x_n),$$

which holds by assumption, in addition with one strict by construction.

Necessity: The above shows that identifying a utility function that satisfies the NIA inequalities is not only sufficient for a RE-PREP, but also necessary for a RE-PREP in which there is only one state of mind per action. The full result folllows from the observation that if we identify a RE-PREP with more than one mental state per decision, then there must exist a RE-PREP with only one state per action: hence allowing for multiple states does not expand the set of EDS for which a RE-PREP exists. To confirm this, suppose that we have identified a RE-PREP (ρ, C, U) of $\mu \in \Delta(\mathcal{F})$ and $P \in$P with the property that there is more than one state such that some given action is taken: say there exists $y \in Y$ and states $s^p \in S(\rho)$ for $1 \le p \le P$ with $P \ge 2$ such that,

$$C(s^p) = y \text{ all } p.$$

Then we can define a new RE-PREP $(\tilde{\rho}, \tilde{S}, \tilde{C}, \tilde{U})$ that is identical to (ρ, C, U) except in three respects: $C^{-1}(y)$ is a singleton with a single state $\tilde{s} \in S$ replacing all s^p; $\rho^f\{\tilde{s}\} = \sum_1^P \rho^f(s^p)$; and

$$\tilde{s}_{mn} = \frac{\sum_1^P \sum_{\{f \in \mathcal{F} | f(y_m) = x_n\}} \mu(f) \rho^f(s^p)}{\sum_1^P \sum_{\{f \in \mathcal{F}\}} \mu(f) \rho^f(s^p)}.$$

To complete the proof, note that this satisfies data matching and rational expectations by construction, as well as optimality,

$$\sum_{n=1}^{N} s_{mn}^{p} U(x_n) \ge \sum_{n=1}^{N} s_{kn}^{p} U(x_n) \ \text{ all } p \Longrightarrow \sum_{n=1}^{N} \tilde{s}_{mn} U(x_n) \ge \sum_{n=1}^{N} \tilde{s}_{kn} U(x_n) \text{ all } 1 \le k \le M.$$

■

5 Two Framing Effects

In most studies in economics and psychology the primary interest is in how likely is each prize to be received as a result of the choices that are made, rather than how often each particular physical location that contains a prize is selected.

Definition 3 *Given (X, Y, μ, P), the* ***stochastic choice function*** $D : \mathcal{F} \longrightarrow \Delta(X)$ *is defined as,*

$$D^f(x) \equiv P^f \{y \in Y | f(y) = x\}.$$

5.1 Layout Effects

The characterization theorem makes clear that the layout may have a significant impact on demand: all that is necessary is that the NIS conditions are satisfied. We now go over an example consistent with the NIS conditions in which there is a particular clarity about the object associated with action 1, for example, which makes it more likely per se to be chosen (as in Geng [2011]).

Example 5 *Suppose that the prizes comprise amounts of money, but that the precise value of the prize is hard to identify.To be concrete, suppose that there are 10 prizes, one of each value \$1 through \$10, and that each of the ten choices is ex ante equally likely to yield each prize. Technically,*

$$\begin{aligned} X &= \{\$1, \$2, ..., \$10\}; \\ Y &= \{y_n | 1 \leq n \leq 10\}; \\ \mu(f) &= \frac{1}{10!} \text{ all } f \in \mathcal{F}. \end{aligned}$$

Now suppose that the process of perception is such that the prize corresponding to action 1 is always seen with perfect clarity, while the other actions are not understood beyond the prior and the noticed identify of the prize corresponding to y_1. In this case, there are 10 subjective states,

$$S(\rho) = \{(x, \Lambda_{-x}) \in X \times \Delta(X)^9 |\},$$

where Λ_{-x} corresponds to $\mu(f)$ conditioned on $y_1 = x$. Even if the DM is risk neutral, it is immediate that action y_1 will be chosen whenever it is seen to contain a prize of \$6 or above, which *is a 50% chance. With risk aversion, this chance will be strictly higher than 50%. Note also that the conditional distribution of prizes based on the selection of action 1 will be entirely different than that associated with any action y_n for $n > 1$.*

5.2 Blind Choice and Symmetry

In certain cases it may be held that the layout is unlikely to impact stochastic choice. Indeed this has been the traditional if implicit assumption in models of stochastic choice. Even when allowance is made for possible confusion, it is typically treated as resulting from similarities between product characteristics (e.g. Natenzon [2011]) and as such as being independent from where exactly prizes are laid out.

Definition 4 (X, Y, μ, P) *is* ***layout-independent*** *if there exists* $D \in \Delta(X)$ *such that,*

$$D^f(x) = D(x) \text{ all } x \in X.$$

Even in the layout-free case, the choice environment may impact stochastic choice. In a RE-PREP, the DM has fully understands the statistical association between subjective perceptions, choices, and prizes. For example, if the first option is a list is always the best, then this is internalized. Hence the DM is able to pick the best object effortlessly by relying on the information content in the decision making environment. This means that existence of a RE-PREP representation is far more restrictive when there is a highly asymmetric choice environment such that a particular prize is always associated with the same physical action. It is far less restrictive in situations in which there is greater symmetry, so that each action is approximately equally likely to be associated with each prize.

In technical terms, overall average expected utility computed with any utility function $U : X \to \mathbb{R}$ that is a component of a RE-PREP representation (ρ, C, U) is always above that available from "blind choice" of $y \in Y$ across all $f \in \mathcal{F}$. The following definition expresses this formally.

Definition 5 *Given* (X, Y, μ) *and* $U : X \to \mathbb{R}$, P *satisfies the* ***No Increasing Blind Strategy*** *(NIBS) Property w.r.t.* (X, Y, μ, U) *if,*

$$\sum_{f \in \mathcal{F}} \sum_{x \in X} D^f(x) U(x) \geq \max_{y \in Y} \sum_{f \in \mathcal{F}} \sum_{x \in X} \mu\{f \in \mathcal{F} | f(y) = x\} U(x).$$

Theorem 1 implies that a necessary condition for (X, Y, μ, P) to have a RE-PREP representation is that there exist $U : X \to \mathbb{R}$ such that P satisfies the NIBS Property w.r.t. (X, Y, μ, U). To prove this, one uses any $U : X \to \mathbb{R}$ for which a RE-PREP exists, noting that if the NIBS property did not hold, then switching always to choice $y \in Y$ whenever it is not chosen would yield a strict increase in utility, contrary to the NIA condition.

Necessity of the NIB constraint implies that the permissible stochastic structure of demand is a function of the information contained in the EDS. For example, when each choice is deterministically associated with a given prize, so that μ assigns probability 1 to a particular element $f \in \mathcal{F}$, the NIB constraint implies that demand must be completely concentrated on goods of maximal utility to be consistent with the model. On the other extreme, the NIBS property is trivially satisfied when there is a complete symmetry in the layout so that μ assigns equal probability to all $f \in \mathcal{F}$. More broadly, one can develop a partial order over the information content of various choice environments $\mu \in \Delta(\mathcal{F})$ based on the convex hull of the lotteries that are delivered by all pure choice strategies, which identifies the NIB constraints. As this convex hull expands, so the set of stochastic demands consistent with the model becomes more restricted. In section 7 we show that, if we treat the layout dependent nature of stochastic choice as unobservable, there is a sense in which the NIBS constraint is not only necessary but also sufficient for existence of a RE-PREP.

6 Nudges, Attention, and Mistakes

The above indicates the key role that the choice environment plays in impacting stochastic choices when our model applies. In fact the key issue is how much additional perceptual effort is put in over and above that which the choice environment provides in an essentially effort-free manner. In applications, this may be the key comparative static: the connection between effortless extraction and the effort required to understand the situation yet better and therefore make a more fully informed choice. We have designed an experiment aimed at just this issue.

If individuals possess imperfect perception, the effect of policy "nudges" on choice quality is ambiguous. Although nudges may provide beneficial information on which layout f individuals are facing through their knowledge of the choice environment μ (i.e. the default retirement plan is likely to be the best plan), nudges may cause subjects to decrease how much effort they put into understanding which layout f they face through perception ρ (i.e. put less time into considering

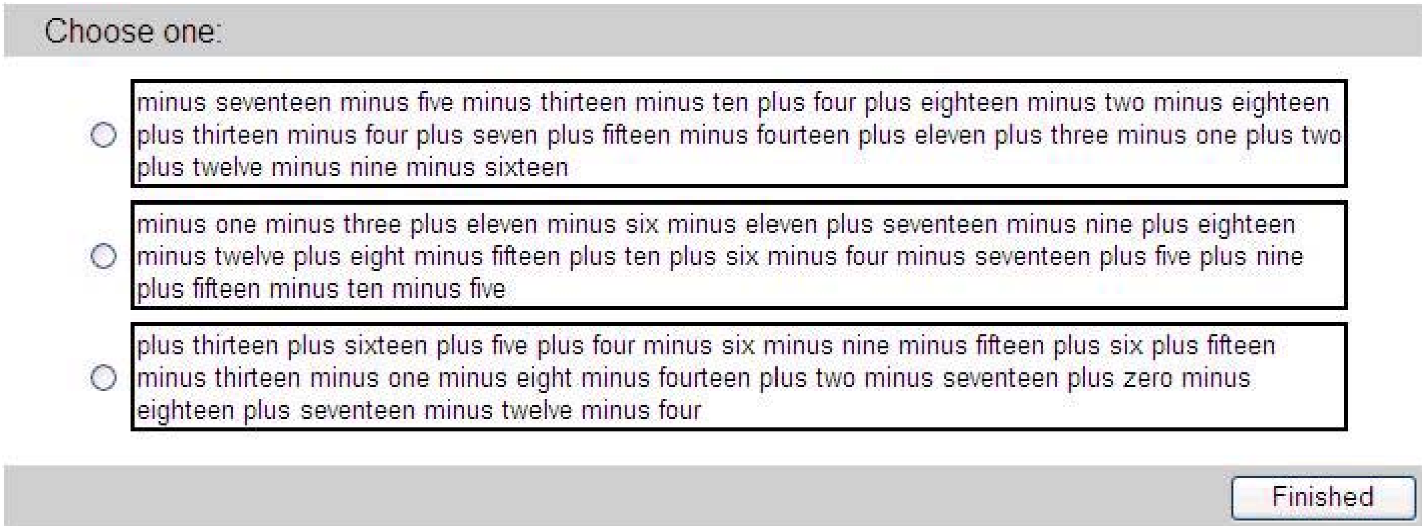

Figure 5: A typical round

each plan).

We have initiated experiments to study this tradeoff experimentally in a setting where imperfect perfection seems highly likely and choice quality is easy to measure. In each round, subjects are presented with three options, each of which is composed of 20 numbers. Figure 5 shows a screen capture of a typical round. The value of each option is the sum of the 20 numbers, and subjects are incentivized to select the object with the highest value. In the baseline treatment ("33%, 33%, 33%"), subjects were informed that all three options were equally likely to be the highest valued option, but in two other treatments, they were nudged towards the first option. In one of the nudge treatments ("40%, 30%, 30%"), subject were informed that the first option was 40% likely to be the highest valued option (the other two were both 30% likely). In the other nudge treament ("45%, 27.5%, 27.5%"), subjects were told that the first option was 45% likely to be the highest valued option (the other two were both 27.5% likely). Subjects completed 12 rounds of each treatment, which were presented in a random order.

The experiment was run with 25 subjects in the CESS laboratory. On average, subjects choose the best (highest valued) option in 54% of rounds, which is higher than the expected percentage under random choice but far below that associated with perfect perception. The fraction of rounds in which the highest valued option is chosen is stable over the course of the experiment – the fraction in the first half of rounds is not significantly different that the fraction in the second half

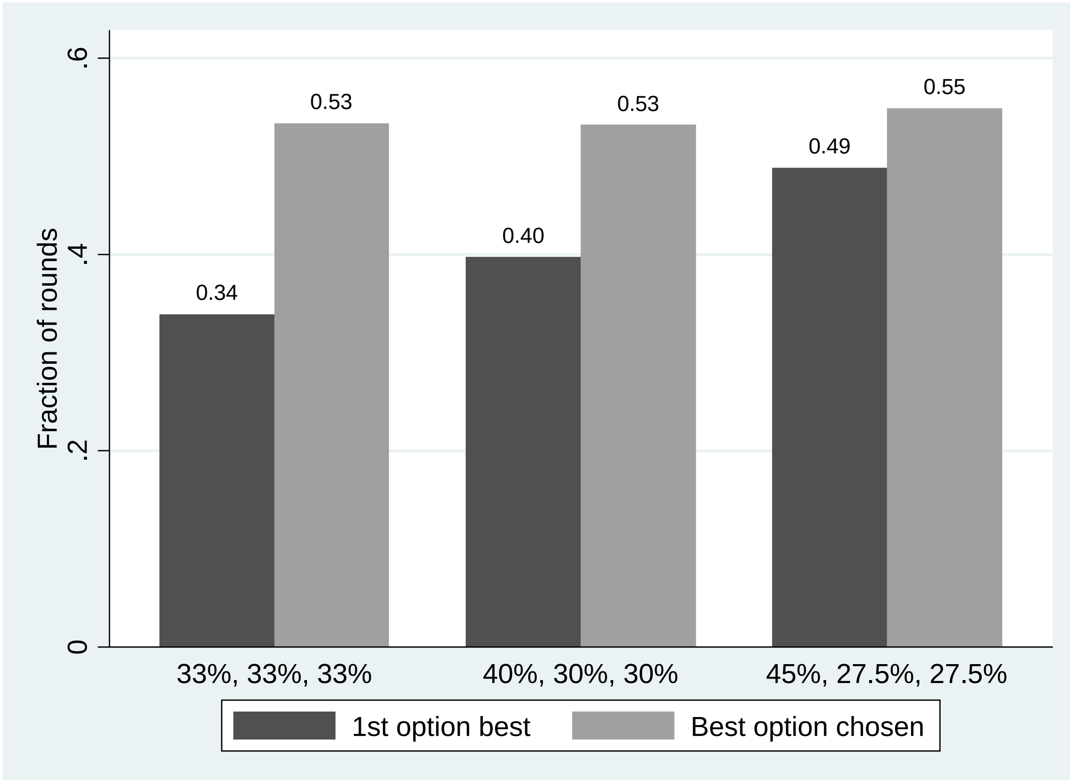

Figure 6: Fraction of rounds in which the 1st option is best and the best option is chosen by treatment

of rounds ($\alpha = .10$). Also, as the figure 6 shows, increasing the likelihood of the first option being the best option does not increase the fraction of rounds in which the best option is chosen. In fact, the fraction is not significantly different between any two treatments ($\alpha = .10$).

Were subjects affected by the nudges? Although performance is similar across treatments, choice behavior is markedly different. The first column of table 1 shows that the percent of rounds in which the first option is chosen increases substantially as the likelihood of the first option being the best option increases and is significant higher in the nudge treatments than the probability of the first option being the best option. This high rate of picking the first option appears to be driven by informational differences, not a predisposition towards picking a particular choice. In the baseline treatment, the data is close to frame free, and the percent of rounds in which the second and third option are chosen is not significantly different in any treatment ($\alpha = .10$).

Table 1. Percent of rounds each option chosen by treatment

Treatment	1st Option	2nd Option	3rd Option
33%, 33%, 33%	36.6%	32.6%	31.9%
40%, 30%, 30%	58.9%	20.2%	20.9%
45%, 27.5%, 27.5%	72.1%	14.1%	13.8%

Thus, it appears that subjects internalized the informational content of the nudge, but reduced their consideration of the alternatives in such a way that their performance was unchanged. In line with this conclusion, and in keeping with the No Improving Switches condition, we see in the table 2 that despite the asymmetry in choice probabilities, the percent of rounds in which the best option is chosen is remarkably similar for all options selected in all treatments.

Table 2. Percent of rounds best option chosen by option chosen and treatment

Treatment	1st Option	2nd Option	3rd Option
33%, 33%, 33%	52.8%	52.6%	54.7%
40%, 30%, 30%	50.9%	58.3%	54.8%
45%, 27.5%, 27.5%	55.1%	54.8%	53.7%

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