

Nancy L. Schwartz Lecture

# ECONOMICS AND LANGUAGE

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**Kellogg**  
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## NANCY L. SCHWARTZ

A dedicated scholar and teacher, Nancy Schwartz was the Morrison Professor of Decision Sciences, Kellogg's first female faculty member appointed to an endowed chair. She joined Kellogg in 1970, chaired the Department of Managerial Economics and Decision Sciences and served as director of the school's doctoral program until her death in 1981. Unwavering in her dedication to academic excellence, she published more than 40 papers and co-authored two books. At the time of her death she was associate editor of *Econometrica*, on the board of editors of the *American Economic Review* and on the governing councils of the American Economic Association and the Institute of Management Sciences.

The Nancy L. Schwartz Memorial Lecture series was established by her family, colleagues and friends in tribute to her memory. The lectures present issues of fundamental importance in current economic theory.

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An author of three books and more than 60 research papers, Professor Rubinstein is one of the world's leading researchers in the areas of economics and game theory. He has done path-breaking work on dynamic strategic interaction, bargaining, interactive epistemology, and bounded rationality, and his papers are published in the leading journals of economics, decision theory and applied mathematics.

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# ECONOMICS AND LANGUAGE

## *Chapter One: Economics and Language*

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### *1.1 A Personal Note*

The psychologist Joel Davitz once wrote: “I suspect that most research in the social sciences has roots somewhere in the personal life of the researcher, though these roots are rarely reported in published papers.” (Davitz, 1976). The first part of this statement is definitely true about this lecture. Although I work in several fields in economics and game theory, all my academic research has been motivated by my childhood desire to understand the way that people argue. In high school, I wanted to study logic. The main reason was that I thought that it would be useful for the political debates, which I was planning to be involved in or in the legal battles against evil, which I hoped to conduct after becoming a solicitor.

Unfortunately, I became neither a lawyer nor a politician. In the meantime, I came to understand that logic is not a very useful tool in those areas in any case.... But I continued to explore formal models of game theory and economic theory, never sustaining any desire to predict human behavior, never wishing to anticipate the stock prices, and never having illusions about capturing all of reality in one tiny model. I simply kept being interested in the arguments people bring in debates and in the reasons motivating decisions. And, I am still puzzled, even fascinated, by the magic of the links between the formal language of the mathematical models and natural language. This brings me to the subject of this lecture, “Economics and Language.”

### *1.2 Economics and Language*

The title of the lecture may be misleading. The caption “economics and language” is somewhat

“attractive” but it is also vague. As a heading, it covers many different subjects that I will not raise here. For example, I will not talk about the language of decision-makers. When decision-makers make deliberate choices, they often formulate their judgments and decisions in words. Were I to speak on this domain, I would present an investigation of the assumption that decision-makers formulate their considerations by using some language. This is a particularly attractive assumption when the “decision maker” is a collective of individuals but it is also appealing when we refer to a decision-maker as an individual. The formalization of this assumption requires the tools of mathematical logic. The analytical task would be to identify the constraints on the set of preferences induced from natural restrictions on the language used by the decision-maker to define his preferences. For example, such constraints make the lexicographic preferences much more appealing when compared to a standard textbook consumer’s utility function, such as  $\log(x_1+1)x_2$ . But today, I leave this subject aside.

And I will not talk about the language of economics. Much can be said about the rhetoric of economic theory and game theory. An economic model is not just a mathematical model. It is a combination of a mathematical model and an interpretation. The investigation of the language we use for attaching interpretations to economic models is, in my opinion, of much interest. If I were to discuss this subject here, I would try to argue that the rhetoric of game theory is actually misleading in creating an impression that it is much more “useful” than it actually is. In particular, I would try to persuade you that basic game theoretical notion of “strategy” is modeled in a way quite different from what is suggested by its meaning in *natural language* and that the term “solution” carries a deterministic flavor which is not appropriate.

But, I will not discuss here the “language of economic agents” and nor the “language of economics.” I will make do with topics within a research

domain, which is probably far away from traditional economics: “*economics about language.*”

Why would economic-type thinking be relevant to some linguistic issues? Economic theory is an attempt to explain regularities in human interaction and the most fundamental nonphysical regularity in human interaction is our natural language. In economic theory we have studied, quite carefully, issues concerning the design of social systems; language is partially a mechanism of communication. Economics tries to explain social institutions as regularities derived from optimizing certain functions which they serve; one can try to use this method regarding language as well. In this lecture I will try to demonstrate what we, economists, can do in this area by presenting two short investigations in which we use “economic” reasoning to address linguistic issues.

And before starting, I owe my audience an apology. Browsing through the literature while preparing these lectures, I came across a short article written by Jacob Marschak called, “Economics of Language” (Marschak, 1965). The article starts with a description of a discussion between engineers and psychologists regarding the design of the communication system of a small fighter plane. After the presentation, Marschak states: “The present writer...apologizes to those of his fellow economists who might prefer to define their field more narrowly, and who would object to...identification of economics with the search of optimality in fields extending beyond, though including, the production and distribution of marketable goods.” He then continues: “Being ignorant of linguistics, he apologizes even more humbly to those linguists who would scorn the designation of a simple dial-and-buttons systems a language.” I believe that like Marschak, I am not really apologizing to economists...but like Marschak, I do feel apologetic toward linguists and philosophers of language because my knowledge of this territory is much too limited.

## Chapter Two: The Functionality of Properties of Binary Relations

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### 2.1 Binary Relations

A binary relation on a set  $\Omega$  specifies a connection between elements within  $\Omega$ . Such binary relations are common in natural language. For example, “person  $x$  knows person  $y$ ,” “tree  $x$  is to the right of tree  $y$ ,” “picture  $x$  is similar to picture  $y$ ,” “chair  $x$  and chair  $y$  have the same color” and so on. I will avoid binary relations like “Professor  $x$  works for university  $y$ ” or “the social security number of  $x$  is  $y$ ,” which specify “relationships” between elements which naturally belong to two distinct sets. Actually, I will also restrict the term a “binary relation” to a binary relation which is irreflexive: no element relates to itself. The reason is that usually, the term “ $x$  relates to  $y$ ” when  $x=y$  is very different than the meaning of the term “ $x$  relates to  $y$ ” when  $x \neq y$ . For example, the statement “ $a$  loves  $b$ ” is very different than the content of the statement that “ $a$  loves himself.”

The nature of many binary relations requires that they satisfy certain properties. For example, the relation “ $x$  is a neighbor of  $y$ ” must, in any acceptable use of this relation, satisfy the symmetry property (if  $x$  is a neighbor of  $y$  then  $y$  is a neighbor of  $x$ ). The relation “ $x$  is to the right of  $y$ ” must be a *linear ordering* (that is satisfying the properties of completeness, asymmetry and transitivity). On the other hand, the nature of many other binary relations, such as the relation “ $x$  loves  $y$ ,” does not imply any specific properties that the relation must satisfy a priori. It may be true that in a particular group of people, whenever “ $x$  loves  $y$ ” then “ $y$  loves  $x$ ” as well. However, there is nothing in our understanding of the relation “ $x$  loves  $y$ ” which makes that symmetry necessary.

In fact, the objects of the investigation here are *properties* of those binary relations which appear in natural language. (Formally, a property of the rela-

tion  $R$  is a sentence in the language of the calculus of predicates which uses a name for the binary relation  $R$ , variable names, connectives and qualifiers, but does not include any individual names of the set of objects  $\Omega$ .) I will refer to a combination of properties as a *structure* of a binary relation.

I am curious about the structures of binary relations which appear in natural language and I look for explanations of why, out of the infinite number of potential properties, we find that only a few properties are common in natural languages. For example, it is difficult to find natural properties of binary relations such as:

A1: For every  $x$  and  $y$ , the number of  $z$  for which  $xRz$  is equal to the number of  $z$  for which  $yRz$ .

A2: If  $xRy$  and  $xRz$  ( $y \neq z$ ), and both  $yRa$  and  $zRa$ , then also  $xRa$ .

Or, it is difficult to conceive examples of natural structures of binary relations which require completeness and asymmetry (being a tournament) but do not require transitivity. One exception which comes to mind is the structure of the relation “ $x$  is in a clockwise direction relative to  $y$  (in the shortest arch connecting  $x$  and  $y$ ).” Is it just a coincidence that only a few structures are familiar in natural language?

The starting point for the following presentation will be that binary relations fulfill some functions in life. One can think about many criteria by which to examine the functionality of binary relations. Here, I will examine only two. I will argue that certain properties, all shared by linear orderings, perform better according to each of these criteria. Of course, other criteria are likely to provide alternative explanations for the frequent use of different common structures such as equivalence and similarity relations.

## 2.2 *Indication-friendliness*

Two parties observe a group of trees; the speaker wants to refer to a certain tree. If the tree, was the only olive tree in the grove, the speaker would just use the term “the olive tree.” If there is no mutually recognized name for the tree and the two parties have a certain binary relation defined on the set of the trees in their mutual vocabulary, the user may use this relation to define the element. For example, the phrase “the third tree on the right” is a way of indicating one tree out of many by using the linear ordering “x stands to the left of y” when the group of trees is well-defined and the relation “being to the left of” is a linear ordering. Similarly, the phrase “the seventh floor” is a way of indicating a location in a building given the linear ordering “floor x is above floor y.” There would be no need to use the phrase if the floor is “the presidential floor.” On the other hand, the relation “line a on the clock is clockwise to line b (at the smallest angle possible)” does not enable the user to indicate a certain line on a numberless clock: any formula which is satisfied by 3 o’clock is satisfied by 4 o’clock as well.

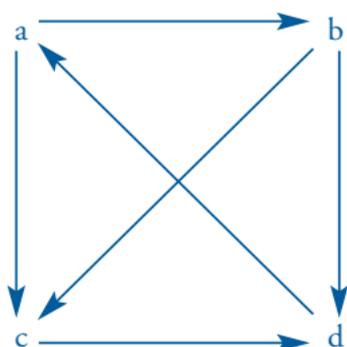
Thus, binary relations are viewed here as tools used for indicating elements out of a set whose objects do not have names. We are looking for a structure of binary relations which guarantees that a binary relation which satisfies it also enables the user to unambiguously single out *any element out of any subset which contains it*. We are led to the following definition:

**Definition:** A binary relation  $R$  on a set  $\Omega$  is *indication-friendly* if for every  $A \subseteq \Omega$ , and every element  $a \in A$ , there is a formula  $f_{a,A}(x)$  (in the language of the calculus of predicates with one binary relation and without individual constants) so that  $a$  is the only element in  $A$  satisfying the formula (when substituting  $a$  in place of the free variable  $x$ ).

Any linear ordering is indication-friendly. If  $R$  is a linear ordering, for  $A \subseteq \Omega$ , the formula

$P_1(x) = \forall y(x \neq y \rightarrow xRy)$  defines the “maximal” element in the set  $A$ . The formula  $P_2(x) = \forall y(x \neq y \wedge \neg P_1(y) \rightarrow xRy)$  defines the “second to the maximal” element, and so on. Note that there are “short cuts” for describing the different elements. For example, the “short cuts” for  $P_1(x)$  and  $P_2(x)$  are “the first” and “the second.”

On the other hand, consider the set  $\Omega = \{a, b, c, d\}$  and the nonlinear binary relation  $R$ , called “beat,” depicted in the diagram (satisfying:  $aRb$ ,  $aRc$ ,  $dRa$ ,  $bRd$ ,  $bRc$  and  $cRd$ ):



Referring to the grand set  $\Omega$ , the element “a” is defined by “it beats two elements, one of which also beats two elements.” The element “b” is defined by “it beats two elements, which beat one element each.” And so on. However, whereas the relation  $R$  allows the user to define any element in the set  $\Omega$ , the relation is not effective in defining elements in the subset  $\{a, b, d\}$ , where the induced relation is cyclical.

We will now demonstrate that if  $\Omega$  is a finite set and  $R$  is a binary relation then  $R$  is indication-friendly if and only if  $R$  is a linear ordering. We have already noted that if  $R$  is a linear ordering on  $\Omega$ , then for every  $A \subseteq \Omega$  and every  $a \in A$ , there is a formula indicating  $a$ . Assume that a binary relation  $R$  is indication-friendly. For any two elements  $a, b \in \Omega$ , in order to indicate the two elements in the two-member set  $A = \{a, b\}$ , it must be that either  $aRb$  or  $bRa$  but not both; thus,  $R$  must be complete and asymmetric.  $R$  must *also* be transitive because for every three elements  $a, b, c \in \Omega$ , in order to indicate each of the elements in the set  $A = \{a, b, c\}$ , it must be that there is no cycle.

**Conclusion 1:** A binary relation enables the user to indicate any element in any subset of the grand set if and only if it is a linear ordering.

### 2.3 *Informativeness*

Another use of binary relations on a set  $\Omega$  is that of a means to transfer or store information concerning a specific relationship between the elements of  $\Omega$ . Consider a case in which the grand set is the set of article authors in some field of research and the speaker is interested in describing the relation “x quotes y in his article.” The speaker may describe the relation by listing the pairs of authors who satisfy the relation. Alternatively, he may use those binary relations which are available in his vocabulary to describe the “x quotes y” relation. If he finds his vocabulary insufficient to describe the relation, he will use a binary relation which best approximates the relation he wants to describe. For example, if the relation “x is younger than y” is well defined, the speaker may use the sentence “any writer who is younger than another writer quotes him” to describe who quotes whom. As this may not be entirely correct, he may add a qualifying statement such as “the exceptions are a who did not mention b (though he is younger), and c who did mention d (though he is not younger).” Those qualified statements are the “loss” incurring from the use of an imprecise relation in order to indicate the “who quotes whom” relation.

Our investigation is on the level of an imaginary “planner” who is able to design only one binary relation at the “initial stage of the world.” Of course, real-life language includes many relations, and the effectiveness of each relation depends on the entire fabric of the language. The assumption that the designer is planning only one binary relation is made here solely for analytical convenience.

The design of one binary relation allows the speaker to select one of four binary relations. For instance, he can say: “everyone is quoted by all authors who are younger,” or “everyone is quoted

by all who are not younger,” and of course he can say also “everyone quotes everyone” and “no one quotes anyone,” two statements which do not require familiarity with any binary relation. Given a relation  $R$ , we will refer to those four relations as the vocabulary spanned by  $R$ , and denote it by  $V(R)$ . (Note that in defining the vocabulary spanned by  $R$ , we ignore other possibilities for defining a binary relation using  $R$ , such as the use of statements of the type “ $xSy$  if there is a  $z$  such that  $xRz$  and  $zRy$ ”).

Now, the speaker, who wants to refer to a binary relation,  $S$ , will use (so we assume) a relation in  $V(R)$  which is the “closest” to the one he really wants to refer to. The loss incurred is measured by the number of differences between the relation, which the speaker wants to describe, and the one he finds in his available vocabulary. The distance between any two binary relations  $R'$  and  $R''$  is taken to be the number of pairs  $(a,b)$  for which it is not true that  $aR'b$  iff  $aR''b$ . Note that by this measure, any pair for which  $R'$  and  $R''$  disagree receives the same weight. Regarding the initial state, it seems proper to put equal weights on all possible “imprecisions.”

The designer’s problem is to minimize the expected loss from optimal use of his vocabulary. It is assumed that from his point of view, all possible binary relations are equally likely to be needed by the speaker. Thus, the designer’s problem is  $\min_R \sum_S \delta(S, V(R))$ , where  $\delta(S, V(R)) = \min_{T \in V(R)} \delta(S, T)$  and  $\delta(S, T)$  is the distance between the relations  $S$  and  $T$ . One can show (see Rubinstein, 1996) that choosing  $R$  so that it will include half of the pairs  $(a,b)$  where  $a \neq b$  is “nearly optimal” for the designer if he wishes to reduce the expected number of imprecisions.

We reach the final point of this section: When planning a binary relation on  $\Omega$ , the designer, so we assume, also considers the possibility that the relation will eventually be used in reference to a subset of  $\Omega$  (this is analogous to the condition

made in the previous section that the speaker will wish to indicate a from any subset of the grand set). Completeness and asymmetry guarantees that if  $R$  has this structure then for every subset  $\Omega' \subseteq \Omega$ , the induced relation  $R_{|\Omega'}$  includes exactly half the pairs in  $\Omega' - \{(\omega, \omega) \mid \omega \in \Omega'\}$ .

**Conclusion 2:** For the task of expressing binary relations as accurately as possible using a vocabulary spanned by a single binary relation, it is nearly optimal that the binary relation will be complete and asymmetric.

## *Chapter Three: Strategic Considerations in Pragmatics*

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### *3.1 Grice's Principles and Game Theory*

The study of language is traditionally (see Levinson, 1983) divided into three domains: syntax, semantics, and pragmatics. Syntax is the study of language as a collection of symbols detached from their interpretation. Semantics studies the rules by which an interpretation is assigned to a sentence independently of the context in which the sentence is uttered. This part touches the third domain, pragmatics, which is the study of the way that the context in which an utterance is made affects the way it is interpreted. It views an utterance as a signal which conveys information within a context: the speaker, the hearer, the place, the time, and so forth. What the hearer thinks about the intentions of the speaker and what the speaker thinks about the presuppositions of the hearer are facts which are relevant for understanding the utterance.

Take, for example, a conversation between A, who converses from home, and his friend, calling from a telephone booth:

A: B, I am about to go for a walk. How's the weather outside?

B: It's not raining heavily.

Normally, A will conclude from B's statement that it is raining but not heavily. This conclusion does not follow from the semantic interpretation of the sentence "it's not raining heavily," which allows for the possibility that it is "raining but not raining heavily" as well as "it's not raining at all." Furthermore, there are circumstances under which the utterance "it's not raining heavily" will indeed be interpreted as not excluding the possibility that it is not raining at all. For example, imagine that B is in a hut and has told A that it is dark outside and

that he is deep under his blanket. In such a case, B would only realize that it is raining if it is “pouring rain” and the rain pounds his roof. In this case, A will indeed infer from B’s statement that it may not be raining at B’s place. Or,

A: “What do you see?”

B: “Well...it’s not a rose.”

Ordinarily, A will understand this statement that B is seeing a flower which is not a rose, though B has not said that he is looking at a flower.

In these examples, the fact that a statement appeared within a conversation between two people made the meaning of each of B’s statements different from that we would have given it had we been asked to interpret the sentence in isolation. A theory which aims to describe the rules for interpreting daily conversational utterances was suggested in the 1960s by the philosopher Paul Grice (see especially Grice, 1989). Grice’s initial point is:

“...while it is no doubt true that the formal devices are especially amenable to systematic treatment by the logician, it remains the case that there are very many inferences and arguments, expressed in natural language and not in terms of these devices, which are nevertheless recognizably valid. So there must be a place for unsimplified...logic of the natural counterparts of these devices;...” (p. 23–4)

I will not enter into a discussion of Grice’s theory. Let me just say that I was amazed how much game theoretical considerations are at the heart of that theory. I will now attempt, by employing work conducted jointly by Kobi Glazer and myself, to demonstrate the possible applications of game theoretical methods in explaining a pragmatics’ phenomenon in the special context of a debate.

### 3.2 *Debates*

By a “debate,” I am referring here to a situation in

which two or more parties who disagree regarding some issue raise arguments in an attempt to persuade a third party of their position. This is, of course, not the only form of debate taking place in life. Sometimes the purpose of the debaters is to argue just for the sake of arguing, and sometimes they aim to influence one another rather than a third party. Debates can be thought of as a special form of conversation in which the parties have different interests. Although debates are very common in real life, debates have rarely been investigated within the economic and game theoretical literature (some exceptions are Lipman and Seppi, 1995; Piketty and Spector, 1996; and Shin, 1994).

The aim of this presentation is to provide an explanation to one phenomenon often observed in debates. At the outset, we should note that the Gricean theory does not apply to debates because his logic of conversation is based on the principle of cooperation, which does not hold in the situation of interest conflict of interests characterizing a debate.

To motivate the discussion, let us start with the results of a survey conducted among several groups of students at Tel Aviv University. The following question was presented to one group of students:

**Question 1:** You are participating in a public debate about the level of education in the world's capitals. You are trying to convince the audience that in most capital cities, the level of education has risen of late. Someone is challenging you by bringing up indisputable evidence showing that the level of education in Bangkok has deteriorated. Now it's your turn to respond. You have similar, indisputable evidence to show that the level of education in Mexico City, Manila, Cairo, and Brussels has gone up. However, because of time constraints, you can argue and present evidence only about one of the four cities mentioned above. Which city would you choose for making the

strongest counter-argument against the Bangkok results?

Another group of subjects was presented with Question 1 but with the sole modification that Bangkok was replaced by Amsterdam. A third, similar group of students was asked to answer Question 2, in which the subjects were asked to select one opening argument by choosing from among the four cities Mexico City, Manila, Cairo and Brussels.

The following table presents the results of their responses to Questions 1 and 2. Manila is the favorite counter-argument to Bangkok; Brussels is the favorite counter-argument to Amsterdam. In contrast, the subjects split quite evenly between the four arguments when answering Question 2.

	<b>QUESTION 1 BANGKOK</b>	<b>QUESTION 1 AMSTERDAM</b>	<b>QUESTION 2</b>
n	38	62	24
Mexico City	19 %	15 %	21 %
Manila	50 %	3 %	21 %
Cairo	11 %	5 %	25 %
Brussels	21 %	78 %	33 %

A puzzling element appears in the survey results. If two arguments contain the same quality of information, why is it that one is considered to be a stronger counter-argument than the other? The fact that Manila is closer to Bangkok than it is to Mexico City seems irrelevant to the substance of the debate, and yet it appears to affect dramatically the choice of the better counter argument.

We believe that this phenomenon is connected to considerations of pragmatics. Within a debate, a responder's counter-argument to "Bangkok," if using anything but Manila, is interpreted as an admission that Manila is also an argument in favor of the opponent's position.

In the rest of the discussion we do not pretend to

fully explain the rules by which people compare arguments. We just would like to point out that the logic of debate should not necessarily include an axiom stating that if an argument  $x$  beats an argument  $y$  (in the sense of being a successful counter-argument against it) then argument  $y$  should not beat argument  $x$ .

### 3.3 *A Model*

We view a debate as a mechanism designed to extract information from debaters. We assume that the aim of the mechanism's design is to increase the probability that the right conclusion will be drawn by the listener subject to the constraints imposed on the debaters and the listener in terms of the time and cognitive abilities they can invest in the process. It will be shown that fulfilling this aim may lead to debating rules in which arguments and counter-arguments are not treated symmetrically.

The setup is very simple: An uninformed listener has to choose between two outcomes,  $O_1$  and  $O_2$ . The "correct" outcome, from his point of view, is determined by five aspects, numbered  $1, \dots, 5$ . An aspect  $i$  may be realized to be either 1 or 2, with the interpretation that if an aspect  $i$  gets the value  $j$ , aspect  $i$  is evidence supporting the outcome  $O_j$ . A state  $\omega = (\omega_j)_{j=1, \dots, 5}$  is a five-tuple of 1s and 2s which describes the realizations of the five aspects. The listener assigns equal weights to all five aspects, and the correct outcome at state  $\omega$ ,  $C(\omega)$ , is the outcome that is supported by the majority of the arguments.

The listener is ignorant of the state but the two debaters, named debater 1 and debater 2, have full information about the state. The "problem" is that there is a conflict of interests between the two debaters and the listener: Each debater  $i$  wishes that outcome  $O_i$  be chosen, whatever the state, whereas the listener wants the correct outcome to be chosen.

A debate is taken to be a mechanism in which each debater reveals pieces of information in order to persuade the listener to choose the debater's

favorite outcome. A debate consists of two elements: The *procedural rules* which specify the order and what sorts of arguments each debater is allowed to raise; the *persuasion rule* which specifies the relationship between the arguments presented and the listener's conclusion.

We need to specify the “language” that the debaters can use: It is assumed that debaters cannot make any moves other than raising arguments of the type “argument  $j$  supports me.” Thus, one debater cannot raise arguments that support the outcome preferred by the other debater. We will assume further that debaters have to prove their claims, namely, debater  $i$  cannot claim that the value of aspect  $j$  is  $i$  unless it is indeed  $i$ .

We want to characterize the optimal debate given an effective constraint about the length of the debate. Of course, if it is possible for three arguments to be raised during the debate, the listener can obtain the correct outcome with certainty. He does so by the request that one of the debaters presents three arguments; that debater will win the debate if and only if he fulfills this task. Thus, to make the length constraint effective, let us assume that the number of arguments, which can be raised, is two.

Formally, we take a debate to be an extensive game form in which

- 1) The set of feasible moves of each debater is a subset of  $\{1, \dots, 5\}$ .
- 2) There can be at most two moves: Either only one of the debaters is allowed to make at most two arguments (the one-speaker debate); or the two debaters move simultaneously, each one making at most one argument (the simultaneous debate); or the debate is a two-stage process where, at each stage, one debater moves *while* making at most one argument (the sequential debate).

- 3) One of the outcomes,  $O_1$  or  $O_2$ , is attached to each terminal history.

A debate  $\Gamma$  and a state  $\omega$  determine a game  $\Gamma(\omega)$ , which will be played by the two debaters. The two-player game form  $\Gamma(\omega)$  is obtained from  $\Gamma$  by deleting, for each debater  $i$ , all aspects which do not support his position in  $\omega$ . (If player  $i$  has to move after a history  $h$  and if at  $\omega$  none of the arguments he is allowed to make at  $h$  support his position, then the history  $h$  in the game  $\Gamma(\omega)$  will become a terminal history and the outcome  $O_j$  is attached to  $h$ .) As to the preferences in  $\Gamma(\omega)$ , debater  $i$  strictly prefers outcome  $O_i$  to the other outcome,  $O_j$ .

The game  $\Gamma(\omega)$  is a two-person zero-sum game (the listener is not a player in the game). The game has a value,  $v(\Gamma, \omega)$ , which is a *lottery* over the set of outcomes. Let  $m(\Gamma, \omega)$  be the probability that  $v(\Gamma, \omega)$  assigns to the *incorrect* outcome. When  $m(\Gamma, \omega)=1$ , we say that debate  $\Gamma$  induces a *mistake* in state  $\omega$ . In debates which are not simultaneous,  $m(\Gamma, \omega)$  is either 0 or 1. In simultaneous debates,  $v(\Gamma, \omega)$  may be a non-degenerate lottery ( $1 > m(\Gamma, \omega) > 0$ ). All mistakes are weighted equally and the optimal debate is taken to be the one which minimizes  $m(\Gamma) = \sum_{\omega} m(\Gamma, \omega)$ .

To demonstrate the calculation of  $m(\Gamma)$ , let us review some examples:

- 1) Let  $\Gamma$  be a debate in which only debater 1 is asked to present two arguments; he wins if he raises two arguments either from the set  $\{1,2,3\}$  or from the set  $\{4,5\}$ .  $M(\Gamma)=4$ : the four mistakes in favor of debater 1 occur in states when only aspects 4 and 5 or two aspects out of the set  $\{1,2,3\}$  support debater 1. Actually, this debate has the least number of mistakes in the set of one speaker's debates.
- 2) Let  $\Gamma$  be a simultaneous debate in which each debater can present at most one argument. Debater 2 wins the debate if debater 1

argues regarding aspect  $x$  and debater 2's argument is regarding aspect  $x+1(\text{mod}5)$  or aspect  $x-1(\text{mod}5)$ . Here, debater 1 rightly wins in any state  $\omega$  where there are three successive aspects (ordered on a circle) in his favor, and he rightly loses in any state where only zero, one, or exactly two nonconsecutive arguments are in his favor. There are ten other states: five of them are the "shift permutation" of  $(1,1,2,1,2)$  and the other five are the "shift permutation" of  $(1,1,2,2,2)$ . In each of these ten states, the value of the induced game is the lottery that selects the two outcomes equally. Thus,  $m(\Gamma)=10(1/2.)=5$ . Actually, one can see that the minimal number of mistakes in the family of simultaneous debates is 5.

In Glazer and Rubinstein (1997), we found that any optimal debate procedure is sequential and the minimal  $m(\Gamma)$  over all debates is three. The following is a debate with the minimal number of mistakes. First, debater 1 and then debater 2 are asked to present an argument. Debater 2 wins if and only if he counter-argues argument  $i$  with an argument regarding an aspect which is listed in the second column in row  $i$ . This debate induces three mistakes, two in favor of debater 1 (in states  $(1,1,2,2,2)$  and  $(2,2,1,1,2)$ ) and one in favor of debater 2 (in state  $(1,2,1,2,1)$ ).

<b>IF DEBATER 1 ARGUES FOR...</b>	<b>DEBATER 2 WINS IF AND ONLY IF HE COUNTER ARGUES WITH...</b>
1	{2}
2	{4,5}
3	{4}
4	{1,5}
5	{2,3}

What does the listener understand from an exchange of arguments where debater 1 argues regarding aspect  $i$  and debater 2 responds with aspect  $j$ ? Is it merely that aspects  $i$  supports debater 1 and aspects  $j$  supports debater 2? We can identify the listener's thoughts by considering the sequential

equilibria of the three-player game constructed from the debate game by adding the listener as a third player who has to choose the outcome of the debate at any terminal history. The above optimal persuasion rule is supported by a sequential equilibrium in which Debater 1's strategy is to raise the first argument  $i$  for which debater 2 does not have a proper counter-argument. If debater 2 has a proper counter-argument for each of his feasible arguments, debater 1 chooses the first argument which supports him. Debater 2's strategy is to respond with a successful counter-argument whenever it is possible. The listener chooses the outcome according to the above persuasion rule. Note, for example, that if debater 1 raises argument 3 and debater 2 raises argument 4, it is optimal for the listener to rule in favor of debater 2, since he concludes that in addition to aspect 4, aspects 1 and 2 are in favor of debater 2.

Note that the three-player sequential debate game has other sequential equilibria as well. One of those is particularly natural: In any state  $\omega$ , debater 1 raises the first argument  $i$  which is in his favor and debater 2 responds with the argument  $j$ , which is the smallest  $j > i$  in his favor. The listener's strategy will be guided by the following logic: Debater 1, in equilibrium, is supposed to raise the first argument in his favor. If he raises argument  $i$ , the listener believes that arguments  $1, 2, \dots, i-1$  are in favor of debater 2. Debater 2, in equilibrium, is supposed to raise the first argument in his favor following argument  $i$ . Hence, if debater 2 raises argument  $j$ , the listener believes that arguments  $i+1, \dots, j-1$  are in favor of debater 1. The listener chooses  $O_1$  if the number of aspects he (the listener) believes are in favor of 1 is larger than those which he believes to support 2. This equilibrium induces six mistakes. To conclude, both equilibria demonstrate the point that the interpretation of the arguments is a matter of equilibrium and not a matter of just the semantic content of the arguments.

An interesting fact about the above optimal persua-

sion rule is that aspect 5 is a persuasive counter-argument against aspect 2 and aspect 2 is a persuasive counter-argument against debater 1's argument regarding aspect 5. This is not coincidental. Actually we showed:

**Conclusion 3:** Any optimal debate is sequential and has a persuasion rule, which does not treat the players symmetrically. That is, there is a pair of aspects,  $i$  and  $j$ , so that when presented in sequence,  $i$  is a persuasive counter-argument against  $j$  and  $j$  is a persuasive counter-argument against  $i$ .

To conclude, in this lecture the rules of debate were treated as tools which were designed to enable the best elicitation of information, given the constraints on the length of the debate and the interests of the debaters. It was shown that in the course of debates, the listener concludes from arguments more than what is said. The content of an argument is an equilibrium phenomenon. In particular it was concluded that it is not necessarily a fallacy that in debates argument  $i$  defeats argument  $j$  and, at the same time, argument  $j$  defeats argument  $i$ . In other words, the "logic of debate" does not "have" to contain a rule that if "p defeats q," "q should not defeat p."

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