

# Mechanism Design and Student Assignment: Some Developments

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February 2011  
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  - ✓ 2009: Chicago

## Provide motivation for study

- Feedback between mechanisms in theory and in the field at heart of 'market design' conceit
- Want to focus on latter developments

# Canonical Model

From Abdulkadirođlu and Sönmez (2003):

## Primitives

1. a set of students  $I = \{i_1, \dots, i_n\}$ ,
2. a set of schools  $S = \{s_1, \dots, s_m\}$ ,
3. a capacity vector  $q = (q_{s_1}, \dots, q_{s_m})$ ,
4. a list of strict student preferences  $P = (P_{i_1}, \dots, P_{i_n})$ , and
5. a list of strict school priorities  $\pi = (\pi_{s_1}, \dots, \pi_{s_m})$ .

**matching**  $\mu : I \rightarrow S$  is a function from the set of students to the set of schools such that no school is assigned to more students than its capacity.

**mechanism**: systematic way to compute a matching for each problem.

# Three Mechanisms

Initial terms of the debate framed by Abdulkadiroğlu and Sönmez

Formally considered three mechanisms

## 1) Boston Mechanism

- ✓ Appears to be the most common mechanism actually used

## 2) Student-optimal stable mechanism

- ✓ Based on Gale and Shapley's student-proposing deferred acceptance algorithm (DA)

## 3) Top trading cycles mechanism (TTC)

- ✓ Adapted from Gale's top trading cycles algorithm for housing market model

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**Some progress, much remains to be done**

# Other issues

## Enlarging scope of the design problem

- What is the point of school choice?
  - ◇ Demand-side competitive pressure on schools?
  - ◇ Better matches?
  - ◇ Prevent exit of wealthy to suburbs?
  - ◇ How does school choice factor into the production of achievement?

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### Spillovers outside of matching theory

- Quasi-experimental variation from assignment provides unique opportunity to advance knowledge on education production
  - ◇ Accountability and Flexibility: Charters and Pilots (2009)
  - ◇ Achievement Effects of Elite Exam Schools (2011)
  - ◇ Small Schools Reform: The Urban Assembly Schools (2011)

# New Developments in the Field



Chicago Sun-Times November 12, 2009  
**8th-graders' shot at elite high schools better**

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*CPS officials said Wednesday they have decided to let any eighth-grader who applied to a college prep for fall 2010 admission re-rank their preferences to better conform with a new selection system.*

*Previously, some eighth-graders were listing the most competitive college preps as their top choice, forgoing their chances of getting into other schools that would have accepted them if they had ranked those schools higher, an official said.*

*Under the new policy, Huberman said, a computer will assign applicants to the highest-ranked school they qualify for on their list.*

*"It's the fairest way to do it." Huberman told Sun-Times.*

# Chicago Public Schools

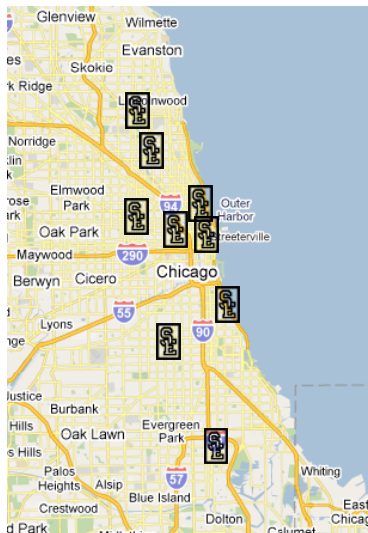
9 selective high schools

Applicants: Any current 8<sup>th</sup> grader in Chicago

Composite test score: entrance exam + 7<sup>th</sup> grade scores

Up to Fall 2009, system worked as follows:

- Take entrance test
- Rank up to 4 schools



# Chicago Selective Enrollment Mechanism

**Round 1:** In Round 1 only the first choices of the students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their composite test score until either there are no seats left or there is no student left who has listed it as her first choice.

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In general, for  $k = 2, \dots, 4$

**Round k:** Consider the remaining students. In Round k only the  $k^{\text{th}}$  choices of these students are considered. For each school with still available seats, consider the students who have listed it as their  $k^{\text{th}}$  choice and assign the remaining seats to these students one at a time following their composite test score until either there are no seats left or there is no student left who has listed it as her  $k^{\text{th}}$  choice.

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  - ✓ Rank up to 4 schools
  - ✓ Students ordered by composite score
  - ✓ The first student obtains her top choice, the second student obtains her top choice among remaining, and so on.

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- Somewhat surprising midstream change, especially given that both mechanisms are manipulable...
- Examined in Pathak and Sönmez (2011): “Admissions Reform in Chicago and England: Comparing Mechanisms by their Vulnerability to Manipulation”



# Framework

- **Players:**  $i = 1, \dots, N$
- **Allocations:**  $A$
- **Preferences:**  $R_i$  over  $A$ , strict version  $P_i$
- **Problem:**  $R = (R_1, \dots, R_N)$
- **Direct Mechanism:**  $\psi$  map from preference profile to outcome

Mechanism  $\psi$  is **manipulable** by player  $i$  at problem  $R$  if there exists a type  $R'_i$  such that

$$\psi(R'_i, R_{-i}) P_i \psi(R).$$

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Equivalent definition: if truth-telling is a Nash equilibrium of the game induced by mechanism  $\psi$ , it is also Nash equilibrium of the game induced by mechanism  $\varphi$  (even though the converse does not hold).

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**Proposition 1.** Suppose there are at least  $k$  schools and let  $k > 1$ . The old Chicago mechanism ( $\text{CHI}^k$ ) is more manipulable than the truncated serial dictatorship Chicago adopted ( $\text{SD}^k$ ) in Fall 2009.

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“I couldn’t believe it,” schools CEO Ron Huberman said. “It’s terrible.”  
suggests that the old mechanism was quite undesirable.
- To make this precise, we need to consider a class of mechanisms:
  - ◇ stable mechanisms?
  - ◇ not satisfied by many school choice mechanisms, including Chicago’s old one

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**Theorem 1.** *The old Chicago mechanism ( $\text{CHI}^k$ ) is at least as manipulable as any weakly stable mechanism.*

## Chicago in 2010-2011

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- Lack of efficiency in the new mechanism in 2009 is apparent
- Possible to have a completely non-manipulable mechanism by considering all choices
- In 2010-11 school year, Chicago decided to consider 6 out of 9 choices, so the mechanism is still manipulable

## Constrained School Choice

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NYC DOE press release on change: “to reduce the amount of gaming families had to undertake to navigate a system with a shortage of good schools” (New York Times, 2003)

- Based on the strategy-proofness of the student-optimal stable mechanism, the following advice was given to students:

You must now rank your **12** choices according to your true preferences.

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**Theorem 2:** *Let  $\ell > k > 0$  and suppose there are at least  $\ell$  schools. The student-optimal stable mechanism where students can rank  $k$  schools is more manipulable than the student-optimal stable mechanism where students can rank  $\ell$  schools.*

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**Corollary:** The 2009 Chicago mechanism ( $SD^4$ ) is more manipulable than the newly adopted 2010 Chicago mechanism ( $SD^6$ ).

# Admissions Reform in England

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Section 2.13: In setting oversubscription criteria the admission authorities for all maintained schools must not:

*give priority to children according to the order of other schools named as preferences by their parents, including 'first preference first' arrangements.*

- A **first preference first system** is any “oversubscription criterion that gives priority to children according to the order of other schools named as a preference by their parents, or only considers applications stated as a first preference” (School Admissions Code, 2007, Glossary, p. 118).

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- Rationale given by DfES (2007):
  - “‘first preference first’ criterion made the system unnecessarily complex to parents”
- Great deal of public discussion throughout England

## Ban of 'Boston' Mechanism in 2007

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  - designed to "make the admissions system fairer" and "create a simpler system for parents"
- Best-known equal preference system is **student-optimal stable mechanism**
- In 2006, Coldron report: 101 LEAs used equal preference, 47 used first preference first

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- ✓ Even though change in Chicago was much more abrupt, **striking** parallel in England
- ✓ Interesting that participants themselves (not matching theorists) re-organized market designs, just like US medical residents did in the early 1950s

**Theorem 3:** *Suppose there are more than  $k$  schools where  $k > 1$ . The Boston mechanism when participants can only rank  $k$  schools is more manipulable than the student-optimal stable mechanism where students can rank  $k$  schools.*

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- ◇ **Corollary:** *The old abandoned Chicago Selective Enrollment mechanism is more manipulable than the new 2009 mechanism.*

# Two-Sided Matching Markets

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  - ◊ Suggests considering incentive problem for one side of the market
  - ◊ Student-optimal stable mechanism is strategy-proof for students
- There is a similar **college-optimal stable mechanism** where the roles of the students and schools change
  - ◊ Roth (1985): there is no stable mechanism where truth-telling is a dominant strategy for each college (in a many-to-one matching model).



# Two-sided Matching Markets

## Stable mechanisms

We can make an even stronger comparison for this case:

Mechanism  $\psi$  is **strongly more manipulable** than mechanism  $\varphi$  if

- ✓ for any problem where  $\psi$  is manipulable,  $\varphi$  is manipulable by any player who can manipulate  $\psi$ , and
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**Theorem 4:** *For colleges, the student-optimal stable mechanism ( $GS^S$ ) is strongly more manipulable than the college-optimal stable mechanism ( $GS^C$ ).*

- NRMP and other clearinghouse reforms
- Williams, Report on Committee Meetings of AMSA (1995):

*“Since it is impossible to remove all incentives for hospitals to misrepresent, it would be best to choose the student-optimal algorithm to remove incentives, at least for students. In other words, within the set of stable algorithms, you either have incentives for both the hospitals and the students to misrepresent their true preferences or only for the hospitals.”*

- According to our definition, reforms made mechanism more manipulable for hospital programs

## Ordering Stable Mechanisms

A similar argument can be used to generalize this result as follows.

Let  $\varphi$  be an arbitrary stable mechanism. Then

- 1)  $\varphi$  is at least as manipulable as  $GS^C$  for colleges,
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One can also generalize the model to allow colleges to report both their preferences and capacities, and obtain the same results

## Conclusion

- Explored an approach to rank mechanisms by their incentive properties
- Other researchers have found other applications
- Results provide some justification for the recent policy changes in Chicago and England
- Many exciting problems emerging from interaction of theory and mechanisms in the field  
e.g., Why a cap in the Chicago mechanism?





## **Proof** (outline)

For simplicity, assume that there are more students than seats and all schools are acceptable. Let  $Q$  be the total school capacity in the economy.

Suppose the old Chicago mechanism is not manipulable; we will show neither is any other weakly stable mechanism

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However, none of the top  $Q$  students has an incentive to manipulate because they all receive their top choices.

Moreover, no other student can manipulate by weak stability.



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Hence, **stability** under  $(Q_c, P_{-c}) \Rightarrow$

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Last property also implies  $c$  obtains the same allocation in any stable matching, including student-optimal one:

$$GS_c^S(Q_c, P_{-c}) = GS_c^C(Q_c, P_{-c})P_cGS_c^S(P).$$