Mechanism Design and Student Assignment: Some Developments

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Student Assignment in Practice

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  - ✓ 2003: New York City
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Provide motivation for study

- Feedback between mechanisms in theory and in the field at heart of 'market design' conceit
- Want to focus on latter developments

Canonical Model

From Abdulkadiroğlu and Sönmez (2003):

Primitives

1. a set of students $I = \{i_1, ..., i_n\}$,
2. a set of schools $S = \{s_1, ..., s_m\}$,
3. a capacity vector $q = (q_{s_1}, ..., q_{s_m})$,
4. a list of strict student preferences $P = (P_{i_1}, ..., P_{i_n})$, and
5. a list of strict school priorities $\pi = (\pi_{s_1}, ..., \pi_{s_m})$.

matching $\mu : I \to S$ is a function from the set of students to the set of schools such that no school is assigned to more students that its capacity.

mechanism: systematic way to compute a matching for each problem.

Three Mechanisms

Initial terms of the debate framed by Abdulkadiroğlu and Sönmez

Formally considered three mechanisms

1) Boston Mechanism
   - ✓ Appears to be the most common mechanism actually used
2) Student-optimal stable mechanism
   - ✓ Based on Gale and Shapley's student-proposing deferred acceptance algorithm (DA)
3) Top trading cycles mechanism (TTC)
   - ✓ Adapted from Gale's top trading cycles algorithm for housing market model

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  - ◇ Quantitative aspects: resolving indifferences, TTC vs. DA
  - ◇ Origins of student preferences

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Some progress, much remains to be done

Other issues

Enlarging scope of the design problem

- What is the point of school choice?
  - ◇ Demand-side competitive pressure on schools?
  - ◇ Better matches?
  - ◇ Prevent exit of wealthy to suburbs?
  - ◇ How does school choice factor into the production of achievement?

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Spillovers outside of matching theory

- Quasi-experimental variation from assignment provides unique opportunity to advance knowledge on education production
  - Accountability and Flexibility: Charters and Pilots (2009)
  - Achievement Effects of Elite Exam Schools (2011)
  - Small Schools Reform: The Urban Assembly Schools (2011)

New Developments in the Field

Chicago Sun-Times November 12, 2009
8th-graders' shot at elite high schools better

Poring over data about eighth-graders who applied to the city's elite college preps, Chicago Public Schools officials discovered an alarming pattern.

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CPS officials said Wednesday they have decided to let any eighth-grader who applied to a college prep for fall 2010 admission re-rank their preferences to better conform with a new selection system.

Previously, some eighth-graders were listing the most competitive college preps as their top choice, forgoing their chances of getting into other schools that would have accepted them if they had ranked those schools higher, an official said.

Under the new policy, Huberman said, a computer will assign applicants to the highest-ranked school they quality for on their list.

"It's the fairest way to do it." Huberman told Sun-Times.

Chicago Public Schools

9 selective high schools

Applicants: Any current 8^{th} grader in Chicago

Composite test score: entrance exam + 7^{th} grade scores

Up to Fall 2009, system worked as follows:

- Take entrance test
- Rank up to 4 schools

Chicago Selective Enrollment Mechanism

Round 1: In Round 1 only the first choices of the students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their composite test score until either there are no seats left or there is no student left who has listed it as her first choice.

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In general, for $k = 2, ..., 4$

Round k: Consider the remaining students. In Round k only the k^{th} choices of these students are considered. For each school with still available seats, consider the students who have listed it as their k^{th} choice and assign the remaining seats to these students one at a time following their composite test score until either there are no seats left or there is no student left who has listed it as her k^{th} choice.

- New Chicago mechanism (SD^4)
  - ✓ Rank up to 4 schools
  - ✓ Students ordered by composite score
  - ✓ The first student obtains her top choice, the second student obtains her top choice among remaining, and so on.

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  - ✓ Students ordered by composite score
  - ✓ The first student obtains her top choice, the second student obtains her top choice among remaining, and so on.
- Somewhat surprising midstream change, especially given that both mechanisms are manipulable...
- Examined in Pathak and Sönmez (2011): "Admissions Reform in Chicago and England: Comparing Mechanisms by their Vulnerabilty to Manipulation"

Framework

- **Players**: $i = 1, ..., N$
- **Allocations**: A
- **Preferences**: R_i over A, strict version P_i
- **Problem**: $R = (R_1, ..., R_N)$
- **Direct Mechanism**: ψ map from preference profile to outcome

Mechanism ψ is **manipulable** by player i at problem R if there exists a type R'_i such that

$$\psi(R'_i, R_{-i}) P_i \psi(R).$$

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Equivalent definition: if truth-telling is a Nash equilibrium of the game induced by mechanism ψ, it is also Nash equilibrium of the game induced by mechanism φ (even though the converse does not hold).

Admissions Reform in Chicago

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- To make this precise, we need to consider a class of mechanisms:
  - ◇ stable mechanisms?
  - ◇ not satisfied by many school choice mechanisms, including Chicago's old one

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Theorem 1. *The old Chicago mechanism* ($\mathrm{C{\scriptsize HI}}^k$) *is at least as manipulable as any weakly stable mechanism.*

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- Lack of efficiency in the new mechanism in 2009 is apparent
- Possible to have a completely non-manipulable mechanism by considering all choices
- In 2010-11 school year, Chicago decided to consider 6 out of 9 choices, so the mechanism is still manipulable

Constrained School Choice

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> NYC DOE press release on change: "to reduce the amount of gaming families had to undertake to navigate a system with a shortage of good schools" (New York Times, 2003)

- Based on the strategy-proofness of the student-optimal stable mechanism, the following advice was given to students:

You must now rank your **12** choices according to your true preferences.

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Corollary: The 2009 Chicago mechanism (SD^4) is more manipulable than the newly adopted 2010 Chicago mechanism (SD^6).

Admissions Reform in England

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Section 2.13: In setting oversubscription criteria the admission authorities for all maintained schools must not:

> *give priority to children according to the order of other schools named as preferences by their parents, including 'first preference first' arrangements.*

- A **first preference first system** is any "oversubscription criterion that gives priority to children according to the order of other schools named as a preference by their parents, or only considers applications stated as a first preference" (School Admissions Code, 2007, Glossary, p. 118).

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- Great deal of public discussion throughout England

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- In 2006, Coldron report: 101 LEAs used equal preference, 47 used first preference first

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- ✓ Even though change in Chicago was much more abrupt, **striking** parallel in England
- ✓ Interesting that participants themselves (not matching theorists) re-organized market designs, just like US medical residents did in the early 1950s

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- **Corollary**: *The old abandoned Chicago Selective Enrollment mechanism is more manipulable than the new 2009 mechanism.*

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  - ◇ Student-optimal stable mechanism is strategy-proof for students

Two-Sided Matching Markets

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- Roth (1982): no mechanism that is stable and strategy-proof
  - Suggests considering incentive problem for one side of the market
  - Student-optimal stable mechanism is strategy-proof <u>for students</u>
- There is a similar **college-optimal stable mechanism** where the roles of the students and schools change
  - Roth (1985): there is no stable mechanism where truth-telling is a dominant strategy for each college (in a many-to-one matching model).

Two-sided Matching Markets

Stable mechanisms

We can make an even stronger comparison for this case:

Mechanism ψ is **strongly more manipulable** than mechanism φ if

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Theorem 4: *For colleges, the student-optimal stable mechanism ($GS^{\mathcal{S}}$) is strongly more manipulable than the college-optimal stable mechanism ($GS^{\mathcal{C}}$).*

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- NRMP and other clearinghouse reforms
- Williams, Report on Committee Meetings of AMSA (1995):

> *"Since it is impossible to remove all incentives for hospitals to misrepresent, it would be best to choose the student-optimal algorithm to remove incentives, at least for students. In other words, within the set of stable algorithms, you either have incentives for both the hospitals and the students to misrepresent their true preferences or only for the hospitals."*

- According to our definition, reforms made mechanism more manipulable for hospital programs

Ordering Stable Mechanisms

A similar argument can be used to generalize this result as follows.

Let φ be an arbitrary stable mechanism. Then

1) φ is at least as manipulable $GS^{\mathcal{C}}$ for colleges,
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One can also generalize the model to allow colleges to report both their preferences and capacities, and obtain the same results

Conclusion

- Explored an approach to rank mechanisms by their incentive properties
- Other researchers have found other applications
- Results provide some justification for the recent policy changes in Chicago and England
- Many exciting problems emerging from interaction of theory and mechanisms in the field

  e.g., Why a cap in the Chicago mechanism?

Proof (outline)
For simplicity, assume that there are more students than seats and all schools are acceptable. Let Q be the total school capacity in the economy.

Suppose the old Chicago mechanism is not manipulable; we will show neither is any other weakly stable mechanism

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Moreover, no other student can manipulate by weak stability.

back

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Suppose c can manipulate $GS^{\mathcal{C}}$ with $\hat{P}_c$, and obtain $\mu(c)$.
manipulability $\Rightarrow$

$$\underbrace{GS_c^{\mathcal{C}}(\hat{P}_c, P_{-c})}_{=\mu(c)} P_c GS_c^{\mathcal{C}}(P).$$

college-optimality $\Rightarrow$

$$GS^{\mathcal{C}}_c(P) R_c GS^{\mathcal{S}}_c(P).$$

Suppose c can manipulate $GS^{\mathcal{C}}$ with $\hat{P}_c$, and obtain $\mu(c)$.

manipulability $\Rightarrow$

$$\underbrace{GS^{\mathcal{C}}_c(\hat{P}_c, P_{-c})}_{=\mu(c)} P_c GS^{\mathcal{C}}_c(P).$$

Consider the preference Q_c where only $\mu(c)$ are acceptable. In problem (Q_c, P_{-c}),

$$GS^{\mathcal{C}}(\hat{P}_c, P_{-c}) \text{ is stable.}$$
$$GS^{\mathcal{C}}(Q_c, P_{-c}) \text{ is stable by definition.}$$

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Fact: Across stable matchings, the number of students assigned to a college is the same.

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Fact: Across stable matchings, the number of students assigned to a college is the same.

College c only ranks $\mu(c)$ under Q_c. Hence, **stability** under $(Q_c, P_{-c}) \Rightarrow$

$$GS_c^{\mathcal{C}}(Q_c, P_{-c}) = GS_c^{\mathcal{C}}(\hat{P}_c, P_{-c}).$$

college-optimality $\Rightarrow$

$$GS_c^{\mathcal{C}}(P) R_c GS_c^{\mathcal{S}}(P).$$

manipulability $\Rightarrow$

$$\underbrace{GS_c^{\mathcal{C}}(\hat{P}_c, P_{-c})}_{=\mu(c)} P_c GS_c^{\mathcal{C}}(P).$$

Hence, **stability** under $(Q_c, P_{-c}) \Rightarrow$

$$GS_c^{\mathcal{C}}(Q_c, P_{-c}) = GS_c^{\mathcal{C}}(\hat{P}_c, P_{-c}).$$

Last property also implies c obtains the same allocation in any stable matching, including student-optimal one:

$$GS_c^{\mathcal{S}}(Q_c, P_{-c}) = GS_c^{\mathcal{C}}(Q_c, P_{-c}) P_c GS_c^{\mathcal{S}}(P).$$

back