Matching Models versus Mechanism Design for Allocating Indivisible Goods

Eric Budish
University of Chicago, Booth School of Business

Matching: Findings, Flaws, and Future.
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Matching "versus" mechanism design

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Mechanism design approach

- Max objective s.t. constraints (technology, incentives)
- Vickrey auction
- Myerson auction

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Matching approach

- Seek a mechanism that satisfies "good properties"
- Gale-Shapley deferred acceptance algorithm
- Gale's Top Trading Cycles algorithm

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  - Maximize proposer-side welfare s.t. stability constraints
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(Else, Alp and Rakesh wouldn't have suggested this topic!)

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Keep in mind: Myerson, Vickrey ... these are the ones that worked!

- If only all problems had such elegant and compelling solutions.

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2. Gale Top Trading Cycles variant

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- "In other applications, the top trading cycles mechanism may be more appealing. . . ."
- "In other cases the choice between the two mechanisms may be less clear and it depends on the policy priorities of the policy makers"

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The fact that we don't know the "optimal" school choice mechanism doesn't mean that we shouldn't discuss "good" school choice mechanisms!

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Other examples: assigning interchangeable workers to tasks or shifts; leads to salespeople; takeoff and landing slots to airlines; shared scientific resources amongst scientists; players to teams

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  - Will sometimes make additional assumptions about preferences (e.g. responsiveness)
- An allocation $\mathbf{x} = (x_i)_{i=1}^N$ is feasible if each $x_i \in 2^{\mathcal{C}}$ and $\sum_{i=1}^N x_{ij} \leq q_j$ for each j

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By contrast, in TU settings the three concepts tend to exactly coincide (e.g. Vickrey auction)

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Essentially no progress on the "constrained Max SWF" problem, for either Bayesian IC or dominant strategy IC

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- Hatfield (2009, p. 514): "[the] results have shown that the only acceptable mechanisms for allocation problems of this sort is a sequential dictatorship, even when we restrict preferences to be responsive (...). Although unfortunate, it seems that in many of these applications, the best procedure (...) may well be a random serial dictatorship."

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- 20 course seats: 10 have "good" professors, 10 have "bad" professors
- Both students agree that any "good" class is better than any "bad" class, and have responsive preferences
- Among the many ex-post Pareto efficient allocations are those in which one student gets all 10 good courses, while the other gets all 10 bad courses.

A mechanism from practice: the "draft"

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  3. Students are allocated courses one at a time, based on their reported preferences and remaining availability.
     - Rounds 1, 3, 5, ...: ascending priority order
     - Rounds 2, 4, 6, ...: descending priority order

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Data (from 2005-2006 academic year)

- Students' actual submitted ROLs (potentially strategic)
- Students' underlying truthful ROLs, from an administration survey (caveats / robustness in paper)

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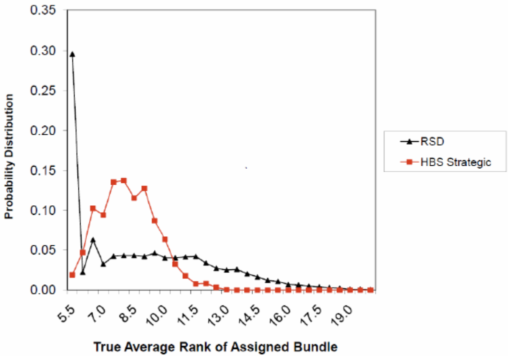

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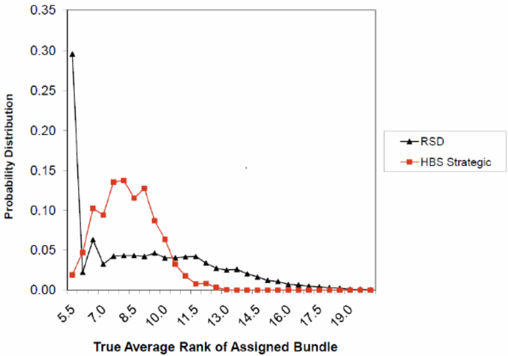

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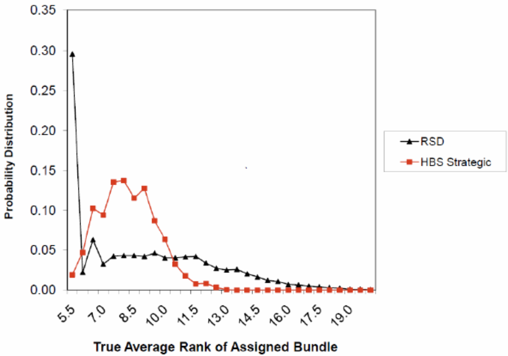

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- Implication: social planner prefers HBS to RSD if students have average-rank preferences and are weakly risk-averse

Why is RSD so Unattractive Ex-Ante? Example

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- Important note: unattractiveness of RSD does not depend on risk preferences. Even risk-neutral agents regard a "win a little, lose a lot" lottery as unappealing.

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- Overall, suggests a nuanced view of the role of strategyproofness in design

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Like the HBS draft, none of these is in the "pure mechanism design" mold, nor in the "pure axiomatization" mold

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No restrictions on preferences: students allowed to have arbitrary preferences over schedules. Allows for scheduling constraints, complementarities, etc.

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- Other extreme: dictatorships can be interpreted as exact CE, but from arbitrarily unequal budgets

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- Two agents. Four objects: two valuable Diamonds (Big, Small) and two ordinary Rocks (Pretty, Ugly). At most two objects per agent.
- Dictatorship?
  - Fairness problems: whoever's first gets both Diamonds.
- CEEI?
  - Existence problems: at any price vector, for any object, either both agents demand it or neither does.
- Approximate CEEI?
  - Randomly assign budgets of 1 and $1 + \beta$, for $\beta \gtrapprox 0$
  - Set the price of the Big Diamond strictly greater than 1
  - Set other prices such that the poorer agent can afford {Small Diamond, Pretty Rock}, wealthier agent gets {Big Diamond, Ugly Rock}

Properties of the Approximate CEEI Mechanism

Efficiency

- *Ex-post efficient, but for small error*

Fairness

- *Symmetric*
- *N+1 Maximin Share Guaranteed*
- *Envy Bounded by a Single Good*

Incentives

- *Strategyproof in the Large*

Approximate CEEI and "Matching versus Mechanism Design"

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A-CEEI is attractive relative to alternatives under either interpretation

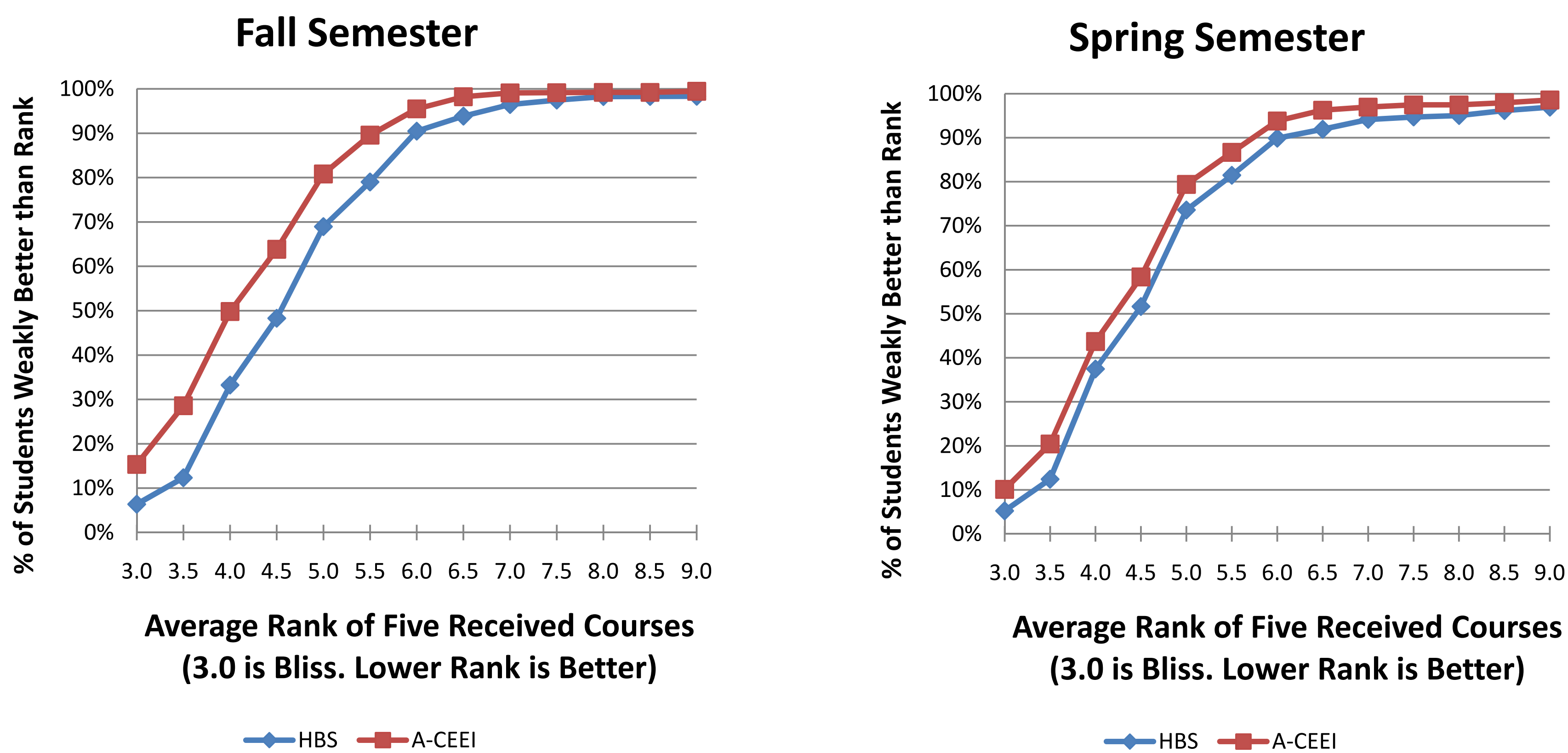
Table 2: Comparison of Alternative Mechanisms

Mechanism	Efficiency (Truthful Play)	Outcome Fairness (Truthful Play)	Procedural Fairness	Incentives	Preference Language
Approximate CEEI Mechanism (A-CEEI)	Pareto Efficient w/r/t Allocated Goods Allocation error is small for practice and goes to zero in the limit	N+1 – Maximin Share Guaranteed Envy Bounded by a Single Good	Symmetric	Strategyproof in the Large	Ordinal over Schedules
A-CEEI v2: Competitive Equilibrium from Equal-as-Possible Incomes (Sec 6.1)	Pareto Efficient	Worst Case: coincides with dictatorship	Symmetric	Strategyproof in the Large	Ordinal over Schedules
A-CEEI v3: A-CEEI with a Pareto-Improving Secondary Market (Sec 6.1)	Pareto Efficient	A bit weaker than N+1 – Maximin Share Guarantee, because prices in the initial allocation may be outside of P(δ,b'). Initial allocation is Envy Bounded by a Single Good. The Pareto-improvement stage may exacerbate envy.	Symmetric	Manipulable in the Large	Ordinal over Schedules
Random Serial Dictatorship (Sec 8.1)	Pareto Efficient	Worst Case: Get k worst Objects	Symmetric	Strategyproof	Ordinal over Schedules
Multi-unit generalization of Hylland Zeckhauser Mechanism (Sec 8.2)	If vNM preferences are described by assignment messages, ex-ante Pareto efficient	If preferences are additive separable, envy bounded by the value of two goods Worst Case: Get Zero Objects	Symmetric	If vNM preferences are described by assignment messages, Strategyproof in the Large	Assignment messages
Bidding Points Mechanism (Sec 8.3)	If preferences are additive-separable, Pareto Efficient but for quota issues described in Unver and Sonmez (forth.)	Worst Case: Get Zero Objects	Symmetric	Manipulable in the Large	Cardinal over Items

Mechanism	Efficiency (Truthful Play)	Outcome Fairness (Truthful Play)	Procedural Fairness	Incentives	Preference Language
Sonmez-Unver (forth.) Enhancement to Bidding Points Mechanism	If preferences are additive-separable, Pareto Efficient	Worst Case: Get Zero Objects	Symmetric	Bidding Phase: Manipulable in the Large Allocation Phase: Strategyproof in the Large	Bidding Phase: Cardinal over Items Allocation Phase: Ordinal over Items
HBS Draft Mechanism (Sec 9.2)	If preferences are responsive, Pareto Efficient with respect to the reported information (i.e., Pareto Possible)	If preferences are responsive and k=2, Maximin Share Guaranteed If preferences are responsive, Envy Bounded by a Single Good	Symmetric	Manipulable in the Large	Ordinal over Items
Bezakova and Dani (2005) Maximin Utility Algorithm	If preferences are additive-separable, ideal fractional allocation is Pareto efficient. Realized integer allocation is close to the fractional ideal.	Worst Case: Get approximately zero objects (if a hedonist and all other agents are depressives)	Symmetric	Manipulable in the Large	Cardinal over items
Brams and Taylor (1996) Adjusted Winner	If preferences are additive-separable, Pareto Efficient	Worst Case: Get Zero Objects	Symmetric	Manipulable in the Large	Cardinal over Items
Herreiner and Puppe (2002) Descending Demand Procedure	Pareto Efficient	Does not satisfy Maximin Share Guarantee or Envy Bounded by a Single Object	Symmetric	Manipulable in the Large	Ordinal over Schedules
Lipton et al (2004) Fair Allocation Mechanism	Algorithm ignores efficiency	If preferences are additive separable, Envy Bounded by a Single Good	Symmetric	Manipulable in the Large	Cardinal over items
UChicago Primal-Dual Linear Programming Mechanism (Graves et al 1993)	Pareto Efficient when preference-reporting limits don't bind	Worst Case: Get Zero Objects	Symmetric	Manipulable in the Large	Cardinal over a Limited Number of Schedules

Figure 3: Ex-Ante Efficiency Comparison

Approximate CEEI Mechanism vs. HBS Draft Mechanism



Description: The Othman, Budish and Sandholm (2010) Approximate CEEI algorithm is run 100 times for each semester of the Harvard Business School course allocation data (456 students, ~50 courses, 5 courses per student). Each run uses randomly generated budgets. For each random budget ordering I also run the HBS Draft Mechanism, using the random budget order as the draft order. The HBS Draft Mechanism is run using students' actual strategic reports under that mechanism. The Approximate CEEI algorithm is run using students' truthful preferences. This table reports the cumulative distribution of outcomes, as measured by average rank, over the 456*100 = 45,600 student-trial pairs. Average rank is calculated based on the student's true preferences. For instance, a student who receives her 1,2,3,4 and 5th favorite courses has an average rank of (1+2+3+4+5)/5 = 3.

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Overall approach of BCKM: see how far we can push the HZ idea in the multi-unit setting

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Also, a timing modification: Essentially, the proxy gets to act *after* learning where the student is in the random priority order, whereas in the HBS draft students submit strategic ROLs *before* learning where they are in the order

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Strategyproofness is too strict a standard. Strategyproof in the large isn't appropriate for all contexts.

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- We still don't know how to maximize ex-ante efficiency in this problem. Budish and Cantillon (2009), Budish (2010), and BCKM (2010) show how to do better on ex-ante efficiency measures under different assumptions on preferences, but the "optimal" mechanism remains unknown.

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We know from Micro 101 that we don't expect most preferences in the world to be lexicographic ...

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  - E.g. in the dictatorship papers, getting exact ex-post Pareto efficiency was treated as more important than having even a modicum of ex-post fairness.

We know from Micro 101 that we don't expect most preferences in the world to be lexicographic ... Perhaps we need new tools to make our preferences over mechanism designs a bit less lexicographic as well.

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- Role for data: sense of magnitudes. Both improvement relative to old mechanisms, and distance versus unconstrained optimum