Auctions with an Expanded Secondary Market

Incentives and Bulow-Klemperer Result

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Google 2004 IPO Auction

Reactions of the Book Builders, the primary buyers

Reactions of the secondary buyers

Intention of Google

Outcome of the Auction

Incentive Issues: Is it a god strategy to take an inclusive approach in selling a good, allowing more secondary buyers to participate in the primary market?

Used Auto Auction

Used Auto Auction in US in which most of them use the exclusive approach:
exclude the final users from the auction, only the dealers are allowed.
Used Auto Auction in Japan

Auctioneers
Auto Dealers
Final Consumers

Issue of Exclusiveness

Does Inclusion Raise Revenues?

Possible benefits of Inclusion:

allowing more "direct" buyers who may be willing to pay more than the

"indirect" buyers

Costs of Inclusion:

lower benefits from resale may depress the bids in the primary market

lower option value

From the IO perspective:

weakened competition during resale vs more competition in the primary market

Framework of the Study

Auctioneer objective: revenue maximization, has no value for the object
Single object for sale
symmetric independent private values for all buyers
Secondary market: the seller, i.e. the winner of the primary auction, uses an optimal mechanism to sell the good

Primary market: any auction format can be used, a revenue equivalence result allowing resale
There are new buyers in the secondary market not present in the primary auction due to entry barriers

Two-Stage Game

There are only two stages, one for the primary auction, the other for resale no discounting between the two stages

A winner of the primary market can always resell to the losers and the new buyers in the secondary market whenever profitable

Assumption of commitment to the auction rules, a seller can only sell once, and keeps the object when it fails.

No second stage of resale if the good is not sold in the primary market.

No more resale at the end of the second stage.

Issue of the information revelation in the primary stage, assumption of no revelation of revealed information.

Direct mechanism

One stage formulation of Myerson (1981) with new buyers, using option value

Two stage formulation of this paper, direct mechanisms first for the primary players only, then

an optimal mechanism is used in the secondary market.

The two-stage model can be formally converted into one-stage model.

Two ways to do this: (I) only direct mechanisms for the primary buyers; (II) A direct mechanism for all buyers

The second method allows decomposition of the revenue contribution of the primary as well as new buyers.

The marginal revenues of the buyers now depend on the total number as well as the composition of the primary and secondary buyers as the assignment of winning probabilities depends on them.

Participation vs Optimal Auction

The Theorem can be thought of a an enhanced incentive property.
Optimal auction with resale, optimal reserve price

reserve value vs. reserve price
dependence of the optimal reserve on the numbers of primary and secondary buyers

Incentive to recruit the new buyers with no reserve value or fixed reserve
Imvalue
Incentive to recruit the new buyers even if you lose the ability to reset the optimal reserve value

Implementation with Bidding Mechanisms

What types of first-stage auction to use:
first-price vs second-price auctions
bidding strategies, existence and uniqueness of equilibrium
symmetry property

Implementation and information revelation:
second-price auctions make it difficult to conceal the information

Condition on F(x)

A condition on $F(.)$ is important for the answer to the incentive issue.

Under Condition (D), we show the incentive of using the inclusive approach.
Under Condition (D), we show that Bulow-Klemperer extension holds.
Without Condition (D), both results may fail.

Condition (D) implies that a switching new buyer:
(a) makes a primary buyer's revenue contribution higher than that of all rival buyers before the switch
(b) increases her own revenue contribution
(c) compensates more than enough for the loss of revenue from the remaining new buyers

The total effect makes the auctioneer revenue higher.
A single crossing property

Comparisons to other Assumptions

Relationship of Condition (D) and the following:

(a) the increasing hazard rate property

(b) the decreasing inverse hazard rate property

(c) the logconcavity property

(d) the convexity property

(e) the increasing virtual value property

(f) the property of the marginal revenue components or virtual option value

The Model

All primary or secondary buyers have the same use value distribution:
$F(v), v \in [0, \beta]$
$N \geq 1$ original buyers
$M \geq 0$ new buyers
Primary market: A direct revelation mechanism
Secondary Market: An Optimal Mechanism
$F(.|w) = \frac{F(.)}{F(w)}$ the conditional distribution of $F(.)$
the updated belief in the secondary market

Optimal Reserve Prices during Resale

Assume increasing virtual value

The conditional virtual value:

$$J(v, w) = v - \frac{F(w) - F(v)}{f(v)}$$

Optimal reserve price in the secondary market: $r(v, w)$ is the unique solution of the equation in x

$$J(x, w) = v.$$

By definition, the inverse function of $r(x, \beta)$ is $J(x, \beta)$.

Option Value of a Primary Buyer

Define the option value of a primary buyer i with use value v as

$$h_M(v) = h(v, M) = v + \int_{r(v,\beta)}^{\beta} (J(x,\beta) - v)\, dF^M(x),$$

defined for real numbers $m : h(v, m), h_m(v)$

Define the virtual option value of our model

$$J_M(v, \beta) = h_M(v) - h'_M(v)\frac{1 - F(v)}{f(v)}. \quad (1)$$

The function $J_m(v, \beta)$ is increasing in m.

Revenue Equivalence Theorem

Theorem

Given v_0, and assume that the primary buyer with the use value v_0 has zero equilibrium payoff and is the highest with zero equilibrium payoff. Under a symmetric winning rule for the primary buyers, the holding probability of a primary buyer $v \geq v_0$ is given by $F^{N-1}(v)F^M(r(v,\beta)), v \geq v_0$, and the auctioneer revenue is the same. The revenue formula one is given by

$$R(N, M, v_0) = \int_{v_0}^{\beta} J_M(x, \beta) dF^N(x).$$

Revenue Decomposition

Revenue Formula II: decomposition into primary and secondary components

$$R(N, M, v_0) = \int_{v_0}^{\beta} J(x, \beta) F^M(r(x, \beta)) dF^N(x)$$

$$+ \int_{r(v_0, \beta)}^{\beta} J(x, \beta) F^N(J(x, \beta)) dF^M(x).$$

Example: $F(x) = x \in [0, 1]$, $N = 1$, $M = 1$, $v_0 = 0$

$$\int_0^1 (2x - 1) \frac{x + 1}{2} dx = \frac{1}{12},$$

$$\int_{\frac{1}{2}}^1 (2x - 1)^2 dx = \frac{1}{6},$$

Total revenue

$$R(1, 1, 0) = h(0) = \frac{1}{4}.$$

The new buyer in the secondary market contributes substantially more revenue than the primary buyer.

Optimal Reserve Value

An optimal reserve value v^* is determined by the first-order condition

$$\frac{h(v)}{h'(v)} - \frac{1 - F(v)}{f(v)} = 0.$$

When there are no new buyers, we have $h(v) = v, h'(v) = 1$, the equation is reduced to setting the virtual value equal to 0.

The optimal reserve value $v^*(m)$ depends on m, and decreasing in m. Often we have $v^*(m) = 0$, meaning that we have optimal revenue as well as (constrained) efficiency.

Incentive Issues

Theorem Any original buyer in the primary market of use value $v > v_0$ has a higher payoff than a new buyer in the secondary market with the same use value.

Theorem A primary buyer has no incentive to move to the secondary market, while a new buyer has the incentive to move to the primary market.

It is not generally true that a primary buyer always contributes more revenue than a secondary buyer.

Condition (D)

$$\frac{d}{dx}\left(\frac{F(r(x,\beta))}{F(x)}\right) \leq 0 \text{ for } x \in [0,\beta]$$

$\frac{f(x)}{F(x)}$ is called the reverse hazard rate,

$\frac{f(x)}{1-F(x)}$ is called the hazard rate.

Logconcavity condition is simply the decreasing reverse hazard rate condition.

If we have increasing hazard rate and decreasing reverse hazard rate, then condition (D) holds.

If $F(.)$ is logconcave and convex, then Condition (D) holds.

Condition (D) holds for the uniform distribution and the power functions.

Main Result

Theorem

Assume that condition (D) holds and $M \geq 1$. When the auctioneer recruits one new buyer from the secondary market into the primary market, the revenue is higher, i.e. $R(N, M, v_0) < R(N+1, M-1, v_0)$

If Condition (D) fails, this need not be true.
Counter example: $F(v) = e^{v^2-1}$ defined over $[0,1]$. Before the move, the revenue is

$$R(1,1,0) = 0.285\,384.$$

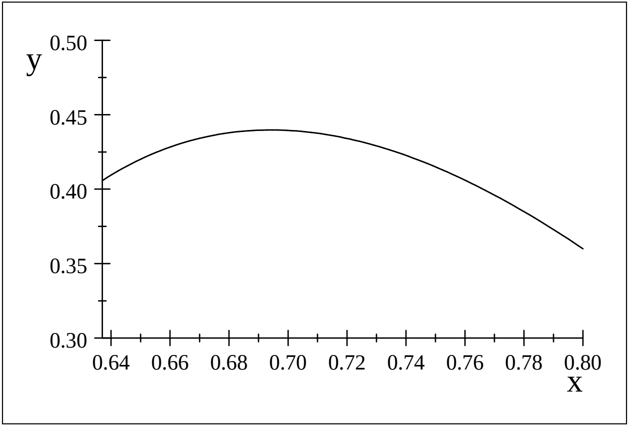
After the move, the revenue is

$$R(2,0,0) = \int_0^1 J(x,\beta)dF^2(x) = 0.243\,835,$$

We have

$$\frac{d}{dx}\ln\left(\frac{F(x)}{F(J(x,\beta))}\right) = \frac{d}{dx}\ln\left(\frac{e^{v^2-1}}{e^{J(v,\beta)^2-1}}\right) = v^2 - J(v,\beta)^2.$$

It is not a decreasing function, violating Condition (D). It is not logconcave.



Main Ideas of the Proof

Before the switch, there are N primary buyers, M new buyers
Step one: Under condition (D), the revenue contribution of a primary buyer after the switch dominates those of all buyers before the switch.

$$P(N, M, v_0) = \int_{v_0}^{\beta} J(x, \beta) F^N(x) F^{M-1}(r(x, \beta)) dF(x) > \int_{v_0}^{\beta} J(x, \beta) F^{N-1}(x)$$

$$\int_{v_0}^{\beta} J(x, \beta) F^N(x) F^{M-1}(r(x, \beta)) dF(x) > \int_{r(v_0,\beta)}^{\beta} J(x, \beta) F^N(J(x, \beta)) F^{M-1}(x$$

Let

$$D(t) = R(1 + t, m - t, v_0) - R(t, m, v_0).$$

Use Step one to show
Step two: A single crossing property $D'(t) > 0$ whenever $D(t) = 0$.

Step one

Case of no resale: If there is one more primary buyer, all rival primary buyers contributes more revenue

Proof using Integration by parts:

$$\frac{R(n,0)}{n} = \int_0^{\beta} J(x,\beta) F^{n-1}(x) dF(x) = \beta - \int_0^{\beta} F^{n-1}(x) dJ(x,\beta)$$

$$\frac{\partial}{\partial n} \frac{R(n,0)}{n} = \int_0^{\beta} (-\ln F(x)) F^{n-1}(x) dJ(x,\beta) > 0.$$

With resale,

$$\frac{R(n,m,0)}{n} = \int_0^\beta J(x,\beta)F^{n-1}(x)F^m(r(x,\beta))dF(x).$$

Not able to show:

$$\frac{\partial}{\partial t}\frac{R(n+t,m-t,0)}{n+t} > 0.$$

Instead we show the following by a single crossing property

$$P(n+1,m-1,v_0) > \max\{P(n,m,v_0),S(n,m,v_0)\}.$$

Bulow-Klemperer Theorem

when buyers are recruited from the secondary market:

Theorem

Assume condition (D) holds and there are N primary buyers, and M new buyers initially, $M \geq 1$. Let v^* be the initial optimal reserve value. The auctioneer revenue is higher if she recruits a new buyer from the secondary market into the primary market with no reserve value, or

Implementation by first-price auctions

uniqueness, symmetry, pure strategy equilibrium
Simple equilibrium strategy when $v_0 = 0$:

$$b(v) = h(v) - \int_0^v F^{N-1}(x|v)dh(x),$$

or

$$b(v) = \int_0^v h(x)dF^{N-1}(x|v).$$

Theorem

Assume that the primary auction is a first-price auction. The equilibrium in the auction with an expanded secondary market must have the symmetry property for all primary buyers.

Corollary

If the primary auction is a first-price auction, then the equilibrium in the auction with an expanded secondary market is unique.

END