1. Introduction

Designing dealer and sales-force incentives have been studied extensively in the literature. The following stair-step incentive scheme is commonly used in the automotive industry: The dealer is paid an additional amount per unit when sales exceed a threshold value; additionally, a fixed bonus may also be offered when sales exceed the threshold. While the traditional literature has considered the impact of such incentives on the manufacturer’s expected profit (or sales), it does not address the impact of incentives on sales variability. This is significant because sales variability also affects the manufacturer’s profit. Even with flexible manufacturing, sales variability increases the manufacturer’s costs. The design of optimal incentive parameters is further complicated when dealerships consolidate and are no longer exclusive to a single manufacturer. Non-exclusive relationships are particularly significant when actual sales are affected by dealer effort, i.e. when the dealer must decide on appropriate allocation of effort across sales for different manufacturers.

In this paper, we consider sales to consist of a market signal and a function of the dealer effort. The growth of sales with respect to the dealer effort is concave and the cost of the effort is convex and increasing. We analyze the impact of incentives on sales variability under two specific scenarios: an exclusive dealership scenario and a non-exclusive (consolidated dealer) dealership scenario. Unlike most existing research that studies the impact of incentives on sales, we study the impact of incentives on both the mean and variance of sales. For the
non-exclusive dealer, we assume that the effort to increase sales is substitutable across the products. In both scenarios the dealer optimizes the additional effort required to increase profits considering the profit margins, incentive parameters, and cost of the effort. Hence, the resulting variation in sales for the manufacturer depends on variance of the market signal and the dealer’s optimal effort response. The manufacturer only observes the final sales and sets incentive parameters with the objective of maximizing profits.

In the case of an exclusive dealership we show that appropriate stair-step incentives, with a fixed bonus on crossing the threshold, not only increase the expected sales, but more importantly, decrease the coefficient of variation of sales. For a given threshold, and bonus payment, we characterize the optimal point at which the coefficient of variance is the least. This happens when the threshold is neither too high nor too low. We also characterize the expected value and variance of sales. Further, we show that if the manufacturer associates a positive cost with sales variance, a stair-step incentive, with a bonus payment, is superior to the scheme without a fixed bonus. For a non-exclusive dealer, however, we show that consolidation reduces the coefficient of variation for the dealer but increases the same for each manufacturer under reasonable conditions. Specifically, we show that the coefficient of variation, for each manufacturer, increases if both offer stair-step incentives. This implies that stair-step incentives decrease the coefficient of variation with exclusive dealers but can increase variability for manufacturers as dealers are no longer exclusive.

2. Model Assumptions

Under the stair-step incentive scheme the dealer gets a profit of $p$ per unit sold. For every additional unit sold above a threshold, $K$, the dealer is offered an additional amount $\Delta$. In addition, the manufacturer may offer a fixed bonus, $D$, when sales exceed the threshold. The input market signal, $x$, determines the sales possible with no extra effort by the dealer and we assume that the expected value of the input market signal, $E(x)$, and the corresponding variance, $V(x)$, are independent of $\Delta$, $K$, and $D$. We assume that the input market signal follows a continuous, and twice differentiable, cumulative distribution function, $F$, with a bounded probability density function $f$. The resulting variation in sales for the manufacturer depends on variance of the market signal and the dealer’s optimal effort response. The manufacturer only observes the final sales. For non-exclusive dealers, we assume that the effort to increase sales is substitutable across products. In both scenarios we assume that the dealer optimizes any additional effort required to increase profit considering profit margins, incentive parameters, and cost of the effort.
The total sales response function depends on the random market signal, $x$, and the dealer effort $e$. As is commonly used in the literature, the growth of sales with respect to the dealer effort ($g(e)$) is concave and the cost of the effort ($c(e)$) is convex. Throughout the analysis we assume that the total sales, $s$, is determined by the following additive form: $s = x + g(e)$.

### 3. The Exclusive Dealer

Given a stair-step incentive plan, the dealer observes the market signal, $x$, for a product and decides to apply an optimal effort to maximize profit from the resulting sales. When $D = 0$ we show that the exclusive dealer exerts effort levels $b_1^*$ and $b_2^*$ such that $c'(b_1^*)/g'(b_1^*) = p$ and $c'(b_2^*)/g'(b_2^*) = p + \Delta$. The dealer makes the maximum profit, when the total sale is below $K$, by exerting an effort $b_1^*$. An effort level of $b_2^*$ ensures that the dealer makes the maximal profit by exceeding $K$. The level of the input signal beyond which the dealer switches the effort level from $b_1^*$ to $b_2^*$ is defined as the cutoff ($co_1$) and is computed as $co_1 = K - g(b_2^*) + \varepsilon$, where $\varepsilon = \frac{1}{\Delta} ([p \ g(b_1^*) - c(b_1^*)] - [p \ g(b_2^*) - c(b_2^*)])$.

The introduction of a positive bonus allows the dealer to exert an additional effort to reach $K$ even before the input signal crosses the cutoff level computed with $D = 0$. This third effort level ($b^K$) is such that $x + g(b^K) = K$ and satisfies the following inequality: $p \ g(b^K) - c(b^K) + D \geq p \ g(b_1^*) - c(b_1^*)$. When $D > 0$ there are two cutoff levels for the input signal: (i) $co_1$ when the dealer switches from $b_1^*$ to $b^K$, and (ii) $co_2$ when the dealer switches from $b^K$ to $b_2^*$. For a given threshold, $K$, these cutoff points are given by $co_2 = K - g(b_2^*)$ and $co_1 = K - cc_1 \ (where \ cc_1 : p \ cc_1 - c(g^{-1}(cc_1)) + D = p \ g(b_1^*) - c(b_1^*))$.

Having computed these optimal effort levels, and cutoff points, we show that the optimal effort levels are nested, i.e. $b^K \geq b_2^* \geq b_1^*$. Denoting $E(s)$ and $V(s)$ as the expected value and variance of the total sales respectively, we prove the following important results. When $D = 0$ the function $E(s) - E(x)$ is constant at $g(b_2^*)$ for $K \in [0, \ g(b_2^*) - \varepsilon]$ and monotonically decreases for larger values of $K$. Similarly, $V(s) - V(x)$ is 0 in the range $[0, \ g(b_2^*) - \varepsilon]$ but increases beyond that until $K$ reaches a critical value, after which the function begins decreasing. The situation is different when the manufacturer offers a bonus. For values of $K \in [0, \ g(b_2^*)]$ the function $E(s) - E(x)$ is constant at $g(b_2^*)$ but begins increasing for values of $K$ beyond $g(b_2^*)$ until $K$ equals $cc_1$. For values of $K \geq cc_1$ the function decreases monotonically. The variance function, $V(s) - V(x)$, is zero for $K \in [0, \ g(b_2^*)]$, but begins decreasing beyond $g(b_2^*)$ until $K = cc_1$. $V(s) - V(x)$ follows a similar inverted-U shape.
for values of $K \geq cc_1$. Figure 3 summarizes the findings for both cases. As shown in the figure, the coefficient of variation ($\sqrt{V(s)/E(s)}$) is the least when $D > 0$ and $K = cc_1$.

These results have important implications for a manufacturer with a high cost associated with sales variance. Suppose $\phi(K, D)$ defines the gap between the expected profit functions for a manufacturer when no bonus is offered ($D = 0$) and a positive bonus is offered ($D > 0$) for different levels of $K$ ($\phi(K, D) = 0$ defines the iso-expected-profit curve). We prove that $\phi(K, D)$ is a decreasing function in the range $g(b_2) \leq K \leq cc_1$. But as seen earlier, the gap in sales variance is simultaneously increasing in the same interval. So, for a manufacturer with a sufficiently high cost of operational sales variance, not offering a bonus is a costly proposition. Such a manufacturer may be able to control profit by offering a bonus.

We conducted several numerical experiments to validate our results. Figure 1 summarizes the results when $x$ follows an Uniform distribution between 0 and 150. The important parameters are shown in the box to the left with the effort and cost function being $\epsilon$ and $e^2/1000$ respectively. The plot to the left shows the decrease in the coefficient of sales variance when a bonus is offered (in this case $cc_1 = 88.7$). The plot to the right shows the

![Figure 1: Exclusive dealer: coefficient of sales variation and manufacturer’s profit with penalty for sales variation.](image)
increase in the objective value (profit) of the manufacturer when a bonus is offered and a penalty for sales variance ($c_v = 50$) is included in the profit objective.

4. The Non-Exclusive Dealer

A non-exclusive dealer has the choice of allocating effort levels across manufacturers to maximize profit from sales of all products. We consider a situation when two manufacturers (1 and 2) sell their products through a consolidated dealer and no bonus is offered on reaching the threshold.

For given levels of input market signals, $x_1$ and $x_2$, the non-exclusive dealer must choose effort levels that maximize profit. First we characterize the optimal effort levels exerted by the dealer for each manufacturer. We study the situation when the threshold value is the same for both the products ($K_1 = K_2 = K$) but the base profits ($p_1, p_2$) and additional incentive profits ($\Delta_1, \Delta_2$) may be different. The analysis is extensible for different threshold values for each of the products. First we prove that, unlike an exclusive dealer, the non-exclusive dealer exerts four levels of effort, $e_{1i}, e_{2i}, e_{3i}, e_{4i}$ for each manufacturer (product) $i$. The introduction of two additional effort levels for a manufacturer is due to dependence on the input signal level of the other manufacturer’s product and the fact that the dealer optimizes effort levels based on profit across both products. In general the effort levels, for the two products 1 and 2, are related as follows:

$$
\frac{\frac{g'(e_{12})}{g(e_{11})}}{\frac{g'(e_{22})}{g(e_{21})}} = \frac{p_1}{p_2}, \quad \frac{\frac{g'(e_{12})}{g'(e_{21})}}{\frac{g'(e_{32})}{g'(e_{31})}} = \frac{p_1 + \Delta_1}{p_2}
$$

and

$$
\frac{\frac{g'(e_{12})}{g'(e_{11})}}{\frac{g'(e_{22})}{g'(e_{21})}} = \frac{p_1}{p_2 + \Delta_2}.
$$

Figure 2: A non-exclusive dealer’s optimal efforts and input signal cutoffs.

Figure 2 shows the optimal efforts for ranges of the input market signals $x_1$ and $x_2$. Each market signal has two cutoff levels (e.g. $co_2$ and $co_4$ for product 1) beyond which the non-exclusive dealer switches effort levels. It is interesting to note that the two dark regions (along the inclined line segment), are a cause of concern for either manufacturer.
Depending on the level of input market signals, the dealer may switch effort levels causing variation in sales for a manufacturer. For example, for a fixed input signal $x_1$ between $c_{o2}$ and $c_{o4}$, the dealer may switch effort level for manufacturer 1 from $e_{31}^*$ to $e_{21}^*$ if input signal $x_2$ (for product 2) rises. In such a situation, though the market signal $x_1$ is high enough, manufacturer 1 sees a sudden dip in sales.

Before we compare the exclusive and non-exclusive scenarios we characterize the conditions when the effort levels are nested; i.e. $b_2^* \geq e_{31}^* \geq e_{11}^* \geq e_{41}^* \geq e_{21}^*$. We then prove the two results comparing exclusive and non-exclusive dealers. First, we prove that the expected sales (and hence profit) are lower for the individual manufacturers dealing with a consolidated dealer and higher for the non-exclusive dealer. Next we show that, under certain probability distribution conditions, the variance of sales increases for manufacturers. Specifically, the condition provides a lower bound on the minimum probability associated with a particular effort level. These results also imply that there are situations when the coefficient of variation of sales is lower for a non-exclusive dealer and significantly higher for the corresponding manufacturers.

Figure 3: Comparing coefficient of sales variation for manufacturers, exclusive dealers, and non-exclusive dealers.

Figure 3 summarizes our results through a sample numerical experiment. The parameters of the experiment are shown in boxes with the effort and cost function being $\sqrt{p}$ and $e^2/1000$ respectively. Both input signals are assumed to be uniformly distributed between 0 and 150. The plot to the left shows that the manufacturer’s coefficient of variation is higher when dealing with a non-exclusive dealer (NED) and lower with an exclusive dealer (ED). The plot to the right shows that a non-exclusive dealer’s coefficient of variation is much lower than an exclusive dealer’s coefficient of variation. The manufacturer’s coefficient of variation is the highest with the non-exclusive dealer.