1. Introduction

With the advancement in information technology facilitating information sharing between the parties in the supply chain and the emergence of more efficient transportation systems, effective management of distribution systems has become even more critical in recent years. Replenishment/ordering, shipment consolidation/coordination policies, different methods to allocate inventory to locations downstream, and effective utilization of information are among the most important supply chain management issues that need to be studied. To reduce the inventory/logistics costs, companies are increasingly moving towards such practices as shipment coordination/consolidation through information sharing. Shipment consolidation means combining multiple shipments into a single group through coordinated replenishment. Among the several methods to handle the flow of goods across the supply chain, cross-docking is considered to be the most efficient one to facilitate consolidation. Under cross-docking, shipments from the suppliers are received at an intermediary point (cross-dock facility) that usually does not hold stock. Upon receipt, they are broken and sorted and then shipped according to customers’ orders. By introducing a warehouse that is a stylized representation of a cross-dock (keeps no inventory) and let it coordinate the ordering, allocation, and distribution of various inventory items to various retailers, we can achieve the potential savings in fixed ordering/shipment costs.
In this paper, we study the impact of coordinated replenishment and shipment in multi-item inventory/distribution systems with multiple identical retailers, one warehouse (with no inventory), and multiple suppliers each with unlimited supply. Both policies (Policies 1 and 2) that we analyze in this paper achieve coordination across retailers. That is all the retailers are replenished simultaneously. But, the coordination in Policy 2, which will be discussed in detail in Section 2, is two-fold: 1) retailers are replenished simultaneously, and 2) synchronization of ordering/shipment of different items is made possible through scheduled ordering. Thus, in this paper, our main goal is to quantify the benefit of coordination across different items via comparing these two policies.

2. Model

We consider a centralized distribution system consisting of $N$ identical retailers, $M$ suppliers, and a warehouse (with no inventory). Each item is procured from a different supplier where the transit times from any supplier to the warehouse (for each item) and from the warehouse to any retailer are assumed to be constant. In addition, all the end demands at the retailers for various items are assumed to be random (Poisson distributed) and independent of one another. All excess demand at retailers is backordered. We assume that the warehouse has full information about demand/inventory status and cost structure at retailers and is in charge of replenishing the stock for the system through outside suppliers who hold ample supply of the particular item. We also assume that the inbound and the outbound trucks to and from the warehouse have capacity limitations and a penalty cost is incurred if the shipment quantity exceeds the truck capacity.

The notation is as follows (we remove the index for retailers since they are assumed to be identical):

$M$ ($N$): Total number of items (retailers) in the system
$L_{0j}$: transit time from the supplier to warehouse for item $j$ ($j=1,...,M$)

$L$: constant transit time from the warehouse to any retailer

$L_{Tj}$: Total lead time for item $j$ ($L_{0j} + L$)

$I\!P_j(t)$ ($IL_j(t)$): Any retailer’s inventory position (level) at time $t$ for item $j$

$D_j(t)$: Demand at any retailer for a period of time $t$ for item $j$.

$\lambda$: Mean demand rate for item $j$

$T_{\eta}$: the time between two consecutive replenishments for item $j$.

$h_j$: Unit inventory holding (backorder) cost/time at a retailer for item $j$.

$K_{0j}$: Fixed ordering/transportation cost per order at the warehouse for item $j$.

$K$: Fixed cost of a shipment to a retailer.

$Z_{0j}$: Shipment quantity of item $j$ from the supplier to the warehouse per order cycle.

$Z_j$: Shipment quantity of item $j$ from the warehouse to a retailer per order cycle.

$Y$: Total shipment quantity (all items combined) from the warehouse to a retailer per order cycle.

$C_{0j}$: Maximum capacity of a truck from supplier to warehouse for item $j$

$C$: Maximum capacity of a truck from warehouse to a retailer

$g(n;C_{0j})$: Penalty cost to the warehouse when the inbound quantity ($Z_{0j}$) is “$n$” units.

$g_r(n;C)$: Penalty cost to a retailer when the outbound quantity ($Z_j$ or $Y$) is “$n$” units.

$CR_i$: Cost rate (cost per time unit) for Policy $i$.

Specifically, we analyze the following two policies:

**Policy 1:** The warehouse orders to raise all the retailers’ inventory position to $S_j$ for item $j$ whenever any retailer’s inventory position for item $j$ drops to $s_j$ or the total demand at all the retailers for item $j$ reaches $Q_j$ (for $j=1,...,M$).
Without loss of generality, we index the items such that $L_{01} \geq L_{02} \geq \ldots \geq L_{0M}$. That is, the item with the largest transportation time from the supplier to the warehouse is item 1. Moreover, let $\delta_{0j} = L_{01} - L_{0j}$ for $j=2,\ldots,M$.

**Policy 2:** The warehouse orders to raise all the retailers’ inventory position to $S_1$ for item 1 whenever any retailer’s inventory position for item 1 drops to $s_j$ or the total demand at all the retailers for item 1 reaches $Q_1$. Moreover, the inventory position of all retailers’ for item $j$ ($j=2,\ldots,M$) are raised to $S_j$ exactly $\delta_{0j}$ time units after every time an order for item 1 is placed at the supplier.

The two policies are based on both the installation and the echelon inventory positions. The system replenishment may be triggered in two ways: (1) installation trigger: whenever any retailer’s inventory position drops to $s$ (which is a special type of can-order policies with the can-order levels being equal to the order-up to levels); or (2) echelon trigger: whenever the echelon inventory position drops to $NS-Q$. The major difference between the two is that Policy 1 replenishes different items independent of one another. Hence, the problem is separable in the sense that the optimal values of the decision variables $(S_i, s_i, Q_i)$ for each item can be determined separately. On the other hand, Policy 2 consolidates the outbound shipments by synchronizing the arrival of the items at the warehouse. Placing an order for item $j$ $\delta_{0j}$ time units after every time an order for item 1 is placed ensures that the inbound trucks carrying different items arrive at the warehouse at the same time. Thus, the warehouse can consolidate the outbound shipments. This enables the warehouse to ship different items in the same truck to the retailers, rather than shipping the items to the retailers at different points in time, which would lead to savings in outbound transportation costs. Also, the system benefits from the reduced inventory costs since the warehouse doesn’t keep any inventory possibly at the expense of higher inventory costs at
retailers. One should note that under Policy 2, the warehouse is not allowed to adjust the size of the order placed at the supplier even though extra demand for item $j$ will probably be realized within $\delta_{0j}$ time units before the shipment is actually made from the supplier. We are currently working on a policy where the timing of the order is the same but the warehouse can adjust the order quantity of item $j$ based on the demand realized within $\delta_{0j}$ time units.

The decision variables for Policy 1 are $S_j$, $s_j$, $Q_j$ (for $j=1,\ldots,M$) whereas for Policy 2 $S_1$, $s_1$, $Q_1$, and $S_j$ (for $j=2,\ldots,M$) are the decision variables. With $\Delta=S_j-s_j$, the cost rates (cost per unit time) will be as follows for the two policies:

$$CR_1 = \sum_{j=1}^{M} \frac{K_{0j} + \sum_{i=2}^{N} K \Pr(Z_j \geq 1)}{E[\tau_j]} + \frac{K + \sum_{i=2}^{N} K \Pr(Z_j \geq 1) + N^* \left((h_j + \pi_j)E[I_{L_j}^{-}] + \pi_j \left(E[D_j(LT_j)] - E[I_{P_j}]\right)\right)}{E[\tau_j]} + \frac{K + \sum_{i=2}^{N} K (1-\beta_i)}{E[\tau_1]} + \frac{1}{E[\tau_1]} \left(\sum_{n=\min(\Delta_j,Q_j)}^{Q_j} g(n;C_0) \Pr(Z_{0j} = n) + \sum_{j=2}^{M} \sum_{n=0}^{\infty} g_i(n;C) \Pr(Y = n)\right).$$

For Policy 2, let $\chi_j$ be the probability that an order will be placed at the supplier for item $j$ ($j\neq 1$), and $\beta_i$ be the probability that there will be no shipment to retailer $i$ in an order cycle. Then the cost rate for Policy 2 is:

$$CR_2 = \frac{K_{0j} + \sum_{j=2}^{M} K_{0j} (1-\chi_j)}{E[\tau_1]} + \frac{K + \sum_{i=2}^{N} K (1-\beta_i)}{E[\tau_1]} + \frac{1}{E[\tau_1]} \left(\sum_{n=\min(\Delta_j,Q_j)}^{Q_j} g(n;C_0) \Pr(Z_{0j} = n) + \sum_{j=2}^{M} \sum_{n=0}^{\infty} g_i(n;C) \Pr(Y = n)\right).$$

3. Numerical Results

In order to see the impact of coordination across different items (in addition to coordination across retailers), we compare the cost rates of the two policies for a distribution system with $N=5$ (identical) retailers and $M=2$ items. We assume, for the numerical tests only, that the items are...
also identical except for the values of $L_{0j}$. We fixed the values of $h_j$, $\lambda_j$, $L$, $K_{0j}$, and the inbound/outbound truck capacities. We also assume that transportation penalty cost is linear in the number of units beyond the capacity. A total of 135 cases were considered based on the following parameter set:

$$\frac{K_{0j}}{K_{00}} \in \{0.2, 0.4, 0.6, 0.8, 1\}, \; L_{01} \in \{0.5, 1, 2\}, \; \frac{L_{02}}{L_{01}} \in \{0.25, 0.5, 1\}, \; \frac{\pi_j}{\pi_h} \in \{0.9, 0.95, 0.99\}.$$

Numerical results indicate that Policy 2 outperforms Policy 1 (95% of all the cases). The average deviation of Policy 1 from Policy 2 is 10.24%, whereas the maximum deviation is as high as 19.6%. We observe that the savings from coordination are higher for larger values of $K$, lower $L_{01}$, higher $L_{02}/L_{01}$, and lower $\pi_j$. The first three observations are as expected, and the last one is probably due to the fact that there is no reorder level for the second item under coordination and this hurts Policy 2 when the unit shortage cost is very high compared to unit holding cost. Surprisingly enough, we also observe that the impact of $L_{01}$ (and $L_{02}/L_{01}$) is not as significant as $K$ and $\pi_j$.

4. Conclusion

In this paper, we analyze a distribution system with multiple retailers and items (suppliers) with a warehouse with no inventory. We analyze the impact of replenishment/shipment coordination across different items in addition to the coordination across retailers. We observe that it pays off to coordinate across different items such that shipments to the retailers are synchronized. The current work in progress is to improve Policy 2 by introducing reorder levels and allowing the warehouse to adjust the order quantity by the actual time the order is placed for all products.