1. Introduction

Traditional inventory and supply-chain models typically assume that materials only flow in a forward direction. In reality, however, supply chains often include reverse flows as well. For example, customers may change their minds about purchases and return the items to the store where they bought them (or perhaps a different location if the store is part of a chain). Reverse flows can also arise when customers lease goods instead of purchasing them. For example, customers subscribing to digital cable television service receive a control box that they then return if they choose to cancel their subscription.

This paper considers a distribution system where such returns are present. There is a single warehouse and N retailers, and stochastic demands and returns arrive at the retailers according to independent Poisson processes. We explore the impact of returns on the system under two different types of control policy – one uses local inventory information, while the other uses centralized information. We develop approaches for approximating system performance and computing heuristic inventory policies, and investigate how the presence of returns affects the accuracy of these approximations. We also compare cost performance of the two approaches.

Our approach to the local policy is a variation on the METRIC model of Sherbrooke (1968), while our approach to the centralized policy builds on the work of Miller (1974) and Eppen and Schrage (1981), and further developments by Federgruen and Zipkin (1984a-c). For a discussion of the distribution system literature, see Federgruen (1993), Axsater (1993) and Zipkin (2000).
For a single-stage system with product returns, Fleischmann et al. (2002) establish the form of the optimal policy and show how to compute it. DeCroix et al. (2005) establish the form of the optimal policy for a series system with returns, and develop both exact and approximate methods for evaluating the performance of any policy. Based on one of the approximations they show how to compute near-optimal policies. For reviews of other research on systems with reverse flows, see Dekker et al. (2004).

2. Model

We consider a system in which a single product is sold by \( N \) retailers. Customer demands arrive at retailer \( j \) according to a Poisson process with rate \( \mu_j \). Each retailer faces a Poisson stream of customer returns with rate \( \lambda_j \) – returned items are immediately available to satisfy future demands. Demand and return streams are independent of each other and across retailers, and unsatisfied demands are backordered. Retailers place orders with a single warehouse (facility 0), which places orders with an outside supplier. Delivery lead times to the warehouse and retailers are constant and denoted by \( L_0 \) and \( L_j \), respectively. The system incurs unit holding costs \( h_j \) at facility \( j = 0, \ldots, N \), and unit backorder costs \( b_j \) at retailer \( j = 1, \ldots, N \).

We consider two different approaches for managing the system. (In describing these approaches we use the notation in Zipkin 2000.) The first is a local policy. Each retailer orders from the warehouse when its local inventory position \( IP_j'(t) \) drops below its base-stock level \( s_j' \). The warehouse ships units immediately if available; if not, backorders at the warehouse are filled on a first-come first-served basis. The warehouse places an order when \( IP_0'(t) \) drops below \( s'_0 \).

The second is a centralized policy. The key difference is that the warehouse has system-wide information and makes its ordering and allocation decisions based on echelon quantities. As a result, backordered retailer demands are not filled on a first-come, first-served basis.
3. **Policy Analysis**

For both policies, the presence of returns alters the inventory and ordering dynamics at the retailers. Instead of having $IP_j' = s_j'$ at all times (as in a system without returns), $IP_j' = s_j' + Z_j$, where $Z_j$ is the number of jobs in an M/M/1 queuing system with arrival rate $\lambda_j$ and service rate $\mu_j$. Demands occurring when $Z_j = 0$ result in an order placed with the warehouse (thus preserving $IP_j' = s_j'$), while demands occurring when $Z_j > 0$ simply reduce $Z_j$ by 1 and do not trigger an order to the warehouse.

For the local policy, this means that the warehouse does not see actual end-product demand. The warehouse’s local net inventory can still be written as $IN_0' = s_0' - D_0$, but $D_0$ represents retailer orders during the warehouse lead time $L_0$, which is no longer a Poisson random variable (as in systems without returns). However, we propose approximating $D_0$ by a Poisson random variable with mean $E(D_0) = L_0 \sum_{j=1}^{N} (\mu_j - \lambda_j)$. Numerical experiments reveal that this approximation may be somewhat inaccurate for small $N$, but becomes increasingly accurate as the number of retailers grows. With this approximation, we can easily compute the distributions of warehouse net inventory $I_0' = [IN_0']^+$ and warehouse backorders $B_0' = [IN_0']^-$. Under the local policy, retailer $j$’s net inventory is $IN_j' = s_j' + Z_j - B_{0j}' - D_j$, where $B_{0j}'$ represents backorders at the warehouse that are owed to retailer $j$, and $D_j$ represents net lead time demand (demands minus returns). Marginal distributions for $Z_j$ and $D_j$ are easy to compute, and $Z_j - B_{0j}'$ is independent of $D_j$. However, two challenges remain: 1) we need to compute the marginal distribution of $B_{0j}'$, and 2) $B_{0j}'$ and $Z_j$ are not independent. Preliminary numerical trials suggest that the (negative) correlation between $B_{0j}'$ and $Z_j$ is not particularly strong, so we assume independence as an approximation. For the marginal distribution of $B_{0j}'$, we use the binomial disaggregation approach – i.e., we treat the conditional random variable $(B_{0j}' | B_0')$ as binomial with parameters $B_0'$ and $(\mu_j - \lambda_j) / (\mu - \lambda)$, where $\mu = \sum_{j=1}^{N} \mu_j$ and $\lambda = \sum_{j=1}^{N} \lambda_j$. Approximate system costs can be computed based on the approximate
performance measures, and can be optimized in a nested fashion – search over $s'_0$, solving for the optimal $s'_j$ for each $s'_0$ (where the latter is easy since for fixed $s'_0$ the system decomposes and costs are convex in each $s'_j$).

The centralized approach uses the balanced assumption (retailer inventories can be rebalanced at any point in the future) and the myopic allocation (allocate scarce units to retailers to minimize current costs), which have been shown to be effective in systems without returns. This facilitates approximation of the distribution system by a two-stage series system. The demand faced by the downstream facility is approximated by a Normal random variable with mean equal to that of aggregate retailer net demand – i.e., $\sum_{j=1}^N L_j (\mu_j - \lambda_j)$ – and standard deviation equal to the sum of the retailer-specific lead-time-demand standard deviations – i.e., $\sum_{j=1}^N \sqrt{L_j (\mu_j + \lambda_j)}$. Notice that returns reduce mean demand but increase variability, resulting in a higher coefficient of variation. With returns present, we also need to capture the influence of the $Z_j$ – we approximate this using the random variable $Z = \sum_{j=1}^N Z_j$. We compute base-stock levels $s_0$ and $s_r$ for this system using the approach presented by DeCroix et al. (2005). Then the system orders to bring the warehouse’s echelon inventory position up to $s_0$ and the aggregate retailer inventory position up to $s_r$. When retailers are identical, this translates to a base-stock policy at each retailer with each $s_j = s_r / N$.

4. Numerical Study

To evaluate the accuracy of our approximations and assess relative performance of the two policies, we conducted numerical trials with identical retailers, $\lambda_j = \alpha \mu_j$ for $\alpha \in \{0.1, 0.3\}$, $N \in \{2, 4, 8\}$ (with aggregate demand and returns held constant), and various cost and lead time parameters. Figure 1 shows the average relative error ([approximate – simulated]/simulated) of each approach for the case of $\alpha = 0.3$ when evaluated at the policies obtained by optimizing the approximate models. Although both approximations tend to underestimate the true cost, the local
approximation becomes more accurate as the number of retailers increases, while the centralized approximation becomes less accurate. However, despite this trend toward reduced accuracy, and despite the increased coefficient of variation caused by returns (recall that the centralized policy tends to perform well in systems without returns when the coefficient of variation of demand is not too high), the centralized policy tended to yield lower system costs even in the cases with a larger number of retailers. For example, for \( \alpha = 0.3 \), the local policy had costs that averaged 4.7%, 6.7% and 4.2% higher than the centralized policy for the cases of \( N = 2, 4 \) and 8, respectively.

![Figure 1. Relative accuracy of approximations](image)

5. **Future Work**

This paper is a work in process – current and future directions for extension of the analysis include the following:

- More extensive testing of the approximation approaches to more completely characterize settings in which they perform well;
- Further refinement of the approximations to improve performance;
- Exploration of the impact of alternative strategies for handling returns, for example, allowing demands and returns directly at the warehouse (as might occur when a retailer has both online and retail sales channels).
References


