1. Introduction

Upselling is the practice of offering an additional product or service to a customer who just made a purchase. Many catalog, online and traditional retailers use upselling. One popular reason for the use of upselling is liquidation of excess inventories. In such cases, the retailer identifies an item as a slow-moving item with excess inventory, and tries to liquidate the inventory of this item by offering it, possibly at a discount, to customers who purchased other items. In this paper, we focus on upselling practices aimed at liquidation of excess inventories.

When a firm is upselling a product, the firm will not only sell the product to customers who purchased other items, but there will also be some customers arriving with the intention of purchasing the product in the first place. There will be an announced price that these customers will see. If the firm is trying to clear the excess inventory of the product, then it would be natural for the firm to adjust this announced price over time as long as it is feasible to do so. Regardless of whether the firm engages in such dynamic pricing or not, the firm can choose to use upselling. In cases where dynamic pricing is used, upselling can be seen as a promotional tool that complements dynamic pricing. We consider both the case in which the firm is adjusting the announced price dynamically over time and the case in which the announced price is fixed in the beginning.
In using upselling to reduce excess inventories, the firm has to make two important decisions: (i) The firm must decide which items to upsell, a decision that essentially requires the firm to decide what items to classify as slow-moving ones with excess inventories. In this paper, we assume that the firm has already chosen a single item for upselling (hereafter, the promotional product). (ii) Once the firm decides which items to upsell, the firm must decide whether the upsell offers should be accompanied by a discount off of the announced price. Firms may be able to tailor these decisions at the individual customer level. A firm using upselling has one valuable piece of information about the customer who is about to receive the upsell offer: The firm knows that this customer just purchased a certain product at a certain price. This information sets the customer in question apart from a generic customer in the population, and allows the firm to make a better-informed decision on whether to offer a discount. This is the decision that we focus on in this paper. In order to address the question, we use a model where the consumer population is divided into two segments for each product: target customers and non-target customers. The firm does not know which segment an arriving customer belongs to, but can use the purchase decision made by the customer in order to update its knowledge of the customer. This updated knowledge is then used when tailoring the upsell offer to the customer.

In some cases, the firm may be able to decide not only whether the upsell offer to an individual customer should include a discount, but also how deep the discount offered to that customer should be. On the other hand, there may exist cases where the firm may be flexible in deciding whether the customer will get a discounted upsell offer, but the size of the discount may be fixed and the same for all customers who do get a discount. We consider both cases.

We describe the model in Section 2 for the case where the announced price is dynamically adjusted and the discount level is customized for each customer. Section 3 presents a discussion of our results for that case. In Section 4, we extend our results to the case where the announced price and the discount level are set in the beginning and remain fixed. We conclude by a discussion of extensions and numerical results in Section 5.

2. Model Description

We consider a firm offering two distinct products. One of these is a well-established product with a year-long, stable demand; one can think of it as a permanent, regular item in the
firm’s product portfolio. We assume that the price of this product is exogenously fixed at \( r \), and it is always available when a customer demands it. The other product, which we refer to as the promotional product, is the one that the firm is upselling. There is a certain deadline after which the demand for the promotional item will cease to exist. The promotional item has a fixed initial inventory, and no replenishment opportunities exist during the product’s lifetime.

For each product, we assume that the customer population is clustered into two segments: the target segment and the non-target segment. The target customers are those who are the primary consumers of the product, while the non-target customers are all the other customers in the population. We will use 1 as the index for the target segment and 2 as the index for the non-target segment. We let \( q_{Ri}, i = 1, 2 \), denote the probability that a customer belongs to segment \( i \) for the regular item. Similarly, \( q_{Pi}, i = 1, 2 \), denotes the probability that a customer belongs to segment \( i \) for the promotional item. We allow for the possibility that a customer may belong to the target segment for a product and the non-target segment for the other product. Let \( \delta_{ij}, i, j = 1, 2 \), denote the probability that a customer belongs to segment \( j \) for the promotional item given that he belongs to segment \( i \) for the regular item.

The customers have stochastic reservation prices for the two products. Let \( F_i \) and \( G_i, i = 1, 2 \), be the cumulative distribution functions (cdf) of the reservation price of segment \( i \) customers for the regular item and the promotional item, respectively. Throughout the paper, we let \( \overline{F}_i(x) := 1 - F_i(x) \) and \( \overline{G}_i(x) := 1 - G_i(x) \). For two cdf’s \( \Phi_1 \) and \( \Phi_2 \), if \( \Phi_1 \) dominates \( \Phi_2 \) in failure rate ordering, then we write \( \Phi_1(x) \geq_{fr} \Phi_2(x) \). We make the following assumptions on the reservation price distributions:

(A1) \( F_i, G_i, i = 1, 2 \), are continuous and they all have the same support.

(A2) \( F_i, G_i, i = 1, 2 \), have increasing generalized failure rates.

(A3) \( F_1(x) \geq_{fr} F_2(x) \) and \( G_1(x) \geq_{fr} G_2(x), \forall x \).

The ordering stated in (A3) can be interpreted as saying that the absolute price elasticity of demand of a target customer is less than that of a non-target customer, i.e., customers in the target segment are less price-sensitive.

We focus our attention on the time horizon during which the promotional item will be sold. We assume that this finite horizon is divided into \( T \) periods, each of which is short enough that either one customer arrives in each period or none at all. Furthermore, each period is divided into two stages. In the first stage, the seller announces \( p_t \), the price for the promotional item that will be in effect during the first stage of period \( t \). Then, one
of three events happens: A customer considering the purchase of the regular item arrives with probability $\lambda_R$; a customer considering the purchase of the promotional item arrives with probability $\lambda_P$, and no customer arrives with probability $1 - \lambda_R - \lambda_P$. A customer considering a product will purchase it if the price of the product is less than or equal to his reservation price for the product. The second stage occurs only if a customer arriving in the first stage purchases the regular item. This is the stage in which the firm makes an upsell offer to a customer who purchased the regular item. Let $d_t$ denote the discount offered to the customer in the second stage of period $t$. Note that $d_t$ may be zero. The customer will purchase the promotional item if $p_t - d_t$ is less than or equal to his reservation price for the promotional item.

In this setting, the firm’s problem is to maximize its total revenue over the $T$-period horizon by optimally choosing $p_t$ and $d_t$, $t = 1, \ldots, T$. In the first stage of period $t$, given that a customer arrived and is considering the purchase of the regular item, let $\beta_R(r)$ denote the probability that the customer will buy the regular item. We can write $\beta_R(r) = q_{R1} F_1(r) + q_{R2} F_2(r)$. Similarly, let $\beta_P(p_t)$ denote the probability that a customer will buy the promotional item, given that the customer arrived in the first stage of period $t$ and is considering the purchase of the promotional item. Then, we can write $\beta_P(p_t) = q_{P1} G_1(p_t) + q_{P2} G_2(p_t)$. If a customer arriving in a period buys the regular product in the first stage, this provides the firm with additional information about the customer, namely that the customer’s reservation price for the regular product is above $r$. Then, in the second stage, before deciding what level of discount to offer to this customer on the promotional item, the firm can use this new information in order to update the probabilities with which the customer belongs to the target and non-target segments for the promotional item. Let $\alpha(p_t, d_t)$ denote the probability that a customer will buy the promotional item in the second stage in period $t$ when the firm offers him a discount $d_t$ off of the announced price $p_t$. One can write an expression for $\alpha(p_t, d_t)$ as a function of the problem parameters. Now, the dynamic programming problem that the firm faces is as follows:

$$V_t(y) = \max_{p_t \geq 0, d_t \geq 0} \{ \lambda_R \beta_R(r) [r + \alpha(p_t, d_t) (p_t - d_t + V_{t-1}(y - 1))] \\
+ \lambda_P \beta_P(p_t) (p_t + V_{t-1}(y - 1)) + (1 - \lambda_R \beta_1(r) \alpha(p_t, d_t) - \lambda_P \beta_P(p_t)) V_{t-1}(y) \}$$
3. Dynamic Price, Dynamic Discount

One question we are interested in answering is whether the firm should accompany upsell offers with discounts on the promotional item and, if so, how deep those discounts should be. The following proposition states that the answer depends only on the values of $\delta_{ij}$’s.

**Proposition 1** Let $p_t^*(y)$ and $d_t^*(y)$ denote, respectively, the optimal announced price and the optimal discount for the promotional item with $y$ units in inventory and $t$ periods to go. Given assumptions (A1) through (A3):

(a) If $\delta_{11} + \delta_{22} < 1$, then $d_t^*(y) > 0$.
(b) If $\delta_{11} + \delta_{22} > 1$, then $d_t^*(y) = 0$.

Note that whether the firm will offer a discount or not has nothing to do with time or the inventory level, but depends completely on the segments’ alignment across the products. The intuition behind the result is that if the segments are inversely aligned to such a degree that $\delta_{11} + \delta_{22} < 1$, then a customer who bought the regular product is less likely to be a target customer for the promotional product compared to a customer chosen from the population at random. Next, we state a result that characterizes the behavior of the optimal announced price and the optimal upsell price in time and inventory level:

**Proposition 2** Let $p_t^*(y)$ and $d_t^*(y)$ denote, respectively, the optimal announced price and the optimal discount for the promotional item with $y$ units in inventory and $t$ periods to go. Then, both $p_t^*(y)$ and $p_t^*(y) - d_t^*(y)$ are non-increasing in $y$ and non-decreasing in $t$.

The first part of the proposition says that both the optimal announced price and the optimal upsell price will be lower if there is more inventory of the promotional item or if there is less time until the end of the horizon.

4. Fixed Price, Fixed Discount

So far, we have been concerned with the case in which both the announced price and the discounts that accompany upsell offers can be adjusted over time. We now turn our attention to the case where the announced price and the discount level is fixed in the beginning. In this case, the announced price is fixed in the beginning so that $p_t = p$ for $t = 1, \ldots, T$. On the other hand, in each period $t$, the firm has two choices regarding the discount that
will accompany the upsell offer: $d$ or 0. The following proposition states our result on the optimal discount levels for this scenario:

**Proposition 3** For a fixed inventory level $y$, if it is optimal to set $d_t = d$ at any time $t = t_0$, then it is also optimal to set $d_t = d$ for any $t < t_0$. On the other hand, at any fixed time $t$, if it is optimal to set $d_t = d$ when there are $y = y_0$ units in inventory, then it is also optimal to set $d_t = d$ when there are $y > y_0$ units in inventory.

The proposition indicates that, in the fixed price and fixed discount case, whether or not a discount will accompany an upsell offer depends critically on the inventory level and time to go. To be more precise, the policy is of the switching curve type: There exists a boundary of price and time-to-go pairs above which the firm will offer a discount along with the upsell offer. Note that this result is markedly different from that obtained when both the announced price and the discount could be dynamically adjusted.

5. **Conclusion**

Our discussion so far focused on the case where there is only one regular product. We have extended some of the results to the case where there are multiple regular products. The insights mostly remain the same with one important difference: With multiple products, in the dynamic-price, dynamic-discount case, there may be situations where the firm will offer a discount to a purchaser of a regular product even when the promotional product and the regular product in question are not inversely aligned. However, it is still true that the firm will continue to offer discounts whenever the two products are inversely aligned.

In addition, we carried out a numerical study to evaluate the improvement in expected profits due to the use of customer purchase information while upselling. Solving 2997 test cases, we find that these benefits can be up to 5%, but in the majority of the test cases, they were less than 1%. As expected, the benefits seem to be especially large when a customer in the target segment of the regular product is very likely to be in the non-target segment of the promotional product. Whether the prices and the discounts are adjusted dynamically does not seem to have a significant influence on these benefits. We also analyzed how the benefits from dynamic pricing depend on whether customer purchase information is used or not. Interestingly, in a small number of cases, we find that using dynamic pricing without the use of customer purchase information can lead to a reduction in expected revenues compared to static pricing.