Advance Selling with Limited Capacity

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1 Introduction

Advance selling has already become a standard way of selling products in many service industries (airline, hotel, entertainment, and personal services), and it has been successfully adopted in some non-service industries including toys, books, and apparels. By selling in advance, sellers offer customers an opportunity to purchase goods or services and guarantee the availability at a time considerably preceding consumption. However, customers are often unsure about their future valuations of a product, especially when consumption is going to take place in future. The future valuation may depend on their health, mood, arrangement conflicts, etc, which are not clear until a time very close to consumption. Customers who choose to purchase in advance have to bear the risk of not valuing the product in the future as high as they expected. In order to make the advance offer attractive, the seller often quotes a discounted price or guarantee the availability at the time of consumption. It is not clear whether advance selling always benefits the seller, especially when the seller has limited capacity and faces a population of heterogenous customers. In this paper, we focus on the impacts of customer uncertain valuation and finite capacity on seller’s joint pricing and capacity-allocation strategy, when advance selling is an option.

2 The Model

Consider a monopolistic risk-neutral seller who can sell an undifferentiated single product in two periods, advance and spot, to potential risk-neutral customers. The seller has capacity $T$ and unit cost $c$. In advance time, seller offers price $p_1$ to $N_1$ customers who arrive at that time. Customers decide whether to buy the product based on their (expected) utility of consumption: $U_i = A + \alpha_i - Bp$ for customer $i$, where $A$, $B$ are constants, $p$ is the price that customer pays, $\alpha_i$ is a nonnegative individual valuation whose value is unknown in advance.
We assume customers’ future valuations $\alpha_i$ are drawn from an identical distribution $G(\alpha)$, which is known to the seller. The customers forecast the spot price and choose whether to buy in advance or wait until spot by comparing their expected utilities. In spot time, additional $N_2$ customers arrive and all customers’ uncertainty on their own valuations ($\alpha_i'$s) is resolved, but these valuations remain private and are not known to the seller. Seller decides $p_2$ and all remaining customers, those who arrive in spot time and those who did not purchase in advance, decide whether to buy the product in spot period.

Although customers have the same prior belief $G(\alpha)$ about their future valuation in advance, their realized valuation may be very different when the valuations are highly dependent on personal factors (e.g. health), or fairly close to each other when certain common factors (e.g. weather) are in dominance. In view of this, we consider three different assumptions for customer behavior in spot: Deterministic Demand, Stochastic Homogeneous Valuations, and Stochastic Heterogeneous Valuations.

• **Deterministic Demand** is a special case of customer interdependency, when the spot demand is a deterministic function of the spot price. Specifically, $G(\alpha)$ represents the proportion of (remaining) population who value the product at $\alpha$ or less in spot. This model could be treated as a reasonable approximation for markets with a large population of customers who have idiosyncratic valuations.

• **Stochastic Homogenous Valuations Model** assumes that the realized valuations of entire population are identical, implying spot demand is either all customers or none. This model represents the ‘herd’ behavior and may apply to markets with highly-conforming customers and homogenously-evaluated goods. Such behavior usually occurs when valuations depend on some exogenous effects which are identical to all customers, such as weather, economic status, government policy, etc. For example, the valuation of a snow-shovelling contract is highly dependent on the actual snowfall, which is unknown in advance of snow season.

• **Stochastic Heterogenous Valuations Model** emphasizes individual heterogeneity and allows for independent realization of customer valuations. This model is most plausible for markets of small or median size, where demand is fairly sensitive to individual or small-group of customers’ decisions. The behavior of customers is characterized by a medium level of variance (higher than in deterministic model, but not as high as in stochastic homogeneous model). Most cases when advanced selling is used in practice fall into this


3 Optimal Strategy

3.1 Unlimited Capacity

In order to illustrate the impact of finite capacity, we first provide results for a benchmark case where seller has unlimited capacity.

Theorem 1. (Equivalence of Three Behavioral Assumptions when Capacity is Unlimited) When capacity is unlimited, the seller’s optimal strategy is the same under the assumption of Deterministic Demand, Stochastic Homogenous Valuations, and Deterministic Heterogenous Valuations.

Thus, when service capacity is infinite, the seller’s optimal strategy is independent of consumer behavior. This seemingly-surprising result is driven by the fact that although spot demand has different distribution, the expected demand is identical.

Further, when seller has an option to sell in advance, we show that discounted advance price is always optimal, i.e. advance price $p_1^*$ does not exceed spot price $p_2^*$.

Theorem 2. (Optimal Prices when Capacity is Unlimited)

$$p_1^* = \frac{1}{B} \{A + E(\alpha) - E(\max(A + \alpha - Bp_2^*, 0))\} \leq p_2^*$$

Furthermore, the profitability of advance selling is determined by seller’s unit cost and the distribution of valuation. While there exist counterexamples, we show that with reasonable cost, advance selling is always better than selling only in spot for most-often used probability distributions such as uniform, exponential and triangular distributions.

3.2 Limited Capacity

• Deterministic Demand:

One of the cited reasons for premium price offered in advance is limited capacity or availability of finished goods. Contrary to the belief that with constrained capacity, customers in advance period may be willing to pay premium to secure his/her chance of obtaining the product, we show that when demand is deterministic, premium advance price is never optimal.
Theorem 3. (Optimal Prices in Deterministic Demand with Limited Capacity)

For any given continuous $G(\alpha)$,

$$p^*_1 = \frac{1}{B} \{ A + E(\alpha) - E(\max(A + \alpha - Bp^*_2, 0)) \} \leq p^*_2$$

i.e. premium advance-selling strategy is never optimal.

Note that when demand is deterministic and capacity is constrained, it is optimal for the seller to charge a spot price such that shortages never occur in spot. In advance period, customers knows that the future availability is always guaranteed, thus they will never pay a premium price.

With limited capacity, in addition to two prices, the seller has to decide how much capacity $S^*_1$ to offer in advance. With IFR distribution and some mild technical assumptions, we characterize the optimal capacity-allocation strategy as illustrated in Figure 1.

![Figure 1: Optimal Capacity-Allocation strategy with Deterministic Demand](image)

As shown in the figure, advance selling is not beneficial with small capacity or high cost. When capacity is very low, selling the product in advance at a discounted price is not beneficial. With high production cost, the seller does not have much flexibility to offer a discounted price to attract enough customers in advance, hence advance selling is not beneficial either. As capacity increases and cost decreases, advance selling becomes more appealing to seller, as it increases sales and revenue over two periods. Furthermore,
note that with medium levels of capacity, it is optimal for the seller to create shortages in advance (by setting $S_1^* < \min(T, N_1)$) and force customers to buy in spot.

- Stochastic Homogenous Valuations

We now examine how the stochasticity of customer behavior influences the seller’s optimal strategy. First we show that the introduction of stochastic demand indeed changes the qualitative features of seller’s behavior.

**Theorem 4. (Optimal Prices in Stochastic Homogeneous Valuations with Limited Capacity)**

$$p_1^* > p_2^* \quad \text{if} \quad T < N_1 + \frac{N_2(A + E(\alpha) - Bp_2^*)}{E(\max(A + \alpha - Bp_2^*, 0))} \quad \text{and} \quad A + E(\alpha) - Bp_2^* > 0$$

Theorem 4 confirms the intuition that premium advance price is optimal when capacity is small. However, we note that it is the joint existence of limited capacity and stochastic demand that allows for a premium price. The stochastic demand in spot prevents the seller from perfectly clearing the market, thus creating potential shortage, which in turn induces the customers to pay premium in advance.

This may partly explain why premium prices may be observed in situations like jazz concerts or football games, where market is relative small and sensitive to many exogenous and homogenous factors, while we seldom observe premium advance prices in airline industry where demand is highly price-dependent and fairly predictable.

The optimal capacity-allocation strategy is illustrated in Figure 2. Figure 2 implies that variability of demand expands the region where seller prefers to sell in advance. We also notice that for a range of costs, the seller is always better off rationing all available capacity to advance market, regardless of his capacity level. Advance selling allows not only to hedge future uncertain demand, but also to obtain higher margins. Furthermore, stochasticity of demand makes advance selling profitable at cost levels which are prohibitive in the cases of infinite capacity or deterministic demand.

- Stochastic Heterogenous Valuations

Numerical results for the heterogeneous valuations indicate that this case can be treated (as expected) as an intermediate one between deterministic and stochastic homogeneous cases. This immediately implies that depending on demand variability a mix of insights from the two above models can coexist.
4 Conclusion

In this paper, we characterize joint optimal pricing and capacity-allocation strategy for a seller whose market consists of customers with uncertain future valuations and who has the option to offer advance selling. Three kinds of customer behaviors are considered: Deterministic Demand, Stochastic Homogenous Valuations, and Stochastic Heterogenous Valuations. We show that advance selling is profitable when seller has sufficiently low cost and large capacity. Furthermore, advance selling can be a useful tool to hedge against stochastic demand, which allows the seller to increase profits, even in some of the cases where deterministic demand is not profitable.

We also show that the idiosyncrasy of customer valuations qualitatively changes the optimal strategy. Premium advance price is acceptable for customers only when demand is stochastic and seller’s capacity is sufficiently small. When customer realized valuations are heterogenous, the seller with medium capacity level is better off by rationing his capacity to intentionally create shortage in advance.