Design Principles for Effective Transshipment Networks

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Introduction

This work studies network design issues in a supply chain where identical retailers use transshipments to match their supply with an uncertain demand. The use of transshipments has already been proven beneficial in previous research, see, for example, Tagaras (1999) and Herer et al. (2004). However, the study of transshipment network design has been limited to special configurations such as complete pooling and grouping. With complete pooling, every retailer is able to send and receive stock from any other retailer, where in a grouping configuration members of the same group can send and receive from each other, but stock cannot be transferred between different groups. Complete pooling may be expensive to implement due to the infrastructure cost of establishing a transshipment network with a large number of connections. Grouping configurations address this problem, but don’t exploit the full potential of transshipments in the network.

In this work we study a new configuration in the transshipment context, namely chaining. The chaining structure has been shown to be successful in production scenarios for product-plant assignment and worker cross-training, see Jordan and Graves (1995) and Iravani and Krishnamurthy (2003). In a transshipment scenario, the chaining structure connects all retailers in a unidirectional or bidirectional loop. Figure 1 illustrates six basic configurations of network design: (a) no transshipment; (b) complete pooling; (c)-(d) grouping; and (e)-(f) chaining configurations. In the figure, each node represents a retailer and each arc represents a transshipment link. It is assumed that all nodes are connected directly to a supplier, which is not shown in the figure.

The number of transshipment links has an important role in the design of transshipment networks. Since each link is associated with a fixed cost which is not considered directly in the model, we compare networks with an identical number of links. We show theoretically and numerically that chaining network configurations are preferred over grouping configurations.
which have the same number of links. Compared to complete pooling configurations, they consist of a smaller number of connections.

Problem Definition
Our model involves $N$ retail locations that face independent customer demand and are assumed to follow an order-up-to policy. Such a policy minimizes transshipment and inventory-related costs, as was proved in Herer et al. (2004). In addition we assume that all locations are identical, meaning that they incur the same costs and observe demand from an identical distribution. Similar characteristics can be observed in settings in which retailers serve near-homogenous populations over moderately sized geographic regions. We assume that the supplier to these locations has sufficient or infinite capacity. The lead time of replenishment from the supplier is one time unit. The cost of replenishment is the same across all configurations considered, and thus will be ignored. Fixed costs of orders and transshipments are negligible.

For each time period, events occur in the following order:
1. Replenishments from the warehouse arrive from orders made in the previous time period; backlogged demands from the previous time period are satisfied. The inventory levels are now equal to the order-up-to levels.
2. Demands are observed.
3. Transshipment decisions are made and occur immediately.
4. Demands are satisfied or backlogged.
5. Inventory levels are updated and replenishment orders are made.

At each location, the order-up-to level and periodic transshipment decisions are made to minimize long-run expected costs, which is the sum of transshipment, inventory holding and shortage costs. Since all locations follow an order-up-to policy the system regenerates itself every time period, therefore minimizing the expected cost for one time period is equivalent to minimizing the long-run expected costs. Herer et al. (2004) demonstrated how to calculate the optimal values of the order-up-to quantities, using a sample-path-based optimization procedure. Then, given the order-up-to quantities, an optimal transshipment policy may be determined using an LP/Network flow framework. We use this procedure to solve this problem for various system configurations which we discuss throughout the paper.

We define the following notation for the model parameters:

- \( d_i \) observed demand at location \( i \) in an arbitrary period
- \( I_i^+ \) net surplus at end of time period (after transshipment) at location \( i \)
- \( I_i^- \) net shortage at end of time period (after transshipment) at location \( i \)
- \( c_t \) cost of transshipping one item along a single arc
- \( c_s \) cost of one unit of shortage
- \( c_h \) cost of holding one unit in inventory

The following notation is defined for the decision variables:

- \( S_i \) order-up-to level at location \( i \)
- \( X_{ij} \) transshipment flow from \( i \) to \( j \)

**Analytical results**

Note first that one unit of transshipment from retailer \( i \) to retailer \( j \) implies one less unit of inventory (\( I_i^+ \)) and one less unit of shortage (\( I_j^- \)). It is then clear that transshipments will decrease total costs if \( c_t < c_s + c_h \). This leads to the concept of profitable “jumps” in transshipment networks.

A **jump** is defined as a move along a single arc in the transshipment network between two connected retailers. A jump is denoted **profitable** when a move along an arc reduces the total
cost, meaning: \( c_t < c_s + c_h \). When the costs satisfy \( c_t < k(c_s + c_h) \) where \( k \) is a positive integer, then the transshipment network allows for \( k \) profitable jumps.

**Observation:** The ratio of transshipment cost to inventory-related costs (holding and shortage) determines the number of profitable jumps in a transshipment network.

It is clear that certain networks benefit more from a high number of profitable jumps than other networks. In particular, this leads to the intuition that network configurations based on the chaining structure benefit more from a high number of profitable jumps and therefore will outperform grouping network configurations with the same number of arcs, see Figure 1. Notice that in the grouping configuration of size 2, retailers are only able to transship to and receive from one other retailer regardless of the number of profitable jumps. In contrast in the unidirectional chain configuration (which has the same number of links) retailers can transship to a number of retailers equal to the number of profitable jumps. A similar comparison holds when comparing the bidirectional chain and grouping of size 3 configurations.

We formalize these claims in the following theorems. Their proofs can be found in Iravani et al. (2005).

**Theorem 1:** Given a network with \( N \) identical locations facing independent demand, the optimal expected cost under a unidirectional chain configuration is lower than or equal to the optimal expected cost under a pairing (grouping of size 2) configuration.

**Theorem 2:** Given a network with \( N \) identical locations facing independent demand, the optimal expected cost under a bidirectional chain configuration is lower than or equal to the optimal expected cost under a grouping of size 3 configuration.

**Numerical Analysis**
In this section we compare the performance of the six configurations from Figure 1 relative to average cost per period. The optimal system cost is obtained using the solution procedure of Herer et al. (2004) as mentioned above. We apply the six configurations to a model with 12 identical retailers and simulate to calculate the average cost. The demand at each retailer is sampled from a normal distribution with mean 100 and standard deviation of 25. The holding
and shortage cost is set at $c_h = 2$ and $c_s = 11$. Each configuration is tested with different ratios of transshipment cost to the sum of holding and shortage costs. Figure 2 presents the results.

![Figure 2: Average transshipment network costs as a function of network configuration and cost ratio](image)

Note that for all cost ratios, the 12-arc unidirectional chaining has lower average costs than the 12-arc grouping configuration of size 2, and the 24-arc bidirectional chaining has lower average costs than the 24-arc grouping configuration of size 3, which is consistent with Theorems 1 and 2, respectively. These cost differences increase as the ratio of transshipment to holding and shortage cost decrease; recall that lower cost ratios implies a higher number of profitable jumps. For cost ratios of .85 and higher, the performance of all the configurations are close to the performance of the complete grouping configuration.

Also note that in Figure 2 for certain cost ratios unidirectional chaining has lower average costs than grouping configuration of size 3, which has double the arcs. This demonstrates that configurations with more arcs do not always outperform configurations with fewer arcs - the design of the network is also important.
**Discussion**

Chaining configurations not only outperform grouping configurations, but they also have the advantage of putting an equal burden on every retailer. Nevertheless, the benefits of chaining decrease as the number of nodes increase because every location is connected directly to only two other locations, and transshipping to locations beyond that is more costly. For this reason we explore other configurations for larger networks that have many of the same properties of chaining. Principles and a possible metric for constructing and identifying efficient network configurations are topics of our current and future research.

**References**


