Transshipment of Inventories Among the Retailers: Myopic vs. 
Farsighted Stability

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1. Introduction

In their recent papers, Anupindi, Bassok and Zemel (hereafter referred to as ABZ) (2001) and (1999), and Granot and Sošić (hereafter referred to as G&S) (2003) have analyzed a distribution problem in which independent retailers that sell an identical product face stochastic demands, and must determine their stocking quantity before the actual demand is known. After demands are realized, some retailers may be left with unsold inventory, while others may have customers with unsatisfied demands. The retailers can gain additional profit by sharing the leftover inventory, that is, by shipping residual supplies to the retailers with residual demands. The papers mentioned above formulate this problem as a decentralized multi-stage distribution model. The first, non-cooperative stage is the same in both the ABZ and G&S models. In this stage, each retailer unilaterally determines her inventory order before the actual demand realization. After demand is realized, each retailer first satisfies her own demand from inventory on hand. In the ABZ model, this is followed by the second, cooperative stage, in which the retailers share their inventories to fulfill the unsatisfied demands and then distribute the corresponding additional profit. It is implicitly assumed by ABZ that the retailers will share all of their unsold inventories and unsatisfied demands in this cooperative stage. G&S modify the ABZ model by adding another non-cooperative stage between the two stages of the ABZ model, in which each retailer unilaterally determines the amount of residuals that she wants to share with other retailers. The third stage of the G&S model thus corresponds to the second stage of the ABZ model. In both models, agreements
for distribution of the additional profit that results from transshipments of leftover inventories are made before the initial stocking quantities are determined. Naturally, in both models the quantities ordered in the first, non-cooperative stage depend upon the allocation rule used in the last, cooperative stage. Because this model is studied in the game-theoretical framework, we next introduce some concepts from game theory that are used in this paper.

We denote by \( N = \{1, 2, \ldots, n\} \) the set of all players. A subset \( S \subseteq N \) is called a coalition, and \( N \) is called the grand coalition. A coalition structure, \( \mathcal{Z} \), is a partition on \( N \). That is, \( \mathcal{Z} = \{Z_1, \ldots, Z_m\}, \cup_{i=1}^m Z_i = N, Z_j \cap Z_k = \emptyset, j \neq k \). \( v(S) \) will denote the value that players in coalition \( S \) can generate by themselves, without participation by players outside \( S \). A mapping \( \Phi \) that assigns to every game a vector \( \varphi = (\varphi_1, \ldots, \varphi_n) \in \mathbb{R}^N \) is called an allocation rule, and \( \varphi \) is called an allocation. Two commonly used allocation rules are the core and the Shapley value. We say that an allocation \( \varphi \) is a member of the core if it satisfies

\[
\sum_{i \in S} \varphi_i \geq v(S) \quad \forall S \subseteq N,
\]

\[
\sum_{i=1}^n \varphi_i(v) = v(N).
\]

The Shapley value is an allocation rule that can be interpreted as follows. Define a marginal contribution of player \( i \) with respect to a particular ordering of players as its marginal worth to the coalition formed by the players before him in the order. Thus, if \( 1, 2, \ldots, i-1 \) are the players preceding \( i \) in the given ordering, then \( i \)'s marginal contribution is

\[
v(\{1, 2, \ldots, i-1, i\}) - v(\{1, 2, \ldots, i-1\}).
\]

The Shapley value is obtained by averaging the marginal contributions for all possible orderings,

\[
\Phi_i(v) = \sum_{\{S: i \in S\}} \frac{(|S|-1)!(n-|S|)!}{n!} (v(S) - v(S \setminus \{i\})).
\]

We now return to our distribution models. The allocation rule in ABZ is based on a dual solution for the transshipment problem. Such an allocation is always in the core of the associated transshipment game, and therefore it encourages the retailers to not defect from the grand coalition and form subcoalitions during the transshipment stage. G&S show that allocations based on dual solutions will not, in general, induce the retailers to share all of their residuals with others. This, in turn, may reduce the additional profit obtained through the sharing of inventories. In the framework of transshipment games, G&S introduce
the definition of value-preserving allocation rules as follows: an allocation rule is value-preserving if it induces all retailers to share their leftover inventories or unsatisfied demands in amounts that maximize the profit from transshipment. G&S show that Shapley value is value-preserving, but is not in the core of the corresponding transshipment game. In addition, they show that there are no value-preserving core allocation rules with an arbitrary number of players. Thus, G&S (2003) reach the following conclusion.

...regardless of which allocation rule is being implemented, in a completely decentralized mode, i.e., without enforceable contracts, management of inventories may result with residual losses. These losses may stem either from the formation of subcoalitions in the transshipment stage, if value-preserving and thus non core allocations are used, or from strategic withholding of residuals, if core and thus non value-preserving allocations are used.

Note that this conclusion holds in a myopic framework. That is, it is assumed by both ABZ and G&S that the retailers who consider possible deviations from the grand coalition look only at immediate consequences of their actions and not at the potential reactions of other coalitions that may be triggered by their move. However, it may very well happen that an initial deviation is followed by a sequence of other defections, before some sort of stability is reached. For instance, consider an arbitrary coalition structure, $\mathcal{Z}$, and suppose that a subset of players, $Z$, can increase their payoffs by deviating and forming a different coalition structure. From a myopic view of stability, this would make $\mathcal{Z}$ unstable. However, we should consider possible further defections from the initial deviation. Another coalition might decide to deviate from the current status quo, which could benefit a possibly different set of players. In fact, any defection could trigger a sequence of further moves, with the result that some players who initially deviated – members of $Z$ – might receive lower payoffs than the ones they would have obtained in $\mathcal{Z}$. Under such a scenario, farsighted players might prefer not to move in the first place, and as a result, coalition structure $\mathcal{Z}$, which initially appeared unstable, could actually prove to be stable. In this paper, we attempt to model such scenarios by incorporating the notion of farsighted stability proposed by Chwe (1994) and the equilibrium process of coalition formation introduced by Konishi and Ray (2003). We briefly review Chwe’s definition of farsighted stability.

Let us denote by $\prec_i$ the players’ strong preference relations, described as follows: for two
coalition structures, $Z_1$ and $Z_2$,

$$Z_1 \prec_i Z_2 \iff u_{i}^{Z_1} < u_{i}^{Z_2},$$

where $u_{i}^{Z}$ is a retailer $i$'s payoff in the coalition structure $Z$. If $Z_1 \prec_i Z_2$ for all $i \in S$, we write $Z_1 \prec S Z_2$. For a given coalition $S$, let $F_S(Z)$ denote the set of coalition structures achievable by a one-step coalitional move by $S$ from $Z$. We say that $Z_1$ is directly dominated by $Z_2$, denoted by $Z_1 < Z_2$, if there exists an $S$ such that $Z_2 \in F_S(Z_1)$, and $Z_1 \prec S Z_2$. We say that $Z_1$ is indirectly dominated by $Z_m$, denoted by $Z_1 \ll Z_m$, if there exist $Z_1, Z_2, Z_3, \ldots, Z_m$ and $S_1, S_2, S_3, \ldots, S_{m-1}$ such that $Z_{i+1} \in F_{S_i}(Z_i)$ and $Z_i \prec_{S_i} Z_m$ for $i = 1, 2, 3, \ldots, m - 1$.

A set $Y$ is called consistent if the following holds: $Z \in Y$ if and only if for all $S$ and all $V \in F_S(Z)$ there is a $B \in Y$, where $V = B$ or $V \ll B$, such that $Z \nprec S B$. Chwe (1994) proves the existence, uniqueness, and non-emptiness of the largest consistent set (LCS). Since every coalition considers the possibility that, once it reacts, another coalition may react, and then yet another, and so on, the LCS incorporates farsighted coalitional stability. The LCS describes all possible stable outcomes and has the merit of "ruling out with confidence": if $Z$ is not contained in the LCS, $Z$ cannot be stable.

2. Analysis and Results

The analysis in this paper has been motivated by the following problem. Suppose that the retailers distribute the profit from inventory sharing according to an allocation rule that does not belong to the core but that induces the retailers to share their leftover inventories and unsatisfied demands in a way that maximizes the profit from transshipments. In other words, assume that the retailers use a value-preserving allocation rule. G&S have shown that, in general, value-preserving allocation rules are not contained in the core. Is it likely, then, that all of the retailers will jointly share their residuals under this scenario? In other words, are there any players who benefit if they defect from the grand coalition?

Our analysis of the problem takes the following approach. We assume that each retailer can individually decide how much of her leftover inventory or unsatisfied demand she wants to share with others, so we adopt the three-stage model introduced by G&S. We further assume that the retailers use Shapley allocations to distribute the profits from inventory sharing. Thus, because of the monotonicity of Shapley value, the retailers share their residuals in amounts that maximize the profit from transshipments (see G&S 2003). As mentioned
before, Shapley value is not a core allocation rule, and thus the grand coalition is, in general, unstable in the myopic sense. We, however, are concerned with its stability in a farsighted framework. We take under consideration various scenarios that describe possible relationships between unsold inventories and unfulfilled demands. Because of the large number of possible scenarios (exponential in number of retailers), we limit our detailed analysis to cases with a small number of retailers. As a result, we completely characterize the set of farsighted stable outcomes only for the models with three and four retailers. Characterization of the entire stable set becomes tedious when the number of retailers increases, and our interest in this analysis is primarily in determining whether the alliance of all retailers is stable or not. Thus, when we extend our analysis to an arbitrary number of retailers, we do it in a more limited scope. Instead of characterizing entire stable sets, we show that the grand coalition always belongs to the set of stable outcomes. This implies that farsighted retailers in the grand coalition who allocate profit from transshipments according to Shapley value do not want to defect and form subcoalitions.

3. Conclusions

While the main message from G&S (2003) is negative in its nature, concluding that one cannot simultaneously achieve stability of the grand coalition and maximize the profit from inventory sharing without enforceable contracts, we show that this result holds only for myopic retailers. Once we assume that the retailers are farsighted, that is, that they consider possible reactions of other retailers to their actions, we are able to reach a positive outcome. Namely, we show that an allocation rule that maximizes the profit from transshipments (Shapley value) induces the retailers in the grand coalition not to defect and form subcoalitions. Moreover, we show that the grand coalition is the only coalition structure that is stable (in a farsighted sense) under all possible relationships between unsold inventories and unsatisfied demands.

References


