On Selecting the Right Order Quantity with Availability-Sensitive Customers

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In a non-monopoly market, customers reaction to a stockout is either accepting delayed delivery (backorder, usually with some incentive) or taking their orders elsewhere (lost sales). When customers take the second action, the retailers suffer not only a loss of sales but also a loss of goodwill. Thus, the future demand is affected by the retailer’s ordering decision. We use attraction model to study the stationary market shares of multiple competing retailers in such a market. With the attraction model, customers make purchasing decisions based on a number of dimensions of the product, such as brand name, price, and after-sales services, as well as the availability level. We study the optimal ordering decisions when retailers actively use availability levels to compete with each other for market shares. To maximize its expected profit, a retailer needs to consider not only the tradeoff between underage cost and overage cost as considered in the classical newsvendor problem, but also the impact of availability level on the market share it gains. We prove that there exists a unique (pure) Nash equilibrium. With identical retailers, we show that the total market value is decreasing in the number of retailers in the whole market. We further provide an upper bound for the percentage of value decrease, which is quadratically decreasing in the number of total retailers. In particular, when the number of retailers goes to infinity, the total market value converges to a positive constant.

Keywords. service-sensitive demand; availability competition; attraction model; fill rate; Nash equilibrium

1. Introduction

This paper studies competition in product availability between two firms. In a non-monopoly market, the immediate customer reaction to a stockout is either accepting the delayed delivery (backorder, often with some incentive) or taking the order elsewhere (lost sales). The standard inventory control theory handles the immediate impact of a stockout satisfactorily from the supplier’s point of view. When the second action is taken, however, the supplier suffers not only a loss of sales but also a loss of goodwill, which will likely affect future demand for the product (Schwartz, 1966). Clearly, two factors should be considered when
suppliers formulate their production and supply strategies in a competitive market: the possible actions and reactions of the competitors; and the impact of a strategy on market share allocation. Issues related to these two factors have been studied extensively in the literature, roughly in two research streams: that considering the impact of a service strategy on future demands; and that considering product substitution and competition.

Schwartz (1966) pointed out that backorder cost for stockout cannot fully describe the impact of loss of goodwill in the future. He proposed a “perturbed demand” model, in which the demand in any period equals the potential demand multiplied by a factor which depends on observed customer fulfillment rate. This research has been followed and extended in a number of works, including Schwartz (1970), Hill (1976), Caine and Plaut (1976) and Robinson (1990) among others.

The literature on product substitution deals with situations in which two or more suppliers with substitutable products compete in the same market. Examples include, but are not limited to, Mcgillivray and Silver (1978), Parlar and Goyal (1984), Parlar (1985), Parlar (1988), Li et al. (1990), Lau (1991), and Wang and Parlar (1994). The focus here is on developing strategies for competition in the current period. Future effects are ignored. Similarly, a number of authors have developed strategies for competition based on product availability in a single period. Li (1992) considered competition in production speed in a buyer’s market, assuming that a demand will be filled by the supplier who produces the next available product first. This line of research has been followed and extended by Ernst and Cohen (1992), Li and Lee (1994), and Lederer and Li (1997). Lippman and McCardle (1997) introduced competition into the standard newsvendor problem. In their model, two firms make ordering decisions at the beginning of a period to compete for the demand in current period. When a shortage happens at one firm, the unmet demand switches to the other firm, and the future market base for the firm won’t change. Along the same line, Netessine et al. (2002) considered a two-firm competition problem in a reorder point system setting. When a stockout occurs at one firm, the unmet demand will either be backordered or switch to a competitor immediately. Stationary optimal ordering strategies are developed under four different scenarios. Since future demand is not affected by current activities, the problem is essentially a one-period problem. Bernstein and Federgruen (2004) considered price- and service-sensitive demands in a one-period setting, using a multiplicative demand model. They showed that the equilibrium in an infinite-horizon setting is the same as in the one-period setting. Hall and Porteus (2000) considered service competition among mul-
tiple firms in a dynamic setting in which product availability in current period affects the expected demand in the next period. Liu et al. (2005) generalized their work by examining the impact of a general demand model on the firm behavior under competition, and showing the relation between dynamic game and stationary game.

The most related existing literature is that by Bernstein and Federgruen (2004). One major assumption in their model is that, the customers decide their patronage based on the nominal fill rates announced by the competing retailers, while the retailers need to guarantee that the realized fill rates are no less than the nominal ones. In another word, the actual order quantities won’t affect the customers’ choice. In our work, we assume that the ordering decisions of the retailers directly affect the attractions of the retailers. In another word, customers have full information on how previous demands were satisfied at different retailers and hence decide which retailer to go based on the realized fill rate. Besides showing the existence and uniqueness of the Nash equilibrium, we analyze the impact of availability competition on the who industry profit.

The remainder of this paper is organized as follows. In §2, we specify the model basics, in particular, on how the availability level of a retailer will affect its customer patronage. We further formulate a profit maximization problem for a single retailer, and show how this problem can be efficiently solved. In §3, we consider a competitive game environment, where the ordering decisions of the retailers are interactive. We show there always exists a unique Nash equilibrium. We further consider the whole industry profit with identical retailers, and provide interesting managerial insights.

2. Model Formulation and Analysis

2.1 The Basic Model

We consider a retailer that sells perishable products to customers who are sensitive to its service level. The retailer needs to make ordering decision to maximize its expected profit. The total demand for this product in the whole market, denoted by $D_0$, is a random variable with known distribution function $F_0(x)$ and probability density function $f_0(x)$. Here we assume that the distribution function is continuous on its support. The retailer can only capture a proportion of customers in the market, depending on its attraction to the customers together with that of its competitors. The retailer can gain more market by increasing its attraction to customers. A retailer’s attraction is measured by the key dimensions of
its products, such as the reputation of the retailer, warranty services, and other factors. Customers are service-sensitive, and the service level will play a key role in determining the retailer’s attraction. To achieve the maximal profit, the retailer should consider two major factors in determining its ordering decision from its supplier. One is the tradeoff between the underage cost and overage cost as that considered in the classical newsvendor problem. The other one is setting the right service level to compete for more customers.

In a product supply context, we usually have two performance measures for the service level. One is the probability that the retailer meets no stockout. The other one is referred to as fill rate, or the fraction of demand that can be satisfied immediately from stock on hand (See Axsäter (2000)). In this paper, we use fill rate to measure the service level. Suppose that the retailer meets a random demand $D$, with $F(x)$ and $f(x)$ as its distribution function and probability density function respectively. When the retailer orders $q$ products from the supplier, its service level, or the fill rate can be expressed by:

$$S(q, p) = \frac{\mathbb{E}\min(q, D)}{\mathbb{E}[D]} = \frac{\int_0^q xdF(x) + q(1 - F(q))}{\mathbb{E}[D]}, \quad (1)$$

where the demand $D$ is treated as a function of $p$ for $p \in [0, 1]$.

We use $U$ to denote the utility of the retailer, which directly measures the attraction of the retailer. The demand of the retailer is determined by its utility function through the following relation: $D = pD_0$, where $p = e^U/(e^U + A)$, $A = \sum e^{U_j} > 0$ and the summation goes over all other retailers in the market. The above market share allocation has been widely used in marketing and operations literature, interested readers can refer to McFadden (1990), Lee and Cohen (1985), Cooper (1993) and Ho and Zheng (2004). Here we assume that the retailer is in a passive competitive environment and only gains a slight proportion of customers in the market. Therefore, the impact of its decision on the remaining market is ignorable, and its competitors will not react to its decision to adjust their order quantities. In §3, we will discuss the competitive interactions among competing retailers when each retailer actively adjusts its ordering strategy based on that of its competitors.

Next, we turn to the specifics of the utility function, and consider the following utility: $U(q) = \alpha + \beta S(q, p)$. In this equation, the constant $\alpha$ summarizes the cumulative effects of other key dimensions of the product, while $\beta S(q, p)$ shows the contribution of service level in obtaining market share. The nonnegative constant $\beta$ reflects customer sensitivity to the service level. We can see that we focus on the impact of service level on the market share
allocation. We allow other product dimensions of the retailer to be different from those of its competitors, while we don’t explicitly study the impact of other product dimensions.

The objective of the retailer is to maximize its expected profit, taking into consideration the customers’ reaction to the service level it provides. We use \( r \) to denote the profit of selling each product, and \( c \) to denote the lose of holding each unsold product. Then the retailer should consider the following optimization problem:

\[
\max_{q>0} \quad rq - (r + c)\mathbb{E}[(q - D)^+] \\
\text{subject to} \quad D = pD_0, \tag{3} \\
p = \frac{e^U}{e^U + A}, \tag{4} \\
U = \alpha + \beta S(q, p). \tag{5}
\]

### 2.2 Profit Maximization

We consider the optimization problem defined by equation (2) to (5). Before finding the optimal ordering decision, we first focus on the relation between the order quantity and the market share the retailer can gain.

**Lemma 1** The order quantity \( q \) determines a unique market gain \( p \in [0, 1] \). Furthermore, \( p \) is increasing on \( q \).

**Proof.** See Appendix.

From Lemma 1, we can see that the retailer can gain more market by raising its order quantity. Therefore, the retailer should be more aggressive in making ordering decision than it should be in a standard newsvendor context. We call \( t = q/p \) the normalized order quantity of the retailer. The convenience of introducing the normalized order quantity lies in that it totally determines the fill rate. While in equation (1), the fill rate is determined by two variables: the order quantity and the proportion of market it gains.

Another important observation from Lemma 1 is that, there is a one-to-one correspondence between the ordering decision and the normalized order quantity. Lemma 1 shows that any order quantity corresponds to a unique fill rate indicator. On the other hand, when the normalized order quantity is given, then the service level and the utility value of the retailer are determined. Then by equation (4), the market proportion \( p \) is unique. Therefore, the order quantity \( q = tp \) is uniquely determined. With this observation, we can transform the
optimization problem into a new one with the normalized order quantity as the decision variable. Then the objective of optimization problem can be rewritten into:

$$\max_{t>0} g(t) = p(t) \left\{ [r - (r + c)F_0(t)]t + (r + c) \int_0^t xF_0(x) \right\},$$  \hspace{1cm} (6)

and the original constraints can be reduced into

$$S(q, p) = S_0(t) = \int_0^t xF_0(x) + t[1 - F_0(t)]$$

$$E[D_0].$$  \hspace{1cm} (7)

and

$$p(t) = \frac{e^{U(t)}}{e^{U(t)} + A},$$  \hspace{1cm} (8)

$$U(t) = \alpha + \beta S_0(t).$$  \hspace{1cm} (9)

The analysis of the basic model is given in the following Theorem.

**Theorem 1** The expected profit is strictly concave in $t$, and the optimal order quantity equals $q^* = t^*p^*$, where $t^*$ is the maximal point of $g(t)$ and $p^* = U(t^*)/[U(t^*) + A]$.

**Proof.** See Appendix.