KELLOGG SCHOOL OF MANAGEMENT
Northwestern University

DECS-452 Midterm Project Answers

Honor Code: These problems should not be discussed with (nor these pages shared with) anyone who might take this course in the future.

A.a,b

Ex ante, Haydon could have found itself in either of two situations: facing high unit costs (H) and needing to set a first-stage price (h or l), or facing low costs (L) and needing to set a price (h or l).

National will also face one of two situations: seeing Haydon setting a high price (h) and needing to decide whether or not to enter the market (I or O), or seeing a low price (l) and needing to make an entry decision (I or O).

If National's cost of entry is $120,000, the strategic representation is:

Hayden's payoffs	hIlI	hIlO	hOlI	hOlO
HhLh	$1,100,000	$1,100,000	$1,500,000	$1,500,000
HhLl	$1,200,000	$1,450,000	$1,350,000	$1,600,000
HlLh	$1,050,000	$1,200,000	$1,300,000	$1,450,000
HlLl	$1,150,000	$1,550,000	$1,150,000	$1,550,000

National's payoffs	hIlI	hIlO	hOlI	hOlO
HhLh	$280,000	$280,000	$0	$0
HhLl	$280,000	$90,000	$190,000	$0
HlLh	$280,000	$190,000	$90,000	$0
HlLl	$280,000	$0	$280,000	$0

Clearly, hIlI (if they see a high price, enter; if they see a low price, enter) is a dominant strategy for National. Therefore, they should enter no matter what price they see Haydon setting. Consequently, Haydon should follow its HhLl (if costs are high, price high; if costs are low, price low) strategy.

A.c

If National's cost of entry is $380,000, the strategic representation is:

Hayden's payoffs	hIlI	hIlO	hOlI	hOlO
HhLh	$1,100,000	$1,100,000	$1,500,000	$1,500,000
HhLl	$1,200,000	$1,450,000	$1,350,000	$1,600,000
HlLh	$1,050,000	$1,200,000	$1,300,000	$1,450,000
HlLl	$1,150,000	$1,550,000	$1,150,000	$1,550,000

National's payoffs	hIlI	hIlO	hOlI	hOlO
HhLh	$20,000	$20,000	$0	$0
HhLl	$20,000	($40,000)	$60,000	$0
HlLh	$20,000	$60,000	($40,000)	$0
HlLl	$20,000	$0	$20,000	$0

Haydon's third strategy is dominated by its second; ruling out this third strategy, National's second and fourth are dominated by its first; finally, ruling them out, Haydon's fourth strategy is dominated by its second. [Haydon should never expect to encounter hOlO, and would only see hIlO if National thinks Haydon would be foolish enough to play HlLh. National should never expect to see HlLh, and would only see HlLl if Haydon thinks National would sometimes play hIlO, expecting Haydon to foolishly play HlLh.]

If Haydon thinks there is at least a 60% chance that National will enter no matter what price Haydon sets, Haydon should use HhLl. But if there's at least a 40% chance that National will stay out when they see a high price, then Haydon should use HhLh.

[The threshold probability of 60% is determined by solving $1.1q + 1.5(1-q) = 1.2q + 1.35(1-q)$.]

Similarly, National should enter no matter what they see if they think there's at least a 66.7% chance that Haydon, facing low costs, would price high; otherwise, National should stay out if they see a high price.

[The previous paragraph is equivalent to saying that National, seeing a high price, should enter only if they now still think there’s at least a 40% chance that unit costs are low. This is, however, a somewhat less clear question to ask of National’s management than the direct question of how likely they think it is that Haydon, facing low costs, would have priced high in order to deter entry.]

A.d

Pricing low is a dominant strategy for each, independent of the unit cost of production. In the high-cost case, the second-stage pricing game is an instance of the Prisoners’ Dilemma.

B1.a

In order to decide how to vote on a proposed division of the loot, the team members will need to ask themselves what they expect to happen if the division is rejected. This means that they need to figure out how the 4-expedition-members case will play out. But how that will play out depends on their expectations for how the 3-expedition-members case would play out, and so on. So, we work out all of the cases, starting with the 1-team-member case.

Proposed share | Vote

Alfred		Blackie		Chuck		Dirk		Edward	
								10	Y
						10	Y	0	N
				9	Y	0	N	1	Y
		9	Y	0	N	1	Y	0	N
8	Y	0	N	1	Y	0	N	1	Y

There's a critical assumption underlying all of this: Binding precommitments aren't possible. If they were, then (for example), Edward could precommit to a vote of "No" in the 3-survivors case, unless he's given all 10 gold pieces. (Chuck would then prefer to give away the 10 and survive, rather than die.) The analysis above presumes that in each round, each is free to vote – and will vote – in his best interest at the time. (And that each believes the others are rational, believes that they believe he is rational, believes the others believe that he believes that they're rational, and believes that they believe that he believes that they believe he's rational.)

B1.b

We take the same approach here:

Proposed share | Vote

Alfred		Blackie		Chuck		Dirk		Edward	
								10	Y
						any	Y	any	N
				10	Y	0	Y	0	N
		8	Y	0	N	1	Y	1	Y
7	Y	0	N	1	Y	2/0	N	0/2	N/Y

The difference springs from the realization that, in the 2-survivors case, Dirk dies no matter what he offers to Edward. Therefore, in the 3-survivors case, Chuck doesn't need to offer anything to Dirk in order to get Dirk's vote. Ultimately, Alfred needs to offer two coins to either Dirk or Edward (and one to Chuck) in order to survive.

B2.a I: three situations, with three choices at each => $3^3 = 27$ pure strategies

(For example, one strategy is “hold A, guess B; hold B, guess B; hold C, guess A.”)

II: nine situations (what he holds and what he hears), with three choices at each => $3^9 = 19{,}683$ pure strategies

(For example, one strategy is “hold A, and hear “A”, guess B; hold A and hear “B”, guess A; hold A and hear “C”, guess B; hold B and hear “A”, guess C; hold B and hear “B”, guess C; hold B and hear “C”, guess B; hold C and hear “A”, guess C; hold C and hear “B”, guess A; hold C and hear “C”, guess A.”)

B2.b In practical terms, it all comes down to:

I: A - guess own card
B - randomly (50:50) guess from other two

II: C - if own guessed, choose randomly from other two; else match I's guess
D - If own guessed, choose randomly from other two; else pick unnamed, unheld card

Expected payoffs (to I):

	C	D
A	-1	0
B	1/4	-1/4

Feeding these payoffs into the “0-sum games” workbook, we find that, optimally,
I randomizes (A:1/3, B:2/3), and II randomizes (C:1/6, D:5/6). The value of the game = -1/6 (to I).

(I does best by not even looking at his card!)