Sample Exam Problems

Question 1

Let A and B be two events. Suppose P(A) = 0.2, P(B) = 0.3 and P(A and B) = 0.1.

 a) Compute P(A or B).

 b) Are A and B independent events? Explain.

 c) Compute P(A | B).

Question 2

The finance department of a particular company has 15 employees, 6 of whom have MBA degrees. Suppose we select three different employees sequentially at random from this department. Determine the probability of the following events.

 a) The first employee has an MBA, given that there is a total of one MBA among all three employees.

 b) There are exactly two employees with an MBA, given that the first employee has an MBA.

 c) The first employee has an MBA, given that there is at least one MBA among all three employees.

Question 3

A stock has a current share price of $100/share. Let X represent the dollar amount by which its price will change over the next one week. Assume X is normally distributed, with E(X)=0.10, and standard deviation of 0.50.

 a) What is the probability that after one week, the stock's value will be between $99.50 and $101.00?

 b) Now examine the price of this stock one year from now (52 weeks). Suppose that each weekly price change of the stock is described by this same normal distribution. In addition, suppose weekly price changes are independent. Find the probability that after one year, the stock's value will be between $99 and $111.

Question 4

A fair coin is tossed 5 times. Each toss is independent of the others.

 a) What is the probability of getting at least one Head?

 b) What is the expected number of Heads?

Question 5

A major credit rating agency rates a bond of a company as CCC if the probability of default is 0.10 in any given year and defaults across years are independent.

What is the probability that a CCC bond defaults in the 4th year (that is, the company goes for 3 years without default, and then defaults in the fourth)?

Question 6

Unoccupied seats on flights cause airlines to lose revenues. A large airline wants to estimate its average number of unoccupied seats per flight over the past year. To accomplish this, the records of 225 flights are randomly selected, and the number of unoccupied seats is noted for each of the flights in the sample. The sample mean is 14.5 seats and the sample standard deviation is s = 8.2 seats.

    a)   Provide a 95% confidence interval for the mean number of unoccupied seats per flight during the past year.

    b)   What is the significance level of the data with respect to the null hypothesis: "The average number of unoccupied seats per flight during the last year was greater than or equal to 15.5."

Question 7

With the plan to develop a chain of boutique pet supply stores, you have obtained data on pet ownership among residents of Chicago's Gold Coast. You are interested in the percentage of households in that area that own at least one pet. Within a random sample of 300 households, 30 of the households own exactly one pet and 12 households own two or more pets. The rest own none.

    a)   What is your estimate of the percentage of Gold Coast households that own at least one pet?

    b)   What is a 95% confidence interval for this estimate?

Question 8

A company sells an insurance policy, against losses incurred during travel, for $15.

40,000 customers buy the insurance policy. The policy pays the full amount claimed up till $5,000. Most policy holders claim nothing, the rest either $500 or $5,000.

Each customer has the following probabilities of claiming a certain amount for damages, independent of the other customers.

claim	$0	$500	$5,000
probability	99%	0.8%	0.2%

Estimate the probability that the company makes a positive net profit across the 40,000 contracts.

Question 9

A survey is carried out by the Chicago Public School District to determine the distribution of household size. They draw a simple random sample of 1,000 households and send interviewers to each household to collect information. After several visits, the interviewers find people at home in only 691 of the sample households. Rather than face such a high non-response rate, the District draws a second batch of households, and uses the first 309 interviews in the second batch to bring the sample up to its planned size of 1,000 households. The District counts 3,121 people in these 1,000 households, and estimates the average household size in the District to be about 3.1 persons.

Is this estimate likely to be too low, too high, or about right? Why?