Homework Assignment 3

PROBLEM 1: The WarmUp Company

As previously noted, your firm, The WarmUp Company, sells only winter hats and coffee mugs, at prices of $20 per hat and $10 per mug. Let H denote the number of hats you will sell this year, and let M denote the number of mugs you will sell this year. As before, you believe that

$$E[H] = 22{,}000 \quad Var(H) = 9{,}000{,}000 \qquad E[M] = 8{,}000 \quad Var(M) = 1{,}000{,}000 .$$

You also believe that H and M have a correlation coefficient of 0.3. (They both covary with the outdoor temperature.)

A. What is the covariance of H and M, Cov(H,M)?

B. What is the standard deviation of your total revenue for the year?

PROBLEM 2: Gipper's Gummies

The Jelly Belly Candy Company sells small bags of Jelly Beans. Each bag is filled using a machine that is supposed to put around 50 Jelly Beans in each bag. However, this is not always achieved because the machine is inexact: it actually puts X Jelly Beans in any one bag, where X has the following probability distribution.

x	Pr(X = x)
48	20%
49	10%
50	20%
51	20%
52	30%

A. Find both the expected value and standard deviation of X.

B. The number of jelly beans in any one bag is independent of the number in any other bags. Suppose we count the total number of jelly beans that the machine puts into the next 100 bags. Denote this number of Jelly Beans by S. Find both the expected value and standard deviation of S.

C. What is the probability that the machine will dispense 5000 or fewer jelly beans into the 100 bags? In other words, what is $\Pr(S \leq 5000)$?

PROBLEM 3: Panhandle Recycling

Panhandle Recycling is a company in Dothan, Alabama that recycles automobile catalytic converters. Catalytic converters are part of a vehicle's exhaust system. Emissions go through a ceramic filter that contains valuable metals, including platinum, palladium, and rhodium. Recyclers extract these metals from catalytic converters of junked cars, and resell them at the metals' current market prices.

Unfortunately for recyclers like Panhandle, there is no way to know exactly how much of each precious metal is inside any particular catalytic converter without actually cutting it apart. (The amount of the metals used to construct catalytic converters has varied across car makers and over time, though there are typical "recipes" of combinations of metals.) On the other hand, Panhandle has extensive data on the contents of previously recycled catalytic converters. Based on this data, and the current market price of platinum, palladium, and rhodium, they estimate that the worth of any one catalytic converter has the following probability distribution.

Worth of Converter	Probability
$1200	35%
$1800	50%
$2400	15%

A local junkyard offers a load of 500 catalytic converters to Panhandle at a particular price. Panhandle needs to assess the worth of the entire load to decide whether to purchase it. Assume that the worth of any one converter is independent of the worth of the others.

A. What is the expected value of the total worth of the 500 catalytic converters?

B. What is the standard deviation of the total worth?

C. To cover its costs of processing and to insure itself a healthy profit in the face of uncertainty, Panhandle typically would not pay a price higher than the 25th percentile of the probability distribution associated with the worth of the load. (In other words, they would agree to a price only if there is a 75% chance that the load is worth more than that price.) Following this rule, what is the maximum price that Panhandle would be willing to pay to the junkyard for the load of 500 catalytic converters?