

Exercises I

PROBLEM 1: Mad Cow Disease

There exists a test for Mad Cow disease. If a tested cow is sick, the test comes back positive with 90% probability. If the cow is well, the test comes back negative with 99% probability.

1. Suppose 3% of cows have Mad Cow disease. If the test comes back positive, what are the chances that the tested cow has Mad Cow disease?
2. Suppose 0.03% of cows have Mad Cow disease. If the test comes back positive, what are the chances that the tested cow has Mad Cow disease?
3. What would the prevalence of the disease need to be for your previous answer to be greater than 50%?

PROBLEM 2: Recruiting

Matthew Billings-Avery (MBA) is heading into a job interview. He thinks there is a 20% chance of getting a job offer from the firm by the end of the week. If he will be getting a job offer, he believes there is a 75% chance the interviewer will call him "Matt" when he leaves the interview. If he won't be getting a job offer, he believes there is a 10% chance the interviewer will call him Matt when he leaves.

As Matthew is leaving the interview, the interviewer says, "It's been great speaking with you, Matt." What are the chances he gets a job offer?

PROBLEM 3: Arrests in Pennsylvania

In Pennsylvania, 16% of all felonies lead to arrests, 27% of those arrests lead to convictions, and 19% of those convictions lead to prison terms. Treat these numbers as, respectively, the probability of arrest for a randomly chosen felony, the probability of conviction given arrest, and the probability of a prison term given conviction. Find the probability that any particular Pennsylvania felony results in:

1. arrest and conviction
2. arrest but not conviction
3. a prison term
4. no arrest at all

PROBLEM 4: Collateralized debt obligations (CDOs)

You sell a CDO that insures the buyer against the event that a particular company (GM, say) declares bankruptcy within a given calendar year.

The likelihood of a declared bankruptcy depends on the state of the auto industry as a whole. It's been forecasted that the state of the industry next year will be 'strong' with probability 0.60, and 'weak' with probability 0.40. The probability of bankruptcy given that the auto industry is strong is 0.10. The probability of bankruptcy, given that the auto industry is weak, increases dramatically to 0.30.

1. What is the probability that the company will declare bankruptcy next year?
2. What is the probability that the auto industry is strong next year, given that the company covered by the CDO does not declare bankruptcy?

PROBLEM 5: Inspector Clouseau

Inspector Clouseau believes the probability someone is trying to kill him equals 0.40. If someone really is trying to kill him, the chances of seeing a piano falling from the sky and missing him by less than 1 inch equals 0.95. If no one is really trying to kill him, the chances of seeing a piano falling from the sky and missing him by less than 1 inch equals 0.02.

1. If Clouseau is walking down the road and a piano falls from the sky and misses him by less than 1 inch, what are the chances someone is trying to kill him?
2. If he is walking down the road and he does not see a piano fall and miss him by less than 1 inch, then what are the chances that no one is trying to kill him?